Quantized Dynamic Output Feedback H_∞ Control for Discrete-time Systems with Quantizer Ranges Consideration

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Abstract The problem of quantized dynamic output feedback H_∞ control for discrete-time linear time-invariant (LTI) systems is investigated in this paper. The quantizer considered is dynamic and composed of an adjustable "zoom" parameter and a static quantizer. Static quantizer ranges are of practical significance and are fully considered. First, taking quantization errors into account, a quantized control strategy is dependent not only on the controller states but also on the system measurement outputs, which is proposed such that the quantized closed-loop system is asymptotically stable and with a prescribed H_∞ performance bound. Then, on the basis of this result, an iterative LMI-based optimization algorithm is developed to optimize the static quantizer ranges to meet H_∞ performance requirements for closed-loop systems. An example is presented to illustrate the effectiveness of the proposed method.

Key words Dynamic output feedback, dynamic quantizer, optimization, H_∞ control, LMI

Recent years have witnessed a growing interest in investigating effects of signal quantization on feedback control systems. This is mainly because of the widespread use in control systems of digital computers that use finite-precision arithmetic. The emerging network based control[1−2], where information between the controller and the plant is exchanged through a network medium with limited capacities has further strengthened the importance of the study on quantized feedback control.

It is well known that a feedback law which globally asymptotically stabilizes a given system in the absence of quantization will in general fail to provide global asymptotic stability of the closed-loop system that arises in the presence of a quantizer with a finite number of values. There are two reasons to account for these changes in the system′s behavior. One reason is saturation and another reason is deterioration of performance near the equilibrium. Because of these phenomena, instead of global asymptotic stability, the so-called practice stability is obtained, where there is a region of attraction in the state and the steady state converges to a small limit cycle[3−8].

When a dynamic quantizer is allowed, it is shown in [9−10] that stabilization of an SISO LTI system can be achieved using only a finite number of quantization levels. On this line, many important results have been reported[9−15]. Reference [15] defines a quantizer that takes value in a finite set and considers quantized feedback stabilization for linear systems. A hybrid control strategy originating in [15] relies on the possibility of making discrete online adjustments of quantizer parameters. Building on the work of [15−16], more general nonlinear systems with general types of quantizers affecting the state of the system, the measured output, or the control input is studied. Recently, [17] extended the result of [16] to continuous-time quantized state feedback H_∞ control systems. And in [9], a dynamic scaling method for the logarithmic quantizer was introduced.

In fact, the dynamic quantizer introduced in [16] can be seen as a device composed of an adjustable "zoom" parameter and a static quantizer. For a static quantizer, its range is of practical significance and cannot be chosen arbitrarily. For example, if a quantizer is used to represent a camera, this corresponds to zooming in and out, i.e., varying the focal length, whereas the number of pixels of course remains fixed and cannot be chosen arbitrarily. In addition, for a quantizer with an especially large quantizer range, the hardware cost is very high and in some situations this point have no economical advantages. But noting that for all the above quantized results, there is no one considers the static quantizer ranges, and they all give the stabilization or performance guaranteed condition under the assumption that the static quantizer ranges can be chosen sufficiently larger. This does not accord with the fact that the static quantizer range is fixed and practically significant. Moreover, among the above mentioned results, there is none that deals with the problem of dynamic output feedback H_∞ controller design for discrete-time systems with the consideration of three quantized signals at the same time, i.e., controller state quantization, control input quantization, and system measurement output quantization.

Motivated by the above reasons and considering the static quantizer ranges, the dynamic output feedback H_∞ control problem for discrete-time LTI systems with dynamic quantizers defined in [16] is studied in this paper. First, on account of the quantization errors, a quantized control strategy, which is dependent not only on the controller states but also on the system measurement outputs, is proposed such that the quantized dynamic output feedback H_∞ control problem can be solved. Then, on the basis of this result, an iterative LMI-based optimization algorithm is developed to optimize the static quantizer ranges to meet H_∞ performance requirements for closed-loop systems.

The outline of this paper is as follows. Section 1 presents the problem under consideration and some preliminaries. Section 2 studies the problem of quantized dynamic output feedback H_∞ control with three quantized signals, i.e., controller state quantization, control input quantization, and measurement output quantization. In Section 3, an example is presented to illustrate the effectiveness of the control

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strategy proposed in this paper. Finally, Section 4 gives some concluding remarks.

Notation. Given a matrix E, E^{T} and E^{-1} denote its transpose and inverse when it exists, respectively. The symbol $*$ within a matrix represents the symmetric entries. For a vector $\boldsymbol{x} \in \mathbf{R}^k$, the 2-norm of $\boldsymbol{x}$ is defined as $|\boldsymbol{x}| = (\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x})^{\frac{1}{2}}$, and for a matrix $Q \in \mathbf{R}^{m\times n}$, $\|Q\|$, $\lambda_{\max}(Q)$, and $\lambda_{\min}(Q)$ are defined as the largest singular value, the maximum eigenvalue, and the minimum eigenvalue of Q, respectively.

1 Problem statement and preliminaries

Consider the discrete-time LTI model described by

$$\begin{aligned} \boldsymbol{x}(k+1) &= A\boldsymbol{x}(k) + B_1\boldsymbol{u}(k) + B_2\boldsymbol{\omega}(k) \\ \boldsymbol{z}(k) &= C_1\boldsymbol{x}(k) + D_{12}\boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= C_2\boldsymbol{x}(k) \end{aligned} \tag{1}$$

where $\boldsymbol{x}(k) \in \mathbf{R}^n$ is the state, $\boldsymbol{y}(k) \in \mathbf{R}^p$ is the measured output, $\boldsymbol{z}(k) \in \mathbf{R}^q$ is the regulated output, $\boldsymbol{u}(k) \in \mathbf{R}^m$ is the control input, and $\boldsymbol{\omega}(k) \in \mathbf{R}^l$ is the disturbance input. A, B_1, B_2, C_1, C_2, and D_{12} are known constant matrices of appropriate dimensions. Assume that the triple (A, B_1, C_2) is stabilizable and detectable. Consider a dynamic output feedback controller described by

$$\begin{aligned} \boldsymbol{x}_c(k+1) &= A_c\boldsymbol{x}_c(k) + B_c\boldsymbol{y}(k) \\ \boldsymbol{u}(k) &= C_c\boldsymbol{x}_c(k) \end{aligned} \tag{2}$$

Applying controller (2) to system (1), the following closed-loop system is obtained.

$$\begin{aligned} \boldsymbol{x}_e(k+1) &= A_e\boldsymbol{x}_e(k) + B_e\boldsymbol{\omega}(k) \\ \boldsymbol{z}_e(k) &= C_e\boldsymbol{x}_e(k) \end{aligned} \tag{3}$$

where $\boldsymbol{x}_e(k) = [\boldsymbol{x}^{\mathrm{T}}(k), \boldsymbol{x}_c^{\mathrm{T}}(k)]^{\mathrm{T}}$ and

$$A_e = \begin{bmatrix} A & B_1C_c \\ B_cC_2 & A_c \end{bmatrix}, B_e = \begin{bmatrix} B_2 \\ 0 \end{bmatrix}, \text{ and } C_e = \begin{bmatrix} C_1 & D_{12}C_c \end{bmatrix}$$

However, in practical network communication or digital control systems, the signals of controller states, measurement outputs, and control inputs are quantized and then coded for transmission.

The quantizer considered here is defined with general form as in [16]. Let $\boldsymbol{z} \in \mathbf{R}^l$ be the variable being quantized. By a quantizer we mean a piecewise constant function $q : \mathbf{R}^l \to \boldsymbol{D}$, where $\boldsymbol{D}$ is a finite subset of $\mathbf{R}^l$. Assume that there are positive real numbers M and Δ such that the following condition holds:

$$\begin{aligned} \text{If} \quad & |\boldsymbol{z}| \le M \\ \text{then} \quad & |q(\boldsymbol{z}) - \boldsymbol{z}| \le \Delta \end{aligned} \tag{4}$$

This condition gives a bound on the quantization error when the quantizer does not saturate. M and Δ represent the range of the quantizer $q(\cdot)$ and the quantization error, respectively. Assume that $q(\boldsymbol{x}) = 0$ for $\boldsymbol{x}$ in some neighborhood of the origin, i.e., the origin lies in the interior of the set $\{\boldsymbol{x} : q(\boldsymbol{x}) = 0\}$.

In the control strategy to be proposed below, the quantized measurement is presented with the following form.

$$q_\mu(\boldsymbol{z}) = \mu q(\frac{\boldsymbol{z}}{\mu}) \tag{5}$$

where $\mu > 0$ is the quantizer′s parameter. The range of this quantizer is $M\mu$ and the quantization error is $\Delta\mu$.

Therefore, the quantized controller considered in this paper is given as

$$\begin{aligned} \boldsymbol{x}_c(k+1) &= A_cq_{\mu_1}(\boldsymbol{x}_c) + B_cq_{\mu_2}(\boldsymbol{y}) = \\ & A_c\mu_1q_1(\frac{\boldsymbol{x}_c}{\mu_1}) + B_c\mu_2q_2(\frac{\boldsymbol{y}}{\mu_2}) \\ \boldsymbol{u}(k) &= q_{\mu_3}(C_cq_{\mu_1}(\boldsymbol{x}_c)) = \mu_3q_3(\frac{C_c\mu_1q_1(\frac{\boldsymbol{x}_c}{\mu_1})}{\mu_3}) \end{aligned} \tag{6}$$

where the quantizers $q_{\mu_1}(\cdot)$, $q_{\mu_2}(\cdot)$, and $q_{\mu_3}(\cdot)$ are dynamic and defined by (5), the quantizers $q_1(\cdot)$, $q_2(\cdot)$, and $q_3(\cdot)$ are static and defined by (4). Here, $q_1(\cdot)$ is a controller state quantizer with range M_1 and error Δ_1, $q_2(\cdot)$ is a measurement output quantizer with range M_2 and error Δ_2, and $q_3(\cdot)$ is a control input quantizer with range M_3 and error Δ_3. For this quantized controller, we consider a strategy always satisfying

$$\mu_3 = \theta\mu_1, \ M_3 = \frac{\|C_c\|}{\theta}(\Delta_1 + M_1) \tag{7}$$

which will be derived in the next section, where θ is a positive constant and should be adjusted in real applications. Applying (6) to (1), the following quantized closed-loop system is obtained.

$$\begin{aligned} \boldsymbol{x}_e(k+1) &= A_e\boldsymbol{x}_e(k) + B_e\boldsymbol{\omega}(k) + \bar{B}_1\bar{\boldsymbol{e}} \\ \boldsymbol{z}_e(k) &= C_e\boldsymbol{x}_e(k) + \bar{D}_1\bar{\boldsymbol{e}} \end{aligned} \tag{8}$$

where A_e, B_e, and C_e are defined in (3), $\bar{B}_1 = \begin{bmatrix} 0 & 0 & B_1 \\ A_c & B_c & 0 \end{bmatrix}$, $\bar{D}_1 = \begin{bmatrix} 0 & 0 & D_{12} \end{bmatrix}$, $\bar{\boldsymbol{e}} = \begin{bmatrix} \mu_1\boldsymbol{e}_1 \\ \mu_2\boldsymbol{e}_2 \\ \mu_3\boldsymbol{e}_3 \end{bmatrix}$, with $\boldsymbol{e}_1 = q_1(\frac{\boldsymbol{x}_c}{\mu_1}) - \frac{\boldsymbol{x}_c}{\mu_1}$, $\boldsymbol{e}_2 = q_2(\frac{\boldsymbol{y}}{\mu_2}) - \frac{\boldsymbol{y}}{\mu_2}$ and $\boldsymbol{e}_3 = q_3(\frac{C_cq_1(\frac{\boldsymbol{x}_c}{\mu_1})}{\mu_3}) - \frac{C_c\boldsymbol{x}_c}{\mu_3}$.

Now, the problem under consideration in this paper is described as follows.

Problem 1. For quantizers $q_1(\cdot)$, $q_2(\cdot)$, and $q_3(\cdot)$, find a method to optimize the quantizer ranges M_1, M_2, and M_3, and then, design a quantized control strategy for system (1) such that the closed-loop system (8) is asymptotically stable and with a prescribed H_∞ performance bound.

2 Quantized dynamic output feedback H_∞ controller design

First, a lemma is presented which will be used in the sequel.

Lemma 1. For plant (8), $\gamma > 0$ is a given scalar. If there are matrices $\hat{A}, \hat{B}, \hat{C}$ and $Y > 0$, $X > 0$ such that

the following LMI holds.

$$\begin{bmatrix} -X & * & * & * & * & * & * & * \\ -I & -Y & * & * & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * & * & * \\ \Gamma_1 & A & B_2 & -X & * & * & * & * \\ \hat{A} & \Gamma_2 & YB_2 & -I & -Y & * & * & * \\ \Gamma_3 & C_1 & 0 & 0 & 0 & -I & * & * \\ 0 & 0 & B_2 & 0 & 0 & 0 & -X & * \\ 0 & 0 & YB_2 & 0 & 0 & 0 & -I & -Y \end{bmatrix} < 0 \tag{9}$$

where

$\Gamma_1 = AX + B_1\hat{C}, \Gamma_2 = YA + \hat{B}C_2, \Gamma_3 = C_1X + D_{12}\hat{C}$

Denote

$$\begin{aligned} A_c &= N^{-1}(\hat{A} - YAX - \hat{B}C_2X - YB_1\hat{C})(M^{\mathrm{T}})^{-1} \\ B_c &= N^{-1}\hat{B}, C_c = \hat{C}(M^{\mathrm{T}})^{-1} \end{aligned} \tag{10}$$

where $MN^{\mathrm{T}} = I - XY$, then there are two positive definite matrices $P > 0$ and

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} > 0 \tag{11}$$

such that the following LMI holds

$$\begin{bmatrix} A_e^{\mathrm{T}}PA_e - P + C_e^{\mathrm{T}}C_e + Q_{11} & * \\ B_e^{\mathrm{T}}PA_e + Q_{12}^{\mathrm{T}} & 2B_e^{\mathrm{T}}PB_e + Q_{22} - \gamma^2 I \end{bmatrix} < 0 \tag{12}$$

where A_e, B_e, and C_e are defined in (3).

Proof. Let $P = \begin{bmatrix} Y & N \\ N^{\mathrm{T}} & W \end{bmatrix}$ and $P^{-1} = \begin{bmatrix} X & M \\ M^{\mathrm{T}} & Z \end{bmatrix}$. First, we prove that the following inequality holds.

$$\begin{bmatrix} A_e^{\mathrm{T}}PA_e - P + C_e^{\mathrm{T}}C_e & * \\ B_e^{\mathrm{T}}PA_e & -\gamma^2 I + 2B_e^{\mathrm{T}}PB_e \end{bmatrix} < 0 \tag{13}$$

By the Schur complement, (13) is equivalent to the following LMI.

$$\begin{bmatrix} -P & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * \\ PA_e & PB_e & -P & * & * \\ C_e & 0 & 0 & -I & * \\ 0 & PB_e & 0 & 0 & -P \end{bmatrix} < 0 \tag{14}$$

Denote $\Gamma_1 = \begin{bmatrix} X & I \\ M^{\mathrm{T}} & 0 \end{bmatrix}$ and $\bar{\Gamma} = \mathrm{diag}\{\Gamma_1, I, \Gamma_1, I, \Gamma_1\}$, performing a congruence transformation with $\bar{\Gamma}$ on (14), and using the gain matrices described by (10), we obtain that (14) is equivalent to (9), which implies that (9) is equivalent to (13). Then, according to (13), it is clear that there always are positive definite matrices Q described by (11) and P such that (12) holds. □

Remark 1. It is well-known that the standard H_∞ controller design for discrete-time systems is reduced to solving the following inequality

$$\begin{bmatrix} A_e^{\mathrm{T}}PA_e - P + C_e^{\mathrm{T}}C_e & A_e^{\mathrm{T}}PB_e \\ * & -\gamma^2 I + B_e^{\mathrm{T}}PB_e \end{bmatrix} < 0 \tag{15}$$

By comparing (12) with (15), we see that except the added matrix variable Q, the block (2, 2) in the left of inequality (12) is with an additional term $B_e^{\mathrm{T}}PB_e$, which reflects the effectiveness of the quantization errors.

By using the gain matrices A_c, B_c, C_c, the scalar γ and the two positive definite matrices P and Q obtained by Lemma 1, we have

Theorem 1. Consider plant (1) controlled by the quantized dynamic output feedback controller (6) and assume that M_1 and M_2 are chosen large enough such that

$$M_1 > \frac{\eta\Delta}{\lambda_{\min}(Q)} \tag{16}$$

$$M_2 > \frac{\eta\Delta_2\|C_2\|}{\lambda_{\min}(Q)} \tag{17}$$

where $\Delta = \sqrt{\Delta_1^2 + (\theta\Delta_3 + \|C_c\|\Delta_1)^2}$, $\eta = \phi + \sqrt{\phi^2 + \varphi\lambda_{\min}(Q)}$ with $\phi = \|C_e^{\mathrm{T}}\bar{D}_1 + A_e^{\mathrm{T}}P\bar{B}_1\|$ and $\varphi = \|\bar{D}_1^{\mathrm{T}}\bar{D}_1 + 2\bar{B}_1^{\mathrm{T}}P\bar{B}_1\|$. Then, the control strategy (6) with updating μ_1 and μ_3 by

$$\mu_1 = \frac{2|\boldsymbol{x}_c|}{M_1 + \dfrac{\eta\Delta}{\lambda_{\min}(Q)}}, \ \mu_3 = \theta\mu_1 \tag{18}$$

and updating μ_2 by

$$\mu_2 = \frac{2|\boldsymbol{y}|}{M_2 + \dfrac{\eta\Delta_2\|C_2\|}{\lambda_{\min}(Q)}} \tag{19}$$

renders the closed-loop system (8) asymptotically stable and with the H_∞ performance bound γ.

Proof. By using the properties of (4) for the quantizer $q_2(\cdot)$, it is easy to check that whenever $|\boldsymbol{y}| \le M_2\mu_2$,

$$|\boldsymbol{e}_2| = |q_2(\frac{\boldsymbol{y}}{\mu_2}) - \frac{\boldsymbol{y}}{\mu_2}| \le \Delta_2 \tag{20}$$

and whenever $|\boldsymbol{x}_c| \le M_1\mu_1$,

$$\begin{aligned} &|\boldsymbol{e}_1| = |q_1(\frac{\boldsymbol{x}_c}{\mu_1}) - \frac{\boldsymbol{x}_c}{\mu_1}| \le \Delta_1 \Longrightarrow \\ &|q_1(\frac{\boldsymbol{x}_c}{\mu_1})| \le |\frac{\boldsymbol{x}_c}{\mu_1}| + \Delta_1 \le M_1 + \Delta_1 \end{aligned} \tag{21}$$

and thus,

$$|\frac{C_c\mu_1 q_1(\dfrac{\boldsymbol{x}_c}{\mu_1})}{\mu_3}| \le \frac{\|C_c\|\mu_1}{\mu_3}|q_1(\frac{\boldsymbol{x}_c}{\mu_1})| \le M_3 \tag{22}$$

Using the properties of (4) again for the quantizer $q_3(\cdot)$, we obtain that whenever $|\boldsymbol{x}_c| \le M_1\mu_1$,

$$\begin{aligned} &|\boldsymbol{e}_3| = |q_3(\frac{C_c\mu_1 q_1(\dfrac{\boldsymbol{x}_c}{\mu_1})}{\mu_3}) - \frac{C_c\boldsymbol{x}_c}{\mu_3}| = \\ &|q_3(\frac{C_c\mu_1 q_1(\dfrac{\boldsymbol{x}_c}{\mu_1})}{\mu_3}) - \frac{C_c\mu_1 q_1(\dfrac{\boldsymbol{x}_c}{\mu_1})}{\mu_3} + \frac{C_c\mu_1 q_1(\dfrac{\boldsymbol{x}_c}{\mu_1})}{\mu_3} - \frac{C_c\boldsymbol{x}_c}{\mu_3}| \le \\ &\Delta_3 + \frac{\|C_c\|\mu_1}{\mu_3}|q_1(\frac{\boldsymbol{x}_c}{\mu_1}) - \frac{\boldsymbol{x}_c}{\mu_1}| \le \Delta_3 + \frac{\|C_c\|}{\theta}\Delta_1 \end{aligned} \tag{23}$$

Then, we can obtain that

$$\begin{aligned} |\bar{\boldsymbol{e}}| &= \sqrt{\mu_1^2|\boldsymbol{e}_1|^2 + \mu_2^2|\boldsymbol{e}_2|^2 + \mu_3^2|\boldsymbol{e}_3|^2} \le \\ &\sqrt{\mu_1^2\Delta_1^2 + \mu_2^2\Delta_2^2 + \mu_1^2(\theta\Delta_3 + \|C_c\|\Delta_1)^2} = \\ &\sqrt{\mu_1^2\Delta^2 + \mu_2^2\Delta_2^2} \end{aligned} \tag{24}$$

Now, consider the Lyapunov function candidate $V(k) = \boldsymbol{x}_e^{\mathrm{T}}(k)P\boldsymbol{x}_e(k)$ for the quantized closed-loop system (8). By using (12), the difference of $V(k)$ along solutions of (8) is computed as

$$\begin{aligned}
&\Delta V(k) = V(k+1) - V(k) = \\
&(A_e\boldsymbol{x}_e + B_e\boldsymbol{\omega} + \bar{B}_1\bar{\boldsymbol{e}})^{\mathrm{T}}P(A_e\boldsymbol{x}_e + B_e\boldsymbol{\omega} + \bar{B}_1\bar{\boldsymbol{e}}) - \boldsymbol{x}_e^{\mathrm{T}}P\boldsymbol{x}_e \le \\
&-\boldsymbol{z}_e^{\mathrm{T}}\boldsymbol{z}_e + \gamma^2\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\omega} + \boldsymbol{x}_e^{\mathrm{T}}(A_e^{\mathrm{T}}PA_e - P + C_e^{\mathrm{T}}C_e)\boldsymbol{x}_e + \\
&2\boldsymbol{x}_e^{\mathrm{T}}A_e^{\mathrm{T}}PB_e\boldsymbol{\omega} + 2\boldsymbol{x}_e^{\mathrm{T}}(A_e^{\mathrm{T}}P\bar{B}_1 + C_e^{\mathrm{T}}D)\bar{\boldsymbol{e}} + \\
&2\boldsymbol{\omega}^{\mathrm{T}}B_e^{\mathrm{T}}PB_e\boldsymbol{\omega} + \bar{\boldsymbol{e}}^{\mathrm{T}}(2\bar{B}_1^{\mathrm{T}}P\bar{B}_1 + \bar{D}_1^{\mathrm{T}}\bar{D}_1)\bar{\boldsymbol{e}} - \gamma^2\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\omega} \le \\
&-\boldsymbol{z}_e^{\mathrm{T}}\boldsymbol{z}_e + \gamma^2\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\omega} - \begin{bmatrix}\boldsymbol{x}_e\\ \boldsymbol{\omega}\end{bmatrix}^{\mathrm{T}} Q \begin{bmatrix}\boldsymbol{x}_e\\ \boldsymbol{\omega}\end{bmatrix} + 2|\boldsymbol{x}_e|\phi|\bar{\boldsymbol{e}}| + |\bar{\boldsymbol{e}}|^2\varphi \le \\
&-\boldsymbol{z}_e^{\mathrm{T}}\boldsymbol{z}_e + \gamma^2\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\omega} - \lambda_{\min}(Q)|\boldsymbol{x}_e|^2 + 2|\boldsymbol{x}_e|\phi|\bar{\boldsymbol{e}}| + |\bar{\boldsymbol{e}}|^2\varphi \le \\
&-\boldsymbol{z}_e^{\mathrm{T}}\boldsymbol{z}_e + \gamma^2\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\omega} - \\
&\lambda_{\min}(Q)(\sqrt{\frac{|\boldsymbol{y}|^2}{\|C_2\|^2} + |\boldsymbol{x}_c|^2} - \frac{\eta|\bar{\boldsymbol{e}}|}{\lambda_{\min}(Q)})\times \\
&(\sqrt{\frac{|\boldsymbol{y}|^2}{\|C_2\|^2} + |\boldsymbol{x}_c|^2} - \frac{(\phi - \sqrt{\phi^2 + \varphi\lambda_{\min}(Q)})|\bar{\boldsymbol{e}}|}{\lambda_{\min}(Q)}) \qquad (25)
\end{aligned}$$

Then, on the one hand, according to (16) and (17), there always is a scalar $\varepsilon \in (0,1)$ such that

$$M_1 > \frac{\eta\Delta}{\lambda_{\min}(Q)(1-\varepsilon)} \qquad (26)$$

$$M_2 > \frac{\eta\Delta_2\|C_2\|}{\lambda_{\min}(Q)(1-\varepsilon)} \qquad (27)$$

In contrast, according to (18), for any nonzero $\boldsymbol{x}_c$, there always is a positive scalar μ_1 such that

$$|\boldsymbol{x}_c| = (M_1 + \frac{\eta\Delta}{\lambda_{\min}(Q)})\frac{\mu_1}{2} \qquad (28)$$

Similarly, according to (19), for any nonzero $\boldsymbol{y}$, there always is a positive scalar μ_2 such that

$$|\boldsymbol{y}| = (M_2 + \frac{\eta\Delta_2\|C_2\|}{\lambda_{\min}(Q)})\frac{\mu_2}{2} \qquad (29)$$

Then, by using (26) and (28), we have

$$\frac{\eta\Delta}{\lambda_{\min}(Q)(1-\varepsilon)}\mu_1 \le |\boldsymbol{x}_c| \le M_1\mu_1 \qquad (30)$$

and by using (27) and (29), we obtain

$$\frac{\eta\Delta_2\|C_2\|}{\lambda_{\min}(Q)(1-\varepsilon)}\mu_2 \le |\boldsymbol{y}| \le M_2\mu_2 \qquad (31)$$

This is also true in the case of $\boldsymbol{x}_c = 0$ and $\boldsymbol{y} = 0$, where we set $\mu_1 = 0$ and $\mu_2 = 0$ as an extreme case and consider the outputs of the quantizers $q_1(\cdot)$ and $q_2(\cdot)$ as zero.

In other words, if we always choose μ_1 and μ_2 satisfying (18) and (19), respectively, then (30) and (31) hold. Thus, according to (25), we obtain

$$\begin{aligned}
\Delta V(k) &\le -\boldsymbol{z}^{\mathrm{T}}(k)\boldsymbol{z}(k) + \gamma^2\boldsymbol{\omega}^{\mathrm{T}}(k)\boldsymbol{\omega}(k) - \varepsilon^2\lambda_{\min}(Q)|\boldsymbol{x}_e|^2 \le \\
&-\boldsymbol{z}^{\mathrm{T}}(k)\boldsymbol{z}(k) + \gamma^2\boldsymbol{\omega}^{\mathrm{T}}(k)\boldsymbol{\omega}(k) - \varepsilon^2\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}V(\boldsymbol{x}_e)
\end{aligned} \qquad (32)$$

First, by setting $\boldsymbol{\omega} = 0$, it is easy to see that $\Delta V(k) < 0$, i.e., the system is asymptotically stable.

Next, for any k,

$$V(k) - V(0) \le -\sum_{k=0}^{n}(\boldsymbol{z}^{\mathrm{T}}(k)\boldsymbol{z}(k) - \gamma^2\boldsymbol{\omega}^{\mathrm{T}}(k)\boldsymbol{\omega}(k)) \qquad (33)$$

Using $V(k) \ge 0$, $\Delta V(k) < 0$, and zero initial condition, the following is obtained

$$\|\boldsymbol{z}(k)\|_2^2 < \gamma^2\|\boldsymbol{\omega}(k)\|_2^2 \qquad (34)$$

which implies that the H_∞ disturbance attention level γ is achieved. □

Remark 2. Theorem 1 offers a quantized control strategy for discrete-time dynamic output feedback systems, such that the required H_∞ performance can be guaranteed. A similar result for continuous-time state feedback systems has been obtained in [17]. The difference between them is that in [17], the controller gain used in the quantized control strategy is firstly designed by the standard state-feedback H_∞ controller design, whereas in this paper, the controller gain used in the quantized control strategy of Theorem 1 is firstly designed by Lemma 1 with the consideration of the effect of the quantization errors. In fact, the additional term $B_e^{\mathrm{T}}PB_e$ in the block (2, 2) of inequality (12), as mentioned in Remark 1, is resulted from the term $2\boldsymbol{\omega}^{\mathrm{T}}B_e^{\mathrm{T}}P\bar{B}_1\bar{\boldsymbol{e}}$ in (25) by using $2\boldsymbol{\omega}^{\mathrm{T}}B_e^{\mathrm{T}}P\bar{B}_1\bar{\boldsymbol{e}} \le \boldsymbol{\omega}^{\mathrm{T}}B_e^{\mathrm{T}}PB_e\boldsymbol{\omega} + \bar{\boldsymbol{e}}^{\mathrm{T}}\bar{B}_1^{\mathrm{T}}P\bar{B}_1\bar{\boldsymbol{e}}$, which renders the effect of the quantization errors involved in the proposed design conditions.

Remark 3. By Theorem 1, quantizer ranges for the existence of a quantized control strategy is obtained naturally. But the static quantizer ranges gained by this direct method might be very large and might not be in accord with practice. So in the following we will give a method to optimize the static quantizer ranges and in turn to give an optimized control strategy, such that the required control objective can be achieved.

According to (16) and (17), in order to make M_1, M_2, and M_3 (M_3 is dependent on M_1) small one should minimize the values of $\eta\Delta/\lambda_{\min}(Q)$ and $\eta\Delta_2\|C_2\|/\lambda_{\min}(Q)$, respectively. Note that there is no direct method to minimize the values of $\eta\Delta/\lambda_{\min}(Q)$ and $\eta\Delta_2\|C_2\|/\lambda_{\min}(Q)$ because of their nonlinear property. But they consist of ϕ, φ, and $\|C_c\|$. Thus, by optimizing ϕ, φ, and $\|C_c\|$, we can optimize the values of $\eta\Delta/\lambda_{\min}(Q)$ and $\eta\Delta_2\|C_2\|/\lambda_{\min}(Q)$ indirectly.

Let $\delta > 0$, $\beta > 0$, and $\alpha > 0$ be scalars, then the minimum values of $\|C_c\|$, ϕ, and φ can be found by optimizing indexes δ, β, and α via inequalities $\|C_c\| < \delta, \phi < \beta$, and $\varphi < \alpha$, respectively. Obviously, the above three inequalities are equivalent to the following inequalities, respectively.

$$\begin{bmatrix}-\delta^2 I & C_c^{\mathrm{T}}\\ * & -I\end{bmatrix} < 0 \qquad (35)$$

$$\begin{bmatrix}-\beta^2 I & \bar{B}_1^{\mathrm{T}}PA_e + \bar{D}_1^{\mathrm{T}}C_e\\ * & -I\end{bmatrix} < 0 \qquad (36)$$

$$\begin{bmatrix}-\alpha^2 I & (\bar{D}_1^{\mathrm{T}}\bar{D}_1 + 2\bar{B}_1^{\mathrm{T}}P\bar{B}_1)^{\mathrm{T}}\\ * & -I\end{bmatrix} < 0 \qquad (37)$$

where $A_e, C_e, \bar{B}_1$, and $\bar{D}_1$ are defined in (8).

To solve Problem 1, we need to combine the inequalities (35)$\sim$(37) with (12). But these inequalities are not

convex, we cannot solve them directly. To facilitate solving this nonconvex problem, the following useful lemma is presented.

Denote

$$\Theta_1 = -A_e^{\mathrm{T}} P A_{e0} - A_{e0}^{\mathrm{T}} P A_e + A_{e0}^{\mathrm{T}} P A_{e0}$$

$$\Theta_2 = -\bar{B}_{10}^{\mathrm{T}} P \bar{B}_1 - \bar{B}_1^{\mathrm{T}} P \bar{B}_{10} + \bar{B}_{10}^{\mathrm{T}} P \bar{B}_{10}$$

where $A_{e0} = \begin{bmatrix} A & B_1 C_{c0} \\ B_{c0} C_2 & A_{c0} \end{bmatrix}$ and $\bar{B}_{10} = \begin{bmatrix} 0 & 0 & B_1 \\ A_{c0} & B_{c0} & 0 \end{bmatrix}$.

Lemma 2. For matrix variables A_c, B_c, C_c, A_{c0}, B_{c0}, C_{c0}, Q_{12} and $P > 0$, $Q_{11} > 0$, $Q_{22} > 0$, the following statements hold.

1) (36) holds if and only if the following inequality holds

$$\begin{bmatrix} -\beta^2 I + \Theta_2 & \bar{D}_1^{\mathrm{T}} C_e & \bar{B}_1^{\mathrm{T}} \\ * & -I + \Theta_1 & A_e^{\mathrm{T}} P \\ * & * & -P \end{bmatrix} < 0 \tag{38}$$

2) (37) holds if and only if the following inequality holds

$$\begin{bmatrix} -\alpha^2 I + 4\Theta_2 & \bar{D}_1^{\mathrm{T}} \bar{D}_1 & 2\bar{B}_1^{\mathrm{T}} P \\ * & -I + \Theta_1 & \bar{B}_1^{\mathrm{T}} P \\ * & * & -P \end{bmatrix} < 0 \tag{39}$$

3) (12) holds if and only if the following inequality holds

$$\begin{bmatrix} -P + Q_{11} & Q_{12} & A_e^{\mathrm{T}} P & C_e^{\mathrm{T}} & 0 \\ * & -\gamma^2 I + Q_{22} & B_e^{\mathrm{T}} P & 0 & B_e^{\mathrm{T}} P \\ * & * & -P & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -P \end{bmatrix} < 0 \tag{40}$$

Proof. 1) First,

$$\begin{bmatrix} -\beta^2 I & \bar{B}_1^{\mathrm{T}} P A_e + \bar{D}_1^{\mathrm{T}} C_e \\ * & -I \end{bmatrix} = \begin{bmatrix} -\beta^2 I & \bar{D}_1^{\mathrm{T}} C_e \\ * & -I \end{bmatrix} + \begin{bmatrix} \bar{B}_1^{\mathrm{T}} \\ 0 \end{bmatrix} P \begin{bmatrix} 0 \\ A_e^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} 0 \\ A_e^{\mathrm{T}} \end{bmatrix} P \begin{bmatrix} \bar{B}_1^{\mathrm{T}} \\ 0 \end{bmatrix}^{\mathrm{T}} \tag{41}$$

Apparently, for any two matrices X, Y, and positive definite matrix $P > 0$, the following equality always holds.

$$XPY + Y^{\mathrm{T}} P X^{\mathrm{T}} = (X + Y^{\mathrm{T}}) P (X + Y^{\mathrm{T}})^{\mathrm{T}} - X P X^{\mathrm{T}} - Y^{\mathrm{T}} P Y \tag{42}$$

Thus, according to (42), (41) is equivalent to

$$\begin{bmatrix} -\beta^2 I & \bar{D}_1^{\mathrm{T}} C_e \\ * & -I \end{bmatrix} + \begin{bmatrix} \bar{B}_1^{\mathrm{T}} \\ A_e^{\mathrm{T}} \end{bmatrix} P \begin{bmatrix} \bar{B}_1^{\mathrm{T}} \\ A_e^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} - \begin{bmatrix} \bar{B}_1^{\mathrm{T}} \\ 0 \end{bmatrix} P \begin{bmatrix} \bar{B}_1^{\mathrm{T}} \\ 0 \end{bmatrix}^{\mathrm{T}} - \begin{bmatrix} 0 \\ A_e^{\mathrm{T}} \end{bmatrix} P \begin{bmatrix} 0 \\ A_e^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} < 0 \tag{43}$$

As is known to all that for any matrix V and positive definite matrix $P > 0$, there always exists a matrix V_0, such that the following inequality holds.

$$(V - V_0) P (V - V_0)^{\mathrm{T}} \geq 0 \tag{44}$$

Thus, there exist A_{c0},B_{c0}, and C_{c0} such that (43) is equivalent to

$$\begin{bmatrix} -\beta^2 I & \bar{D}_1^{\mathrm{T}} C_e \\ * & -I \end{bmatrix} + \begin{bmatrix} \bar{B}_1^{\mathrm{T}} \\ A_e^{\mathrm{T}} \end{bmatrix} P \begin{bmatrix} \bar{B}_1^{\mathrm{T}} \\ A_e^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} \Theta_2 \\ 0 \end{bmatrix} \begin{bmatrix} I \\ 0 \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} 0 \\ \Theta_1 \end{bmatrix} \begin{bmatrix} 0 \\ I \end{bmatrix}^{\mathrm{T}} < 0 \tag{45}$$

In fact, on the one hand, according to (44), if (45) holds, then (43) holds. On the other hand, when $A_{c0} = A_c, B_{c0} = B_c$, and $C_{c0} = C_c$, (45) holds if (43) holds.

Now, by the Schur complement, (45) is equivalent to (38).

2) It is similar to the proof of 1) and is omitted here.

3) Obviously, by the Shur complement, (12) is equivalent to (40). □

Denote

$$Q(\xi) = \begin{bmatrix} Q_{11} - \xi I & Q_{12} \\ * & Q_{22} - \xi I \end{bmatrix}$$

Now, on the basis of Lemma 2, in order to solve Problem 1, we need to solve inequalities (35)∼(37), (12), and $Q(\xi) > 0$ or equivalently, we need to solve inequalities (35), (38)∼(40), and $Q(\xi) > 0$. Inequalities (36), (37), and (12) are not convex with respect to the matrix variables A_c, B_c, and C_c, it is difficult to solve them directly. However, when A_c, B_c, and C_c are given, inequalities (35)∼(37), (12), and $Q(\xi) > 0$ become convex with respect to the matrix variables $P, Q_{11}, Q_{12}, Q_{22}$, and the scalars $\beta^2, \delta^2, \alpha^2$, and can be solved by using Matlab via the LMI control toolbox[18]. Similarly, inequalities (38)∼(40) are not convex with respect to the matrix variables A_{c0}, B_{c0}, C_{c0}, and $P > 0$, but when A_{c0}, B_{c0}, C_{c0}, and $P > 0$ are given, inequalities (35), (38)∼(40) and $Q(\xi) > 0$ become convex with respect to the matrix variables A_c, B_c, C_c, Q_{11}, Q_{12}, and Q_{22} and the scalars β^2, δ^2, and α^2. On the basis of the property, an iterative LMI-based optimization algorithm is developed in the following algorithm to optimize the quantizer ranges M_1, M_2, and M_3.

Let $\rho = c_0\delta^2 + c_1\beta^2 + c_2\alpha^2$, where c_0, c_1, and c_2 are constants to be chosen. Now, the following algorithm is presented to minimize ρ.

Algorithm 1. Let $\xi > 0$ and $\gamma > 0$ be given scalars and $\varepsilon > 0$ be a given small constant specifying a convergence criterion.

Step 1. By Lemma 1, we obtain the initial solutions A_c^{ini}, B_c^{ini}, and C_c^{ini}. Let $\rho_0 = -1$.

Step 2. Let $A_c = A_c^{ini}, B_c = B_c^{ini}$, and $C_c = C_c^{ini}$; minimize ρ subject to LMIs $P > 0$, (35)∼(37), (12), and $Q(\xi) > 0$; P_{opt}, Q_{11opt}, Q_{12opt}, Q_{22opt}, δ_{opt}, β_{opt}, and α_{opt} denote the optimal solutions.

Step 3. Let $A_{c0} = A_c^{ini}, B_{c0} = B_c^{ini}, C_{c0} = C_c^{ini}$, and $P = P_{opt}$; minimize ρ subject to LMIs (35), (38)∼(40), and $Q(\xi) > 0$; Q_{11opt}, Q_{12opt}, Q_{22opt}, A_{copt}, B_{copt}, C_{copt}, δ_{opt}, β_{opt}, and α_{opt} denote the optimal solutions. Let $\rho_{opt} = c_0\delta_{opt}^2 + c_1\beta_{opt}^2 + c_2\alpha_{opt}^2$. If $\|\rho_{opt} - \rho_0\| < \varepsilon$, then stop and $Q_{opt} = \begin{bmatrix} Q_{11opt} & Q_{12opt} \\ * & Q_{22opt} \end{bmatrix}$; else, let $A_c^{ini} = A_{copt}, B_c^{ini} = B_{copt}, C_c^{ini} = C_{copt}$, and $\rho_0 = \rho_{opt}$, return to Step 2.

By using the obtained matrix variables Q_{opt}, P_{opt}, A_{copt}, B_{copt}, C_{copt}, and with the scalar γ given in Algorithm 1, the following theorem is presented to give the optimized quantizer ranges, and furthermore an optimized control strategy is given such that the H_∞ control objective is guaranteed.

To facilitate the presentation of Theorem 2, we denote

$$A_{eopt}=\begin{bmatrix}A & B_1C_{copt}\\ B_{copt}C_2 & A_{copt}\end{bmatrix}, C_{eopt}=\begin{bmatrix}C_1 & D_{12}C_{copt}\end{bmatrix}$$

$$\bar{B}_{1opt}=\begin{bmatrix}0 & 0 & B_1\\ A_{copt} & B_{copt} & 0\end{bmatrix}$$

Theorem 2. Consider plant (1) controlled by the quantized dynamic output feedback controller (6), if only M_1 and M_2 are chosen satisfying

$$M_1>\frac{\eta_{opt}\Delta_{opt}}{\lambda_{\min}(Q_{opt})} \tag{46}$$

$$M_2>\frac{\eta_{opt}\Delta_2\|C_2\|}{\lambda_{\min}(Q_{opt})} \tag{47}$$

where $\Delta_{opt}=\sqrt{\Delta_1^2+(\theta\Delta_3+\|C_{copt}\|\Delta_1)^2}$, $\eta_{opt}=\phi_{opt}+\sqrt{\phi_{opt}^2+\varphi_{opt}\lambda_{\min}(Q_{opt})}$ with $\phi_{opt}=\|C_{eopt}^{\mathrm{T}}\bar{D}_1+A_{eopt}^{\mathrm{T}}P_{opt}\bar{B}_{1opt}\|$ and $\varphi_{opt}=\|\bar{D}_1^{\mathrm{T}}\bar{D}_1+2\bar{B}_{1opt}^{\mathrm{T}}P_{opt}\bar{B}_{1opt}\|$. Then, the control strategy (6) with updating μ_1 and μ_3 by

$$\mu_1=\frac{2|\boldsymbol{x}_c|}{M_1+\frac{\eta_{opt}\Delta_{opt}}{\lambda_{\min}(Q_{opt})}}, \mu_3=\theta\mu_1 \tag{48}$$

and updating μ_2 by

$$\mu_2=\frac{2|\boldsymbol{y}|}{M_2+\frac{\eta_{opt}\Delta_2\|C_2\|}{\lambda_{\min}(Q_{opt})}} \tag{49}$$

renders the closed-loop system (8) asymptotically stable and with the H_∞ performance bound γ.

Proof. It is similar to the proof of Theorem 1 and is omitted here. □

Remark 4. Because $\lambda_{\min}(Q)$ has a significant effect on the value of $\eta\Delta/\lambda_{\min}(Q)$, the condition $Q(\xi)>0$ is introduced to restrict the value of $\lambda_{\min}(Q)$, such that $\lambda_{\min}(Q)\geq\xi$.

Remark 5. On the one hand, in Algorithm 1, by means of optimizing the index ρ, we obtain the optimized solutions A_{copt}, B_{copt}, C_{copt}, P_{opt}, and Q_{opt}, such that the values of $\eta_{opt}\Delta_{opt}/\lambda_{\min}(Q_{opt})$ and $\eta_{opt}\Delta_2\|C_2\|/\lambda_{\min}(Q_{opt})$ are minimum. Thus, the ranges M_1, M_2, and M_3 can be optimized indirectly. On the other hand, by Theorem 2, when $M_1>\eta_{opt}\Delta_{opt}/\lambda_{\min}(Q_{opt})$ and $M_2>\eta_{opt}\Delta_2\|C_2\|/\lambda_{\min}(Q_{opt})$, controller (6) with gain matrices A_{copt}, B_{copt}, and C_{copt}, by updating μ_1 , μ_3 according to (48) and updating μ_2 according to (49) renders the closed-loop system (8) asymptotically stable and with the H_∞ performance bound γ. So we have given an optimization method to solve Problem 1.

3 Example

In this section, an example is presented to illustrate the effectiveness of the proposed method.

Example 1. Consider the system of form (1) with

$$A=\begin{bmatrix}-0.5 & 0\\ -1 & 1.5\end{bmatrix}, \boldsymbol{B}_1=\begin{bmatrix}-0.5\\ 1\end{bmatrix}, C_1=\begin{bmatrix}-1 & -1\\ 0 & 0\end{bmatrix}$$

$$\boldsymbol{B}_2=\begin{bmatrix}0.5\\ 0\end{bmatrix}, \boldsymbol{C}_2=\begin{bmatrix}1 & 1\end{bmatrix}, \boldsymbol{D}_{12}=\begin{bmatrix}0\\ -1\end{bmatrix}$$

By solving LMI (9), a controller is obtained with the gain matrices

$$A_c^{ini}=\begin{bmatrix}0.0213 & -1.6320\\ 0.9711 & -1.3363\end{bmatrix}, \boldsymbol{B}_c^{ini}=\begin{bmatrix}3.1451\\ 5.0328\end{bmatrix}$$
$$\boldsymbol{C}_c^{ini}=\begin{bmatrix}-0.0040 & -0.2317\end{bmatrix}$$

correspondingly, the value of γ is obtained as $\gamma_{au}=3.6421$.

Let $\theta=2$ and the quantization errors $\Delta_1=\Delta_2=\Delta_3=0.1$.

On the one hand, by Lemma 1 and $Q(\xi)>0$ with the above gain matrices $A_c^{ini}, \boldsymbol{B}_c^{ini}, \boldsymbol{C}_c^{ini}$, and $\gamma=4.3346$ and $\xi=0.01$, we obtain matrices Q_{lem} and P_{lem}.

Apparently, $\lambda_{\min}(Q_{lem})=0.01$. By Theorem 1, it is easy to compute $\eta\Delta/\lambda_{\min}(Q_{lem})=279.2494$ and $\eta\Delta_2\|C_2\|/\lambda_{\min}(Q_{lem})=663.0357$. Let $M_1=280>279.2494$ and $M_2=664>663.0357$. According to (7), the range of the quantizer $q_3(\cdot)$ can be computed as $M_3=7.5208$.

In contrast, by Algorithm 1, with $A_{c0}=A_c^{ini}, \boldsymbol{B}_{c0}=\boldsymbol{B}_c^{ini}, \boldsymbol{C}_{c0}=\boldsymbol{C}_c^{ini}$, $\varepsilon=0.0001, c_0=1, c_1=100, c_2=10\,000$, and $\gamma=4.3346$ and $\xi=0.01$, we obtain optimized matrices Q_{opt}, P_{opt}, and the controller gain matrices

$$A_{copt}=\begin{bmatrix}-0.0179 & -1.4231\\ 0.9454 & -1.1833\end{bmatrix}, \boldsymbol{B}_{copt}=\begin{bmatrix}2.6540\\ 4.6790\end{bmatrix},$$
$$\boldsymbol{C}_{copt}=\begin{bmatrix}-0.0040 & -0.2317\end{bmatrix}.$$

By Theorem 2, we can obtain $\eta_{opt}\Delta_{opt}/\lambda_{\min}(Q_{opt})=244.3255$ and $\eta_{opt}\Delta_2\|C_2\|/\lambda_{\min}(Q_{opt})=577.7594$. Let $M_1=245>244.3255$ and $M_2=578>577.7594$. And according to (7), the range of the quantizer $q_3(\cdot)$ can be computed as $M_3=6.5810$.

The quantizer ranges obtained by Theorem 1 and by Theorem 2 will be compared in Table 1.

Table 1 Comparison of the quantizer ranges

	Theorem 1	Theorem 2
M_1	280	245
M_2	664	578
M_3	7.5208	6.5810

From Table 1, it is clear that the ranges of the quantizers $q_1(\cdot), q_2(\cdot)$, and $q_3(\cdot)$ obtained by Theorem 2 are much more improved than the corresponding one obtained by Theorem 1, by reducing 12.58%, 12.19%, and 12.50%, respectively.

The initial system state and controller state are chosen as $\boldsymbol{x}_0=[5,-4]$ and $\boldsymbol{x}_{c0}=[10,-10]$, respectively. Let the disturbance input $\boldsymbol{\omega}=10\,randn+5$ for $k\in[15,25]$, and let the disturbance input $\boldsymbol{\omega}=0$ for all the other k, where $randn$ is a normal distribution with mean zero, variance one, and standard deviation one. Then, Fig. 1 shows the regulated output responses of system (8). In Fig. 1, the solid lines show the results obtained by Theorem 1 and the dashed curves show the results obtained by Theorem 2. From this figure, we can see that the results obtained from Theorem 2 is much more improved than the corresponding one obtained from Theorem 1.

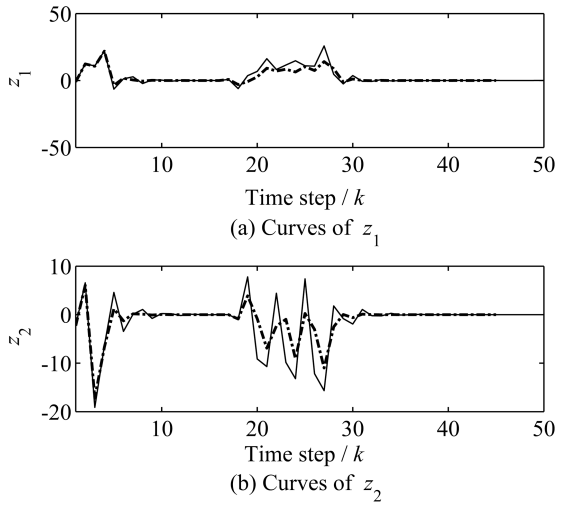

Fig. 1 Trajectories of regulated outputs

4 Conclusion

We have studied the quantized dynamic output feedback H_∞ control problem for discrete-time LTI systems with dynamic quantizers. Dynamic quantizers considered here use dynamic scaling in conjunction with a static quantizer. Furthermore, the problem of optimizing static quantizer ranges is considered. First, a quantized control strategy dependent not only on the controller states but also on the system measurement outputs is proposed, such that the quantized closed-loop system is asymptotically stable and with a prescribed H_∞ performance bound. Then, on the basis of this result, an iterative LMI-based optimization algorithm is developed to optimize the static quantizer ranges, and in turn, an optimized control strategy is derived to guarantee the desired system performance. The effectiveness of the proposed optimization method has been illustrated by an example.

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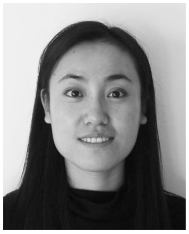

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