H_∞ Controller Design for Networked Control Systems via Active-varying Sampling Period Method

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Abstract This paper studies the problem of designing H_∞ controllers for networked control systems (NCSs) with both network-induced time delay and packet dropout by using an active-varying sampling period method, where the sampling period switches in a finite set. A novel linear estimation-based method is proposed to compensate packet dropout, and H_∞ controller design is also presented by using the multi-objective optimization methodology. The simulation results illustrate the effectiveness of the active-varying sampling period method and the linear estimation-based packet dropout compensation.

Key words Networked control systems (NCSs), active-varying sampling period, linear matrix inequalities (LMIs)

The use of network as a media to interconnect different components in industrial control systems is rapidly increasing. However, the insertion of the communication network will inevitably lead to time delay and data packet dropout, which might be potential sources of instability and poor performance of NCSs.

Many researchers have studied stability, controller design, and performance of NCSs in the presence of network-induced delay[1−3]. In [4], a novel model-predictive-control strategy with a timeout scheme and p-step-ahead state estimation was presented to overcome the adverse influences of packet dropout. For other methods dealing with time delay and packet dropout, see [5−10].

There have been considerable research efforts on H_∞ control of systems with time delay. [11] was concerned with the design of robust H_∞ controllers for uncertain NCSs with the effects of both network-induced delay and data packet dropout taken into consideration. For other results on H_∞ control of systems with delay, see [12−14].

Network-induced time delay and packet dropout might lead to instability and poor performance of NCSs, so it is significant to overcome the adverse influences of time delay and packet dropout. However, the problem of time delay and packet dropout compensation is seldom considered except in [4]. This paper proposes a novel linear estimation-based method to compensate time delay and packet dropout, and H_∞ controller design is also presented.

In NCSs, constant sampling period is usually adopted. If constant sampling period h is adopted, h is usually large enough to avoid network congestion when the network is occupied by the most users.

Recently, there are a number of papers considering the problem of varying sampling period of control systems[15−16]. Unlike the one proposed in this paper (so-called active-varying sampling period), the varying sampling period considered in [15−16] was induced by some external factors, and [15−16] did not study the problems of H_∞ controller design and packet dropout compensation.

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Using the novel active-varying sampling period method, this paper eliminates the probability of packet disordering, which greatly simplifies the analysis and design of NCSs. On the other hand, the proposed active-varying sampling period method can ensure the sufficient use of network bandwidth when the network is idle. A linear estimation-based compensation method is also proposed to compensate packet dropout.

This paper is organized as follows. Section 1 presents the model of NCSs with an active-varying sampling period. A novel linear estimation-based compensation method is proposed in Section 2 to compensate packet dropout, and H_∞ controller design is also presented by using LMI-based method. The results of numerical simulation are presented in Section 3. Conclusions are stated in Section 4.

1 Preliminaries and problem statement

Consider a linear time-invariant plant described by

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + B_1\boldsymbol{u}(t) + B_2\boldsymbol{\omega}(t)\\ \boldsymbol{z}(t) &= C_1\boldsymbol{x}(t) + D_1\boldsymbol{u}(t)\end{aligned} \tag{1}$$

where $\boldsymbol{x}(t)$, $\boldsymbol{u}(t)$, $\boldsymbol{z}(t)$, and $\boldsymbol{\omega}(t)$ are the state vector, control input vector, controlled output, and disturbance input, respectively, and $\boldsymbol{\omega}(t)$ is a piecewise constant. A, B_1, B_2, C_1, and D_1 are known constant matrices. Throughout this paper, matrices, if not explicitly stated, are assumed to have appropriate dimensions.

For NCSs, the shorter the sampling period, the better the system performance. However, a short sampling period will increase the possibility of network congestion. If constant sampling period h is adopted, h should be large enough to avoid network congestion, so network bandwidth cannot be sufficiently used when the network is idle.

In the following, we will propose the active-varying sampling period method to make full use of the network bandwidth, which is one of the motivations of this paper, and the other motivation of this paper is to simplify the analysis and design of NCSs with time delay and packet dropout.

Define τ_{sc} and τ_{ca} as the sensor-to-controller and controller-to-actuator network transmission time delays, respectively, and $\tau_{\rm sum}$ as the sum of the data processing times of the sensor, controller, and actuator. When the network is idle, denote $\tau_{sc}+\tau_{ca}+\tau_{\rm sum}$ as d_1; and when the network is occupied by the most users, define $\tau_{sc}+\tau_{ca}+\tau_{\rm sum}$ as d_2. If constant sampling period is adopted, d_2 can be chosen as the sampling period to avoid network congestion when the network is occupied by the most users.

Suppose t_k is the latest sampling instant, and that τ_k is the time delay of the control input $\boldsymbol{u}_k$ ($\boldsymbol{u}_k$ is on the basis of plant state at instant t_k). Also suppose that the instant at which the control input $\boldsymbol{u}_k$ reaches the actuator is $\tilde{k}$, and that h_k is the length of the kth sampling period. Considering the definition of d_1, we have $\tilde{k} \geq t_k + d_1$. Suppose both controller and actuator are event-driven.

The main idea of the active-varying sampling period method (using a both clock-driven and event-driven sensor) is as follows. Partition $[d_1, d_2]$ into l equidistant small intervals (l is a positive integer); then the next sampling instant $\hat{k}$ ($\hat{k}$ is the first sampling instant after t_k) can be chosen as follows.

$$\hat{k} = \begin{cases} a_1, & \tilde{k} = a_1\\ a_2, & \tilde{k} \in (a_1,\ a_2]\\ t_k + d_2, & \tilde{k} \geq t_k + d_2\end{cases} \tag{2}$$

where $a_1 = t_k + d_1 + p(d_2 - d_1)/l$, $a_2 = t_k + d_1 + (p+1)(d_2 - d_1)/l$, and $p = 0,\ 1,\ \cdots,\ (l-1)$. Then,

$$h_k = \hat{k} - t_k = d_1 + \frac{b(d_2 - d_1)}{l}, \quad b = 0,\ 1,\ \cdots,\ l \tag{3}$$

that is, the sampling period h_k switches in the finite set $\vartheta = \{d_1,\ d_1 + (d_2 - d_1)/l,\ \cdots,\ d_2\}$. If $\tau_k > d_2$, the control input $\boldsymbol{u}_k$ will not be used even if it reaches the actuator eventually (considering the definition of d_2, it is reasonable to drop $\boldsymbol{u}_k$ if $\tau_k > d_2$), and the latest available control input will be used.

If l is sufficiently large, $(d_2 - d_1)/l$ is very small compared with $d_1 + b(d_2 - d_1)/l$. So we can think, approximately, that the control input used in the interval $[t_k,\ \hat{k})$ is $\boldsymbol{u}_{k-i_k}$, where $i_k = 1,\ 2,\ \cdots,\ L$, and $L-1$ is the maximum number of consecutive packet dropout. So the discrete time representation of (1) can be described as follows

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \Phi_k \boldsymbol{x}_k + \Gamma_k \boldsymbol{u}_{k-i_k} + \widetilde{\Gamma}_k \boldsymbol{\omega}_k \\ \boldsymbol{u}_{k-i_k} &= -K\boldsymbol{x}_{k-i_k} \end{aligned} \tag{4}$$

where $\Phi_k = \mathrm{e}^{Ah_k}$, $\Gamma_k = \int_0^{h_k} \mathrm{e}^{As}\mathrm{d}sB_1$, and $\widetilde{\Gamma}_k = \int_0^{h_k} \mathrm{e}^{As}\mathrm{d}sB_2$.

Because the sampling period h_k switches in the finite set ϑ; Φ_k, Γ_k and $\widetilde{\Gamma}_k$ used in (4) also switch in finite sets. Then, the problem of H_∞ controller design for (1) can be reduced to the corresponding problem for (4).

Remark 1. As shown in Fig. 1, the control input $\boldsymbol{u}_{k-3}$ does not reach the actuator at the instant $t_{k-3} + d_2$ for a long time delay, the sensor will sample plant′s state at $t_{k-3} + d_2$, and the delayed control input will be dropped. The control input $\boldsymbol{u}_{k-1}$ is dropped, and the sensor will also begin the next sampling at the instant $t_{k-1} + d_2$. The sampling begins only when the former sampled packet reaches or is dropped, so packet disordering cannot occur.

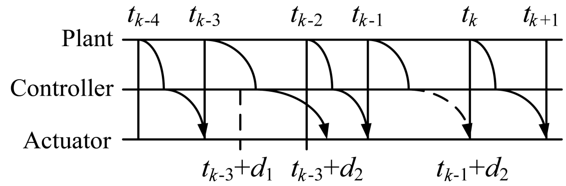

Fig. 1 Both clock-driven and event-driven sampling

Remark 2. To use the active-varying sampling period method, $[d_1,\ d_2]$ should be partitioned into l equidistant small intervals, and a large l can ensure the computational precision, but it might lead to frequent switching of sampling period, so l can be chosen on the basis of $d_2 - d_1$. If $d_2 - d_1$ is large, l should be large, otherwise, l should be small.

The following lemma will be used in the sequel.

Lemma 1[7]**.** Suppose $\boldsymbol{a} \in \mathbf{R}^n$, $\boldsymbol{b} \in \mathbf{R}^m$, and $G \in \mathbf{R}^{n\times m}$. Then, for any $X \in \mathbf{R}^{n\times n}$, $Y \in \mathbf{R}^{n\times m}$, and $Z \in \mathbf{R}^{m\times m}$ satisfying

$$\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geq 0$$

the following inequality holds

$$-2\boldsymbol{a}^{\mathrm{T}}G\boldsymbol{b} \leq \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X & Y - G \\ Y^{\mathrm{T}} - G^{\mathrm{T}} & Z \end{bmatrix} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}$$

2 H_∞ controller design with packet dropout compensation

To compensate the packet dropout of NCSs, a linear estimator might be added into the system to estimate the dropped control input packets. Suppose the states $\boldsymbol{x}_0$, $\boldsymbol{x}_{k_1}$, $\boldsymbol{x}_{k_2}$, $\cdots$, $\boldsymbol{x}_{k_j}$, $\cdots$, and the corresponding control inputs on the basis of these states are transferred to the actuator successfully, and suppose $L-1$ is the maximum number of consecutive packets dropout. Then, the estimated values of the dropped control inputs are as follows.

$$\begin{aligned} &\hat{\boldsymbol{u}}_{k_j+1} = \boldsymbol{u}_{k_j} - \frac{1}{L}\boldsymbol{u}_{k_j} = (1 - \frac{1}{L})\boldsymbol{u}_{k_j} \\ &\hat{\boldsymbol{u}}_{k_j+2} = \hat{\boldsymbol{u}}_{k_j+1} + (\hat{\boldsymbol{u}}_{k_j+1} - \boldsymbol{u}_{k_j}) = (1 - \frac{2}{L})\boldsymbol{u}_{k_j} \\ &\hat{\boldsymbol{u}}_{k_j+3} = \hat{\boldsymbol{u}}_{k_j+2} + (\hat{\boldsymbol{u}}_{k_j+2} - \hat{\boldsymbol{u}}_{k_j+1}) = (1 - \frac{3}{L})\boldsymbol{u}_{k_j} \\ &\qquad\cdots \\ &\hat{\boldsymbol{u}}_{k_{j+1}-1} = \hat{\boldsymbol{u}}_{k_{j+1}-2} + (\hat{\boldsymbol{u}}_{k_{j+1}-2} - \hat{\boldsymbol{u}}_{k_{j+1}-3}) = \\ &\qquad (1 - \frac{k_{j+1} - k_j - 1}{L})\boldsymbol{u}_{k_j} \end{aligned} \tag{5}$$

where L is a predefined positive scalar. Without loss of generality, by supposing the disturbance inputs $\boldsymbol{\omega}_{k_j} = \boldsymbol{\omega}_{k_j+1} = \cdots = \boldsymbol{\omega}_{k_{j+1}-1}$ for every k_j and using the packet dropout compensation method given above, the evolution of plant states can be described as follows.

$$\begin{aligned} &\boldsymbol{x}_{k_j+1} = \Phi_{h_{k_j}}\boldsymbol{x}_{k_j} + \Gamma_{h_{k_j}}\hat{\boldsymbol{u}}_{k_j-1} + \widetilde{\Gamma}_{h_{k_j}}\boldsymbol{\omega}_{k_j} = \\ &\quad \Phi_{h_{k_j}}\boldsymbol{x}_{k_j} - (1 - \frac{k_j - k_{j-1} - 1}{L})\Gamma_{h_{k_j}}K\boldsymbol{x}_{k_{j-1}} + \widetilde{\Gamma}_{h_{k_j}}\boldsymbol{\omega}_{k_j} \\ &\boldsymbol{x}_{k_j+2} = \Phi_b\boldsymbol{x}_{k_j+1} + \Gamma_b\boldsymbol{u}_{k_j} + \widetilde{\Gamma}_b\boldsymbol{\omega}_{k_j} = \\ &\quad (\Phi_b\Phi_{h_{k_j}} - \Gamma_bK)\boldsymbol{x}_{k_j} + (\Phi_b\widetilde{\Gamma}_{h_{k_j}} + \widetilde{\Gamma}_b)\boldsymbol{\omega}_{k_j} - \\ &\quad (1 - \frac{k_j - k_{j-1} - 1}{L})\Phi_b\Gamma_{h_{k_j}}K\boldsymbol{x}_{k_{j-1}} \\ &\boldsymbol{x}_{k_j+3} = \Phi_b\boldsymbol{x}_{k_j+2} + \Gamma_b\hat{\boldsymbol{u}}_{k_j+1} + \widetilde{\Gamma}_b\boldsymbol{\omega}_{k_j} = \\ &\quad [{\Phi_b}^2\Phi_{h_{k_j}} - \Phi_b\Gamma_bK - (1 - \frac{1}{L})\Gamma_bK]\boldsymbol{x}_{k_j} - \\ &\quad (1 - \frac{k_j - k_{j-1} - 1}{L}){\Phi_b}^2\Gamma_{h_{k_j}}K\boldsymbol{x}_{k_{j-1}} + \\ &\quad (\Phi_b^2\widetilde{\Gamma}_{h_{k_j}} + \Phi_b\widetilde{\Gamma}_b + \widetilde{\Gamma}_b)\boldsymbol{\omega}_{k_j} \\ &\qquad\cdots \\ &\boldsymbol{x}_{k_{j+1}} = \widetilde{A}_j\boldsymbol{x}_{k_j} + \widetilde{B}_j\boldsymbol{x}_{k_{j-1}} + \widetilde{D}_j\boldsymbol{\omega}_{k_j} \end{aligned} \tag{6}$$

where $\Phi_{h_{k_j}} = \mathrm{e}^{Ah_{k_j}}$, $\Phi_b = \mathrm{e}^{Ad_2}$, $\Gamma_{h_{k_j}} = \int_0^{h_{k_j}} \mathrm{e}^{As}\mathrm{d}sB_1$, $\widetilde{\Gamma}_{h_{k_j}} = \int_0^{h_{k_j}} \mathrm{e}^{As}\mathrm{d}sB_2$, $\Gamma_b = \int_0^{d_2} \mathrm{e}^{As}\mathrm{d}sB_1$, $\widetilde{\Gamma}_b = \int_0^{d_2} \mathrm{e}^{As}\mathrm{d}sB_2$, and

$$\begin{aligned} &\widetilde{A}_j = {\Phi_b}^{k_{j+1}-k_j-1}\Phi_{h_{k_j}} - {\Phi_b}^{k_{j+1}-k_j-2}\Gamma_bK - \\ &\qquad (1 - \frac{1}{L}){\Phi_b}^{k_{j+1}-k_j-3}\Gamma_bK - \\ &\qquad (1 - \frac{2}{L}){\Phi_b}^{k_{j+1}-k_j-4}\Gamma_bK - \cdots - \sigma_1\Gamma_bK \\ &\widetilde{B}_j = -\sigma_2{\Phi_b}^{k_{j+1}-k_j-1}\Gamma_{h_{k_j}}K \\ &\widetilde{D}_j = {\Phi_b}^{k_{j+1}-k_j-1}\widetilde{\Gamma}_{h_{k_j}} + {\Phi_b}^{k_{j+1}-k_j-2}\widetilde{\Gamma}_b + \cdots + \widetilde{\Gamma}_b \\ &\sigma_1 = (1 - \frac{k_{j+1} - k_j - 2}{L}) \\ &\sigma_2 = (1 - \frac{k_j - k_{j-1} - 1}{L}) \end{aligned} \tag{7}$$

Define $\boldsymbol{x}_{k_{j+1}}$, $\boldsymbol{x}_{k_j}$, $\boldsymbol{x}_{k_{j-1}}$, and $\boldsymbol{\omega}_{k_j}$ as $\boldsymbol{\xi}_{j+1}$, $\boldsymbol{\xi}_j$, $\boldsymbol{\xi}_{j-1}$, and $\boldsymbol{\omega}_j$, respectively. Then,

$$\boldsymbol{\xi}_{j+1} = \widetilde{A}_j\boldsymbol{\xi}_j + \widetilde{B}_j\boldsymbol{\xi}_{j-1} + \widetilde{D}_j\boldsymbol{\omega}_j \tag{8}$$

We are now in a position to design the feedback gain K, which can make system (8) asymptotically stable with the H_∞ norm bound $\gamma_{h_{k_j}}$ ($\gamma_{h_{k_j}}$ is the H_∞ norm bound corresponding to the sampling period h_{k_j}).

Theorem 1. If there exist symmetric positive definite matrices M, $\widetilde{R}$, and $\widetilde{Z}$, and matrices $\widetilde{X}$, $\widetilde{Y}$, N, and scalars $\gamma_{h_{k_j}} > 0$, such that the following LMIs (9) and (10) hold for every feasible values of $k_{j+1} - k_j$ and h_{k_j} ($k_{j+1} - k_j = 1, \cdots, L$, $h_{k_j} \in \vartheta$)

$$\begin{bmatrix} \Psi_0 & -\widetilde{Y} & 0 & MC_1^{\mathrm{T}} & \Psi_1^{\mathrm{T}} & \Psi_1^{\mathrm{T}} - M \\ * & -\widetilde{R} & 0 & -\sigma_2 N^{\mathrm{T}} D_1^{\mathrm{T}} & \Psi_2^{\mathrm{T}} & \Psi_2^{\mathrm{T}} \\ * & * & -\gamma_{h_{k_j}} I & 0 & \widetilde{D}_j^{\mathrm{T}} & \widetilde{D}_j^{\mathrm{T}} \\ * & * & * & -\gamma_{h_{k_j}} I & 0 & 0 \\ * & * & * & * & -M & 0 \\ * & * & * & * & * & \widetilde{Z} - 2M \end{bmatrix} < 0 \tag{9}$$

$$\begin{bmatrix} \widetilde{X} & \widetilde{Y} \\ \widetilde{Y}^{\mathrm{T}} & \widetilde{Z} \end{bmatrix} \geq 0 \tag{10}$$

where

$$\begin{aligned} \Psi_0 &= -M + \widetilde{X} + \widetilde{Y} + \widetilde{Y}^{\mathrm{T}} + \widetilde{R} \\ \Psi_1 &= {\Phi_b}^{k_{j+1}-k_j-1} \Phi_{h_{k_j}} M - {\Phi_b}^{k_{j+1}-k_j-2} \Gamma_b N - \\ &\quad (1 - \frac{1}{L}) {\Phi_b}^{k_{j+1}-k_j-3} \Gamma_b N - \\ &\quad (1 - \frac{2}{L}) {\Phi_b}^{k_{j+1}-k_j-4} \Gamma_b N - \cdots - \sigma_1 \Gamma_b N \\ \Psi_2 &= -\sigma_2 {\Phi_b}^{k_{j+1}-k_j-1} \Gamma_{h_{k_j}} N \end{aligned}$$

then, with the control law

$$\boldsymbol{u}_j = -K \boldsymbol{\xi}_j, \quad K = NM^{-1}$$

the system described by (8) is asymptotically stable with H_∞ norm bounds $\gamma_{h_{k_j}}$.

Proof. Let us consider the following Lyapunov function

$$\begin{aligned} V_j &= V_{1j} + V_{2j} + V_{3j} \\ V_{1j} &= \boldsymbol{\xi}_j^{\mathrm{T}} P \boldsymbol{\xi}_j \\ V_{2j} &= (\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1})^{\mathrm{T}} Z (\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1}) \\ V_{3j} &= \boldsymbol{\xi}_{j-1}^{\mathrm{T}} R \boldsymbol{\xi}_{j-1} \end{aligned}$$

where P, Z, and R are symmetric positive definite matrices. Noticing that

$$\boldsymbol{\xi}_{j+1} = (\widetilde{A}_j + \widetilde{B}_j) \boldsymbol{\xi}_j - \widetilde{B}_j (\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1}) + \widetilde{D}_j \boldsymbol{\omega}_j \tag{11}$$

So, the difference of function V_j along the trajectory of (11) is given by

$$\begin{aligned} \Delta V_{1j} = &\boldsymbol{\xi}_{j+1}^{\mathrm{T}} P \boldsymbol{\xi}_{j+1} - \boldsymbol{\xi}_j^{\mathrm{T}} P \boldsymbol{\xi}_j = \\ &\boldsymbol{\xi}_j^{\mathrm{T}} (\widetilde{A}_j + \widetilde{B}_j)^{\mathrm{T}} P (\widetilde{A}_j + \widetilde{B}_j) \boldsymbol{\xi}_j - \boldsymbol{\xi}_j^{\mathrm{T}} P \boldsymbol{\xi}_j - \\ &2\boldsymbol{\xi}_j^{\mathrm{T}} (\widetilde{A}_j + \widetilde{B}_j)^{\mathrm{T}} P \widetilde{B}_j (\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1}) + \\ &(\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1})^{\mathrm{T}} \widetilde{B}_j^{\mathrm{T}} P \widetilde{B}_j (\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1}) + \\ &2\boldsymbol{\xi}_j^{\mathrm{T}} (\widetilde{A}_j + \widetilde{B}_j)^{\mathrm{T}} P \widetilde{D}_j \boldsymbol{\omega}_j - \\ &2(\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1})^{\mathrm{T}} \widetilde{B}_j^{\mathrm{T}} P \widetilde{D}_j \boldsymbol{\omega}_j + \boldsymbol{\omega}_j^{\mathrm{T}} \widetilde{D}_j^{\mathrm{T}} P \widetilde{D}_j \boldsymbol{\omega}_j \end{aligned} \tag{12}$$

Defining $\boldsymbol{a} = \boldsymbol{\xi}_j$, $G = (\widetilde{A}_j + \widetilde{B}_j)^{\mathrm{T}} P \widetilde{B}_j$, $\boldsymbol{b} = (\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1})$, and using Lemma 1, for any X, Y, and Z satisfying $\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geq 0$, we have

$$\begin{aligned} &-2\boldsymbol{\xi}_j^{\mathrm{T}} (\widetilde{A}_j + \widetilde{B}_j)^{\mathrm{T}} P \widetilde{B}_j (\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1}) \leq \\ &\boldsymbol{\xi}_j^{\mathrm{T}} X \boldsymbol{\xi}_j + 2\boldsymbol{\xi}_j^{\mathrm{T}} [Y - (\widetilde{A}_j + \widetilde{B}_j)^{\mathrm{T}} P \widetilde{B}_j] (\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1}) + \\ &(\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1})^{\mathrm{T}} Z (\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1}) \end{aligned} \tag{13}$$

$$\begin{aligned} \Delta V_{2j} = &(\boldsymbol{\xi}_{j+1} - \boldsymbol{\xi}_j)^{\mathrm{T}} Z (\boldsymbol{\xi}_{j+1} - \boldsymbol{\xi}_j) - \\ &(\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1})^{\mathrm{T}} Z (\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1}) = \\ &[(\widetilde{A}_j - I)\boldsymbol{\xi}_j + \widetilde{B}_j \boldsymbol{\xi}_{j-1} + \widetilde{D}_j \boldsymbol{\omega}_j]^{\mathrm{T}} Z [(\widetilde{A}_j - I)\boldsymbol{\xi}_j + \\ &\widetilde{B}_j \boldsymbol{\xi}_{j-1} + \widetilde{D}_j \boldsymbol{\omega}_j] - (\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1})^{\mathrm{T}} Z (\boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1}) \end{aligned} \tag{14}$$

$$\Delta V_{3j} = \boldsymbol{\xi}_j^{\mathrm{T}} R \boldsymbol{\xi}_j - \boldsymbol{\xi}_{j-1}^{\mathrm{T}} R \boldsymbol{\xi}_{j-1}$$

Combining (12)$\sim$(14) together, we have

$$\Delta V_j = \Delta V_{1j} + \Delta V_{2j} + \Delta V_{3j} = \tilde{\boldsymbol{\xi}}_j^{\mathrm{T}} \Lambda_j \tilde{\boldsymbol{\xi}}_j \tag{15}$$

where

$$\tilde{\boldsymbol{\xi}}_j = \begin{bmatrix} \boldsymbol{\xi}_j \\ \boldsymbol{\xi}_{j-1} \\ \boldsymbol{\omega}_j \end{bmatrix}, \ \Lambda_j = \begin{bmatrix} \Lambda_j^{11} & \Lambda_j^{12} & \Lambda_j^{13} \\ * & \Lambda_j^{22} & \Lambda_j^{23} \\ * & * & \Lambda_j^{33} \end{bmatrix}$$

$$\begin{aligned} \Lambda_j^{11} &= -P + X + Y + Y^{\mathrm{T}} + R + \widetilde{A}_j^{\mathrm{T}} P \widetilde{A}_j + \\ &\quad (\widetilde{A}_j - I)^{\mathrm{T}} Z (\widetilde{A}_j - I) \\ \Lambda_j^{12} &= -Y + \widetilde{A}_j^{\mathrm{T}} P \widetilde{B}_j + (\widetilde{A}_j - I)^{\mathrm{T}} Z \widetilde{B}_j \\ \Lambda_j^{13} &= \widetilde{A}_j^{\mathrm{T}} P \widetilde{D}_j + (\widetilde{A}_j - I)^{\mathrm{T}} Z \widetilde{D}_j \\ \Lambda_j^{22} &= -R + \widetilde{B}_j^{\mathrm{T}} P \widetilde{B}_j + \widetilde{B}_j^{\mathrm{T}} Z \widetilde{B}_j \\ \Lambda_j^{23} &= \widetilde{B}_j^{\mathrm{T}} P \widetilde{D}_j + \widetilde{B}_j^{\mathrm{T}} Z \widetilde{D}_j \\ \Lambda_j^{33} &= \widetilde{D}_j^{\mathrm{T}} P \widetilde{D}_j + \widetilde{D}_j^{\mathrm{T}} Z \widetilde{D}_j \end{aligned}$$

If the packet dropout compensation method proposed in (5) is used, the available control input at instant k_j is $-\sigma_2 K \boldsymbol{\xi}_{j-1}$, that is, $z_j = C_1 \boldsymbol{\xi}_j - \sigma_2 D_1 K \boldsymbol{\xi}_{j-1}$. For any nonzero $\boldsymbol{\xi}_j$, we have

$$\gamma_{h_{k_j}}^{-1} \boldsymbol{z}_j^{\mathrm{T}} \boldsymbol{z}_j - \gamma_{h_{k_j}} \boldsymbol{\omega}_j^{\mathrm{T}} \boldsymbol{\omega}_j = \tilde{\boldsymbol{\xi}}_j^{\mathrm{T}} \Xi \tilde{\boldsymbol{\xi}}_j$$

where

$$\Xi = \begin{bmatrix} \gamma_{h_{k_j}}^{-1} {C_1}^{\mathrm{T}} C_1 & -\sigma_2 \gamma_{h_{k_j}}^{-1} {C_1}^{\mathrm{T}} D_1 K & 0 \\ * & \sigma_2^2 \gamma_{h_{k_j}}^{-1} K^{\mathrm{T}} D_1^{\mathrm{T}} D_1 K & 0 \\ * & * & -\gamma_{h_{k_j}} I \end{bmatrix}$$

So,

$$\gamma_{h_{k_j}}^{-1} \boldsymbol{z}_j^{\mathrm{T}} \boldsymbol{z}_j - \gamma_{h_{k_j}} \boldsymbol{\omega}_j^{\mathrm{T}} \boldsymbol{\omega}_j + \Delta V_j = \tilde{\boldsymbol{\xi}}_j^{\mathrm{T}} \widetilde{\Lambda}_j \tilde{\boldsymbol{\xi}}_j$$

where $\widetilde{\Lambda}_j = \Lambda_j + \Xi$. In the following, we will prove that $\gamma_{h_{k_j}}^{-1} \boldsymbol{z}_j^{\mathrm{T}} \boldsymbol{z}_j - \gamma_{h_{k_j}} \boldsymbol{\omega}_j^{\mathrm{T}} \boldsymbol{\omega}_j + \Delta V_j < 0$, that is, $\widetilde{\Lambda}_j < 0$. By the Schur complement, $\widetilde{\Lambda}_j < 0$ is equivalent to

$$\begin{bmatrix} \Upsilon & -Y & 0 & C_1^{\mathrm{T}} & \widetilde{A}_j^{\mathrm{T}} & (\widetilde{A}_j - I)^{\mathrm{T}} \\ * & -R & 0 & -\sigma_2 K^{\mathrm{T}} D_1^{\mathrm{T}} & \widetilde{B}_j^{\mathrm{T}} & \widetilde{B}_j^{\mathrm{T}} \\ * & * & -\gamma_{h_{k_j}} I & 0 & \widetilde{D}_j^{\mathrm{T}} & \widetilde{D}_j^{\mathrm{T}} \\ * & * & * & -\gamma_{h_{k_j}} I & 0 & 0 \\ * & * & * & * & -P^{-1} & 0 \\ * & * & * & * & * & -Z^{-1} \end{bmatrix} < 0 \tag{16}$$

where $\Upsilon = -P + X + Y + Y^{\mathrm{T}} + R$.

Pre- and post-multiplying (16) by $\mathrm{diag}\{P^{-1},\ P^{-1},\ I,\ I,\ I,\ I\}$ and $\mathrm{diag}\{P^{-1},\ P^{-1},\ I,\ I,\ I,\ I\}$, and defining $P^{-1}XP^{-1} = \widetilde{X}$, $P^{-1}YP^{-1} = \widetilde{Y}$, $P^{-1}ZP^{-1} = \widetilde{Z}$, $P^{-1}RP^{-1} = \widetilde{R}$, $KP^{-1} = N$, and $P^{-1} = M$. We can see that (16) is equivalent to

$$\begin{bmatrix} \widetilde{\Upsilon} & -\widetilde{Y} & 0 & MC_1^{\mathrm{T}} & M\widetilde{A}_j^{\mathrm{T}} & \mathscr{S} \\ * & -\widetilde{R} & 0 & -\sigma_2 N^{\mathrm{T}} D_1^{\mathrm{T}} & M\widetilde{B}_j^{\mathrm{T}} & M\widetilde{B}_j^{\mathrm{T}} \\ * & * & -\gamma_{h_{k_j}} I & 0 & \widetilde{D}_j^{\mathrm{T}} & \widetilde{D}_j^{\mathrm{T}} \\ * & * & * & -\gamma_{h_{k_j}} I & 0 & 0 \\ * & * & * & * & -M & 0 \\ * & * & * & * & * & -Z^{-1} \end{bmatrix} < 0 \tag{17}$$

where $\widetilde{\Upsilon} = -M + \widetilde{X} + \widetilde{Y} + \widetilde{Y}^{\mathrm{T}} + \widetilde{R}$, and $\mathscr{S} = M(\widetilde{A}_j - I)^{\mathrm{T}}$.

For symmetric positive definite matrices P and Z, we have $(P - Z)Z^{-1}(P - Z) \geq 0$, which is equivalent to $PZ^{-1}P - 2P + Z \geq 0$. Pre- and post-multiplying $PZ^{-1}P - 2P + Z \geq 0$ by P^{-1} and P^{-1}, we have $-Z^{-1} \leq P^{-1}ZP^{-1} - 2P^{-1} = \widetilde{Z} - 2M$. So if (9) is satisfied, (17) is also feasible. On the other hand, to ensure that (13) holds, we should have $\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geq 0$. Pre- and post-multiplying $\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geq 0$ by $\mathrm{diag}\{P^{-1},\ P^{-1}\}$ and $\mathrm{diag}\{P^{-1},\ P^{-1}\}$, we can see that $\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geq 0$ is equivalent to $\begin{bmatrix} \widetilde{X} & \widetilde{Y} \\ \widetilde{Y}^{\mathrm{T}} & \widetilde{Z} \end{bmatrix} \geq 0$. That is, if (9) and (10) are feasible, we have $\gamma_{h_{k_j}}^{-1} \boldsymbol{z}_j^{\mathrm{T}} \boldsymbol{z}_j - \gamma_{h_{k_j}} \boldsymbol{\omega}_j^{\mathrm{T}} \boldsymbol{\omega}_j + \Delta V_j < 0$.

Since $\gamma_{h_{k_j}}^{-1} \boldsymbol{z}_j^{\mathrm{T}} \boldsymbol{z}_j - \gamma_{h_{k_j}} \boldsymbol{\omega}_j^{\mathrm{T}} \boldsymbol{\omega}_j + \Delta V_j < 0$, we have

$$\gamma_{h_{k_j}}^{-1} \boldsymbol{z}_j^{\mathrm{T}} \boldsymbol{z}_j - \gamma_{h_{k_j}} \boldsymbol{\omega}_j^{\mathrm{T}} \boldsymbol{\omega}_j < -\Delta V_j$$

Summing up $\boldsymbol{z}_j^{\mathrm{T}} \boldsymbol{z}_j$, $\boldsymbol{\omega}_j^{\mathrm{T}} \boldsymbol{\omega}_j$, and ΔV_j in the above inequality for $j = 0$ to $j = n$, and using the zero initial condition and the character that the disturbance input $\boldsymbol{\omega}_j$ has limited energy, we have

$$\sum_{j=0}^{n} ||\boldsymbol{z}_j||^2 < \gamma_{h_{k_j}}^2 \sum_{j=0}^{n} ||\boldsymbol{\omega}_j||^2 - \gamma_{h_{k_j}} V_{n+1}$$

for all n. Let $n \to \infty$, we have

$$||\boldsymbol{z}||_2^2 < \gamma_{h_{k_j}}^2 ||\boldsymbol{\omega}||_2^2$$

If the disturbance input $\boldsymbol{\omega}_j = 0$, (9) and (10) can ensure the asymptotic stability of the system described by (8), and if $\boldsymbol{\omega}_j \neq 0$, we have $||\boldsymbol{z}||_2^2 < \gamma_{h_{k_j}}^2 ||\boldsymbol{\omega}||_2^2$. So if the LMIs (9) and (10) are feasible, the system described by (8) with $K = NM^{-1}$ is asymptotically stable with H_∞ norm bounds $\gamma_{h_{k_j}}$. □

If the sampling period h_{k_j} is constant, to stabilize the system described by (8), the LMIs (9) and (10) should be satisfied for the specific h_{k_j}, which can be described as the following corollary.

Corollary 1. For the specific sampling period h_{k_j}, if there exist symmetric positive definite matrices M, $\widetilde{R}$, and $\widetilde{Z}$, and matrices $\widetilde{X}$, $\widetilde{Y}$, N, and scalar $\gamma > 0$, such that the following LMIs hold for every feasible value of $k_{j+1} - k_j$ $(k_{j+1} - k_j = 1,\ \cdots,\ L)$, namely,

$$\begin{bmatrix} \Psi_0 & -\widetilde{Y} & 0 & MC_1^{\mathrm{T}} & \Psi_1^{\mathrm{T}} & \Psi_1^{\mathrm{T}} - M \\ * & -\widetilde{R} & 0 & -\sigma_2 N^{\mathrm{T}} D_1^{\mathrm{T}} & \Psi_2^{\mathrm{T}} & \Psi_2^{\mathrm{T}} \\ * & * & -\gamma I & 0 & \widetilde{D}_j^{\mathrm{T}} & \widetilde{D}_j^{\mathrm{T}} \\ * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & -M & 0 \\ * & * & * & * & * & \widetilde{Z} - 2M \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} \widetilde{X} & \widetilde{Y} \\ \widetilde{Y}^{\mathrm{T}} & \widetilde{Z} \end{bmatrix} \geq 0 \tag{19}$$

where

$$\begin{aligned} \Psi_0 &= -M + \widetilde{X} + \widetilde{Y} + \widetilde{Y}^{\mathrm{T}} + \widetilde{R} \\ \Psi_1 &= {\Phi_b}^{k_{j+1}-k_j-1} \Phi_{h_{k_j}} M - {\Phi_b}^{k_{j+1}-k_j-2} \Gamma_b N - \\ &\quad (1 - \frac{1}{L}) {\Phi_b}^{k_{j+1}-k_j-3} \Gamma_b N - \\ &\quad (1 - \frac{2}{L}) {\Phi_b}^{k_{j+1}-k_j-4} \Gamma_b N - \cdots - \sigma_1 \Gamma_b N \\ \Psi_2 &= -\sigma_2 {\Phi_b}^{k_{j+1}-k_j-1} \Gamma_{h_{k_j}} N \end{aligned}$$

then, with the control law

$$\boldsymbol{u}_j = -K\boldsymbol{\xi}_j, \quad K = NM^{-1}$$

the system described by (8) is asymptotically stable with H_∞ norm bound γ.

Remark 3. Just as shown in Theorem 1, it is difficult to optimize all the $\gamma_{h_{k_j}}$ simultaneously. The linear weighted sum $\gamma_{\rm sum}$ of $\gamma_{h_{k_j}}$ might be introduced to optimize $\gamma_{h_{k_j}}$. Define $\gamma_{\rm sum} > \alpha_1\gamma_{h_1} + \alpha_2\gamma_{h_2} + \cdots + \alpha_{l+1}\gamma_{h_{l+1}}$, where h_m $(m = 1,\ 2,\ \cdots,\ l+1)$ are the feasible values of sampling period h_{k_j}, α_m are the weighting coefficients, and $\alpha_m > 0$. The optimal $\gamma_{h_{k_j}}$ can be obtained by optimizing $\gamma_{\rm sum}$.

3 Numerical example

To illustrate the merits of the proposed active-varying sampling period method and the linear estimation-based packet dropout compensation, we present an open loop unstable system as follows.

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \begin{bmatrix} -0.3901 & 0.8855 \\ 1.4900 & -0.9821 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} -0.5359 \\ 1.0727 \end{bmatrix} \boldsymbol{u}(t) + \\ &\quad \begin{bmatrix} 0.3105 \\ 0.3144 \end{bmatrix} \boldsymbol{\omega}(t) \\ \boldsymbol{z}(t) &= \begin{bmatrix} -1.3659 & 0.1823 \end{bmatrix} \boldsymbol{x}(t) + 0.5485\boldsymbol{u}(t) \end{aligned} \tag{20}$$

Suppose the minimum sampling period of sensor is 0.05 s, and the maximum sampling period is 0.2 s, and suppose the feasible values of sampling period are $h_1 = 0.05$ s, $h_2 = 0.1$ s, and $h_3 = 0.2$ s, and the initial state of the system is $\boldsymbol{x}_0 = [1\ -1]^{\mathrm{T}}$. For simplicity, suppose the packets at the sampling instants 0, 3, 6, $\cdots$ are transferred to the actuator successfully, that is, 2 packets are dropped among every 3 packets, which means that $L = 3$.

If the method proposed in Theorem 1 is used to design H_∞ controllers, we might choose $\alpha_1 = 4$, $\alpha_2 = 3$, and $\alpha_3 = 5$ (denoted as Case 1), or $\alpha_1 = 8$, $\alpha_2 = 8$, and $\alpha_3 = 12$ (denoted as Case 2). The H_∞ norm bounds corresponding to sampling periods h_m $(m = 1,\ 2,\ 3)$ are shown in Table 1, and the controller gains corresponding to Case 1 and Case 2 are $\boldsymbol{K} = [21.6662\ 13.0813]$ and $\boldsymbol{K} = [21.6330\ 13.0642]$, respectively.

If constant sampling period is adopted, h_3 should be chosen as the sampling period to avoid network congestion when the network is occupied by the most number of users. The corresponding H_∞ norm bound is 7.1330, and the controller gain obtained by Corollary 1 is $\boldsymbol{K}$ = [21.6542 13.0743].

From Table 1, we can see that short sampling periods might provide better H_∞ norm bounds than constant sampling period h_3, and that one can choose appropriate weighting coefficients α_1, α_2, α_3 to obtain the desired H_∞ norm bounds. Table 1 also illustrates the effectiveness of the proposed packet dropout compensation method.

Table 1 The H_∞ norm bounds

γ_{h_m}	γ_{h_1}	γ_{h_2}	γ_{h_3}
Case 1	6.8027	6.5762	8.3125
Case 2	6.9335	6.6963	8.1406

If constant sampling period h_3 and the controller gain by Corollary 1 are used ($\boldsymbol{K}$ = [21.6542 13.0743]), the disturbance inputs $\sin(j)$ (j = 1, 2, $\cdots$, 20) are added into the system during the time interval [3, 15) s; the plant state response and controlled output are illustrated in Fig. 2 (a). If the initial sampling period is h_1, and the disturbance inputs $\sin(j)$ (j = 1, 2, $\cdots$, 20) are added into the system during the time interval [3, 6) s, then, at instant 6 s, the sampling period switches to h_3. And other disturbance inputs $\sin(j)$ (j = 1, 2, $\cdots$, 20) are added into the system during the time interval [6.6, 18.6) s, and the controller gain $\boldsymbol{K}$ = [21.6330 13.0642] is used; the plant state response and controlled output are illustrated in Fig. 2 (b).

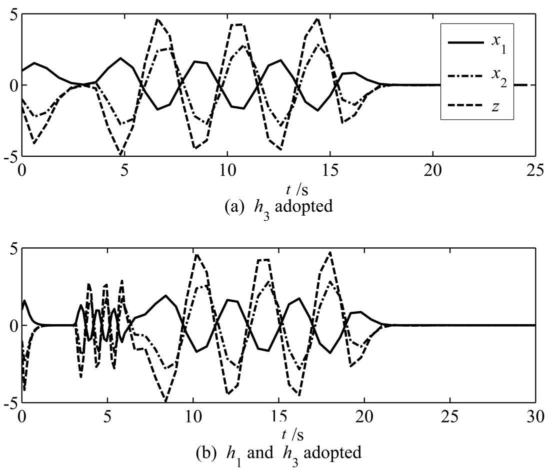

Fig. 2 Curves of states response and controlled outputs

Just as shown in Fig. 2, if $\boldsymbol{\omega}(t) \neq 0$ and there exists switching of sampling periods, short sampling periods might provide better H_∞ performance than constant sampling period h_3. Fig. 2 also illustrates the effectiveness of the proposed packet dropout compensation method.

4 Conclusion

This paper is concerned with the problem of designing robust H_∞ controllers for NCSs with both network-induced time delay and packet dropout by using the active-varying sampling period method. H_∞ controller design for NCSs with packet dropout compensation is presented by using LMI-based method. The simulation results have illustrated the effectiveness of the newly proposed active-varying sampling period method and the linear estimation-based packet dropout compensation.

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