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A Novel Nonlinear Approximation Approach—Enhanced Hinge-Finding Algorithm[1)]

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Abstract Nonlinear approximation has been widely applied to many fields. As a kind of simple and effective approach, the hinge-finding algorithm has particular advantage. The algorithm uses the hinging hyperplanes model that uses hinging hyperplanes as basis functions in expansion. By means of theoretically analyzing it is shown that the representation capability of the model is deficient. The deficiency causes the model's inability to achieve optimal approximation. In this paper the hinging hyperplanes model is extended and the deficiency is remedied at a two-dimensional space. The extended model has enough representation capability, which theoretically ensures the possibility to achieve optimal approximation. In simulation, with the same number of parameters, the new algorithm gets better approximation precision and less prediction error than those of the hinge-finding algorithm.

Key words Piecewise linear, nonlinear approximation, hinging hyperplanes, hinge-finding algorithm

1 Introduction

In essence, nonlinear approximation can be described as fitting the basis functions expansion to the original nonlinear functions. Generally, the basis functions can be obtained by parameterizing a single mother basis function. The typical mother basis functions include the Gaussian bell, the trigonometric function, the unit interval indicator function, the unit step function and the sigmoid function. These mother basis functions achieve good results respectively in different applications. When taking some piecewise linear (PWL) functions as mother basis function, one has the PWL expansion.

The PWL models have been proven very helpful in the nonlinear circuits analysis, not only from computational point of view but also because they are much more amenable to analysis than general nonlinear equations, and make it easier to get insight into the behavior of these nonlinear systems. Moreover, in contrast with other nonlinear functions, PWL functions are preserved under the inverse and composition operations that are useful in nonlinear circuit analysis.

Recently, some PWL approximation methods have been presented[1,2,3]. Among them the hinge-finding algorithm proposed by Breiman is a kind of simple and practical nonlinear approximation approach. The hinge-finding algorithm uses the hinging hyperplanes model that regards the hinging hyperplanes as the basis functions in expansion[1]. Theoretically, the hinging hyperplane model can serve as a universal approximant of all continuous functions at any accuracy[4]. However, according to theoretical analyzing the representation capability of the model is deficient and this makes it impossible to achieve optimal approximation with the model[5]. In this paper the hinging hyperplanes model is extended and the deficiency of the model is remedied at a two-dimensional space. The extended model's representation capability ensures that it is possible to achieve optimal approximation theoretically. With the same number of parameters, the new algorithm named enhanced hinge-finding algorithm gets better approximation precision and less prediction error than the hinge-finding algorithm in simulation.

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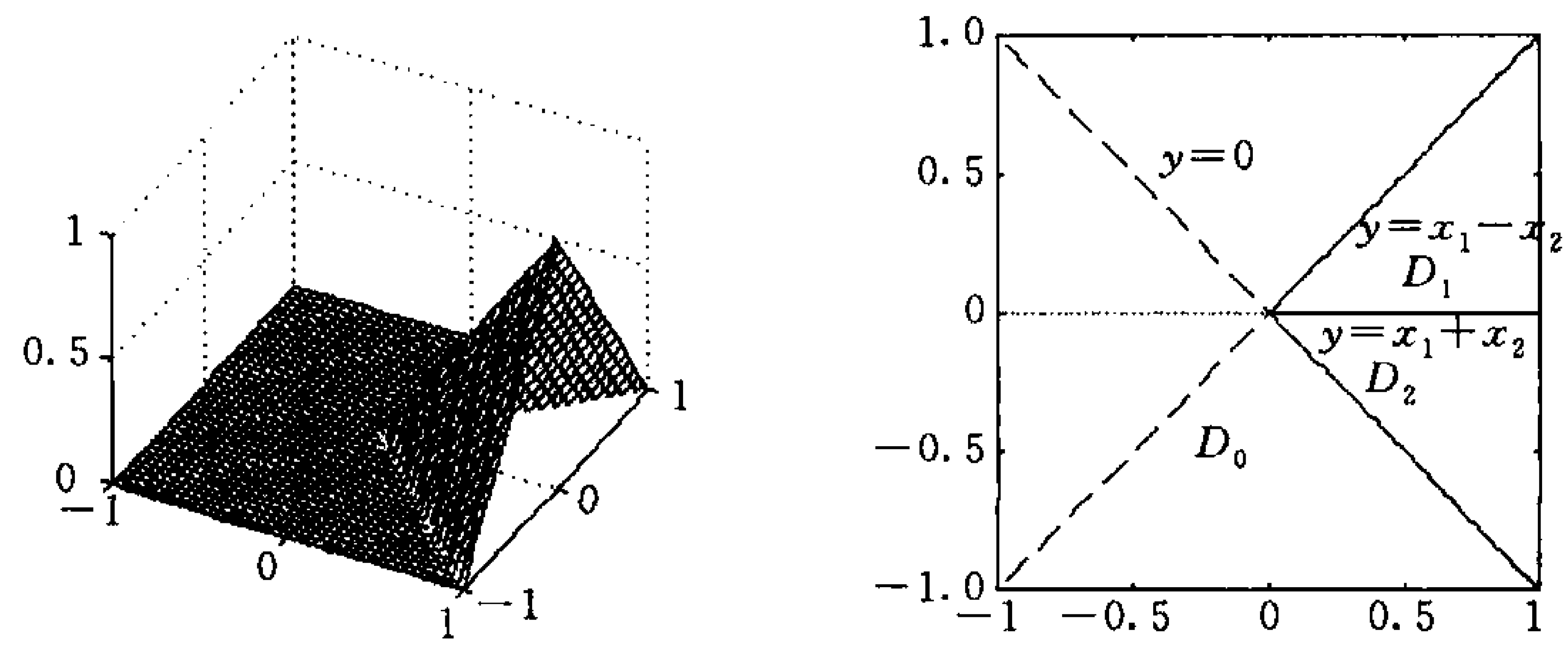
2 Deficiency of the Hinging Hyperplanes Model

The hinging hyperplanes model takes the following three functions as mother basis functions.

$$\boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{x}+\beta,\ \max(\boldsymbol{\xi}_1^{\mathrm{T}}\boldsymbol{x}+\zeta_1, \boldsymbol{\xi}_2^{\mathrm{T}}\boldsymbol{x}+\zeta_2),\ \min(\boldsymbol{\xi}_1^{\mathrm{T}}\boldsymbol{x}+\zeta_1, \boldsymbol{\xi}_2^{\mathrm{T}}\boldsymbol{x}+\zeta_2)$$

where $\boldsymbol{\alpha}, \boldsymbol{\xi}_1, \boldsymbol{\xi}_2 \in R^2$, $\beta, \zeta_1, \zeta_2 \in R$.

Actually, the hinging hyperplanes model is identified with the canonical model proposed by Kang and Chua[6] and it is proven in [5] that the canonical model cannot represent all two-dimensional continuous PWL functions. In fact, the model presented by Breiman only covers a part of two-dimensional continuous PWL functions that possess the consistent variation property and is far from representing all two-dimensional continuous PWL functions. For example, the PWL function shown in Fig. 1(a) cannot be represented by the model. Fig. 1(b) indicates the domain of the function. However, the model given in this paper can represent this function as $y=\frac{1}{2}\{(x_1-|x_2|)+|x_1-|x_2||\}$.



(a) function (b) domain

Fig. 1 The PWL function that cannot be represented by the hinging hyperplanes model

The limitation of the hinging hyperplanes model results in that the hinge-finding algorithm cannot achieve the optimum approximation. The algorithm proposed in this paper is based on the model possessing sufficient representation capability and can realize the optimum approximation theoretically.

3 Enhanced Model

First, it is necessary to review the canonical PWL model proposed by Kang and Chua[6] and to determine the deficiency of the model. The canonical PWL model possesses the following form:

$$f(\boldsymbol{x}) = \boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{x}+\beta+\sum_i(-1)^{\sigma_i}\ |\boldsymbol{\alpha}_i^{\mathrm{T}}\boldsymbol{x}+\beta_i| \tag{1}$$

where $\boldsymbol{\alpha}, \boldsymbol{\alpha}_i \in R^2$, $\beta, \beta_i \in R$, $\sigma_i=0,1$.

Let

$$g^0(\boldsymbol{x}) = \boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{x}+\zeta$$
$$g^1(\boldsymbol{x}) = g_1^0(\boldsymbol{x})+g_2^0(\boldsymbol{x})+(-1)^{\sigma}\ |g_2^0(\boldsymbol{x})|,\ \sigma=0,1$$
$$g^2(\boldsymbol{x}) = g^0(\boldsymbol{x})+g^1(\boldsymbol{x})+(-1)^{\sigma}\ |g^1(\boldsymbol{x})|,\ \sigma=0,1$$

Then (1) can be represented as $f^1(\boldsymbol{x})=g^0(\boldsymbol{x})+\sum_i g_i^1(\boldsymbol{x})$. It is evident that $g^0(\boldsymbol{x})$ is a plane and $g^1(\boldsymbol{x})$ a hinging plane. A sort of $g^2(\boldsymbol{x})$ is shown in Fig. 1. What should be noticed is that there is a nesting of two levels of absolute value in the expression of $g^2(\boldsymbol{x})$.

Definition. Let a 2-D continuous PWL function consist of the following m planes:

$$\begin{aligned} y &= a_{11}x_1+a_{12}x_2+b_1 \\ y &= a_{21}x_1+a_{22}x_2+b_2 \\ &\cdots \\ y &= a_{m1}x_1+a_{m2}x_2+b_m \end{aligned},\quad m\geqslant 2$$

If Equation group (2) has solutions, the continuous PWL function corresponding to

(2) is called a fundamental structure of PWL function and denoted by $y=f^2(\boldsymbol{x})$.

$$\begin{cases} y = a_{11}x_1 + a_{12}x_2 + b_1 \\ y = a_{21}x_1 + a_{22}x_2 + b_2 \\ \quad \cdots \\ y = a_{m1}x_1 + a_{m2}x_2 + b_m \end{cases} \tag{2}$$

Let $$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \cdots & \cdots \\ a_{m1} & a_{m2} \end{bmatrix}, \quad \boldsymbol{e} = \begin{bmatrix} 1 \\ 1 \\ \cdots \\ 1 \end{bmatrix}_{m\times 1}, \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Lemma 1. If $\mathrm{rank}(A,-\boldsymbol{e})=3$, $y=f^2(\boldsymbol{x})=g^0(\boldsymbol{x})+\sum_i g_i^1(\boldsymbol{x})+\sum_j g_j^2(\boldsymbol{x})$.

The proof is in Appendix.

Lemma 2. If $\mathrm{rank}(A,-\boldsymbol{e})=2$, $y=f^2(\boldsymbol{x})=g^0(\boldsymbol{x})+g^1(\boldsymbol{x})$.

The proof is in Appendix.

Lemma 3. If $\mathrm{rank}(A,-\boldsymbol{e})=1$, $y=f^2(\boldsymbol{x})=g^0(\boldsymbol{x})$.

The conclusion is evidently true.

Since any continuous PWL function can be represented as the superposition of some fundamental structures, we arrive at the following conclusion.

Theorem 1. Any 2-D continuous PWL function can be represented as follows:

$$y = \sum_i f_i^2(\boldsymbol{x}) = g^0(\boldsymbol{x}) + \sum_i g_i^1(\boldsymbol{x}) + \sum_j g_j^2(\boldsymbol{x}).$$

The following theorem converts the absolute value representation to a minimum-maximum form to make the comparison with the hinging hyperplanes model easy.

Theorem 2. Any 2-D continuous PWL function can be represented as the superposition of following seven kinds of functions:

$$\boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{x}+\beta,\ \max(\boldsymbol{\xi}_1^{\mathrm{T}}\boldsymbol{x}+\zeta_1,\ \boldsymbol{\xi}_2^{\mathrm{T}}\boldsymbol{x}+\zeta_2),\ \min(\boldsymbol{\xi}_1^{\mathrm{T}}\boldsymbol{x}+\zeta_1,\ \boldsymbol{\xi}_2^{\mathrm{T}}\boldsymbol{x}+\zeta_2),$$
$$\max(\boldsymbol{\alpha}_1^{\mathrm{T}}\boldsymbol{x}+\beta_1,\ \max(\boldsymbol{\alpha}_2^{\mathrm{T}}\boldsymbol{x}+\beta_2,\ \boldsymbol{\alpha}_3^{\mathrm{T}}\boldsymbol{x}+\beta_3)),\ \max(\boldsymbol{\alpha}_1^{\mathrm{T}}\boldsymbol{x}+\beta_1,\ \min(\boldsymbol{\alpha}_2^{\mathrm{T}}\boldsymbol{x}+\beta_2,\ \boldsymbol{\alpha}_3^{\mathrm{T}}\boldsymbol{x}+\beta_3)),$$
$$\min(\boldsymbol{\alpha}_1^{\mathrm{T}}\boldsymbol{x}+\beta_1,\ \min(\boldsymbol{\alpha}_2^{\mathrm{T}}\boldsymbol{x}+\beta_2,\ \boldsymbol{\alpha}_3^{\mathrm{T}}\boldsymbol{x}+\beta_3)),\ \min(\boldsymbol{\alpha}_1^{\mathrm{T}}\boldsymbol{x}+\beta_1,\ \max(\boldsymbol{\alpha}_2^{\mathrm{T}}\boldsymbol{x}+\beta_2,\ \boldsymbol{\alpha}_3^{\mathrm{T}}\boldsymbol{x}+\beta_3)),$$

where $\boldsymbol{\alpha},\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2,\boldsymbol{\alpha}_3,\boldsymbol{\xi}_1,\boldsymbol{\xi}_2\in R^2$, $\beta,\beta_1,\beta_2,\beta_3,\zeta_1,\zeta_2\in R$.

The proof is in Appendix.

Apparently, the hinging hyperplanes model only uses three of the above seven mother basis functions. Therefore the representation capability of the model is not sufficient and it is impossible to obtain optimum approximation with the model though the model can approximate any continuous nonlinear functions at any accuracy.

4 Simulation Experiment

Here, the enhanced hinge-finding algorithm still employs the iterative process used in the hinge-finding algorithm to finding the hinge, updating the hinge and adding the hinge. However, the hinge functions not only include the three kinds of functions in the hinging hyperplanes model but also include the four kinds of functions supplied in Theorem 2. In fact, this algorithm employs all mother basis functions of the two-dimensional continuous PWL functions. In other word, the model used in the algorithm possesses sufficient representation capability. So it is theoretically ensured that the algorithm can achieve optimum approximation. The simulation experiment shows that the new algorithm is superior to the hinge-finding algorithm (with the same number of parameters, the new algorithm gets better approximation precision and less prediction error than the hinge-finding algorithm).

In the simulation experiment the enhanced hinge-finding algorithm and the hinge-finding algorithm are respectively applied to fitting the PWL functions to the sample data of the nonlinear function $y=G(\boldsymbol{x})=e^{-\delta_1\|\boldsymbol{x}-0.25\boldsymbol{e}\|_2^2}-e^{-\delta_2\|\boldsymbol{x}-0.75\boldsymbol{e}\|_2^2}$ in the rectangle area $[0,1]\times[0,1]\subset R^2$, where $\boldsymbol{e}=\begin{bmatrix}1\\1\end{bmatrix}$.

In order to fully examine the approximation effects under the different nonlinear degrees, (δ_1, δ_2) is assigned three values and the corresponding three experiments are respectively fulfilled.

The sample data $x_t (1 \leqslant t \leqslant N)$ are randomly generated in the rectangle area $[0,1] \times [0,1] \subset R^2$. The signal-to-noise and the number of parameters are respectively fixed at 3.28 and 36.

The approximation precision and the prediction error are all calculated by formula $E = \frac{\sum_{t=1}^{N} (\hat{y}_t - y_t)^2}{\sum_{t=1}^{N} (y_t - \bar{y})^2}$, where $\hat{y}_t$ is the approximation result (namely the value of PWL function) of the original nonlinear function at x_t, $\bar{y} = \frac{1}{N} \sum_{t=1}^{N} y_t$.

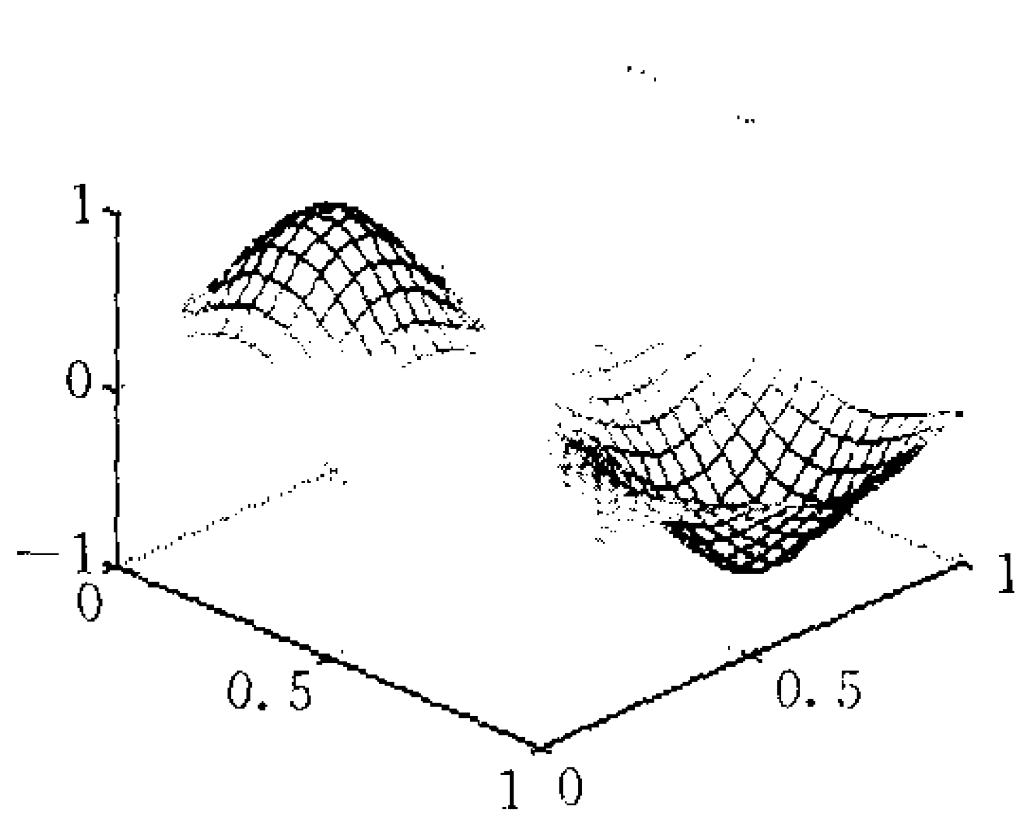
When the approximation error is calculated, let $N = 500$ and $y_t = G(x_t) + e_t$, where e_t is stochastic noise.

When the prediction error is calculated, let $N = 4900$, $y_t = G(x_t)$ and the sample data will be regenerated. The results of simulation experiment are listed in Table 1.

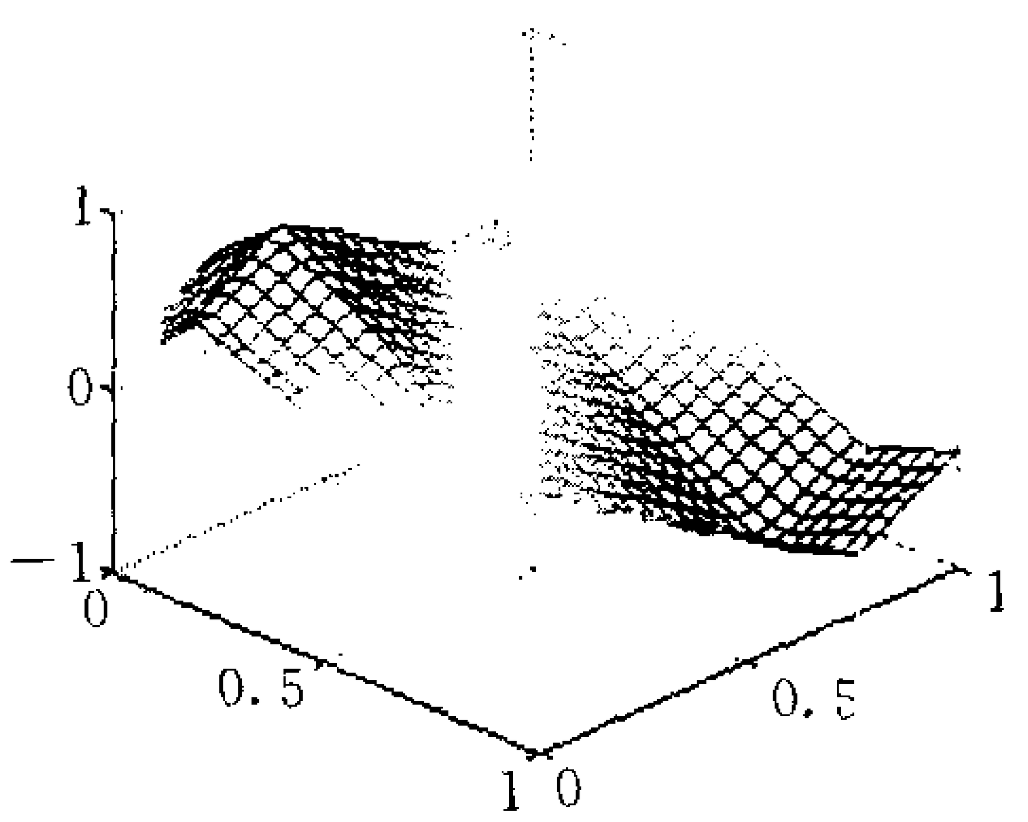
Table 1. Simulation Results of Two Algorithms

δ_1	δ_2	approximation error of new algorithm	approximation error of old algorithm	prediction error of new algorithm	prediction error of old algorithm
16	16	0.0242	0.0787	0.0296	0.0841
9	9	0.0086	0.0636	0.0117	0.0809
4	4	0.0063	0.0134	0.0084	0.0160

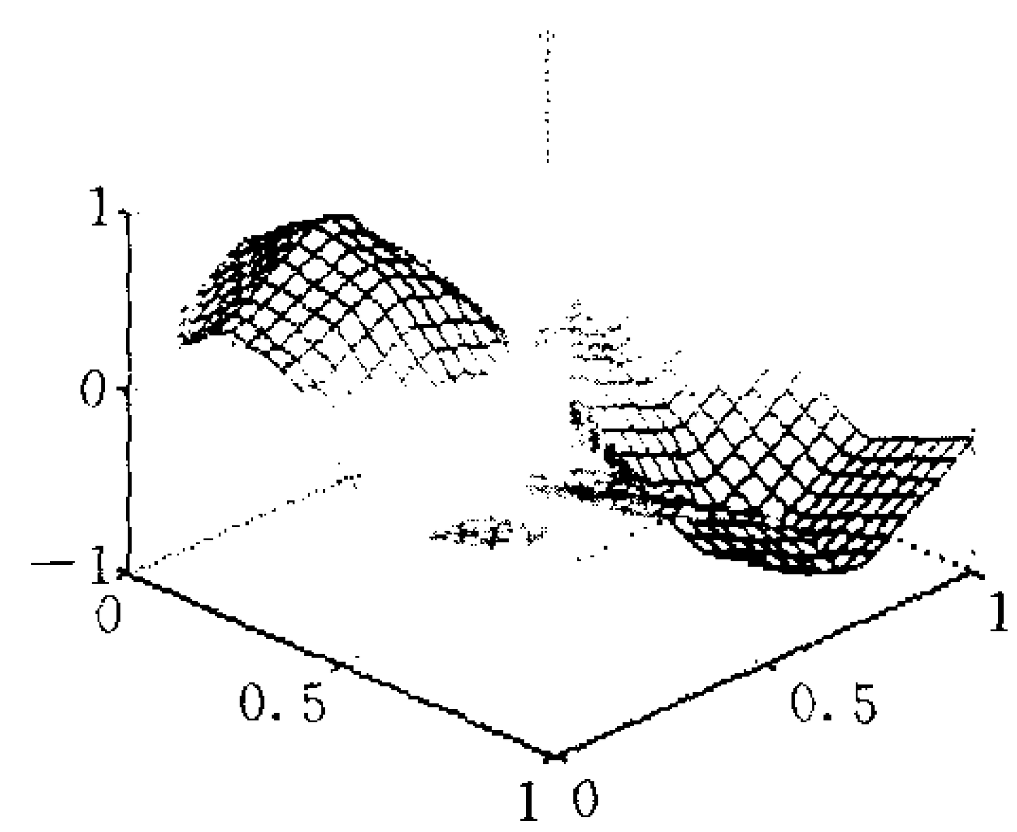
Figs. 2(a), 2(b) and 2(c) respectively show the original function, the result of the hinge-finding algorithm and the result of the enhanced hinge-finding algorithm.



(a) the original function



(b) the result of old algorithm



(c) the result of new algorithm

Fig. 2 The comparison among the original function and the results of both algorithms

5 Conclusion

As a kind of simple and effective nonlinear approximation method, the hinge-finding algorithm has particular advantage. However, The limitation of the hinging hyperplanes model results in that the hinge-finding algorithm can only achieve the optimum approximation on one-dimensional space. In order to obtain desirable approximation precision on high-dimensional space it is inevitable to increase the pieces on the PWL function, namely, to increase the parameters. In this paper the hinging hyperplanes model is improved on two-dimensional space and the new PWL approximation algorithm——enhanced hinge-finding algorithm——is constructed by means of the model improved. In the theoretical analysis and the simulation experiment it is shown that with the same number of parameters, the new algorithm gets better approximation precision and less prediction error than the hinge-finding algorithm presented by Breiman. Meantime, the improvement contributes to the idea of constructing the PWL approximation algorithm on high-dimensional space.

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Appendix

Proof of Lemma 1. Here, the PWL function represented by (2) consists of m ($m\geqslant 3$) planes that intersect at one point.

If x_1 is regarded as a constant, according to the conclusion in Kang and Chua[1], we have

$$g_{x_1}^0(x_2)=\alpha(x_1)x_2+\beta(x_1)=\alpha(x_1)x_2+g^0(x_1)+\sum_l g_l^1(x_1)=$$
$$\alpha(x_1)x_2+\xi x_1+\zeta+\sum_l(\varphi_l x_1+\psi_l+(-1)^{\sigma_l}\mid\varphi_l x_1+\psi_l\mid),\quad \sigma_l=0,1$$

Because the function is piecewise linear,

$$g_{x_1}^0(x_2)=[\xi,\alpha]\begin{bmatrix}x_1\\x_2\end{bmatrix}+\zeta+\sum_l\left([\varphi_l,0]\begin{bmatrix}x_1\\x_2\end{bmatrix}+\psi_l+(-1)^{\sigma_l}\left|[\varphi_l,0]\begin{bmatrix}x_1\\x_2\end{bmatrix}+\psi_l\right|\right),\quad \sigma_l=0,1$$

Since the PWL function consists of m planes intersecting at one point, all the lines that partition the domain have one common point.

So $\sum_l(-1)^{\sigma_l}\left|[\varphi_l,0]\begin{bmatrix}x_1\\x_2\end{bmatrix}+\psi_l\right|=\sigma\left|[1,0]\begin{bmatrix}x_1\\x_2\end{bmatrix}+\tau\right|$, $\sigma_l=0,1$, $\sigma=0,1$. Otherwise, some lines that partition the domain will be parallel. This is inconsistent with that all the lines partitioning the domain have one common point. Thus

$$g_{x_1}^0(x_2)=g_1^0(\boldsymbol{x})+g_2^0(\boldsymbol{x})+(-1)^\sigma\mid g_2^0(\boldsymbol{x})\mid=g^1(\boldsymbol{x}),\ \sigma=0,1$$
$$g_{x_1}^0(x_2,x_3,\cdots,x_n)=g_1^0(\boldsymbol{x})+g_2^0(\boldsymbol{x})+(-1)^\sigma\mid g_2^0(\boldsymbol{x})\mid=g^1(\boldsymbol{x})$$
$$g_{x_1}^1(x_2)=g_{x_1,1}^0(x_2)+g_{x_1,2}^0(x_2)+(-1)^\sigma\mid g_{x_1,2}^0(x_2)\mid=g_1^1(\boldsymbol{x})+g_2^1(\boldsymbol{x})\pm\mid g_2^1(\boldsymbol{x})\mid=g^2(\boldsymbol{x}),\quad \sigma=0,1$$

According to the conclusion in Kang and Chua[1] again we have

$$y=f^2(\boldsymbol{x})=f_{x_1}^2(x_2)=g_{x_1}^0(x_2)+\sum_j g_{x_1,j}^1(x_2)=g^1(\boldsymbol{x})+\sum_j g_j^2(\boldsymbol{x})=g^0(\boldsymbol{x})+\sum_i g_i^1(\boldsymbol{x})+\sum_j g_j^2(\boldsymbol{x})$$

Therefore the conclusion is right. □

Proof of Lemma 2. Because only fundamental structure (Equation group (2) has solutions and the function is monodrome one) will be considered, it is inevitable that $m=2$ and rank$(A)=2$.

Here A is a nonsingular matrix. Thus $A\boldsymbol{x}=\boldsymbol{e}$ has a unique solution $\tilde{\boldsymbol{p}}_1$.

Let $\|\tilde{\boldsymbol{p}}_1\|_2=\frac{1}{\delta}$ and $\boldsymbol{p}_1=\tilde{\boldsymbol{p}}_1\delta$.

We expend $\boldsymbol{p}_1$ to an orthogonal matrix $P=[\boldsymbol{p}_1,\boldsymbol{p}_2]$ (only by taking one standard orthogonal solutions of $\boldsymbol{p}_1^{\mathrm{T}}\boldsymbol{x}=0$ as $\boldsymbol{p}_2$).

Then an orthogonal transformation $\boldsymbol{x}=P\boldsymbol{\zeta}$ is constructed, where $\boldsymbol{\zeta}=\begin{bmatrix}\zeta_1\\\zeta_2\end{bmatrix}$.

So $A\boldsymbol{x}=AP\boldsymbol{\zeta}=[\boldsymbol{e}\delta,\boldsymbol{C}]\boldsymbol{\zeta}$, where $\boldsymbol{C}=\begin{bmatrix}c_{12}\\c_{22}\end{bmatrix}$.

Thus the two planes in (2) can be represented as follows:

$$\begin{cases}y=\zeta_1\delta+c_{12}\zeta_2+b_1\\y=\zeta_1\delta+c_{22}\zeta_2+b_2\end{cases}\tag{A1}$$

If we use orthogonal transformation $\begin{bmatrix}y\\\zeta_1\end{bmatrix}=\begin{bmatrix}\frac{1}{\sqrt{1+\delta^2}} & \frac{\delta}{\sqrt{1+\delta^2}}\\-\frac{\delta}{\sqrt{1+\delta^2}} & \frac{1}{\sqrt{1+\delta^2}}\end{bmatrix}\begin{bmatrix}y'\\\zeta_1'\end{bmatrix}$ again, (A1) can be represented as follows:

$$\begin{cases} y' = \dfrac{1}{\sqrt{1+\delta^2}}(c_{12}\zeta_2 + b_1) \\ y' = \dfrac{1}{\sqrt{1+\delta^2}}(c_{22}\zeta_2 + b_2) \end{cases} \tag{A2}$$

Considering that all transformations used are orthogonal transformations, the function represented by (A2) is identical with that represented by (2).

Now (A2) can be denoted by $y'=h(\zeta_2)$ and we have $y'=h(\zeta_2)=g^0(\zeta_2)+g^1(\zeta_2)$.

So $y=\zeta_1\delta+\sqrt{1+\delta^2}g^0(\zeta_2)+\sqrt{1+\delta^2}g^1(\zeta_2)$.

Applying $\boldsymbol{\zeta}=P^{\mathrm{T}}\boldsymbol{x}$, the inverse transformation of $\boldsymbol{x}=P\boldsymbol{\zeta}$ to this function, we have the conclusion.

$$y = f^2(\boldsymbol{x}) = \delta \boldsymbol{p}_1^{\mathrm{T}}\boldsymbol{x} + \sqrt{1+\delta^2}g^0(\boldsymbol{p}_2^{\mathrm{T}}\boldsymbol{x}) + \sqrt{1+\delta^2}g^1(\boldsymbol{p}_2^{\mathrm{T}}\boldsymbol{x}) = g^0(\boldsymbol{x}) + g^1(\boldsymbol{x}). \qquad \square$$

Proof of Theorem 2. Note that $\max(a,b)=\dfrac{a+b+|a-b|}{2}$ and $\min(a,b)=\dfrac{a+b-|a-b|}{2}$. It is evident that $g^1(\boldsymbol{x})=g_1^0(\boldsymbol{x})+g_2^0(\boldsymbol{x})+(-1)^\sigma|g_2^0(\boldsymbol{x})|(\sigma=0,1)$ corresponds to the following two minimum-maximum functions:

$\max\left(\dfrac{g_1^0(\boldsymbol{x})+2g_2^0(\boldsymbol{x})}{2},\dfrac{g_1^0(\boldsymbol{x})}{2}\right)$ and $\min\left(\dfrac{g_1^0(\boldsymbol{x})+2g_2^0(\boldsymbol{x})}{2},\dfrac{g_1^0(\boldsymbol{x})}{2}\right)$.

Since the linear combination of functions of the form of $g^0(\boldsymbol{x})$ is still a function of the form of $g^0(\boldsymbol{x})$, the two minimum-maximum functions mentioned above can be reduced as $\max(g_1^0(\boldsymbol{x}), g_2^0(\boldsymbol{x}))$ and $\min(g_1^0(\boldsymbol{x}),g_2^0(\boldsymbol{x}))$.

Similarly, $g^2(\boldsymbol{x})=g^0(\boldsymbol{x})+g^1(\boldsymbol{x})+(-1)^\sigma|g^1(\boldsymbol{x})|(\sigma=0,1)$ corresponds to the two minimum-maximum functions $\max(g^1(\boldsymbol{x}),g^0(\boldsymbol{x}))$ and $\min(g^1(\boldsymbol{x}),g^0(\boldsymbol{x}))$.

Thus the proposed theorem is correct. □

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"增强找链接"算法——一种新的非线性逼近方法

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摘　要　非线性逼近在许多方面有着广泛的应用."找链接"算法作为一种简洁有效的非线性逼近方法有其独特的优势,该算法使用的"链接超平面"模型,也就是以"链接超平面"作为基函数.理论分析表明使用这种基函数的模型表示能力不足,这使"找链接"算法不可能达到最佳逼近.本文在二维空间上弥补了"链接超平面"模型的局限性,经扩充后的模型在二维空间上有充分的表示能力,从理论上保证了扩充后的模型可以达到最佳逼近.仿真实验表明,在参数个数相同的情况下,使用新模型的逼近算法在逼近精度与预测误差两方面都优于使用原模型.

关键词　分片线性,非线性逼近,链接超平面,找链接

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