



关于非线性 Morgan 问题的几点注记

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摘要 对输入数等于输出数加1情形的非线性Morgan问题, Glumineau & Moog曾给出一个构造性解. 这一结果最近被指出是错误的. 文章给出上述结果的一个修正, 并构造出10阶系统, 该系统具有唯一的解耦反馈, 且解耦后的系统比原系统具有更大的本性阶.

关键词 非线性系统, Morgan问题, Singh算法, 齐次阶系统.

1 引言及预备知识

Morgan问题是现代控制理论中的经典问题之一. 这一问题至今没有得到很好地解决. 文献[1]研究了非方系统的非线性Morgan问题, 给出当系统输出数比输入数小1时问题的解. 最近, 夏小华举例指出文献[1]的主要结果(Theorem 6)是错误的. 本文对上述结果加以修正, 并举例表明非线性Morgan问题具有非常复杂的特性.

考虑带输出的仿射非线性系统

$$\begin{cases} \dot{x} = f(x) + \sum_{i=1}^m g_i(x)u_i = f(x) + g(x)u, x \in R^n, u \in R^m, \\ y = h(x), y \in R^p. \end{cases} \quad (1)$$

设在式(1)中, 向量 $f(x)$, $g(x)$, $h(x)$ 的分量是变元 x 的解析函数. 并设 $\text{rank } g(x) = m$, 且系统(1)右可逆.

文献[1]给出了非线性Morgan问题系统的提法, 这里不再赘述. 记系统(1)的(代数)无穷结构(infinite structure)为 $\{n'_1, \dots, n'_p\}$, 相应于输出 y_i 的本性阶(essential order)为 n_{ie} . 详细的定义参见文献[2].

记 $n_e = \max \{n_{ie} : i = 1, \dots, p.\}$, 及 $l_i = n_e - n_{ie}, i = 1, \dots, p.$ 定义系统(1)的齐次阶系统为

$$\dot{\bar{x}} = \begin{bmatrix} O_{l_1-1} \\ h_1 \\ \vdots \\ O_{l_p-1} \\ h_p \\ f \end{bmatrix} + \begin{bmatrix} O_{(\sum_{i=1}^p l_i) \times m} \\ g \end{bmatrix} u, \quad (2a)$$

$$\bar{y}_i = \begin{cases} h_i & l_i = 0, \\ \bar{x}_{\sum_{k=1}^{i-1} t_k + 1}, & l_i > 0. \end{cases} \quad (2b)$$

可以验证, 齐次阶系统(2)的所有本性阶相等(等于 n_e).

对系统(2)实施 Singh 算法(见文献[3])(必要时交换输出变元的顺序), 可得

$$\left\{ \begin{array}{l} \bar{y}_1^{(\bar{r}_1)} = \bar{a}_1(\bar{x}) + \bar{b}_1(\bar{x})u, \\ \vdots \\ \bar{y}_k^{(\bar{r}_k)} = \bar{a}_k(\bar{x}, \bar{y}_j^{(l)}, j < k, \bar{r}_j \leq l \leq \bar{r}_k) + \bar{b}_k(\bar{x}, \bar{y}_j^{(l)}, j < k, \bar{r}_j \leq l \leq \bar{r}_k - 1)u, \\ \vdots \\ \bar{y}_p^{(\bar{r}_p)} = \bar{a}_p(\bar{x}, \bar{y}_j^{(l)}, j < p, \bar{r}_j \leq l \leq \bar{r}_p) + \bar{b}_p(\bar{x}, \bar{y}_j^{(l)}, j < p, \bar{r}_j \leq l \leq \bar{r}_p - 1)u. \end{array} \right. \quad (3)$$

易知在上述表达式右侧中, 不出现 $\bar{y}_i^{(l)}, i=1, \dots, p, l \leq n_e - n_{ie}$, 因而(3)可以重新表述为

$$\left\{ \begin{array}{l} y_1^{(r_1)} = a_1(x) + b_1(x)u, \\ \vdots \\ y_k^{(r_k)} = a_k(x, y_j^{(l)}, j < k, r_j \leq l \leq r_k + n_{je} - n_{ke}) + \\ \quad b_k(x, y_j^{(l)}, j < k, r_j \leq l \leq r_k + n_{je} - n_{ke} - 1)u, \\ \vdots \\ y_p^{(r_p)} = a_p(x, y_j^{(l)}, j < p, r_j \leq l \leq r_p + n_{je} - n_{pe}) + \\ \quad b_p(x, y_j^{(l)}, j < p, r_j \leq l \leq r_p + n_{je} - n_{pe} - 1)u. \end{array} \right. \quad (4)$$

可以看到, 在表达式(4)中, 若 $r_i < n_{ie}$, 则在 $y_i^{(r_i)}$ 式的右侧不出现 $y_j^{(n_{je})}, j=1, \dots, p$. 于是由交互圈(interactor)秩(见文献[4])的定义, 知集 $\{i : r_i < n_{ie}\}$ 的元素个数为 $p - r^*$, 其中 r^* 是系统(1)的交互圈的本性秩.

2 非构造解

在 $m=p+1$ 的情形, 只有当系统交互圈的本性秩 r^* 不小于 $p-1$ 时, 系统才可能解耦. 此时, 对系统实施齐次化的 Singh 算法, 可得(参阅式(4))

$$\left\{ \begin{array}{l} y_1^{(n_{1e})} = a_1(x) + b_1(x)u, \\ \vdots \\ y_{k-1}^{(n_{(k-1)e})} = a_{k-1}(x) + b_{k-1}(x)u, \\ y_k^{(r_k)} = a_k(x) + b_k(x)u, \\ y_{k+1}^{(n_{(k+1)e})} = a_{k+1}(x, y_k^{(j)}, r_k < j \leq n_{ke}) + b_{k+1}(x, y_k^{(j)}, r_k < j \leq n_{ke} - 1)u, \\ \vdots \\ y_p^{(n_{pe})} = a_p(x, y_k^{(j)}, r_p < j \leq n_{ke}) + b_p(x, y_k^{(j)}, r_p < j \leq n_{ke} - 1)u, \end{array} \right. \quad (5)$$

其中 $r_k = \sum_{i=1}^p n_{ie} - \sum_{i=1}^p n'_i$.

设阵 $D(x) : (m-1) \times m$ 使得矩阵 (D^T, b_k^T) 非异, 作正则反馈变换

$$\begin{cases} v_1 = a_k(x) + b_k(x)u, \\ \vdots \\ v_m \end{cases} = D(x)u,$$

为简便, 记在此变换下的系统(1)为

$$\begin{cases} \dot{x} = f(x) + \sum_{i=1}^m g_i(x)v_m = f(x) + g(x)v, \\ y = h(x). \end{cases} \quad (6)$$

定理 1. 考虑右可逆系统(1). 设 $m=p+1$, 则系统(1)可行对行解耦的充分且必要条件是

- (i) $r^* \geq p-1$,
- (ii) 存在一实函数 $\phi(x)$, 和一整数 $\delta \geq \sum_{i=1}^m n_{ie} - \sum_{i=1}^m n'_i$, 使得

$$L_{g^2} L_{f+\phi g_1}^i \phi = 0, \quad i = 1, \dots, \delta - 2,$$

$$d(L_{g^2} L_{f+\phi g_1}^{\delta-1} \phi) \notin \varepsilon_n,$$

这里 $g^2 = (g_2, \dots, g_m)$, $\varepsilon_n = \text{span}\{dx, dy, \dots, dy^{(n)}\}$.

证. 只需考虑 $r^* = p-1$ 的情形.

充分性. 记 $v^2 = (v_2, \dots, v_m)^T$. 令 $v_1 = \phi(x)$, 于是系统(6)化为

$$\dot{x} = f + \phi g_1 + g^2 v^2. \quad (7)$$

由题设(ii)可得

$$\begin{aligned} \phi^{(i)} &= L_{f+\phi g_1}^i \phi, \quad i = 1, \dots, \delta - 1, \\ \phi^{(\delta)} &= L_{f+\phi g_1}^\delta \phi + (L_{g^2} L_{f+\phi g_1}^{\delta-1} \phi) v^2. \end{aligned}$$

注意到式(5), 由(ii)知系统(7)可正则解耦. 于是系统(6)可(非正则)解耦.

必要性. 设在反馈 $v = \alpha(x) + \beta(x)w$ 下, 系统(6)满足

$$y_i^{(l_i)} = w_i, \quad l_i \geq n_{ie}, \quad i = 1, \dots, p.$$

于是 $v_1 = \alpha_1(x)$. 此时系统(6)形为

$$\begin{cases} \dot{x} = f + \sum_{i=1}^m \alpha_i g_i + \sum_{i=2}^m g'_i w_{i-1}, \\ y = h(x). \end{cases} \quad (8)$$

易知 $\text{span}\{g_2, \dots, g_m\} = \text{span}\{g'_2, \dots, g'_m\} \stackrel{\text{def}}{=} G$, 及 $d w_k \notin \varepsilon_n$.

计算

$$\dot{y}_k = L_{f+\sum_{i=1}^m \alpha_i g_i} y_k + \sum_{i=2}^m (L_{g'_i} y_k) w_{i-1},$$

由 $\dot{y}_k = \dot{y}_k(x)$, 知 $L_G y_k = 0$, 由此可得

$$L_{f+\sum_{i=1}^m \alpha_i g_i} y_k = L_{f+\alpha_1 g_1} y_k.$$

类似地, 由 $y_k^{(i)} = y_k^{(i)}(x), i = 2, \dots, l_k - 1$, 可得

$$L_G L_{f+a_1 g_1}^{i-1} y_k = 0, \quad L_{f+\sum_{i=1}^m a_i g_i}^i y_k = L_{f+a_1 g_1}^i y_k, \quad i = 2, \dots, l_k - 1. \quad (9)$$

由 $y_k^{(l_k)} = w_k$ 知,

$$dL_G L_{f+a_1 g_1}^{l_k-1} y_k = \text{span}\{(0, \dots, d w_k, \dots, 0)\}. \quad (10)$$

令 $\delta = l_k - r_k, \phi = a_1$. 注意到

$$y_k^{(r_k)} = L_{f+a_1 g_1}^{r_k} y_k = a_1,$$

由关系式(9,10)容易得到本定理的结论(ii). 证毕.

定理 1 是对文献[1] Theorem 6 的修正. 可以看出, 文献[1] Theorem 6 的 ii) 蕴含定理 1 的条件(ii), 但反之不然. 但对线性系统, 二者是等价的, 因而文献[1]中 Theorem 6 对线性系统仍是成立的(参见文献[5]).

例 1. 考察系统

$$\begin{cases} \dot{x}_1 = u_1, \dot{x}_2 = u_2, \dot{x}_3 = x_4 + u_1, \dot{x}_4 = x_5, \dot{x}_5 = x_6, \dot{x}_6 = u_3, \\ \dot{x}_7 = x_8, \dot{x}_8 = x_9 + x_2 x_7 + x_2 u_1, \dot{x}_9 = x_{10}, \dot{x}_{10} = u_4, \\ y_1 = x_1, y_2 = x_2, y_3 = x_3. \end{cases} \quad (11)$$

可以验证上述系统不满足文献[1] Theorem 6 的题设. 但简单的计算表明, $\delta = 4$, $\phi = x_7$ 是满足定理 1 的唯一的解, 因此, 该系统可解耦. 它的唯一的解耦反馈是

$$u_1 = -x_7, u_2 = w_2, u_3 = x_{10} + w_3, u_4 = -w_1. \quad (12)$$

可以看出, 系统(11)在反馈变换(12)下的闭环系统的本性阶为 $\{5, 1, 4\}$, 大于原系统(11)的本性阶 $\{4, 1, 4\}$.

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SOME NOTES ABOUT NONLINEAR MORGAN'S PROBLEM

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Abstract For nonlinear Morgan's problem with one more inputs than outputs, Glumineau & Moog presented a constructive solution. This result was lately pointed out to be incorrect. In this paper, a modification of the Glumineau-Moog Theorem is given and a 10-dimensional system is also presented. This system has only one decoupling-feedback, and the essential orders of the decoupled closed-loop system are higher than those of the original system.

Key words Nonlinear control systems, morgan's problem, singh's algorithm, shifted systems

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