Output-feedback Adaptive Control for a Class of Nonlinear Systems with Unknown Control Directions

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Abstract In this paper, output-feedback adaptive stabilization is investigated for a class of nonlinear systems with unknown control directions. First, through a linear state transformation, the unknown control coefficients are lumped together and the original system is transformed to a new system for which control design becomes feasible. Then, after the introduction of an observer and an estimator for state and parameter estimates, respectively, a constructive design procedure is given for the output-feedback adaptive stabilizing controller using integrator backstepping and tuning function techniques. It is shown that the controller designed ensures the original system state converges to the origin whereas all the other closed-loop system states are bounded. Simulation results are illustrated to show the effectiveness of the proposed approach.

Key words Nonlinear systems, unknown control directions, output-feedback, adaptive control, backstepping

1 Introduction

Ever since the introduction of backstepping design[1], extensive research has been investigated on the constructive control design[1∼5], incorporated with nonlinear damping[6], tuning functions, MT filters[2], neural networks[7], Nussbaum gain function[8∼11], and other methods.

In the last two decades, the control problem of systems with unknown control directions has received much attention[8∼16]. When the signs of control coefficients are unknown, the control problem becomes much more difficult, because in this case, we cannot decide the direction along which the control operates. This control problem had remained open till the early 1980s. The breakthrough solution was originally given for a class of first-order linear systems by introducing the Nussbaum function[17]. Using Nussbaum gain methods, adaptive control was given for first-order nonlinear systems in [12].

As the first step toward higher order systems, backstepping with Nussbaum function was then developed for second-order systems in [9]. Later, backstepping with Nussbaum function was successfully developed for arbitrary finite order of nonlinear systems in the triangular structure, with constant control coefficient in [10, 11], where nonlinear systems and linear minimum phase systems were studied separately, and with time varying control coefficients[13,15]. Recently, the results have also been extended to nonlinear systems with general unknown nonlinear functions, rather than the standard assumption of linear-in-the-parameters form made by combining adaptive and neural network parametrization techniques[18].

In comparison with the plentiful research results in the state-feedback controls[9∼11,13,15,19], there are only a few results on output-feedback control for nonlinear systems[14,16,19], and many problems still remain open. The unknown signs of control coefficients will cause much greater technical difficulty in observer design in an effort to do output-feedback control as compared with state-feedback design. To the best of our knowledge, output-feedback control has only been investigated for a very special class of nonlinear systems with only one unknown control coefficient, the so-called high-frequency gain[14,16,19].

In this paper, we will consider the output-feedback adaptive stabilization control design for a large class of nonlinear relative degree n systems in strict feedback form without zero dynamics but with unknown perturbations and unknown control directions, and generalize the result in [16] where only one control coefficient is unknown. First, we introduce a linear state transformation, through which the unknown control coefficients distributed in the system can be lumped together for ease of manipulation. The original system is transformed into a new nonlinear system, for which the control design becomes feasible. Then, an observer and an estimator are designed to give the estimates of the state and parameter, respectively. Finally, a constructive design procedure is presented for output-feedback adaptive stabilization control using the integrator backstepping. Moreover, the tuning function method is utilized to remove the overparameterization. The main result shows that under some mild conditions the controller designed ensures the original system states asymptotically converge to zero whereas all the other closed-loop system states are bounded.

2 Preliminary knowledge

In this paper, the control coefficient is said to be unknown, if both its value and sign are unknown.

Definition 1. A continuous function, $N : [0, \infty) \to \mathbf{R}$, is called a Nussbaum function if it satisfies

$$\begin{cases} \lim\limits_{s\to\infty} \sup \left(\dfrac{1}{s}\displaystyle\int_0^s N(\upsilon)\mathrm{d}\upsilon\right) = +\infty \\ \lim\limits_{s\to\infty} \inf \left(\dfrac{1}{s}\displaystyle\int_0^s N(\upsilon)\mathrm{d}\upsilon\right) = -\infty \end{cases}$$

For example, $\upsilon \mapsto \mathrm{e}^{\upsilon^2}\cos(\pi\upsilon/2)$, $\upsilon \mapsto \ln(\upsilon+1)\cos(\sqrt{\ln(\upsilon+1)})$, and $\upsilon \mapsto \upsilon^2\cos(\pi\upsilon/2)$ are Nussbaum functions[8,10].

From Definition 1, it is easy to see that the Nussbaum-type function is not restricted to be even or odd. However, throughout this paper, we restrict $\zeta \mapsto N(\zeta) = \mathrm{e}^{\zeta^2}\cos(\pi\zeta/2)$ as an even Nussbaum function, and in the following lemma.

Lemma 1[13]**.** Let $t_f \in [0, \infty)$, and $V : [0, t_f) \to \mathbf{R}_+$, $\zeta : [0, t_f) \to \mathbf{R}$ be smooth functions, and $N : [0, t_f) \to \mathbf{R}$ be an even Nussbaum function. If the following inequality holds, *i.e.*,

$$V(t) \le c + \int_0^t (gN(\zeta(\upsilon)) + 1)\mathrm{d}\zeta(\upsilon), \quad \forall\, t \in [0, t_f)$$

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where $g \in \mathbf{R}$ is a nonzero constant and $c \in \mathbf{R}$ represents some suitable constant, then $t \mapsto \zeta(t)$, $t \mapsto V(t)$, and $t \mapsto \int_0^t N(\zeta(\upsilon))\mathrm{d}\zeta(\upsilon)$ are bounded on $[0,\ t_f)$.

3 System model and control problem

3.1 System model

Consider the nonlinear system in the following form

$$\begin{cases} \dot{x}_i &= g_i x_{i+1} + \boldsymbol{\theta}_i^{\mathrm{T}} \boldsymbol{\phi}_i(y), \quad i = 1,\ \cdots,\ n-1 \\ \dot{x}_n &= g_n u + \boldsymbol{\theta}_n^{\mathrm{T}} \boldsymbol{\phi}_n(y) \\ y &= x_1 \end{cases} \tag{1}$$

where $\boldsymbol{x} = [x_1,\ x_2,\ \cdots,\ x_n]^{\mathrm{T}} \in \mathbf{R}^n$, $u \in \mathbf{R}$, $y \in \mathbf{R}$ are system states, control input, and output, respectively; the initial states $\boldsymbol{x}(0) = \boldsymbol{x}_0$; $g_i,\ i = 1,\ \cdots,\ n$, called the control coefficients, are unknown constants; $\boldsymbol{\theta}_i \in \mathbf{R}^{r_i},\ r_i \in \mathbf{N},\ i = 1,\ \cdots,\ n$ are unknown but constant system parameters; $\boldsymbol{\phi}_i :\ \mathbf{R} \to \mathbf{R}^{r_i},\ i = 1,\ \cdots,\ n$ are known regressor vector-valued functions of system output y only.

It is easy to see that system (1) satisfies the triangular structure. This suffices to use integrator backstepping in control design. When for all $i = 1, \cdots, n$, $g_i = 1$, system (1) is in the parametric strict-feedback form, which have been intensely investigated in [2]. However, when g_is are unknown, especially the signs are unknown, the control problems of such systems will become much difficult and challenging. It has been shown that when the control directions of the systems are unknown, the method based on Nussbaum function is an effectual tool without worrying too much about their practical applications, and various results have been obtained, but most are for state feedback. It is worth mentioning that system (1) is more general and includes that in [16] as a special case, where only one control coefficient is unknown.

Assumption 1. $\boldsymbol{\phi}_i \in \mathbf{C}^\infty(\mathbf{R};\ \mathbf{R}^{r_i})$ with $\boldsymbol{\phi}_i(0) = \mathbf{0}$, and $g_i \neq 0$ for $i = 1,\ \cdots,\ n$.

Remark 1. Assumption 1 is much standard for such control problems, which implies that the origin is the equilibrium point of system (1), and g_i's are positive or negative but not zero, to satisfy the controllability condition of the system.

3.2 Control objective

The objective of this paper is to develop a dynamic output-feedback adaptive stabilizing controller as follows.

$$\begin{cases} \dot{\boldsymbol{x}}_c = \boldsymbol{f}_c(y,\ \boldsymbol{x}_c) \\ u\ = h_c(y,\ \boldsymbol{x}_c) \end{cases}$$

such that the state $\boldsymbol{x}$ of the closed-loop system with the initial condition $y(0) = y_0$, $\boldsymbol{x}_c(0) = \boldsymbol{x}_{co}$ asymptotically converges to zero while all the other signals are bounded, where $\boldsymbol{f}_c,\ h_c$ are continuously differential, and $\boldsymbol{x}_c$ is composed of observer state $\hat{\boldsymbol{\xi}}$, parameter estimate $\hat{\boldsymbol{\vartheta}}$, and variable ζ as will be detailed in Section 4.

4 Output-feedback adaptive control design

Because of the existence of unknown control coefficients, g_i, *i.e.*, both their values and signs are unknown, output-feedback control system design is very hard, even impossible. The best result available thus far in the literature is probably that in [16], which, however, is applicable to the class of systems with only one unknown high-frequency control coefficient.

In this paper, we can show that through a linear state transformation the system can be easily transformed into a system without the said difficulty, and make the output-feedback control feasible. By carefully examining system (1), we find that the difficulty of unknown control coefficient, g_i, disappears if the corresponding equation is divided by g_i itself with the scaled state being defined as the new state. This simple observation motivates us to introduce a novel linear state transformation to convert the original difficult system into a new system in which there are no unknown control coefficients, and the original unknown control coefficients are lumped together in the new states for which closed-loop observer design can be used to re-construct them for stable output-feedback control design.

The output-feedback adaptive control is developed in three steps as shown in this section. First, a linear state transformation is introduced to transform the original system to a new system with determinate control coefficients. Then, observer design is briefly presented for the transformed system using the most recent results. Finally, with the observer dynamics in the loop, output-feedback control design is given for the overall system in the lower triangular structure for which backstepping design is applied.

4.1 State transformation

Let $\boldsymbol{\Theta}_i = \boldsymbol{\theta}_i / \prod_{j=i}^n g_j$ and $\xi_i = x_i / \prod_{j=i}^n g_j$ for $i = 1, \cdots, n$. The dynamics of $\boldsymbol{\xi} = [\xi_1, \cdots, \xi_n]^{\mathrm{T}}$ are then given by

$$\begin{cases} \dot{\xi}_i &= \xi_{i+1} + \boldsymbol{\Theta}_i^{\mathrm{T}} \boldsymbol{\phi}_i(y), \quad i = 1,\ \cdots,\ n-1 \\ \dot{\xi}_n &= u + \boldsymbol{\Theta}_n^{\mathrm{T}} \boldsymbol{\phi}_n(y) \end{cases} \tag{2}$$

in which all control coefficients are known, *i.e.*, $g_i = 1,\ i = 1, \cdots, n$, the system is in the low triangular form, and the uncertainties $\boldsymbol{\Theta}_i^{\mathrm{T}} \boldsymbol{\phi}_i(y),\ i = 1, \cdots, n$ are functions of system output y only.

For system (2), if states ξ_is are available, control design can be carried out easily. But, because of the existence of unknown control coefficients in the linear state transformation, each of states ξ_is is unavailable, and a full-order observer is necessary for control design.

4.2 Observer design

Let us consider the observer designed in [16] for system (2):

$$\begin{cases} \dot{\hat{\xi}}_i &= \hat{\xi}_{i+1} - k_i \hat{\xi}_1, \quad i = 1,\ \cdots,\ n-1 \\ \dot{\hat{\xi}}_n &= u - k_n \hat{\xi}_1 \end{cases} \tag{3}$$

where k_is are constant design parameters to be determined and the initial condition is $\hat{\xi}_i(0) = \hat{\xi}_{0i}$ for any $i = 1, \cdots, n$. Denote $\hat{\boldsymbol{\xi}} = [\hat{\xi}_1,\ \cdots,\ \hat{\xi}_n]^{\mathrm{T}}$ and let $\tilde{\boldsymbol{\xi}} = \boldsymbol{\xi} - \hat{\boldsymbol{\xi}}$ be the observer error. For convenience, define $\hat{\xi}_{n+1} = u$ and $\tilde{\xi}_{n+1} = 0$.

It should be noticed that the above observer is of full-order. In fact, with the way of [20] in hand, it is easy to design a reduced-order observer, and the control design and stability analysis are mostly the same as those in the latter.

Combining (2) and (3) together, and bearing in mind the definition of $\tilde{\xi}_{n+1}$, we get the observer error dynamics of $\tilde{\boldsymbol{\xi}}$ as follows

$$\dot{\tilde{\boldsymbol{\xi}}} = A\tilde{\boldsymbol{\xi}} + \Theta^{\mathrm{T}} \boldsymbol{\Phi}(y) + \boldsymbol{K}\xi_1$$

where $\Theta = \mathrm{diag}[\boldsymbol{\Theta}_1, \cdots, \boldsymbol{\Theta}_n]$, $\boldsymbol{\Phi} = [\boldsymbol{\phi}_1^{\mathrm{T}}, \cdots, \boldsymbol{\phi}_n^{\mathrm{T}}]^{\mathrm{T}}$, and

$$A = \begin{bmatrix} -k_1 & & \\ \cdots & I_{n-1} & \\ -k_n & 0 \ \cdots \ 0 & \end{bmatrix}, \ \boldsymbol{K} = \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix}$$

Choose positive constants $k_1, \cdots, k_n$ such that A is asymptotically stable; hence there exists a $P = P^{\mathrm{T}} > 0$ such that

$$A^{\mathrm{T}}P + PA = -I \tag{4}$$

Next subsection turns to the control design.

4.3 Output-feedback adaptive control design

In this subsection, we begin the design procedure of the output-feedback adaptive stabilization control for the following overall system with the observer in the loop

$$\begin{cases} \dot{\tilde{\boldsymbol{\xi}}} &= A\tilde{\boldsymbol{\xi}} + \Theta^{\mathrm{T}}\boldsymbol{\Phi}(y) + \boldsymbol{K}\xi_1 \\ \dot{y} &= \overline{g}(\hat{\xi}_2 + \tilde{\xi}_2) + \boldsymbol{\theta}_1^{\mathrm{T}}\boldsymbol{\phi}_1(y), \ \ \overline{g} = \prod_{i=1}^n g_i \\ \dot{\hat{\xi}}_i &= \hat{\xi}_{i+1} - k_i\hat{\xi}_1, \ \ i = 2, \ 3, \ \cdots, \ n-1 \\ \dot{\hat{\xi}}_n &= u - k_n\hat{\xi}_1 \end{cases} \tag{5}$$

With this system as the starting point, we begin the stabilizing control design, which is addressed in a step-by-step manner by using integrator backstepping approach together with tuning function technique, and a Nussbaum-type function to overcome the mentioned difficulty. The design procedure given below is in a domain of the initial condition and in a small time interval $[0, \ t_f)$.

Step 0. This step can be viewed as the initial assignment for the entire design procedure. Define $V_0 : \ \mathbf{R}^n \to \mathbf{R}$ by $V_0(\tilde{\boldsymbol{\xi}}) = \tilde{\boldsymbol{\xi}}^{\mathrm{T}}P\tilde{\boldsymbol{\xi}}, \ \boldsymbol{\xi} \in \mathbf{R}^n$, where $P = P^{\mathrm{T}} > 0$ satisfies (4). Its time derivative satisfies

$$\dot{V}_0 = -\|\tilde{\boldsymbol{\xi}}\|^2 + 2\tilde{\boldsymbol{\xi}}^{\mathrm{T}}P\left(\Theta^{\mathrm{T}}\boldsymbol{\Phi}(y) + \boldsymbol{K}\xi_1\right), \quad \forall t \in [0, \ t_f) \tag{6}$$

For the second term, by Assumption 1, there exists $\boldsymbol{\Phi}(y) = y\overline{\boldsymbol{\Phi}}(y)$, $\forall y \in \mathbf{R}$, where $\overline{\boldsymbol{\Phi}}(\cdot)$ is a smooth matrix-valued function defined by $y \mapsto \overline{\boldsymbol{\Phi}}(y) = \int_0^1 \frac{\partial \boldsymbol{\Phi}(s)}{\partial s}|_{s=ay}\mathrm{d}a$; hence

$$2\tilde{\boldsymbol{\xi}}^{\mathrm{T}}P\Theta^{\mathrm{T}}\boldsymbol{\Phi}(y) \le \frac{1}{4}\|\tilde{\boldsymbol{\xi}}\|^2 + 4\|P\|^2 r_\Theta^2 \left\|\overline{\boldsymbol{\Phi}}(y)\right\|^2 y^2 \tag{7}$$

where $r_\Theta > 0$ is an unknown number such that $\|\Theta\| \le r_\Theta$.

From the foregoing definition of ξ_1, we know that $y = x_1 = \xi_1 \prod_{i=1}^n g_i$. By this equation together with Assumption 1, we have

$$2\tilde{\boldsymbol{\xi}}^{\mathrm{T}}P\boldsymbol{K}\xi_1 \le \frac{1}{4}\|\tilde{\boldsymbol{\xi}}\|^2 + \frac{4\|P\boldsymbol{K}\|^2}{\prod_{i=1}^n g_i^2}y^2, \ \forall \tilde{\boldsymbol{\xi}} \in \mathbf{R}^n, \ \forall y \in \mathbf{R} \tag{8}$$

Substituting (7) and (8) into (6) concludes that

$$\dot{V}_0 \le -\frac{1}{2}\|\tilde{\boldsymbol{\xi}}\|^2 + \nu(1 + \|\overline{\boldsymbol{\Phi}}(y)\|^2)y^2, \quad \forall t \in [0, \ t_f) \tag{9}$$

where $\nu = 4\max\left\{\|P\|^2 r_\Theta^2, \ \frac{\overline{g}^2}{8\varepsilon_1}, \ \frac{\|P\boldsymbol{K}\|^2}{\prod_{i=1}^n g_i^2}\right\}$ and ε_1 satisfies $0 < \varepsilon_1 < \min\left\{0.5, \ \frac{1}{2(n-1)}\right\}$.

The following turns to the recursive design procedure, which is divided into n steps in all.

Step 1. Let $z_1 = y$ and $z_2 = \hat{\xi}_2 - \alpha_1(y, \ \zeta, \ \hat{\boldsymbol{\vartheta}})$, where $\alpha_1(\cdot)$ is a smooth function to be determined, $\hat{\boldsymbol{\vartheta}}$ denotes the estimate of $\boldsymbol{\vartheta} = [\nu, \ \overline{g}, \ \boldsymbol{\theta}_1^{\mathrm{T}}]^{\mathrm{T}}$ whose updating law is postponed until the last step (See (19)), and $\tilde{\boldsymbol{\vartheta}}$ denotes the estimation error of $\boldsymbol{\vartheta}$, that is $\tilde{\boldsymbol{\vartheta}} = \boldsymbol{\vartheta} - \hat{\boldsymbol{\vartheta}}$.

Choose Lyapunov function candidate $V_1 = V_0 + \frac{1}{2}\tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\Gamma_{\boldsymbol{\vartheta}}^{-1}\tilde{\boldsymbol{\vartheta}} + \frac{1}{2}z_1^2$, where $\Gamma_{\boldsymbol{\vartheta}} = \Gamma_{\boldsymbol{\vartheta}}^{\mathrm{T}} > 0$ is called the adaptive gain matrix.

Noting (9) and $\dot{\tilde{\boldsymbol{\vartheta}}} = -\dot{\hat{\boldsymbol{\vartheta}}}$, the time derivative of V_1 along the trajectories of system (5) is as follows.

$$\begin{aligned} \dot{V}_1 \le & -\frac{1}{2}\|\tilde{\boldsymbol{\xi}}\|^2 + \nu(1 + \|\overline{\boldsymbol{\Phi}}(y)\|^2)y^2 - \tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\Gamma_{\boldsymbol{\vartheta}}^{-1}\dot{\hat{\boldsymbol{\vartheta}}} + \\ & z_1\left(\overline{g}(\alpha_1 + z_2 + \tilde{\xi}_2) + \boldsymbol{\theta}_1^{\mathrm{T}}\boldsymbol{\phi}_1(y)\right), \ t \in [0, \ t_f) \end{aligned} \tag{10}$$

The term $\overline{g}\tilde{\xi}_2 z_1$ on the right-hand side of (10) has unknown constant gain $\overline{g}$. Then, noting the definition of the unknown constant ν earlier, we have

$$\overline{g}z_1\tilde{\xi}_2 \le \varepsilon_1\tilde{\xi}_2^2 + \frac{\overline{g}^2}{4\varepsilon_1}z_1^2 \le \varepsilon_1\|\tilde{\boldsymbol{\xi}}\|^2 + 0.5\nu z_1^2$$

From the above equation and the definition of $\boldsymbol{\vartheta}$, it follows that

$$\begin{aligned} & \nu(1 + \|\overline{\boldsymbol{\Phi}}(y)\|^2)y^2 + \overline{g}z_1\tilde{\xi}_2 + z_1\boldsymbol{\theta}_1^{\mathrm{T}}\boldsymbol{\phi}_1(y) \le \\ & \varepsilon_1\|\tilde{\boldsymbol{\xi}}\|^2 + z_1\boldsymbol{\vartheta}^{\mathrm{T}}\boldsymbol{\Psi}_1(y) \end{aligned}$$

where $\boldsymbol{\Psi}_1(y) = [0.5y + (1 + \|\overline{\boldsymbol{\Phi}}(y)\|^2)y, \ 0, \ \boldsymbol{\phi}_1^{\mathrm{T}}(y)]^{\mathrm{T}}$. Substituting this equation into (10), we have

$$\begin{aligned} \dot{V}_1 \le & -c_1\|\tilde{\boldsymbol{\xi}}\|^2 - \tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\Gamma_{\boldsymbol{\vartheta}}^{-1}\dot{\hat{\boldsymbol{\vartheta}}} + z_1\overline{g}\alpha_1 + \\ & z_1\boldsymbol{\vartheta}^{\mathrm{T}}\boldsymbol{\Psi}_1(y) + \overline{g}z_1 z_2, \quad t \in [0, \ t_f) \end{aligned} \tag{11}$$

where $c_1 = \frac{1}{2} - \varepsilon_1$, which will be positive if $\varepsilon_1 < \frac{1}{2}$.

Because of the presence of the unknown parameter, $\overline{g}$, we have to use the Nussbaum technique in designing the control law[7] as follows.

$$\begin{cases} \alpha_1 &= N(\zeta)\eta(z_1, \ \hat{\boldsymbol{\vartheta}}) \\ \dot{\zeta} &= z_1\eta(z_1, \ \hat{\boldsymbol{\vartheta}}) \\ \eta &= \beta_1 z_1 + \hat{\boldsymbol{\vartheta}}^{\mathrm{T}}\boldsymbol{\Psi}_1(y) \end{cases} \tag{12}$$

where $\beta_1 > 0$ is a design constant, the function $N(\cdot)$ is smooth, even and Nussbaum-type, chosen as $\zeta \mapsto \mathrm{e}^{\zeta^2}\cos(\pi\zeta/2)$ throughout this paper. Clearly, α_1 is a smooth function of $(y, \ \zeta, \ \hat{\boldsymbol{\vartheta}})$ and vanishes when $y = 0, \ \zeta = 0$. It should be mentioned that there are various methods to construct the Nussbaum-type functions, for example, in [11] two slightly different ways are presented for linear minimum-phase systems.

Substituting (12) into (11), and after some algebraic manipulations, we have

$$\begin{aligned} \dot{V}_1 \le & -c_1\|\tilde{\boldsymbol{\xi}}\|^2 - \beta_1 z_1^2 - \tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\Gamma_{\boldsymbol{\vartheta}}^{-1}(\dot{\hat{\boldsymbol{\vartheta}}} - \boldsymbol{\tau}_1) + \\ & (\overline{g}N(\zeta) + 1)\dot{\zeta} + \overline{g}z_1 z_2 \end{aligned}$$

where $\boldsymbol{\tau}_1 = \Gamma_{\boldsymbol{\vartheta}}\boldsymbol{\Psi}_1 z_1$ and $\boldsymbol{\tau}_i (i = 2, \ \cdots, \ n)$ in the latter steps are called the tuning functions[2], which are usually used to avoid overparameterization in adaptive control design.

Remark 2. Note that the first and second items are stabilizing, the third item can be made stabilizing by designing tuning law later, and the fourth item can be analyzed using Lemma 1 as will be detailed soon. The difficulty lies in

presence of the last term "$\overline{g}z_1z_2$" because it has unknown gain $\overline{g}$ and the standard technique for known sign case is no longer applicable here. At first glance, it may seem to be difficult. In fact, it can be easily solved by grouping $\overline{g}$ into one of the parameter $\boldsymbol{\vartheta}$ so that the parameter estimator using tuning functions can be used to solve the problem with ease which will be explained later.

Step 2. Define $z_3 = \hat{\xi}_3 - \alpha_2(y,\ \bar{\hat{\boldsymbol{\xi}}}_2,\ \zeta,\ \hat{\boldsymbol{\vartheta}})$, where $\bar{\hat{\boldsymbol{\xi}}}_2 = [\hat{\xi}_1,\ \hat{\xi}_2]^{\mathrm{T}}$. Then from (5), the dynamic of z_2 is easily obtained.

Define $V_2 :\ \mathbf{R}^n \times \mathbf{R}^2 \to \mathbf{R}$ by $V_2 = V_1 + \frac{1}{2}z_2^2$. Its time derivative of V_2 satisfies

$$\begin{aligned}\dot{V}_2 \leq & -c_1\|\tilde{\boldsymbol{\xi}}\|^2 - \tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\Gamma_{\boldsymbol{\vartheta}}^{-1}(\dot{\hat{\boldsymbol{\vartheta}}} - \boldsymbol{\tau}_1) - \\ & \beta_1 z_1^2 + (\overline{g}N(\zeta)+1)\dot{\zeta} + \overline{g}z_1z_2 + \\ & z_2\Big(z_3 + \alpha_2 - k_2\hat{\xi}_1 - \frac{\partial\alpha_1}{\partial y}\Big(\overline{g}(\hat{\xi}_2+\tilde{\xi}_2) + \\ & \boldsymbol{\theta}_1^{\mathrm{T}}\boldsymbol{\phi}_1(y)\Big) - \frac{\partial\alpha_1}{\partial\zeta}\dot{\zeta} - \frac{\partial\alpha_1}{\partial\hat{\boldsymbol{\vartheta}}}\dot{\hat{\boldsymbol{\vartheta}}}\Big), \quad t\in[0,\ t_f) \end{aligned} \tag{13}$$

Note that $\overline{g}z_1z_2$, $-\frac{\partial\alpha_1}{\partial y}\overline{g}\hat{\xi}_2z_2$ and $-\frac{\partial\alpha_1}{\partial y}\overline{g}\tilde{\xi}_2z_2$ have the same unknown coefficient $\overline{g}$. It is clear that $\overline{g}z_1z_2 - \frac{\partial\alpha_1}{\partial y}\hat{\xi}_2\overline{g}z_2 = \overline{g}\left(z_1 - \frac{\partial\alpha_1}{\partial y}\hat{\xi}_2\right)z_2$. The term $-\frac{\partial\alpha_1}{\partial y}\overline{g}\tilde{\xi}_2z_2$ satisfies the following inequality

$$-\frac{\partial\alpha_1}{\partial y}\overline{g}\tilde{\xi}_2z_2 \leq \varepsilon_1\|\tilde{\boldsymbol{\xi}}\|^2 + \frac{\nu}{2}\left(\frac{\partial\alpha_1}{\partial y}\right)^2 z_2^2$$

Then, noting the definition of $\boldsymbol{\vartheta}$ earlier, we have

$$\begin{aligned}&\overline{g}z_1z_2 - \frac{\partial\alpha_1}{\partial y}\overline{g}\hat{\xi}_2z_2 - \frac{\partial\alpha_1}{\partial y}\overline{g}\tilde{\xi}_2z_2 - \frac{\partial\alpha_1}{\partial y}\boldsymbol{\theta}_1^{\mathrm{T}}\boldsymbol{\phi}_1(y)z_2 \leq \\ &\varepsilon_1\|\tilde{\boldsymbol{\xi}}\|^2 + z_2\boldsymbol{\vartheta}^{\mathrm{T}}\boldsymbol{\Psi}_2 \end{aligned} \tag{14}$$

where $\boldsymbol{\Psi}_2 = \left[\frac{1}{2}\left(\frac{\partial\alpha_1}{\partial y}\right)^2 z_2,\ z_1 - \frac{\partial\alpha_1}{\partial y}\hat{\xi}_2,\ -\frac{\partial\alpha_1}{\partial y}\boldsymbol{\phi}_1^{\mathrm{T}}(y)\right]^{\mathrm{T}}$ is smooth with respect to each of it arguments.

Design the virtual control α_2 as follows.

$$\alpha_2 = -\beta_2z_2 + k_2\hat{\xi}_1 - \hat{\boldsymbol{\vartheta}}^{\mathrm{T}}\boldsymbol{\Psi}_2 + \frac{\partial\alpha_1}{\partial\zeta}\dot{\zeta} + \frac{\partial\alpha_1}{\partial\hat{\boldsymbol{\vartheta}}}\boldsymbol{\tau}_2 \tag{15}$$

where $\boldsymbol{\tau}_2 = \boldsymbol{\tau}_1 + \Gamma_{\boldsymbol{\vartheta}}\boldsymbol{\Psi}_2z_2$. From (15), it is easy to see that α_2 can be rewritten into the function of $(y, \bar{\hat{\boldsymbol{\xi}}}_2,\ \zeta, \hat{\boldsymbol{\vartheta}})$ by the definitions of $z_1,\ z_2$.

Letting $c_2 = c_1 - \varepsilon_1$ and substituting (14) and (15) into (13) and after some algebraic manipulations, we have

$$\begin{aligned}\dot{V}_2 \leq & -c_2\|\tilde{\boldsymbol{\xi}}\|^2 - \left(\tilde{\boldsymbol{\vartheta}} + \Gamma_{\boldsymbol{\vartheta}}\left(\frac{\partial\alpha_1}{\partial\hat{\boldsymbol{\vartheta}}}\right)^{\mathrm{T}}z_2\right)^{\mathrm{T}} \times \\ & \Gamma_{\boldsymbol{\vartheta}}^{-1}(\dot{\hat{\boldsymbol{\vartheta}}} - \boldsymbol{\tau}_2) - \beta_1z_1^2 - \beta_2z_2^2 + \\ & (\overline{g}N(\zeta)+1)\dot{\zeta} + z_2z_3, \quad t\in[0,\ t_f)\end{aligned}$$

Step 2 is thus completed. The remaining steps can be recursively carried out.

Step i ($i = 3,\ 4,\ \cdots,\ n$). Assume that Lyapunov function candidates V_j ($j = 2,\ 3,\ \cdots,\ i-1$) correspond to Step j, respectively, and V_{i-1} satisfy the following inequality

$$\begin{aligned}\dot{V}_{i-1} \leq & -c_{i-1}\|\tilde{\boldsymbol{\xi}}\|^2 - \left(\tilde{\boldsymbol{\vartheta}} + \Gamma_{\boldsymbol{\vartheta}}\sum_{j=2}^{i-1}\left(\frac{\partial\alpha_{j-1}}{\partial\hat{\boldsymbol{\vartheta}}}\right)^{\mathrm{T}}z_j\right)^{\mathrm{T}} \\ & \Gamma_{\boldsymbol{\vartheta}}^{-1}\left(\dot{\hat{\boldsymbol{\vartheta}}} - \boldsymbol{\tau}_{i-1}\right) - \sum_{j=1}^{i-1}\beta_jz_j^2 + \\ & (\overline{g}N(\zeta)+1)\dot{\zeta} + z_{i-1}z_i \quad t\in[0,\ t_f)\end{aligned} \tag{16}$$

Define $z_{i+1} = \hat{\xi}_{i+1} - \alpha_i(y,\ \bar{\hat{\boldsymbol{\xi}}}_i,\ \zeta,\ \hat{\boldsymbol{\vartheta}})$, where α_i is called the ith virtual control and $\bar{\hat{\boldsymbol{\xi}}}_i = [\hat{\xi}_1, \cdots, \hat{\xi}_i]^{\mathrm{T}},\ i = 1, \cdots, n$.

From (5), the dynamic of z_i is easily obtained. Then, for $-\frac{\partial\alpha_{i-1}}{\partial y}\overline{g}\hat{\xi}_2z_i$ and $-\frac{\partial\alpha_{i-1}}{\partial y}\overline{g}\tilde{\xi}_2z_i$ in $z_i\dot{z}_i$, which have the same unknown constant gain $\overline{g}$, bearing in mind the definition of ν, we have $z_i\frac{\partial\alpha_{i-1}}{\partial y}\overline{g}\tilde{\xi}_2 \leq \varepsilon_1\|\tilde{\boldsymbol{\xi}}\|^2 + \frac{\nu}{2}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_i^2$. From this and the definition of $\boldsymbol{\vartheta}$, it follows that

$$\begin{aligned}&-\frac{\partial\alpha_{i-1}}{\partial y}\overline{g}\hat{\xi}_2z_i - \frac{\partial\alpha_{i-1}}{\partial y}\overline{g}\tilde{\xi}_2z_i - \frac{\partial\alpha_{i-1}}{\partial y}\boldsymbol{\theta}_1^{\mathrm{T}}\boldsymbol{\phi}_1(y)z_i \leq \\ &\varepsilon_1\|\tilde{\boldsymbol{\xi}}\|^2 + \boldsymbol{\vartheta}^{\mathrm{T}}\boldsymbol{\Psi}_iz_i\end{aligned} \tag{17}$$

where $\boldsymbol{\Psi}_i = \left[\frac{1}{2}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_i,\ -\frac{\partial\alpha_{i-1}}{\partial y}\hat{\xi}_2,\ -\frac{\partial\alpha_{i-1}}{\partial y}\boldsymbol{\phi}_1^{\mathrm{T}}(y)\right]^{\mathrm{T}}$.

Substituting (17) into (16), and after some algebraic manipulations, we have for any $t\in[0,\ t_f)$

$$\begin{aligned}\dot{V}_i \leq & -c_i\|\tilde{\boldsymbol{\xi}}\|^2 - \left(\tilde{\boldsymbol{\vartheta}} + \Gamma_{\boldsymbol{\vartheta}}\sum_{j=2}^{i-1}\left(\frac{\partial\alpha_{j-1}}{\partial\hat{\boldsymbol{\vartheta}}}\right)^{\mathrm{T}}z_j\right)^{\mathrm{T}} \times \\ & \Gamma_{\boldsymbol{\vartheta}}^{-1}(\dot{\hat{\boldsymbol{\vartheta}}} - \boldsymbol{\tau}_{i-1}) - \sum_{j=1}^{i-1}\beta_jz_j^2 + (\overline{g}N(\zeta)+1)\dot{\zeta} + \\ & z_i\Bigg(\alpha_i + z_{i-1} - k_i\hat{\xi}_1 + \boldsymbol{\vartheta}^{\mathrm{T}}\boldsymbol{\Psi}_i - \\ & \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\xi}_j}\left(\hat{\xi}_{j+1} - k_j\hat{\xi}_1\right) - \frac{\partial\alpha_{i-1}}{\partial\zeta}\dot{\zeta} - \\ & \frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{\vartheta}}}\dot{\hat{\boldsymbol{\vartheta}}}\Bigg) + z_iz_{i+1}, \quad t\in[0,\ t_f)\end{aligned}$$

where $c_i = c_{i-1} - \varepsilon_1$.

Thus, we can choose α_i as follows.

$$\begin{aligned}\alpha_i = & -\beta_iz_i - z_{i-1} + k_i\hat{\xi}_1 - \hat{\boldsymbol{\vartheta}}^{\mathrm{T}}\boldsymbol{\Psi}_i + \sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\xi}_j}(\hat{\xi}_{j+1} - \\ & k_j\hat{\xi}_1) + \frac{\partial\alpha_{i-1}}{\partial\zeta}\dot{\zeta} - \sum_{j=2}^{i}\frac{\partial\alpha_{j-1}}{\partial\hat{\boldsymbol{\vartheta}}}z_j\boldsymbol{\Psi}_i + \frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{\vartheta}}^{\mathrm{T}}}\boldsymbol{\tau}_i\end{aligned} \tag{18}$$

where $\boldsymbol{\tau}_i = \boldsymbol{\tau}_{i-1} + \Gamma_{\boldsymbol{\vartheta}}\boldsymbol{\Psi}_iz_i$. From (18) and the definition of $\bar{z}_i$, it is clear that α_i can be expressed as the function of $(y,\ \bar{\hat{\boldsymbol{\xi}}}_i,\ \zeta, \hat{\boldsymbol{\vartheta}})$.

So far, the actual adaptive controller and the updating law of $\hat{\boldsymbol{\vartheta}}$ are obtained and given by

$$\begin{cases}u(y,\ \hat{\boldsymbol{\xi}},\ \zeta,\ \hat{\boldsymbol{\vartheta}}) = \alpha_n(y,\ \hat{\boldsymbol{\xi}},\ \zeta,\ \hat{\boldsymbol{\vartheta}}) \\ \dot{\hat{\boldsymbol{\vartheta}}} = \boldsymbol{\tau}_n, \quad \hat{\boldsymbol{\vartheta}}(0) = \hat{\boldsymbol{\vartheta}}_0, \quad t\in[0,\ t_f)\end{cases} \tag{19}$$

and $V_n = V_0 + \frac{1}{2}\sum_{j=1}^{n} z_j^2$ satisfies for any $t \in [0,\ t_f)$

$$\dot{V}_n \leq -c_n\|\tilde{\boldsymbol{\xi}}\|^2 - \sum_{i=1}^{n}\beta_i z_i^2 + (\overline{g}N(\zeta)+1)\dot{\zeta} \qquad (20)$$

where $\boldsymbol{\tau}_n$, α_n, and c_n can be be easily written out by above argument.

At this stage, we have completed the design procedure of the output-feedback adaptive control using backstepping and tuning function techniques. It is worth mentioning that Step 2 cannot be included in Step i. This is because that in Step 2, "$\overline{g}z_1z_2$" should be stabilized and hence $\boldsymbol{\Psi}_2$ is not a special case of $\boldsymbol{\Psi}_i$.

The main results contained in the following theorem summarizes this section.

Theorem 1. Consider system (1) satisfying Assumptions 1. If the design parameters ε_1 and $\beta_i,\ i = 1,\ \cdots,\ n$ are appropriately chosen, such as $\varepsilon_1 = \frac{1}{2n},\quad \beta_1 = \beta_2 = \cdots = \beta_n = 1$, then, for any initial condition $\boldsymbol{x}(0) = \boldsymbol{x}_0, \hat{\boldsymbol{\xi}}(0) = \hat{\boldsymbol{\xi}}_0, \hat{\zeta}(0) = \hat{\zeta}_0$ and $\hat{\boldsymbol{\vartheta}}(0) = \hat{\boldsymbol{\vartheta}}_0$, the application of the observer (3) in conjunction with (12), (18) and the control (19) into system (1) ensures that all the closed-loop system states are well-defined and bounded for any $t \in [0,\ \infty)$. Moreover,

$$\lim_{t\to\infty}\boldsymbol{x}(t) = 0, \qquad \lim_{t\to\infty}\boldsymbol{z}(t) = 0$$

Proof. In the following, we will first prove that the closed-loop solution is bounded and well-defined on $[0,\ \infty)$, and then, prove the asymptotical stability of the system states $\boldsymbol{x}(t)$ and $\boldsymbol{z}(t)$ as $t \to \infty$.

It is easy to see that the right-hand sides of closed-loop system are locally Lipschitz in a domain of the initial condition, and hence the system has a unique solution $(\boldsymbol{z}(t), \hat{\boldsymbol{\xi}}(t), \hat{\zeta}(t), \hat{\boldsymbol{\vartheta}}(t))$ on a small interval $[0,\ t_f)$. Let $[0,\ T_f)$ be its maximal interval of the existence and uniqueness. We are now ready to show that $T_f = \infty$. For this, we first prove the boundedness of all the closed-loop states starting from the given initial condition on $[0,\ T_f)$. Define $W(\tilde{\boldsymbol{\xi}},\ \boldsymbol{z}) = -c_n\|\tilde{\boldsymbol{\xi}}\|^2 - \sum_{j=1}^{n}\beta_j z_j^2$, which is negative definite with respect to $\tilde{\boldsymbol{\xi}}$ and $\boldsymbol{z}$. From this, (20) and Lemma 1, it follows that ζ and V are bounded on $[0,\ T_f)$. This means that $(\tilde{\boldsymbol{\xi}}(t), \boldsymbol{z}(t), \tilde{\boldsymbol{\vartheta}}(t))$ are bounded for any t in $[0,\ T_f)$; namely, $\exists M_1 > 0$, independent of T_f, for any $t \in [0,\ T_f)$, $\|\tilde{\boldsymbol{\xi}}(t)\| \leq M_1,\quad \|\boldsymbol{z}(t)\| \leq M_1,\quad \|\tilde{\boldsymbol{\vartheta}}(t)\| \leq M_1$. From this together with the boundedness of $\boldsymbol{\vartheta}$, it is easy to see that $\hat{\boldsymbol{\vartheta}}$ is bounded on $[0,\ T_f)$; that is, $\exists M_2 > 0$, independent of T_f, for any $t \in [0,\ T_f)$, $\|\hat{\boldsymbol{\vartheta}}(t)\| < M_2$. Let us next prove the boundedness of $\hat{\boldsymbol{\xi}}$ on $[0,\ T_f)$. First from the boundedness of $\tilde{\boldsymbol{\xi}}(t)$ for any t in $[0,\ T_f)$, the definition $\hat{\xi}_2 = z_2 + \alpha_1$ and the smoothness of α_1, the boundedness of $\hat{\xi}_2(t)$ can be ensured for any t in $[0,\ T_f)$. Therefore, from (3); that is, $\dot{\hat{\xi}}_1 = \hat{\xi}_2 - k_1\hat{\xi}_1,\ k_1 > 0$, we immediately conclude that $\hat{\xi}_1(t)$ is bounded for any t in $[0,\ T_f)$; that is, $\exists M_3 > 0$, independent of T_f, for any t in $[0,\ T_f)$, $|\hat{\xi}_1(t)| \leq M_3$. Using inductive argument, the boundedness of $\hat{\xi}_i(t),\ i = 2, \cdots, n$, as well as $\boldsymbol{\xi}(t)$ for any t in $[0,\ T_f)$, can further be established by the boundedness of $\hat{\xi}_1(t)$ and $\hat{\boldsymbol{\vartheta}}(t)$ for any t in $[0,\ T_f)$ and the smoothness of $\alpha_i,\ i = 1, \cdots, n-1$. So far, all the closed-loop system states are upper bounded by $c = \max\{M_1,\ M_2,\ M_3\}$ on $[0,\ T_f)$. Because c is independent of T_f, it is natural that T_f can be maximized to $+\infty$. Otherwise, finite time escape will happen, namely, the solution will escape from any compact set as $t \to T_f$. By this together with Theorem 3.3 of [21], we conclude that the closed-loop system has a unique bounded solution that is well-defined for all $t \geq 0$.

The rest is to prove the asymptotical stability of the system states $\boldsymbol{z}$, $[y,\ \hat{\xi}_1,\ \hat{\xi}_2,\ \cdots,\ \hat{\xi}_n]^{\mathrm{T}}$ and $\boldsymbol{\xi}$ (as well as $\boldsymbol{x}$). Again from (20) and by Lemma 1, it follows that $\tilde{\boldsymbol{\xi}}(t)$ and $\boldsymbol{z}(t)$ are L_2; that is, there exists a constant $M_4 > 0$, such that

$$\int_0^\infty \|\tilde{\boldsymbol{\xi}}(t)\|^2\mathrm{d}t < M_4, \quad \int_0^\infty \|\boldsymbol{z}(t)\|^2\mathrm{d}t < M_4 \qquad (21)$$

Then, from the smoothness of the right-hand sides of the closed-loop system, the boundedness of $\tilde{\boldsymbol{\xi}}(t)$, $\zeta(t)$, $\hat{\boldsymbol{\xi}}$, $\boldsymbol{z}(t)$ and $\hat{\boldsymbol{\vartheta}}(t)$ on $[0,\ \infty)$, it follows that the time-derivatives of z_i's are bounded for all $t \geq 0$. This, in turn, implies that $z_i,\ i = 1, \cdots, n$ are uniformly continuous, and further, together with (21) and the well-known Barbalat's Lemma, leads to

$$\lim_{t\to\infty}\boldsymbol{z}(t) = 0 \qquad (22)$$

Let us next show that $[y,\ \hat{\xi}_1,\ \hat{\xi}_2,\ \cdots,\ \hat{\xi}_n]^{\mathrm{T}}$ is asymptotically stable indeed, and then prove that $\boldsymbol{\xi}$, and $\boldsymbol{x}$, is so.

From (22) and $z_1 = y = x_1$, it follows that y, and hence x_1 is asymptotically stable. By the boundedness of ζ and $\hat{\boldsymbol{\vartheta}}$, the smoothness of α_1, the expressions of α_1 and η by (12), and Assumption 1 on $\boldsymbol{\phi}_i$s, we know that $\lim_{y\to 0}\alpha_1(y,\ \zeta,\ \hat{\boldsymbol{\vartheta}}) = 0$. From this together with (22) and $\hat{\xi}_2 = z_2 + \alpha_1(y,\ \zeta,\ \hat{\boldsymbol{\vartheta}})$, we conclude that $\hat{\xi}_2$ is asymptotically stable. Therefore, by $\dot{\hat{\xi}}_1 = \hat{\xi}_2 - k_1\hat{\xi}_1,\ k_1 > 0$, we immediately obtain the asymptotical stability of $\hat{\boldsymbol{\xi}}$. Suppose that $[\hat{\xi}_1,\ \hat{\xi}_2,\ \cdots,\ \hat{\xi}_{k-1}]^{\mathrm{T}}$ is asymptotically stable for any k $(k = 3,\ 4,\ \cdots, n)$. Then, by $\hat{\xi}_k = z_k + \alpha_{k-1}(y,\ \hat{\xi}_1,\ \hat{\xi}_2,\ \cdots,\ \hat{\xi}_{k-1},\ \zeta,\ \hat{\boldsymbol{\vartheta}})$, the boundedness of ζ and $\hat{\boldsymbol{\vartheta}}$, the smoothness and expression of α_{k-1}, Assumption 1 and (22), we know that $\hat{\xi}_k$, and hence, $[y, \hat{\xi}_1, \hat{\xi}_2, \cdots, \hat{\xi}_k]^{\mathrm{T}}$ is asymptotically stable. Therefore, by induction argument, $[y,\ \hat{\xi}_1, \hat{\xi}_2, \cdots, \hat{\xi}_n]^{\mathrm{T}}$ is asymptotically stable. By this together with the boundedness of $\tilde{\boldsymbol{\xi}}$ and $\hat{\boldsymbol{\vartheta}}$ on $[0,\ \infty)$, we obtain the boundedness of the time-derivative of $\tilde{\boldsymbol{\xi}}$. From this together with (21) and Barbalat's Lemma, it follows that $\tilde{\boldsymbol{\xi}}$ is asymptotically stable and so is $\boldsymbol{\xi}$. This together with the relation between $\boldsymbol{\xi}$ and $\boldsymbol{x}$ defined above leads immediately to the global asymptotic stability of $\boldsymbol{x}$. □

Remark 3. Theorem 1 shows that the output-feedback control (19) regulates the closed-loop system state $\boldsymbol{x}$ as well as $\boldsymbol{z}$ to zero, and at the same time, guarantees the boundedness of all the other closed-loop system states.

5 Simulation example

Consider the second-order system:

$$\dot{x}_1 = g_1x_2 + \theta_1y^2,\quad \dot{x}_2 = g_2u + \theta_2y,\quad y = x_1$$

Suppose that $g_1 = 0.2,\ g_2 = 0.4,\ \theta_1 = \theta_2 = 1$.

After the transformation $\boldsymbol{\xi} = [\xi_1,\ \xi_2]^{\mathrm{T}} = G\boldsymbol{x}$, where $G = \mathrm{diag}[g_1^{-1}g_2^{-1},\ g_2^{-1}]$, we have

$$\dot{\xi}_1 = \xi_2 + \Theta_1y^2,\quad \dot{\xi}_2 = u + \Theta_2y \qquad (23)$$

Design the observer for system (23) as follows.

$$\dot{\hat{\xi}}_1 = \hat{\xi}_2 - \hat{\xi}_1,\quad \dot{\hat{\xi}}_2 = u - \hat{\xi}_1$$

By virtue of the design procedure given in Section 4, we can easily obtain the useful signals $\hat{\boldsymbol{\vartheta}}$, ζ and hence the controller u.

With the initial conditions: $x_1(0) = 0$, $x_2(0) = 0.2$, $\hat{\xi}_1(0) = \hat{\xi}_2(0) = \zeta(0) = 0$, $\hat{\vartheta}_1(0) = 10$, $\hat{\vartheta}_2(0) = 0.5$, $\hat{\vartheta}_3(0) = 0.9$, the simulation results are shown in Figs 1∼6. Fig. 1 shows the convergence of the estimate $\hat{\xi}_1$ (dashdotted line) toward the state ξ_1 (solid line), whereas Fig. 2 gives the convergence of the estimate $\hat{\xi}_2$ (dashdotted line) toward the state ξ_2 (solid line). Fig. 3 verifies the boundedness of variable ζ which is introduced to identify the direction of the control gain implicitly. Fig. 4 shows the boundedness of the control signal u. Fig. 5 depicts the boundedness of the parameter estimates $\hat{\vartheta}_1$, $\hat{\vartheta}_2$, and $\hat{\vartheta}_3$. Fig. 6 gives the convergence of states x_1 and x_2 of the original system. The large transient responses were expected because of the lack of knowledge about the systems, and control signals need to scan around before finding the best signal.

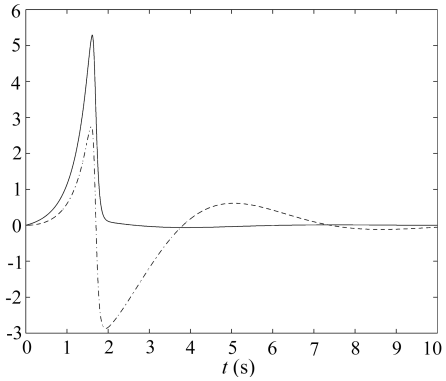


Fig. 1 ξ_1 (solid) and $\hat{\xi}_1$ (dashdotted)

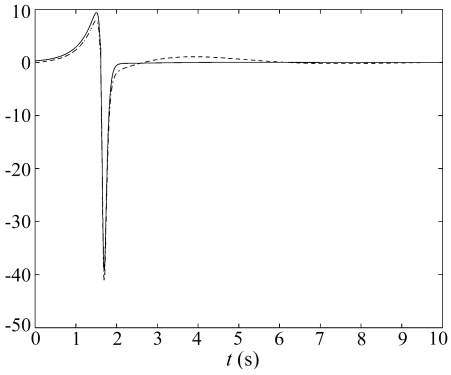


Fig. 2 ξ_2 (solid) and $\hat{\xi}_2$ (dashdotted)

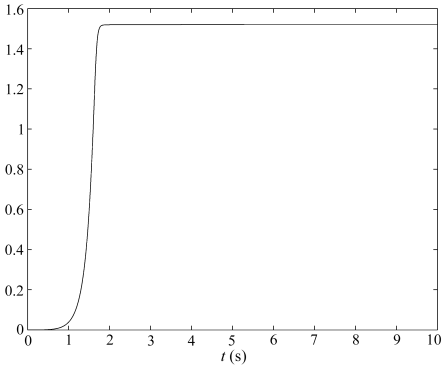


Fig. 3 Variable ζ

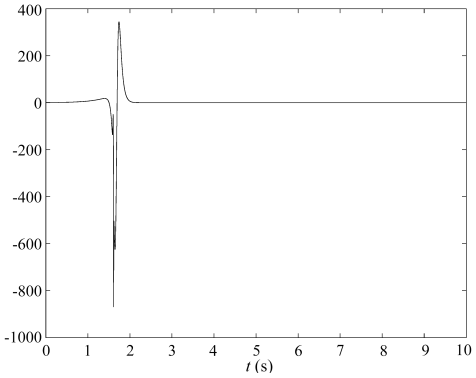
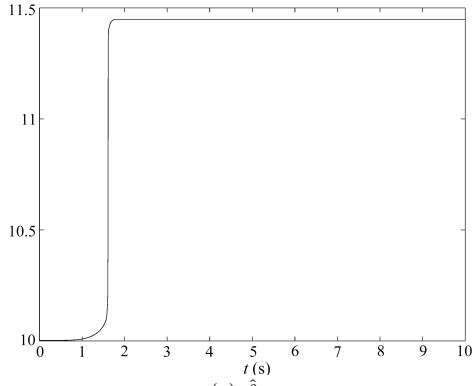
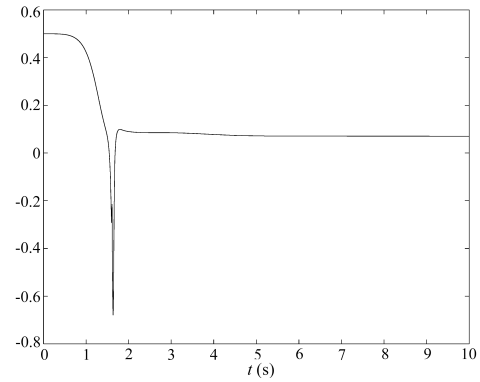


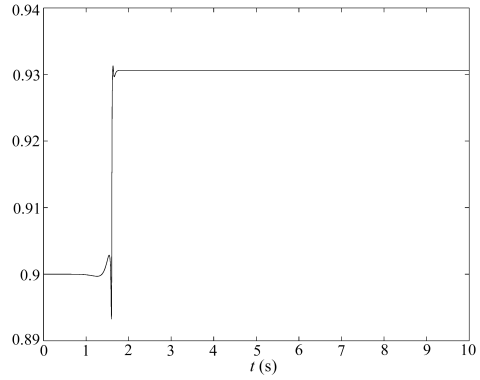
Fig. 4 Control input u



(a) $\hat{\vartheta}_1$



(b) $\hat{\vartheta}_2$



(c) $\hat{\vartheta}_3$

Fig. 5 Parameter estimates $\hat{\vartheta}_1$, $\hat{\vartheta}_2$, and $\hat{\vartheta}_3$

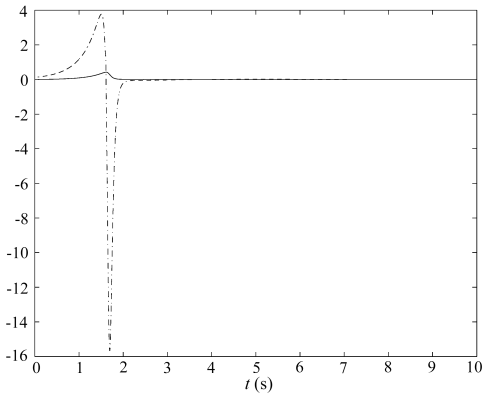


Fig. 6 x_1 (solid) and x_2 (dashdotted)

6 Concluding remarks

In this paper, output-feedback adaptive control design has been investigated for a class of nonlinear systems with unknown control directions. Through a linear state transformation, the unknown control coefficients are lumped together and the original system is transformed to the new system for which control design becomes feasible. Then, the state observer and parameter estimator are introduced, which make output-feedback adaptive control design feasible using integrator backstepping and tuning function techniques. The controller guarantees the boundedness of all the closed-loop system states, and asymptotical stability of the original system state and the observer errors.

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