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动态奇异系统的主从策略

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摘　　要

本文讨论奇异系统的主从对策问题,给出系统开环主从策略的存在条件,推导了开环主从策略.

关键词: 奇异系统,对策论,主从策略.

一、引　　言

由于奇异系统能够描述许多正常系统所不能处理的实际问题(如动态投入产出模型、奇异系统中的慢子系统模型等都是奇异系统的实例),所以,近几十年来,奇异系统引起许多学者的关注,研究不断深入,与传统的状态系统理论相平行,线性奇异系统理论已初步形成.如奇异系统的可解性、可控性、可观性[1]、反馈与极点配置[2]等都得到了一些研究,尤其是奇异系统的最优控制问题已取得了满意的结果[3,4].

至今为止,包括微分对策、主从对策在内的动态对策理论基本是以正常系统为基础的[5,6],因此能否将正常系统的动态对策理论推广到奇异系统中去,是一个很值得研究的问题.

二、开环主从策略存在条件

考虑下述线性定常奇异系统

$$Ex(k+1) = Ax(k) + B^1u^1(k) + B^2u^2(k),\quad Ex(0) = Ex_0, \tag{2.1}$$

其中 $x(\cdot)\in R^n$, $u^1(\cdot)\in R^{m^1}$ 及 $u^2(\cdot)\in R^{m^2}$ 分别是从方及主方的决策变量. $\operatorname{rank}E = r \leqslant n$ 且 $|sE - A| \neq 0$. 令主方及从方的目的分别是使 J_2 及 J_1 达到最小,其中

$$J_i(u^1,u^2) = \frac{1}{2}x(K)^TE^TQ^i(K)Ex(K) + \frac{1}{2}\sum_{k=0}^{K-1}[x(k)^TQ^ix(k) + u^1(k)^TR^{1i}u^1(k) + u^2(k)^TR^{2i}u^2(k)]. \tag{2.2}$$

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为了推导对策(2.1),(2.2)式存在开环主从策略的必要条件，首先应确定从方的合理响应存在的必要条件[6]，即给出在(2.1)式约束下 J_1 存在最优控制的条件. 推广文献[3]的结论可得如下条件:

$$Ex(k+1) = Ax(k) + B^1u^1(k) + B^2u^2(k),\ Ex(0) = Ex_0, \tag{2.3a}$$

$$E^Tp^1(k) = Q^1x(k) + A^Tp^1(k+1),\ E^Tp^1(K) = E^TQ^1(K)Ex(K), \tag{2.3b}$$

$$0 = R^{11}u^1(k) + B^{1T}p^1(k+1). \tag{2.3c}$$

而开环主从策略必须是 J_2 在(2.3)式约束下的最优解[6]. 引进拉格朗日乘子 $p^2(k)$,$n(k)$,$m(k)$ 和 $n(K)$,作泛函如下:

$$J_2(u^1,u^2) = \frac{1}{2}x(K)^TE^TQ^2(K)Ex(K) + n(K)^TE^T[Q^1(K)Ex(K) - p^1(K)]$$

$$+ \sum_{k=0}^{K-1}[H_2(k) - p^2(k+1)Ex(k+1) - n(k)^TE^Tp^1(k)].$$

其中

$$H_2(k) = \frac{1}{2}[x(k)^TQ^2x(k) + u^1(k)^TR^{21}u^1(k) + u^2(k)^TR^{22}u^2(k)]$$

$$+ p^2(k+1)^T[Ax(k) + B^1u^1(k) + B^2u^2(k)] + n(k)^T[Q^1x(k)$$

$$+ A^Tp^1(k+1)] + m(k)^T[R^{11}u^1(k) + B^{1T}p^1(k+1)].$$

由变分法知开环主从策略存在的必要条件为

$$Ex(k+1) = Ax(k) + B^1u^1(k) + B^2u^2(k),\ Ex(0) = Ex_0; \tag{2.4a}$$

$$E^Tp^1(k) = Q^1x(k) + A^Tp^1(k+1),\ E^Tp^1(K) = E^TQ^1(K)Ex(K); \tag{2.4b}$$

$$0 = R^{11}u^1(k) + B^{1T}p^1(k+1); \tag{2.4c}$$

$$E^Tp^2(k) = Q^2x(k) + A^Tp^2(k+1) + Q^1n(k),$$

$$E^Tp^2(K) = E^T[Q^2(K)Ex(K) + Q^1(K)En(K)]; \tag{2.4d}$$

$$En(k+1) = An(k) + B^1m(k),\ En(0) = 0; \tag{2.4e}$$

$$0 = R^{21}u^1(k) + B^{1T}p^2(k+1) + R^{11}m(k); \tag{2.4f}$$

$$0 = R^{22}u^2(k) + B^{2T}p^2(k+1). \tag{2.4g}$$

三、开环主从策略的推导

对任意秩为 r 的 n 阶方阵 E，总存在 n 阶非奇异方阵 U 和 V 及 r 阶单位阵 I，使得 $UEV = \mathrm{diag}\{I,0\}$,因此为了不失一般性,假设

$$\{E|A|B^i\} = \left\{\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\middle|\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\middle|\begin{bmatrix} B_1^i \\ B_2^i \end{bmatrix}\right\}, \tag{3.1a}$$

$$\{Q^i(K)|Q^i\} = \left\{\begin{bmatrix} Q_{11}^i(K) & Q_{12}^i(K) \\ Q_{12}^i(K)^T & Q_{22}^i(K) \end{bmatrix}\middle|\begin{bmatrix} Q_{11}^i & Q_{12}^i \\ (Q_{12}^i)^T & Q_{22}^i \end{bmatrix}\right\}. \tag{3.1b}$$

令 $x_1(k)$ 与 $x_2(k)$,$p_1^i(k)$ 与 $p_2^i(k)$ 和 $n_1(k)$ 与 $n_2(k)$ 分别是 $x(k)$,$p^i(k)$ 和 $n(k)$ 的相应分块. 有了(3.1)式的假设,(2.4)式便可等价地写成

$$\bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{B}\bar{u}(k),\ \bar{x}(0)^T = (x_{10}^T,0); \tag{3.2a}$$

$$\bar{p}(k) = \bar{Q}\bar{x}(k) + \bar{A}\bar{p}(k+1) + \bar{S}\bar{u}(k),\ \bar{p}(K) = \bar{Q}(K)\bar{x}(K); \tag{3.2b}$$

$$0 = \bar{S}\bar{x}(k) + \bar{B}^T\bar{p}(k+1) + \bar{R}\bar{u}(k). \tag{3.2c}$$

其中

$$A = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{11} \end{bmatrix},\ \bar{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & A_{12} & B_1^1 & B_1^2 \\ 0 & A_{12} & B_1^1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{Q} = \begin{bmatrix} Q_{11}^2 & Q_{11}^1 \\ Q_{11}^1 & 0 \end{bmatrix},\ \bar{Q}(K) = \begin{bmatrix} Q_{11}^2(K) & Q_{11}^1(K) \\ Q_{11}^1(K) & 0 \end{bmatrix},$$

$$\bar{S} = \begin{bmatrix} A_{21}^T & Q_{12}^1 & 0 & 0 & Q_{12}^2 & 0 & 0 \\ 0 & 0 & 0 & A_{21}^T & Q_{12}^1 & 0 & 0 \end{bmatrix},$$

$$\bar{R} = \begin{bmatrix} 0 & 0 & 0 & 0 & A_{22} & B_2^1 & B_2^2 \\ 0 & 0 & 0 & A_{22}^T & Q_{22}^1 & 0 & 0 \\ 0 & 0 & 0 & B_2^{1T} & 0 & R^{11} & 0 \\ 0 & A_{22} & B_2^1 & 0 & 0 & 0 & 0 \\ A_{22}^T & Q_{22}^1 & 0 & 0 & Q_{22}^2 & 0 & 0 \\ B_2^{1T} & 0 & R^{11} & 0 & 0 & R^{21} & 0 \\ B_2^{2T} & 0 & 0 & 0 & 0 & 0 & R^{22} \end{bmatrix}.$$

$$\bar{x}(k)^T = [x_1(k)^T, n_1(k)^T],\ \bar{p}(k)^T = [p_1^2(k)^T, p_1^1(k)^T], \tag{3.3a}$$

$$\bar{u}(k)^T = [p_2^2(k+1)^T, n_2(k)^T, m(k)^T, p_2^1(k+1)^T, x_2(k)^T, u^1(k)^T, u^2(k)^T]. \tag{3.3b}$$

对于奇异系统(3.2)式，若条件(1)(A_{22}, B_2^1) 行满秩，(2)$Q_{22}^1 > 0$，$Q_{22}^2 \geqslant 0$，$R^{11} > 0$，$R^{21} \geqslant 0$，$R^{22} > 0$ 都满足由式(3.2c)可求出 $\bar{u}(k)$（此时 $\bar{R}$ 可逆[7]）

$$\bar{u}(k) = -\bar{R}^{-1}\bar{S}^T\bar{x}(k) - \bar{R}^{-1}\bar{B}^T\bar{p}(k+1). \tag{3.4}$$

将此式代入式(3.2a)及(3.2b)可推得

$$\bar{x}(k+1) = (\bar{A} - \bar{B}\bar{R}^{-1}\bar{S}^T)\bar{x}(k) - \bar{B}\bar{R}^{-1}\bar{B}^T\bar{p}(k+1),\ \bar{x}(0)^T = (x_{10}^T, 0); \tag{3.5a}$$

$$\bar{p}(k) = (\bar{Q} - \bar{S}\bar{R}^{-1}\bar{S})\bar{x}(k) + (\bar{A} - \bar{B}\bar{R}^{-1}\bar{S}^T)^T\bar{p}(k+1),\ \bar{p}(K) = \bar{Q}(K)\bar{x}(K). \tag{3.5b}$$

现在令 $\bar{p}(k) = P(k)x(k)$，并代入(3.5a)式得

$$[I + \bar{B}\bar{R}^{-1}\bar{B}^T P(k+1)]\bar{x}(k+1) = [\bar{A} - \bar{B}\bar{R}^{-1}\bar{S}^T]\bar{x}(k). \tag{3.6a}$$

如果 $L(k) = [I + \bar{B}\bar{R}^{-1}\bar{B}^T P(k+1)]$ 对于 $0 \leqslant k \leqslant K-1$ 均可逆，则

$$\bar{x}(k+1) = [I + \bar{B}\bar{R}^{-1}\bar{B}^T P(k+1)]^{-1}(\bar{A} - \bar{B}\bar{R}^{-1}\bar{S}^T)\bar{x}(k),\ \bar{x}(0)^T = (x_{10}^T, 0); \tag{3.6b}$$

$$P(k) = (\bar{Q} - \bar{S}\bar{R}^{-1}\bar{S}) + (\bar{A} + \bar{B}\bar{R}^{-1}\bar{S}^T)^T[I + \bar{B}\bar{R}^{-1}\bar{B}^T P(k+1)]^{-1}(\bar{A} - \bar{B}\bar{R}^{-1}\bar{S}^T)$$

$$P(K) = \bar{Q}(K). \tag{3.6c}$$

由式(3.3b)知开环主从策略(若存在)$u^1(k)$及 $u^2(k)$ 应是 $\bar{u}(k)$ 的第 6,7 个子向量，又由式(3.4)知，$\bar{u}(k)$ 应为

$$\bar{u}(k) = -\bar{R}^{-1}(\bar{S}^T + \bar{B}^T P(k+1))\bar{x}(k). \tag{3.7}$$

其中 $P(k+1)$ 及 $\bar{x}(k)$ 分别由(3.6c)及(3.6b)确定。

例.

$$x_1(k+1) = x_2(k) + u^1(k) + u^2(k),\ x_{10} = x_0,$$

$$0 = x_1(k) + u^1(k) + u^2(k).$$

$$J_1 = \sum_{k=0}^{2}\left\{2[x_1(k)]^2 + \frac{1}{2}[x_2(k)]^2 + \frac{1}{2}[u^1(k)]^2\right\} + \frac{1}{2}[x_1(3)]^2,$$

$$J_2=\sum_{k=0}^{2}\left\{[x_1(k)]^2+\frac{1}{2}[x_2(k)]^2+\frac{1}{2}[u^2(k)]^2\right\}+[x_1(3)]^2,$$

其中 $x(k)^T=(x_1(k),x_2(k))$. 该例中 $\bar{A}=0$

$$\bar{Q}=\begin{pmatrix}2&4\\4&0\end{pmatrix},\quad \bar{Q}(3)=\begin{pmatrix}2&1\\1&0\end{pmatrix},$$

$$\bar{B}=\begin{pmatrix}0&0&0&0&1&1&1\\0&1&1&0&0&0&0\end{pmatrix},\quad \bar{S}=\begin{pmatrix}1&0&0&0&0&0&0\\0&0&0&1&0&0&0\end{pmatrix},$$

$$\bar{R}=\begin{bmatrix}0&0&0&0&0&1&1\\0&0&0&0&1&0&0\\0&0&0&1&0&1&0\\0&0&1&0&0&0&0\\0&1&0&0&1&0&0\\1&0&1&0&0&0&0\\1&0&0&0&0&0&1\end{bmatrix},\quad \bar{R}^{-1}=\begin{bmatrix}0&0&0&-1&0&1&0\\0&-1&0&0&1&0&0\\0&0&0&1&0&0&0\\-1&0&1&1&0&-1&1\\0&1&0&0&0&0&0\\1&0&0&-1&0&1&-1\\0&0&0&1&0&-1&1\end{bmatrix}.$$

由式(3.4)可算出

$$u^1(k)=x_1(k)-n_1(k),\quad u^2(k)=n_1(k).$$

而由(3.6)式可得出

$$L(3)^{-1}=\begin{pmatrix}\frac{2}{4}&0\\-\frac{1}{4}&\frac{2}{4}\end{pmatrix},\quad L(2)^{-1}=\frac{1}{184}\begin{pmatrix}30&4\\11&26\end{pmatrix},\quad L(1)^{-1}=\frac{1}{9311}\begin{pmatrix}1446&188\\563&1258\end{pmatrix},$$

$$P(2)=\frac{1}{4}\begin{pmatrix}11&22\\22&-4\end{pmatrix},\quad P(1)=\frac{1}{184}\begin{pmatrix}511&1074\\1074&-188\end{pmatrix},\quad P(0)=\frac{1}{9311}\begin{pmatrix}25924&54420\\54420&-9499\end{pmatrix},$$

$$x_1(1)=-\frac{1446}{9311}x_{10},\quad x_1(2)=\frac{248}{9311}x_{10},$$

$$n_1(1)=-\frac{563}{9311}x_{10},\quad n_1(2)=\frac{166}{9311}x_{10}.$$

因此有

$$u^1(0)=x_{10},\quad u^1(1)=-\frac{833}{9311}x_{10},\quad u^1(2)=\frac{82}{9311}x_{10},$$

$$u^2(0)=0,\quad u^2(1)=-\frac{563}{9311}x_{10},\quad u^2(2)=\frac{166}{9311}x_{10}.$$

结 语

本文的结果可推广到连续奇异系统的对策问题中[7]；另外还可以推广到一主多从及多级奇异系统动态对策问题中[8-10].

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LEADER-FOLLOWER STRATEGIES FOR SINGULAR DYNAMIC SYSTEMS

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Abstract

The paper discusses the problem of leader-follower game for singular systems. Conditions for the existence of open-loop leader-follower strategies are given, and the open-loop leader-follower strategies are derived.

Key words: Singular systems; Stackelberg Strategies; game theory.