# Robust Stability of Uncertain Takagi-Sugeno Fuzzy Systems with Time-varying Input-delay

LUO Yan-Bin[1, 2] CAO Yong-Yan[2] SUN You-Xian[2]

**Abstract** This paper aims to design a controller to robustly stabilize uncertain Takagi-Sugeno fuzzy systems with time-varying input delay. Based on Lyapunov-Krasovskii functional approach, the sufficient conditions for robust stabilization of such systems are given in the form of linear matrix inequalities. The controller design does not have to require that the time-derivative of time-varying input delay be smaller than one. A numeric example is given to show that the proposed results are effective and less conservative.

**Key words** Uncertain Takagi-Sugeno fuzzy systems, linear matrix inequalities (LMIs), robust stability, input delay, Lyapunov-Krasovskii functional approach

Input delays are often encountered in various practical control systems, such as chemical process, biological systems and economical systems. The existence of input delays may deteriorate the performance of the system and make it difficult to stabilize the resulting closed-loop system rather. As a result, the control design problem of input delayed systems has attracted considerable attention recently[1−5].

Since Takagi and Sugeno[6] proposed the Takagi-Sugeno fuzzy model in 1985, fuzzy model control has been extensively studied[7−13] because Takagi-Sugeno (T-S) fuzzy models can provide an effective representation of complex nonlinear systems in terms of fuzzy sets, and fuzzy reasoning has been applied to a set of linear input-output submodels[9]. Up to now, the stabilization problem of uncertain T-S fuzzy systems with time-varying input delay has not been fully investigated, and it remains a challenging problem.

The approaches to solving the control systems with time delay can be classified into two classes: one is Lyapunov-Krasovskii functional method, and the other is Lyapunov-Razumikhin functional method. The stabilization problem is thus reduced to the problem of finding solutions to Lyapunov or Riccati equations, or solving linear matrix inequalities (LMIs). In [13], the Lyapunov-Razumikhin functional technique makes some improvements such as avoiding the constraint that the upper bound of the time-derivative of the time delay must be less than one, however, it requires some other supplementary assumptions in the process of completing the proof of the theorem, therefore the results obtained in [13] are still too conservative.

In this paper, by constructing an appropriate Lyapunov-Krasovskii functional, we provide sufficient conditions for robust stability of uncertain T-S fuzzy systems with time-varying input delay, without using any property of the time-derivative of the input delay. The results obtained in the present paper are less conservative than that obtained in

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1. Institue of Automation, Hangzhou Dianzi University, Hangzhou 310018, P. R. China 2. State Key Laboratory of Industrial Control Technology, Zhejiang University, Hangzhou 310027, P. R. China
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[13].

This paper is organized as follows. In Section 1, we give a general description of uncertain T-S fuzzy systems with time-varying input delay and review some results to be used in this paper. The sufficient conditions for robust stability of uncertain T-S fuzzy systems with time-varying input delay and time-invariant input delay are proposed in Section 2. In Section 3, a numeric example is given to show that the proposed results are effective and less conservative. Some conclusions are drawn in Section 4.

## 1 Problem formulation

Consider the uncertain Taksgi-Sugeno fuzzy model with time-varying input delay. The $i$-th rule is described as

Plant rule $i$: IF $z_1(t)$ is $M_1^i$ and $\cdots$ and $z_p(t)$ is $M_p^i$, THEN

$$\begin{aligned}\dot{\boldsymbol{x}}(t) \;=\; & (A_i + \triangle A_i(t))\boldsymbol{x}(t) + (B_i + \triangle B_i(t))\boldsymbol{u}(t) + \\ & (B_{di} + \triangle B_{di}(t))\boldsymbol{u}(t-\tau(t)) \end{aligned} \tag{1}$$

$$\boldsymbol{x}(0) = \boldsymbol{x}_0, \boldsymbol{u}(t) = \boldsymbol{\phi}(t), t \in [-\tau, 0] \tag{2}$$

where $M_j^i$ $(i = 1, 2, \cdots, r, j = 1, 2, \cdots, p)$ is a fuzzy set, $\boldsymbol{x}(t) \in \mathbf{R}^n$ is a state vector, $\boldsymbol{x}_0 \in \mathbf{R}^n$ is the initial condition of state, $\boldsymbol{u}(t) \in \mathbf{R}^m$ is an input vector, the initial condition of the input $\boldsymbol{\phi}(t)$ is a continuous vector-valued function over $[-\tau, 0]$, $r$ is the number of IF-THEN rules, and $z_1(t), \cdots, z_n(t)$ are the premise variables. It is assumed in this paper that the premise variables do not depend on the input variables $\boldsymbol{u}(t)$, and $\tau(t)$ is a time-varying delay in the input satisfying

$$\tau(t) \in [\tau_0 - \delta, \tau_0 + \delta] \tag{3}$$

where $\tau_0$ and $\delta$ are scalars, $\tau_0 \geq \delta$. $A_i, B_i$, and $B_{di}$ are constant matrices of compatible dimensions, $\triangle A_i(t), \triangle B_i(t)$, and $\triangle B_{di}(t)$ are time-varying matrix functions with compatible dimensions, which can be described as

$$\begin{aligned}&\triangle A_i(t) = D_{ai}F_{ai}(t)E_{ai}, \triangle B_i(t) = D_{bi}F_{bi}(t)E_{bi} \\ &\triangle B_{di}(t) = D_{bdi}F_{bdi}(t)E_{bdi}\end{aligned} \tag{4}$$

$i = 1, 2, \cdots, r$, where $E_{ai}, E_{bi}, E_{bdi}$ and $D_{ai}, D_{bi}, D_{bdi}$ are known constant real matrices of appropriate dimensions and $F_{ai}, F_{bi}, F_{bdi}$ are unknown time-varying matrix functions with Lebesgue measurable elements bounded by

$$F_{ai}^{\mathrm{T}}(t)F_{ai}(t) \leq I, F_{bi}^{\mathrm{T}}(t)F_{bi}(t) \leq I, F_{bdi}^{\mathrm{T}}(t)F_{bdi}(t) \leq I \tag{5}$$

By using the fuzzy inference method with a singleton fuzzifier, product inference, and center average defuzzifier, fuzzy model (1) can be expressed as

$$\dot{\boldsymbol{x}}(t) = \frac{\left(\begin{array}{c}\sum\limits_{i=1}^{r}\mu_i(\boldsymbol{z}(t))\Big(\big(A_i + \triangle A_i(t)\big)\boldsymbol{x}(t) + \big(B_i + \\ \triangle B_i(t)\big)\boldsymbol{u}(t) + \big(B_{di} + \triangle B_{di}(t)\big)\boldsymbol{u}(t-\tau(t))\Big)\end{array}\right)}{\sum\limits_{j=1}^{r}\mu_j(\boldsymbol{z}(t))}$$

where $\boldsymbol{z}(t) = [z_1(t), \cdots, z_p(t)]$, $\mu_i(\boldsymbol{z}(t)) = \prod_{j=1}^{p} M_j^i(\boldsymbol{z}(t))$, $h_i(\boldsymbol{z}(t)) = \frac{\mu_i(\boldsymbol{z}(t))}{\sum_{j=1}^{r}\mu_j(\boldsymbol{z}(t))}$, and $M_j^i(z_j(t))$ is the grade of membership of $z_j(t)$ in $M_j^i$. As usual, assume that $\mu_i(\boldsymbol{z}(t)) \geq 0$, $i = 1, 2, \cdots, r$, and $\sum_{i=1}^{r}\mu_i(\boldsymbol{z}(t)) > 0$ for all $t$ such that $h_i(\boldsymbol{z}(t)) \geq 0$, $i = 1, 2, \cdots, r$, and $\sum_{i=1}^{r} h_i(\boldsymbol{z}(t)) = 1$ for all $t$.

Then

$$\begin{aligned}\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\Big(&\big(A_i + \triangle A_i(t)\big)\boldsymbol{x}(t) + \big(B_i + \\ &\triangle B_i(t)\big)\boldsymbol{u}(t) + \big(B_{di} + \triangle B_{di}(t)\big)\boldsymbol{u}(t-\tau(t))\Big)\end{aligned} \tag{6}$$

By the concept of parallel distributed compensation (PDC)[6], we design fuzzy controllers to stabilize fuzzy systems (1).

**Rule** $i$: IF $z_1(t)$ is $M_1^i$ and $\cdots$ and $z_p(t)$ is $M_p^i$, THEN $\boldsymbol{u}(k) = K_i\boldsymbol{x}(t), i = 1, 2, \cdots, r$.

Hence, the fuzzy controller is

$$\boldsymbol{u}(t) = \frac{\sum\limits_{i=1}^{r}\mu_i(\boldsymbol{z}(t))K_i\boldsymbol{x}(t)}{\sum\limits_{i=1}^{r}\mu_i(\boldsymbol{z}(t))} = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))K_i\boldsymbol{x}(t) \tag{7}$$

Combining (6) and (7), the closed-loop fuzzy system can be expressed as follows

$$\begin{aligned}\dot{\boldsymbol{x}}(t) \;=\; & \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t))\Big(\Big((A_i + \triangle A_i(t)) + \\ & (B_i + \triangle B_i(t))K_j\Big)\boldsymbol{x}(t) + (B_{di} + \triangle B_{di}(t)) \times \\ & K_j\boldsymbol{x}(t-\tau(t))\Big)\end{aligned} \tag{8}$$

Using the Newton-Leibniz formula, one obtains

$$\boldsymbol{x}(t) - \boldsymbol{x}(t-\tau_0) \;=\; \int_{t-\tau_0}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s \tag{9}$$

$$\boldsymbol{x}(t-\tau_0) - \boldsymbol{x}(t-\tau(t)) \;=\; \int_{t-\tau(t)}^{t-\tau_0}\dot{\boldsymbol{x}}(s)\mathrm{d}s \tag{10}$$

## 2 Main results

In the following, we first give a sufficient condition for the asymptotical stability of the T-S fuzzy system (1) when the input-delay is time-invariable, i.e., $\tau(t) = \tau$.

**Lemma 1.** Given a scalar $\tau > 0$, if there exist matrices $X > 0, \tilde{P} > 0, \tilde{Q} > 0$, $\tilde{R} > 0, \tilde{W}_{kij}$ and $Y_i$, for $i, j = 1, 2, \cdots, r, k = 1, 2, 3$, of compatible dimensions, and three scalars $\mu_1, \mu_2$, and $\mu_3$ such that

$$\left[\begin{array}{ccc}\Theta_{ii} & * & * \\ \Psi_{ii} & -\tau\tilde{R} & * \\ \Phi_{ii} & 0 & \tilde{I}\end{array}\right] < 0 \tag{11}$$

$$\left[\begin{array}{ccccc}\Theta_{ij}+\Theta_{ji} & * & * & * & * \\ \Psi_{ij} & -\tau\tilde{R} & * & * & * \\ \Psi_{ji} & 0 & -\tau\tilde{R} & * & * \\ \Phi_{ij} & 0 & 0 & \tilde{I} & * \\ \Phi_{ji} & 0 & 0 & 0 & \tilde{I}\end{array}\right] < 0$$

$$1 \leq i < j \leq r \tag{12}$$

where $*$ denotes the transposed element in the symmetric positions,

$$\Theta_{ij} = \begin{bmatrix} \Gamma_{11}^{ij} & * & * \\ \Gamma_{21}^{ij} & \Gamma_{22}^{ij} & * \\ \Gamma_{31}^{ij} & \Gamma_{32}^{ij} & \Gamma_{33}^{ij} \end{bmatrix}$$

$$\Psi_{ij} = \begin{bmatrix} \tau\tilde{W}_{1ij} & \tau\tilde{W}_{2ij} & \tau\tilde{W}_{3ij} \end{bmatrix}$$

$$\Phi_{ij} = \begin{bmatrix} E_{ai}X^{\mathrm{T}} & 0 & 0 & 0 \\ 0 & 0 & E_{bdi}Y_j & 0 \\ E_{bi}Y_j & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{I} = \begin{bmatrix} -\mu_1 I & * & * \\ 0 & -\mu_2 I & * \\ 0 & 0 & -\mu_3 I \end{bmatrix}$$

and

$$\begin{aligned}
\Gamma_{11}^{ij} &= \tilde{Q} + \tilde{W}_{1ij} + \tilde{W}_{1ij}^{\mathrm{T}} + A_i X^{\mathrm{T}} + X A_i^{\mathrm{T}} + B_i Y_j + \\
&\quad Y_j^{\mathrm{T}} B_i^{\mathrm{T}} + \mu_1 D_{ai} D_{ai}^{\mathrm{T}} + \mu_2 D_{bi} D_{bi}^{\mathrm{T}} + \mu_3 D_{bdi} D_{bdi}^{\mathrm{T}} \\
\Gamma_{21}^{ij} &= \tilde{W}_{2ij} + \lambda_2 A_i X^{\mathrm{T}} + \lambda_2 B_i Y_j + \mu_1\lambda_2 D_{ai} D_{ai}^{\mathrm{T}} + \\
&\quad \mu_2\lambda_2 D_{bi} D_{bi}^{\mathrm{T}} + \mu_3\lambda_2 D_{bdi} D_{bdi}^{\mathrm{T}} \\
\Gamma_{22}^{ij} &= \tilde{Q} - \tilde{W}_{2ij} - \tilde{W}_{2ij}^{\mathrm{T}} + \lambda_2 B_{di} Y_j + \lambda_2 Y_j^{\mathrm{T}} B_{di}^{\mathrm{T}} + \\
&\quad \mu_1\lambda_2^2 D_{ai} D_{ai}^{\mathrm{T}} + \mu_2\lambda_2^2 D_{bi} D_{bi}^{\mathrm{T}} + \mu_3\lambda_2^2 D_{bdi} D_{bdi}^{\mathrm{T}} \\
\Gamma_{31}^{ij} &= \tilde{P} + \tilde{W}_{3ij} - X^{\mathrm{T}} + \lambda_3 A_i X^{\mathrm{T}} + \lambda_3 B_i Y_j + \\
&\quad \mu_1\lambda_3 D_{ai} D_{ai}^{\mathrm{T}} + \mu_2\lambda_3 D_{bi} D_{bi}^{\mathrm{T}} + \mu_3\lambda_3 D_{bdi} D_{bdi}^{\mathrm{T}} \\
\Gamma_{32}^{ij} &= -\tilde{W}_{3ij} - \lambda_2 X^{\mathrm{T}} + \lambda_3 B_{di} Y_j + \mu_1\lambda_2\lambda_3 D_{ai} D_{ai}^{\mathrm{T}} + \\
&\quad \mu_2\lambda_2\lambda_3 D_{bi} D_{bi}^{\mathrm{T}} + \mu_3\lambda_2\lambda_3 D_{bdi} D_{bdi}^{\mathrm{T}} \\
\Gamma_{33}^{ij} &= \tau\tilde{R} - \lambda_3 X^{\mathrm{T}} - \lambda_3 X + \mu_1\lambda_3^2 D_{ai} D_{ai}^{\mathrm{T}} + \\
&\quad \mu_2\lambda_3^2 D_{bdi} D_{bdi}^{\mathrm{T}} + \mu_3\lambda_3^2 D_{bdi} D_{bdi}^{\mathrm{T}}
\end{aligned}$$

then fuzzy system (1) with the control

$$\boldsymbol{u}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t)) Y_i (X^{-1})^{\mathrm{T}} \boldsymbol{x}(t) \tag{13}$$

is asymptotically stable for the time-invariant input delay $\tau$.

**Proof.** Construct the Lyapunov-Krasovskii functional as

$$V(\boldsymbol{x}_t) = \boldsymbol{x}^{\mathrm{T}}(t) P \boldsymbol{x}(t) + \int_{t-\tau}^{t} \boldsymbol{x}^{\mathrm{T}}(s) Q \boldsymbol{x}(s)\mathrm{d}s + \int_{t-\tau}^{t}\int_{s}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(v) R \dot{\boldsymbol{x}}(v)\mathrm{d}v\mathrm{d}s \tag{14}$$

Taking the derivative of $V(\boldsymbol{x}_t)$ yields

$$\begin{aligned}
&\dot{V}(\boldsymbol{x}_t) = 2\boldsymbol{x}^{\mathrm{T}}(t) P \dot{\boldsymbol{x}}(t) + \boldsymbol{x}^{\mathrm{T}}(t) Q \boldsymbol{x}(t) - \boldsymbol{x}^{\mathrm{T}}(t-\tau) \times \\
&Q\boldsymbol{x}(t-\tau) + \tau \dot{\boldsymbol{x}}^{\mathrm{T}}(t) R \dot{\boldsymbol{x}}(t) - \int_{t-\tau}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) R \dot{\boldsymbol{x}}(s)\mathrm{d}s + \\
&2\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t)) h_j(\boldsymbol{z}(t)) \boldsymbol{\xi}^{\mathrm{T}}(t) W_{ij} \times \\
&\Big(\boldsymbol{x}(t) - \boldsymbol{x}(t-\tau) - \int_{t-\tau}^{t} \dot{\boldsymbol{x}}(s)\mathrm{d}s\Big) + \\
&2\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t)) h_j(\boldsymbol{z}(t)) \boldsymbol{\xi}^{\mathrm{T}}(t) H \times \\
&\Big(\Big(\big(A_i + D_{ai}F_{ai}(t)E_{ai}\big) + \big(B_i + D_{bi}F_{bi}(t)E_{bi}\big)K_j\Big)\boldsymbol{x}(t) + \\
&\big(B_{di} + D_{bdi}F_{bdi}(t)E_{bdi}\big)K_j\boldsymbol{x}(t-\tau) - \dot{\boldsymbol{x}}(t)\Big)
\end{aligned} \tag{15}$$

where

$$\begin{aligned}
\boldsymbol{\xi}^{\mathrm{T}}(t) &= [\boldsymbol{x}^{\mathrm{T}}(t) \;\; \boldsymbol{x}^{\mathrm{T}}(t-\tau) \;\; \dot{\boldsymbol{x}}^{\mathrm{T}}(t)]^{\mathrm{T}} \\
W_{ij}^{\mathrm{T}} &= [W_{1ij}^{\mathrm{T}} \;\; W_{2ij}^{\mathrm{T}} \;\; W_{3ij}^{\mathrm{T}}] \\
H^{\mathrm{T}} &= [H_1^{\mathrm{T}} \;\; H_2^{\mathrm{T}} \;\; H_3^{\mathrm{T}}]
\end{aligned}$$

Using the matrix inequality properties proposed in [4−5], we can get the following inequalities:

$$\begin{aligned}
&-2\boldsymbol{\xi}^{\mathrm{T}}(t) W_{ij} \int_{t-\tau}^{t} \dot{\boldsymbol{x}}(s)\mathrm{d}s \le \tau \boldsymbol{\xi}^{\mathrm{T}}(t) W_{ij} R^{-1} W_{ij}^{\mathrm{T}} \boldsymbol{\xi}(t) + \\
&\int_{t-\tau}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) R \dot{\boldsymbol{x}}(s)\mathrm{d}s
\end{aligned} \tag{16}$$

$$\begin{aligned}
&2\boldsymbol{\xi}^{\mathrm{T}}(t) H D_{ai} F_{ai}(t) E_{ai} \boldsymbol{x}(t) \le \mu_1 \boldsymbol{\xi}^{\mathrm{T}}(t) H D_{ai} D_{ai}^{\mathrm{T}} H^{\mathrm{T}} \boldsymbol{\xi}(t) + \\
&\mu_1^{-1} \boldsymbol{x}^{\mathrm{T}}(t) E_{ai}^{\mathrm{T}} E_{ai} \boldsymbol{x}(t)
\end{aligned} \tag{17}$$

$$\begin{aligned}
&2\boldsymbol{\xi}^{\mathrm{T}}(t) H D_{bi} F_{bi}(t) E_{bi} K_j \boldsymbol{x}(t) \le \mu_2 \boldsymbol{\xi}^{\mathrm{T}}(t) H D_{bi} D_{bi}^{\mathrm{T}} H^{\mathrm{T}} \boldsymbol{\xi}(t) + \\
&\mu_2^{-1} \boldsymbol{x}^{\mathrm{T}}(t) K_j^{\mathrm{T}} E_{bi}^{\mathrm{T}} E_{bi} K_j \boldsymbol{x}(t)
\end{aligned} \tag{18}$$

$$\begin{aligned}
&\boldsymbol{\xi}^{\mathrm{T}}(t) H D_{bdi} F_{bdi}(t) E_{bdi} K_j \boldsymbol{x}(t-\tau) \le \\
&\mu_3 \boldsymbol{\xi}^{\mathrm{T}}(t) H D_{bdi} D_{bdi}^{\mathrm{T}} H^{\mathrm{T}} \boldsymbol{\xi}(t) + \\
&\mu_3^{-1} \boldsymbol{x}^{\mathrm{T}}(t-\tau) K_j^{\mathrm{T}} E_{bdi}^{\mathrm{T}} E_{bdi} K_j \boldsymbol{x}(t-\tau)
\end{aligned} \tag{19}$$

Assume that there exist $\lambda_2$ and $\lambda_3$ such that $H_2 = \lambda_2 H_1$, $H_3 = \lambda_3 H_1, \lambda_3 \neq 0$ and $H_1$ is nonsingular matrix. Denote $X = H_1^{-1}$, $\tilde{P} = XPX^{\mathrm{T}}, \tilde{Q} = XQX^{\mathrm{T}}$, $\tilde{R} = XRX^{\mathrm{T}}, \tilde{W}_{kij} = XW_{kij}X^{\mathrm{T}}$ and $Y_j = K_j X^{\mathrm{T}}, k = 1,2,3,\ i,j = 1,2,\cdots,r$. Then, combining (15) $\sim$ (19), one obtains

$$\begin{aligned}
\dot{V}(\boldsymbol{x}_t) \;\le\; &\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t)) h_j(\boldsymbol{z}(t)) \boldsymbol{\xi}^{\mathrm{T}}(t) X^{-1} \Big(\Theta_{ij} + \\
&\tau\tilde{W}_{ij}\tilde{R}^{-1}\tilde{W}_{ij}^{\mathrm{T}} - \Phi_{ij}^{\mathrm{T}}\tilde{I}^{-1}\Phi_{ij}\Big)(X^{-1})^{\mathrm{T}}\boldsymbol{\xi}(t) = \\
&\sum_{i=1}^{r} h_i^2(\boldsymbol{z}(t)) \boldsymbol{\xi}^{\mathrm{T}}(t) \Big(\Theta_{ii} + \tau\tilde{W}_{ii}\tilde{R}^{-1}\tilde{W}_{ii}^{\mathrm{T}} - \\
&\Phi_{ii}^{\mathrm{T}}\tilde{I}^{-1}\Phi_{ii}\Big)(X^{-1})^{\mathrm{T}}\boldsymbol{\xi}(t) + \\
&\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t)) h_j(\boldsymbol{z}(t)) \boldsymbol{\xi}^{\mathrm{T}}(t) X^{-1} \times
\end{aligned}$$

$$\Big(\Theta_{ij}+\Theta_{ji}+\tau\tilde{W}_{ij}\tilde{R}^{-1}\tilde{W}_{ij}^{\mathrm{T}}+\tau\tilde{W}_{ji}\tilde{R}^{-1}\tilde{W}_{ji}^{\mathrm{T}}-\Phi_{ij}^{\mathrm{T}}\tilde{I}^{-1}\Phi_{ij}-\Phi_{ji}^{\mathrm{T}}\tilde{I}^{-1}\Phi_{ji}\Big)(X^{-1})^{\mathrm{T}}\boldsymbol{\xi}(t) \tag{20}$$

Using Schur complements, we can verify that

$$\Theta_{ii}+\tau\tilde{W}_{ii}\tilde{R}^{-1}\tilde{W}_{ii}^{\mathrm{T}}-\Phi_{ii}^{\mathrm{T}}\tilde{I}^{-1}\Phi_{ii}<0$$

if and only if (11) holds, and

$$\Theta_{ij}+\Theta_{ji}+\tau\tilde{W}_{ij}\tilde{R}^{-1}\tilde{W}_{ij}^{\mathrm{T}}+\tau\tilde{W}_{ji}\tilde{R}^{-1}\tilde{W}_{ji}^{\mathrm{T}}-\Phi_{ij}^{\mathrm{T}}\tilde{I}^{-1}\Phi_{ij}-\Phi_{ji}^{\mathrm{T}}\tilde{I}^{-1}\Phi_{ji}<0$$

if and only if (12) holds. Therefore, T-S fuzzy system (1) with control law (13) is asymptotically stable when the input delay $\tau(t)=\tau$. The proof is completed.

In what follows, based on the proof of Lemma 1 we give a sufficient condition for the asymptotical stability of the T-S fuzzy system (1) with time-varying input-delay.

**Theorem 1.** Given two scalars $\tau_0>0$ and $\delta>0$, where $\tau_0\geq\delta$, and if there exist matrices $X>0,\tilde{P}>0,\tilde{Q}>0$, $\tilde{R}_1>0,\tilde{R}_2>0,\tilde{T}_{kij},\tilde{N}_{kij}$ and $Y_i$, for $i,j=1,2,\cdots,r,k=1,2,3,4$, of compatible dimensions, and three scalars $\epsilon_1,\epsilon_2$, and $\epsilon_3$, such that

$$\begin{bmatrix}\Lambda_{ii} & * & *\\ \Delta_{ii} & R & *\\ \Phi_{ii} & 0 & \tilde{I}\end{bmatrix}<0 \tag{21}$$

$$\begin{bmatrix}\Lambda_{ij}+\Lambda_{ji} & * & * & * & *\\ \Delta_{ij} & R & * & * & *\\ \Delta_{ji} & 0 & R & * & *\\ \Phi_{ij} & 0 & 0 & \tilde{I} & *\\ \Phi_{ji} & 0 & 0 & 0 & \tilde{I}\end{bmatrix}<0, 1\leq i<j\leq r \tag{22}$$

where $*$ denotes the transposed element in the symmetric positions,

$$\Lambda_{ij}=\begin{bmatrix}\Upsilon_{11}^{ij} & * & * & *\\ \Upsilon_{21}^{ij} & \Upsilon_{22}^{ij} & * & *\\ \Upsilon_{31}^{ij} & \Upsilon_{32}^{ij} & \Upsilon_{33}^{ij} & *\\ \Upsilon_{41}^{ij} & \Upsilon_{42}^{ij} & \Upsilon_{43}^{ij} & \Upsilon_{44}^{ij}\end{bmatrix}$$

$$\Delta_{ij}=\begin{bmatrix}\tau_0\tilde{N}_{1ij} & \tau_0\tilde{N}_{2ij} & \tau_0\tilde{N}_{3ij} & \tau_0\tilde{N}_{4ij}\\ \delta\tilde{T}_{1ij} & \delta\tilde{T}_{2ij} & \delta\tilde{T}_{3ij} & \delta\tilde{T}_{4ij}\end{bmatrix}$$

$$R=\begin{bmatrix}-\tau_0\tilde{R}_1 & *\\ 0 & -\delta\tilde{R}_2\end{bmatrix}$$

$$\Phi_{ij}=\begin{bmatrix}E_{ai}X^{\mathrm{T}} & 0 & 0 & 0\\ 0 & 0 & E_{bdi}Y_j & 0\\ E_{bi}Y_j & 0 & 0 & 0\end{bmatrix}$$

$$\tilde{I}=\begin{bmatrix}-\epsilon_1 I & * & *\\ 0 & -\epsilon_2 I & *\\ 0 & 0 & -\epsilon_3 I\end{bmatrix}$$

and

$$\begin{aligned}
\Upsilon_{11}^{ij} &= \tilde{Q}+\tilde{N}_{1ij}+\tilde{N}_{1ij}^{\mathrm{T}}+A_iX^{\mathrm{T}}+XA_i^{\mathrm{T}}+B_iY_j+Y_j^{\mathrm{T}}B_i^{\mathrm{T}}+\epsilon_1D_{ai}D_{ai}^{\mathrm{T}}+\epsilon_2D_{bi}D_{bi}^{\mathrm{T}}+\epsilon_3D_{bdi}D_{bdi}^{\mathrm{T}}\\
\Upsilon_{21}^{ij} &= \tilde{N}_{2ij}-\tilde{N}_{1ij}^{\mathrm{T}}+\tilde{T}_{1ij}^{\mathrm{T}}+\rho_2A_iX^{\mathrm{T}}+\rho_2B_iY_j+\epsilon_1\rho_2D_{ai}D_{ai}^{\mathrm{T}}+\epsilon_2\rho_2D_{bi}D_{bi}^{\mathrm{T}}+\epsilon_3\rho_2D_{bdi}D_{bdi}^{\mathrm{T}}\\
\Upsilon_{22}^{ij} &= -\tilde{Q}-\tilde{N}_{2ij}-\tilde{N}_{2ij}^{\mathrm{T}}+\tilde{T}_{2ij}+\tilde{T}_{2ij}^{\mathrm{T}}+\epsilon_1\rho_2^2D_{ai}D_{ai}^{\mathrm{T}}+\epsilon_2\rho_2^2D_{bi}D_{bi}^{\mathrm{T}}+\epsilon_3\rho_2^2D_{bdi}D_{bdi}^{\mathrm{T}}\\
\Upsilon_{31}^{ij} &= \tilde{N}_{3ij}-\tilde{T}_{1ij}^{\mathrm{T}}+\rho_3A_iX^{\mathrm{T}}+\rho_3B_iY_j+\epsilon_1\rho_3D_{ai}D_{ai}^{\mathrm{T}}+\epsilon_2\rho_3D_{bi}D_{bi}^{\mathrm{T}}+\epsilon_3\rho_3D_{bdi}D_{bdi}^{\mathrm{T}}\\
\Upsilon_{32}^{ij} &= -\tilde{N}_{3ij}+\tilde{T}_{3ij}-\tilde{T}_{2ij}^{\mathrm{T}}+\rho_2B_{di}Y_j+\epsilon_1\rho_2\rho_3D_{ai}D_{ai}^{\mathrm{T}}+\epsilon_2\rho_2\rho_3D_{bi}D_{bi}^{\mathrm{T}}+\epsilon_3\rho_2\rho_3D_{bdi}D_{bdi}^{\mathrm{T}}\\
\Upsilon_{33}^{ij} &= -\tilde{T}_{3ij}-\tilde{T}_{3ij}^{\mathrm{T}}+\rho_3B_{di}Y_j+\rho_3Y_j^{\mathrm{T}}B_{di}^{\mathrm{T}}+\epsilon_1\rho_3^2D_{ai}D_{ai}^{\mathrm{T}}+\epsilon_2\rho_3^2D_{bi}D_{bi}^{\mathrm{T}}+\epsilon_3\rho_3^2D_{bdi}D_{bdi}^{\mathrm{T}}\\
\Upsilon_{41}^{ij} &= \tilde{P}+\tilde{N}_{4ij}-\rho_4A_iX^{\mathrm{T}}+\rho_4B_iY_j+\epsilon_1\rho_4D_{ai}D_{ai}^{\mathrm{T}}+\epsilon_2\rho_4D_{bi}D_{bi}^{\mathrm{T}}+\epsilon_3\rho_4D_{bdi}D_{bdi}^{\mathrm{T}}\\
\Upsilon_{42}^{ij} &= -\tilde{N}_{4ij}+\tilde{T}_{4ij}+\epsilon_1\rho_2\rho_4D_{ai}D_{ai}^{\mathrm{T}}+\epsilon_2\rho_2\rho_4D_{bi}D_{bi}^{\mathrm{T}}+\epsilon_3\rho_2\rho_4D_{bdi}D_{bdi}^{\mathrm{T}}\\
\Upsilon_{43}^{ij} &= \tilde{T}_{4ij}-\rho_3X+\rho_4B_{di}Y_j+\epsilon_1\rho_3\rho_4D_{ai}D_{ai}^{\mathrm{T}}+\epsilon_2\rho_3\rho_4D_{bi}D_{bi}^{\mathrm{T}}+\epsilon_3\rho_3\rho_4D_{bdi}D_{bdi}^{\mathrm{T}}\\
\Upsilon_{44}^{ij} &= \tau_0\tilde{R}_1+2\delta\tilde{R}_2-\rho_4X^{\mathrm{T}}-\rho_4X+\epsilon_1\rho_4^2D_{ai}D_{ai}^{\mathrm{T}}+\epsilon_2\rho_4^2D_{bdi}D_{bdi}^{\mathrm{T}}+\epsilon_3\rho_4^2D_{bdi}D_{bdi}^{\mathrm{T}}
\end{aligned}$$

then fuzzy system (1) under the following controller

$$\boldsymbol{u}(t)=\sum_{i=1}^{r}h_i(\boldsymbol{z}(t))Y_i(X^{-1})^{\mathrm{T}}\boldsymbol{x}(t) \tag{23}$$

is asymptotically stable for the time-varying input delay $\tau(t)$ satisfying (3).

**Proof.** Construct the Lyapunov-Krasovskii functional as

$$V(\boldsymbol{x}(t-\beta),\beta\in[\tau_0-\delta,\tau_0+\delta])=V_1+V_2 \tag{24}$$

where

$$V_1=\boldsymbol{x}^{\mathrm{T}}(t)P\boldsymbol{x}(t)+\int_{t-\tau_0}^{t}\boldsymbol{x}^{\mathrm{T}}(s)Q\boldsymbol{x}(s)\mathrm{d}s+\int_{t-\tau_0}^{t}\int_{s}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(v)R_1\dot{\boldsymbol{x}}(v)\mathrm{d}v\mathrm{d}s \tag{25}$$

$$V_2=2\delta\int_{t-\tau_0+\delta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)R_2\dot{\boldsymbol{x}}(s)\mathrm{d}s+\int_{t-\tau_0-\delta}^{t-\tau_0+\delta}\int_{s}^{t-\tau_0+\delta}\dot{\boldsymbol{x}}^{\mathrm{T}}(v)R_2\dot{\boldsymbol{x}}(v)\mathrm{d}v\mathrm{d}s \tag{26}$$

Taking the derivatives of $V_1$ and $V_2$ yields

$$\dot{V}_1=2\boldsymbol{x}^{\mathrm{T}}(t)P\dot{\boldsymbol{x}}(t)+\boldsymbol{x}^{\mathrm{T}}(t)Q\boldsymbol{x}(t)-\boldsymbol{x}^{\mathrm{T}}(t-\tau_0)Q\boldsymbol{x}(t-\tau_0)+\tau_0\dot{\boldsymbol{x}}^{\mathrm{T}}(t)R_1\dot{\boldsymbol{x}}(t)-\int_{t-\tau_0}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)R_1\dot{\boldsymbol{x}}(s)\mathrm{d}s \tag{27}$$

$$\dot{V}_2=2\delta\dot{\boldsymbol{x}}^{\mathrm{T}}(t)R_2\dot{\boldsymbol{x}}(t)-\int_{t-\tau_0-\delta}^{t-\tau_0+\delta}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)R_2\dot{\boldsymbol{x}}(s)\mathrm{d}s \tag{28}$$

Combining (8), (27), and (28), we can obtain

$$\begin{aligned}
\dot{V}=&2\boldsymbol{x}^{\mathrm{T}}(t)P\dot{\boldsymbol{x}}(t)+\boldsymbol{x}^{\mathrm{T}}(t)Q\boldsymbol{x}(t)-\boldsymbol{x}^{\mathrm{T}}(t-\tau_0)Q\boldsymbol{x}(t-\tau_0)+\\
&\tau_0\dot{\boldsymbol{x}}^{\mathrm{T}}(t)R_1\dot{\boldsymbol{x}}(t)-\int_{t-\tau_0}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)R_1\dot{\boldsymbol{x}}(s)\mathrm{d}s+\\
&2\delta\dot{\boldsymbol{x}}^{\mathrm{T}}(t)R_2\dot{\boldsymbol{x}}(t)-\int_{t-\tau_0-\delta}^{t-\tau_0+\delta}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)R_2\dot{\boldsymbol{x}}(s)\mathrm{d}s+\\
&2\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t))\boldsymbol{\eta}^{\mathrm{T}}(t)N_{ij}\times
\end{aligned}$$

$$\Big(\boldsymbol{x}(t)-\boldsymbol{x}(t-\tau_0)-\int_{t-\tau_0}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\Big)+$$
$$2\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t))\boldsymbol{\eta}^{\mathrm{T}}(t)T_{ij}\times$$
$$\Big(\boldsymbol{x}(t-\tau_0)-\boldsymbol{x}(t-\tau(t))-\int_{t-\tau(t)}^{t-\tau_0}\dot{\boldsymbol{x}}(s)\mathrm{d}s\Big)+$$
$$2\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t))\boldsymbol{\eta}^{\mathrm{T}}(t)M\times$$
$$\Big(\Big(\big(A_i+D_{ai}F_{ai}(t)E_{ai}\big)+\big(B_i+D_{bi}F_{bi}(t)E_{bi}\big)\times$$
$$K_j\Big)\boldsymbol{x}(t)+\big(B_{di}+D_{bdi}F_{bdi}(t)E_{bdi}\big)K_j\boldsymbol{x}(t-\tau(t))-$$
$$\dot{\boldsymbol{x}}(t)\Big) \tag{29}$$

where

$$\begin{aligned}\boldsymbol{\eta}^{\mathrm{T}}(t) &= [\boldsymbol{x}^{\mathrm{T}}(t)\ \ \boldsymbol{x}^{\mathrm{T}}(t-\tau_0)\ \ \boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\ \ \dot{\boldsymbol{x}}^{\mathrm{T}}(t)]\\ N_{ij}^{\mathrm{T}} &= [N_{1ij}^{\mathrm{T}}\ \ N_{2ij}^{\mathrm{T}}\ \ N_{3ij}^{\mathrm{T}}\ \ N_{4ij}^{\mathrm{T}}]\\ T_{ij}^{\mathrm{T}} &= [T_{1ij}^{\mathrm{T}}\ \ T_{2ij}^{\mathrm{T}}\ \ T_{3ij}^{\mathrm{T}}\ \ T_{4ij}^{\mathrm{T}}]\\ M^{\mathrm{T}} &= [M_1^{\mathrm{T}}\ \ M_2^{\mathrm{T}}\ \ M_3^{\mathrm{T}}\ \ M_4^{\mathrm{T}}]\end{aligned}$$

Using the matrix inequality properties proposed in [4−5], it is routine to get the following inequalities:

$$-\boldsymbol{\eta}^{\mathrm{T}}(t)N_{ij}\int_{t-\tau_0}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\le\tau_0\boldsymbol{\eta}^{\mathrm{T}}(t)N_{ij}R_1^{-1}N_{ij}^{\mathrm{T}}\boldsymbol{\eta}(t)+\int_{t-\tau_0}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)R_1\dot{\boldsymbol{x}}(s)\mathrm{d}s \tag{30}$$

$$-\boldsymbol{\eta}^{\mathrm{T}}(t)T_{ij}\int_{t-\tau(t)}^{t-\tau_0}\dot{\boldsymbol{x}}(s)\mathrm{d}s\le\delta\boldsymbol{\eta}^{\mathrm{T}}(t)T_{ij}R_2^{-1}T_{ij}^{\mathrm{T}}\boldsymbol{\eta}(t)+\int_{t-\tau_0-\delta}^{t-\tau_0+\delta}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)R_2\dot{\boldsymbol{x}}(s)\mathrm{d}s \tag{31}$$

$$\boldsymbol{\eta}^{\mathrm{T}}(t)MD_{ai}F_{ai}(t)E_{ai}\boldsymbol{x}(t)\le\epsilon_1\boldsymbol{\eta}^{\mathrm{T}}(t)MD_{ai}D_{ai}^{\mathrm{T}}M^{\mathrm{T}}\boldsymbol{\eta}(t)+\epsilon_1^{-1}\boldsymbol{x}^{\mathrm{T}}(t)E_{ai}^{\mathrm{T}}E_{ai}\boldsymbol{x}(t) \tag{32}$$

$$\boldsymbol{\eta}^{\mathrm{T}}(t)MD_{bi}F_{bi}(t)E_{bi}K_j\boldsymbol{x}(t)\le\epsilon_2\boldsymbol{\eta}^{\mathrm{T}}(t)MD_{bi}D_{bi}^{\mathrm{T}}M^{\mathrm{T}}\boldsymbol{\eta}(t)+\epsilon_2^{-1}\boldsymbol{x}^{\mathrm{T}}(t)K_j^{\mathrm{T}}E_{bi}^{\mathrm{T}}E_{bi}K_j\boldsymbol{x}(t) \tag{33}$$

$$\boldsymbol{\eta}^{\mathrm{T}}(t)MD_{bdi}F_{bdi}(t)E_{bdi}K_j\boldsymbol{x}(t-\tau(t))\le\epsilon_3\boldsymbol{\eta}^{\mathrm{T}}(t)MD_{bdi}D_{bdi}^{\mathrm{T}}M^{\mathrm{T}}\boldsymbol{\eta}(t)+\epsilon_3^{-1}\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))K_j^{\mathrm{T}}E_{bdi}^{\mathrm{T}}E_{bdi}K_j\boldsymbol{x}(t-\tau(t)) \tag{34}$$

Assume that there exist $\rho_2$, $\rho_3$ and $\rho_4$ such that $M_2=\rho_2M_1$, $M_3=\rho_3M_1$, $M_4=\rho_4M_1$, $\rho_4\neq0$, and $M_1$ is a nonsingular matrix. Denote $X=M_1^{-1}$, $\tilde{P}=XPX^{\mathrm{T}}$, $\tilde{Q}=XQX^{\mathrm{T}}$, $\tilde{R}_1=XR_1X^{\mathrm{T}}$, $\tilde{R}_2=XR_2X^{\mathrm{T}}$, $\tilde{N}_{kij}=XN_{kij}X^{\mathrm{T}}$, $\tilde{T}_{kij}=XT_{kij}X^{\mathrm{T}}$ and $Y_j=K_jX^{\mathrm{T}}$, $k=1,2,3,4$, $i,j=1,2,\cdots,r$. Then, with (29) ∼ (34), we can get

$$\begin{aligned}\dot{V}\ \le\ &\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t))\boldsymbol{\eta}^{\mathrm{T}}(t)X^{-1}\times\\ &\Big(\Lambda_{ij}+\tau_0\tilde{N}_{ij}\tilde{R}_1^{-1}\tilde{N}_{ij}^{\mathrm{T}}+\delta\tilde{T}_{ij}\tilde{R}_2^{-1}\tilde{T}_{ij}^{\mathrm{T}}-\\ &\Phi_{ij}^{\mathrm{T}}\tilde{I}^{-1}\Phi_{ij}\Big)(X^{-1})^{\mathrm{T}}\boldsymbol{\eta}(t)=\\ &\sum_{i=1}^{r}h_i^2(\boldsymbol{z}(t))\boldsymbol{\eta}^{\mathrm{T}}(t)X^{-1}\Big(\Lambda_{ii}+\tau_0\tilde{N}_{ii}\tilde{R}_1^{-1}\tilde{N}_{ii}^{\mathrm{T}}+\\ &\delta\tilde{T}_{ii}\tilde{R}_2^{-1}\tilde{T}_{ii}^{\mathrm{T}}-\Phi_{ii}^{\mathrm{T}}\tilde{I}^{-1}\Phi_{ii}\Big)(X^{-1})^{\mathrm{T}}\boldsymbol{\eta}(t)+\\ &\sum_{i=1}^{r-1}\sum_{j>i}^{r}h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t))\boldsymbol{\eta}^{\mathrm{T}}(t)X^{-1}\Big(\Lambda_{ij}+\Lambda_{ji}+\\ &\tau_0\tilde{N}_{ij}\tilde{R}_1^{-1}\tilde{N}_{ij}^{\mathrm{T}}+\tau_0\tilde{N}_{ji}\tilde{R}_1^{-1}\tilde{N}_{ji}^{\mathrm{T}}+\delta\tilde{T}_{ij}\tilde{R}_2^{-1}\tilde{T}_{ij}^{\mathrm{T}}+\\ &\delta\tilde{T}_{ji}\tilde{R}_2^{-1}\tilde{T}_{ji}^{\mathrm{T}}-\Phi_{ij}^{\mathrm{T}}\tilde{I}^{-1}\Phi_{ij}-\Phi_{ji}^{\mathrm{T}}\tilde{I}^{-1}\Phi_{ji}\Big)\times\\ &(X^{-1})^{\mathrm{T}}\boldsymbol{\eta}(t)\end{aligned} \tag{35}$$

Using Schur complements, we can easily show that

$$\Lambda_{ii}+\tau_0\tilde{N}_{ii}\tilde{R}_1^{-1}\tilde{N}_{ii}^{\mathrm{T}}+\delta\tilde{T}_{ii}\tilde{R}_2^{-1}\tilde{T}_{ii}^{\mathrm{T}}-\Phi_{ii}^{\mathrm{T}}\tilde{I}^{-1}\Phi_{ii}<0 \tag{36}$$

if and only if inequality (21) holds, and

$$\Lambda_{ij}+\Lambda_{ji}+\tau_0\tilde{N}_{ij}\tilde{R}_1^{-1}\tilde{N}_{ij}^{\mathrm{T}}+\tau_0\tilde{N}_{ji}\tilde{R}_1^{-1}\tilde{N}_{ji}^{\mathrm{T}}+\delta\tilde{T}_{ij}\tilde{R}_2^{-1}\tilde{T}_{ij}^{\mathrm{T}}+\delta\tilde{T}_{ji}\tilde{R}_2^{-1}\tilde{T}_{ji}^{\mathrm{T}}-\Phi_{ij}^{\mathrm{T}}\tilde{I}^{-1}\Phi_{ij}-\Phi_{ji}^{\mathrm{T}}\tilde{I}^{-1}\Phi_{ji}<0 \tag{37}$$

if and only if inequality (22) holds. Therefore, T-S fuzzy system (1) under controller (23) is asymptotically stable. □

## 3 A numeric example

Consider the following nonlinear mass-spring-damper mechanical system[13]:

$$M\ddot{\theta}(t)+D(\dot{\theta}(t))\dot{\theta}(t)+k\theta(t)=\phi F(t-\tau(t))$$

where $\theta(t)$ is the relative position of the mass, $F(t-\tau(t))$ the delayed external force, $M=1$ the mass of this system, $k=0.1$ the stiffness of the spring, and $\phi=1$ the input coefficient. The damping coefficient of the nonlinear damper is assumed to be $D(\dot{\theta}(t))=0.5+0.75\dot{\theta}^2(t)$. Furthermore, assume that $k$ is unknown but bounded within 0.1 of this nominal value.

Choose the state as $\boldsymbol{x}(t)=[\dot{\theta}(t)\ \ \theta(t)]^{\mathrm{T}}$ and the input variable $u(t)$ as $F(t)$. We can get the following state-space differential equation:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}-0.75x_1^3(t)-0.5x_1(t)+\\ 0.1(1+0.1\zeta(t))x_2(t)+u(t-\tau(t))\\ x_1(t)\end{bmatrix} \tag{38}$$

where $|\zeta(t)|^2\le1$. As in [13], the T-S fuzzy system of (38) can be constructed as follows:

$R^1$: IF $x_1(t)$ is about $\Gamma_1$, THEN $\dot{\boldsymbol{x}}(t)=(A_1+\triangle A_1)\boldsymbol{x}(t)+(B_{d1}+\triangle B_{d1})\boldsymbol{u}(t-\tau(t))$,

$R^2$: IF $x_1(t)$ is about $\Gamma_2$, THEN $\dot{\boldsymbol{x}}(t)=(A_2+\triangle A_2)\boldsymbol{x}(t)+(B_{d2}+\triangle B_{d2})\boldsymbol{u}(t-\tau(t))$,

where $x_1(t)\in[-\Omega,\Omega]$, $\Omega=0.8165$, $\Gamma_1(\boldsymbol{x}(t))=1-\frac{x_1^2(t)}{\Omega^2}$, $\Gamma_2(\boldsymbol{x}(t))=1-\Gamma_1(\boldsymbol{x}(t))$, $A_1=\begin{bmatrix}-0.5 & 0.1\\ 1 & 0\end{bmatrix}$, $A_2=\begin{bmatrix}-1 & 0.1\\ 1 & 0\end{bmatrix}$, $\triangle A_1=\triangle A_2=\begin{bmatrix}0 & 0.1\zeta(t)\\ 0 & 0\end{bmatrix}$, $B_{d1}=B_{d2}=[1\ \ 0]^{\mathrm{T}}$, $\triangle B_{d1}=\triangle B_{d2}=[0\ \ 0]^{\mathrm{T}}$, $D_{a1}=D_{a2}=\begin{bmatrix}0 & 0.1\\ 0 & 0\end{bmatrix}$, $E_{a1}=E_{a2}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, and $D_{bd1}=D_{bd2}=E_{bd1}=E_{bd2}=[0]_{2\times2}$.

Choosing $\rho_2=0.1,\rho_3=-0.1,\rho_4=1.0$, and applying the Matlab LMI toolbox to solve the LMIs (21) and (22)

in Theorem 1, we can get $\boldsymbol{K}_1 = [-0.7295 \quad -0.3385]$, $\boldsymbol{K}_2 = [-0.2295 \quad -0.3385]$, and $\tau = 9.02$. Then, the above T-S fuzzy system is robustly stable when the time-varying input delay $\tau(t) \in [0, 9.02]$. Let the initial state $\boldsymbol{x}(0) = \boldsymbol{x}_0 = [0 \quad 1]^{\mathrm{T}}$. The simulation result is shown in Fig 1. However, the maximum input delay $\tau$ obtained in [13] is 0.4223. Obviously, our result is less conservative than that obtained using the method in [13].

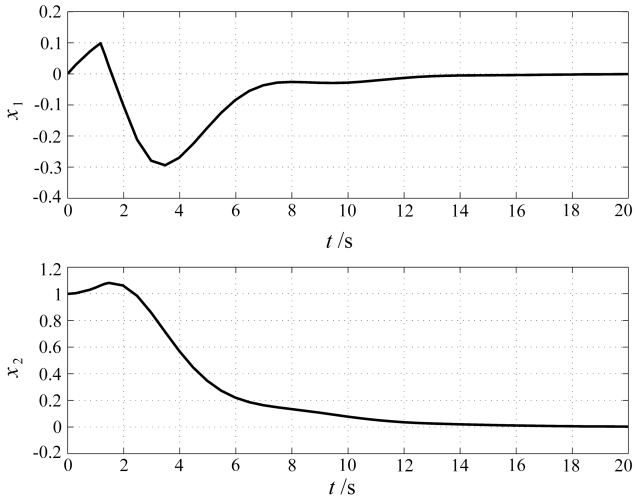


Fig. 1 The trajectories of a system under the feedback gains $K_1$ and $K_2$

## 4 Conclusion

In this paper, we have discussed the robust stability of uncertain T-S fuzzy systems with time-varying input delay. A new method for designing a fuzzy controller to stabilize uncertain T-S fuzzy systems with time-varying input delay is established by introducing some relaxation matrices and turning parameters. The numeric example demonstrated that the proposed results are effective and less conservative.

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**LUO Yan-Bin** Received her Ph. D. degree in control science and engineering from Zhejiang University in 2007. She is a lecturer at Institute of Automation, Hangzhou Dianzi University. Her research interest covers constrained control, time-delay systems, and fuzzy control. Corresponding author of this paper. E-mail: ybluo@hdu.edu.cn

**CAO Yong-Yan** Received his Ph. D. degree from Zhejiang University in 1996. He is a professor at Institute of Industrial Process Control, Zhejiang University. His research interest covers robust and nonlinear control, constrained control, time-delay systems, and fuzzy control.

**SUN You-Xian** Professor at Zhejiang University, an academician of the Chinese Academy of Engineering. His research interest covers robust and nonlinear control, fault tolerance control, and industrial optimization and control.