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Output Regulation of Singular Nonlinear Systems Via Output Dynamic Feedback[1)]

ZHU Jian-Dong CHENG Zhao-Lin
(*Department of Mathematics, Shandong University, Jinan* 250100)
(E-mail: zhu@math.sdu.edu.cn)

Abstract The output regulation problem for singular nonlinear systems is considered. Without the assumption of normalizability, an output dynamic feedback reduced-order normal controller is designed. Necessary and sufficient conditions for the output regulation problem to be solvable are derived.

Key words Singular nonlinear systems, output regulation, output dynamic feedback

1 Introduction

Singular systems are more general than normal state space systems, which are also called descriptor systems, generalized systems, or differential-algebraic systems. There are many practical systems modeled by singular systems such as electrical net[1], constrained robots[2] and so on. The singular linear system theory has been mature and widely used in practice . For singular nonlinear system theory, there have been many papers[2~4], which are paid much attention. But there are still a lot of open problems for singular nonlinear system theory.

The output regulation problem is an important problem, which is used to track reference outputs and reject a class of disturbances. It is well-known that the output regulation problem of linear systems can be solved by regulator equations. In 1990, famous scholars Isidori and Byrnes extended the above result to nonlinear systems[5]. Under some conditions, the output regulation problem of nonlinear systems can be solved by nonlinear regulator equations. For singular linear system theory, a singular form of regulator equations is given by [6]. Inspired mainly by [5,6], the authors of [3] researched the output regulation of singular nonlinear systems and gave some excellent results, such as generalized version of the centre manifold theorem and regulator equations. Since it is difficult to realize singular controller physically, it is desirable to design normal controllers. Under the assumption of normalizability, in [3] a full-order normal controller is designed. But in fact, normalizability may not be satisfied. It is told in [6] that the assumption of normalizability can be removed for singular linear systems. Inspired by [3,6], we try to remove the assumption of normalizabilty and design reduced-order normal controllers in this paper. Applying the results of this paper to singular linear systems, we may get yet a new controller different from that in [6]. The way to remove the assumption of normalizability in [6] is the system decomposition proposed by [1], which is dependent on the assumption of regularity. In this paper, we do not assume the singular linear system is regular. So even for singular linear systems, this paper is also meaningful.

This paper is organized as follows. In Section 2, the output regulation problem is stated and basic assumptions are given. In Section 3, some basic lemmas and the main result are given. Section 4 is the conclusion.

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Notations. Throughout the paper, the superscript "T" stands for matrix transposition, R^n denotes the n-dimensional Euclidean space, $R^{n\times m}$ is the set of $n\times m$ real matrices, $\sigma(\cdot)$ denotes the set composed by all the eigenvalues of a matrix, $\sigma(E, A)$ denotes the set composed by all the finite eigenvalues of a matrix pencil $\lambda E-A$, C^- denotes $\{\lambda \mid \mathrm{Re}\lambda<0\}$, and C^+ denotes $\{\lambda \mid \mathrm{Re}\lambda\geqslant 0\}$.

2 Statement of problem and basic assumptions

The complete statement of output regulation problem is given in [3]. In this paper, we focus on designing reduced-order normal controllers to solve output regulation problem by output feedback. So the problem in this paper is stated as the following.

Consider the singular nonlinear system

$$\begin{aligned} E\dot{x}(t) &= f(x(t), u(t), w(t)), \quad Ex(0) = x_0 \\ e(t) &= h(x(t), w(t)), \quad t\geqslant 0 \end{aligned} \tag{1}$$

and an exosystem $\dot{w}(t)=s(w(t))$, $w(0)=w_0$, where f, h, s are smooth mappings; $x\in R^n$ is the vector of state variables; $u\in R^m$ is the vector of input variables; $e\in R^p$ is the vector of output variables; $w\in R^q$ is the disturbance signal; $E\in R^{n\times n}$ is a singular constant matrix; $\mathrm{rank}E=r<n$, $f(0,0,0)=\mathbf{0}$, $h(0,0)=\mathbf{0}$. We try to design the kind of reduced-order normal controller: $\dot{z}(t)=g(z(t), e(t))$, $u(t)=\beta(z(t), e(t))$, where $z\in R^k$, $k<n+q$, $g(0,0)=\mathbf{0}$, $\beta(0,0)=\mathbf{0}$, such that the closed-loop system

$$\begin{aligned} E_c\dot{x}_c(t) &= f_c(x_c(t), w(t)), \; E_cx_c(0) = x_{c0} \\ \dot{w}(t) &= s(w(t)), \; w(0) = w_0 \\ e(t) &= h_c(x_c(t), w(t)) \end{aligned} \tag{2}$$

where $E_c = \mathrm{blockdiag}\{E, I_k\}$, $f_c(x_c, w) = [f^{\mathrm{T}}(x,\beta(z,h(x,w)),w) \quad g^{\mathrm{T}}(z,h(x,w))]^{\mathrm{T}}$, $h_c(x_c,w)=h(x,w)$, $x_c=[x^{\mathrm{T}} \quad z^{\mathrm{T}}]^{\mathrm{T}}$, has the following properties

R1). The singular system $E_c\dot{x}_c(t)=A_cx_c$ is strongly stable, i. e., (E_c, A_c) is regular, impulse-free and $\sigma(E_c, A_c)\subset C^-$ (see [1]), where $E_c\dot{x}_c(t)=A_cx_c$ is the linearization of the system $E_c\dot{x}_c(t)=f_c(x_c,\mathbf{0})$.

R2). The trajectory $(x_c(t), w(t))$ starting from every sufficiently small initial value (x_{c0}, w_0) satisfies $\lim\limits_{t\to\infty}e(t)=\lim\limits_{t\to\infty}h_c(x_c(t), w(t))=\mathbf{0}$.

We denote the linearization of (1) and the exosystem by

$$\begin{aligned} E\dot{x} &= Ax + Bu + Pw \\ e &= Cx + Qw \\ \dot{w} &= Sw \end{aligned} \tag{3}$$

where E, $A\in R^{n\times n}$, $B\in R^{n\times m}$, $P\in R^{n\times q}$, $C\in R^{p\times n}$, $Q\in R^{p\times q}$, $S\in R^{q\times q}$.

In order to introduce the basic hypotheses, we give some concepts.

Definition 1. The singular linear system (E, A, B) is impulse controllable if there is a matrix K such that $(E, A+BK)$ is regular and impulse-free.

Definition 2. The singular linear system (E, A, B) is R-stabilizable if

$$\mathrm{rank}[\lambda E - A \quad B]= n, \; \forall \lambda \in C^+.$$

Definition 3. The singular linear system (E, A, C) is impulse observable if there is a matrix G such that $(E, A+GC)$ is regular and impulse-free.

Definition 4. The singular linear system (E, A, C) is R-detectable if

$$\mathrm{rank}[\lambda E^{\mathrm{T}} - A^{\mathrm{T}} \quad C^{\mathrm{T}}]= n, \; \forall \lambda \in C^+.$$

Remark 1. The above definitions do not require that (E, A) be regular, i. e., it is not needed to assume that there exists a complex number s_0 such that $\det(s_0E-A)\neq 0$. From [1], it is easy to see that the above definitons are equivalent to those in [1] when (E, A) is regular. Therefore, we may continue to use the concepts of "impulse controllable" and "impulse observable" for non-regular systems. But the concept of "impulse model" for non-regular systems does not exist.

Basic hypotheses.

H1). $w=0$ is a stable equilibrium of the exosystem, and there exists a neighborhood W of the origin of R^q with the property: each initial value $w_0 \in W$ is stable in the sense of Poisson.

H2). (E, A, B) is strongly stabilizable, i. e., impulse controllable and R-stabilizable.

H3). $\left(\begin{bmatrix} E & 0 \\ 0 & I_q \end{bmatrix}, \begin{bmatrix} A & P \\ 0 & S \end{bmatrix}, [C \quad Q]\right)$ is strongly detectable, i. e., impulse observable and R-detectable.

Remark 2. H1)~H3) are the same as those in [3]. H1) is a standard assumption introduced by Isidori and Byrnes[5]. H2), H3) are made to ensure the fulfilment of R1).

Remark 3. Another assumption H4), i. e., (E, B) is normalizable, is given in [3] in order to design normal controllers. In this paper, it is removed.

3 Main results

In [3], a main lemma is given which is derived from the generalized vision of the centre manifold theorem. The lemma is a tool to deal with the output regulation problem. Now, we state it as follows.

Lemma 1[3]. Assume H1) holds, and suppose there exists a control law (state or output feedback) such that the closed-loop system (2) satisfies R1). Then the closed-loop system also satisfies R2) if and only if there exists a sufficiently smooth function $\boldsymbol{\chi}_c(\boldsymbol{w})$, $\boldsymbol{\chi}_c(\mathbf{0})=\mathbf{0}$, locally defined in a neighborhood W of the origin such that

$$E_c \frac{\partial \boldsymbol{\chi}_c(\boldsymbol{w})}{\partial \boldsymbol{w}} \boldsymbol{s}(\boldsymbol{w}) = \boldsymbol{f}_c(\boldsymbol{\chi}_c(\boldsymbol{w}), \boldsymbol{w}) \tag{4}$$

$$\mathbf{0} = \boldsymbol{h}_c(\boldsymbol{\chi}_c(\boldsymbol{w}), \boldsymbol{w}) \tag{5}$$

Since rank $E=r<n$, there exist nonsingular matrices M, $N \in R^{n\times n}$ such that $MEN=$ blockdiag$\{I_r, 0\}$. Let

$$\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix} = N^{-1}\boldsymbol{x}, \quad MAN = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad M[B \quad P] = \begin{bmatrix} B_1 & P_1 \\ B_2 & P_2 \end{bmatrix}$$

$$CN = [C_1 \quad C_2], \quad \begin{bmatrix} \breve{\boldsymbol{f}}_1(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{u}, \boldsymbol{w}) \\ \breve{\boldsymbol{f}}_2(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{u}, \boldsymbol{w}) \end{bmatrix} = M\boldsymbol{f}\left(N\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix}, \boldsymbol{u}, \boldsymbol{w}\right)$$

Then system (1) is restricted system equivalent (r. s. e.) to

$$\dot{\boldsymbol{x}}_1 = \breve{\boldsymbol{f}}_1(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{u}, \boldsymbol{w}) = A_{11}\boldsymbol{x}_1 + A_{12}\boldsymbol{x}_2 + B_1\boldsymbol{u} + P_1\boldsymbol{w} + \boldsymbol{f}_1(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{u}, \boldsymbol{w}) \tag{6}$$

$$\mathbf{0} = \breve{\boldsymbol{f}}_2(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{u}, \boldsymbol{w}) = A_{21}\boldsymbol{x}_1 + A_{22}\boldsymbol{x}_2 + B_2\boldsymbol{u} + P_2\boldsymbol{w} + \boldsymbol{f}_2(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{u}, \boldsymbol{w}) \tag{7}$$

$$\boldsymbol{e} = \breve{\boldsymbol{h}}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{w}) = C_1\boldsymbol{x}_1 + C_2\boldsymbol{x}_2 + Q\boldsymbol{w} + \boldsymbol{h}_1(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{w}) \tag{8}$$

where $\boldsymbol{f}_1$, $\boldsymbol{f}_2$, $\boldsymbol{h}_1$ are the nonlinear parts. From H2) H3), we know that (E,A,B,C) is impulse controllable and impulse observable, i. e., $[A_{22} \quad B_2]$ and $[A_{22}^{\mathrm{T}} \quad C_2^{\mathrm{T}}]$ both have full row rank (see [1]). By Lemma 3. 5. 1 in [1], there exists a matrix K such that $A_{22}+B_2KC_2$ is nonsingular. Let $\boldsymbol{u}=K\boldsymbol{e}+\boldsymbol{v}$, then the system is transformed to

$$\dot{\boldsymbol{x}}_1 = \bar{A}_{11}\boldsymbol{x}_1 + \bar{A}_{12}\boldsymbol{x}_2 + B_1\boldsymbol{v} + \bar{P}_1\boldsymbol{w} + \bar{\boldsymbol{f}}_1(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{v}, \boldsymbol{w}) \tag{9}$$

$$\mathbf{0} = \bar{A}_{21}\boldsymbol{x}_1 + \bar{A}_{22}\boldsymbol{x}_2 + B_2\boldsymbol{v} + \bar{P}_2\boldsymbol{w} + \bar{\boldsymbol{f}}_2(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{v}, \boldsymbol{w}) \tag{10}$$

$$\boldsymbol{e} = C_1\boldsymbol{x}_1 + C_2\boldsymbol{x}_2 + Q\boldsymbol{w} + \boldsymbol{h}_1(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{w}) \tag{11}$$

where $\bar{\boldsymbol{f}}_i(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{v}, \boldsymbol{w}) = \boldsymbol{f}_i(\boldsymbol{x}_1, \boldsymbol{x}_2, K\breve{\boldsymbol{h}}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{w}) + \boldsymbol{v}, \boldsymbol{w}) + B_iK\boldsymbol{h}_1(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{w})$, $\bar{A}_{ij} = A_{ij} + B_iKC_j$, $\bar{P}_i = P_i + B_iKQ$, $\bar{A}_{22}$ is nonsingular, i, $j=1,2$. It is easily seen that the Jacobi matrix of $\bar{\boldsymbol{f}}_i$ at the origin is zero matrix. Since $\bar{A}_{22}$ is nonsingular, by the Implicit Function Theorem, (10) determines a unique smooth function

$$x_2 = \alpha(x_1, v, w), \quad \alpha(0, 0, 0) = 0 \tag{12}$$

defined in a neighborhood of the origin of $R^n \times R^m \times R^q$. Substituting (12) into (10) gives

$$\bar{A}_{21} x_1 + \bar{A}_{22}\alpha(x_1, v, w) + B_2 v + \bar{P}_2 w + \bar{f}_2(x_1, \alpha(x_1, v, w), v, w) \equiv 0 \tag{13}$$

i. e.,

$$\breve{f}_2(x_1, \alpha(x_1, v, w), K\breve{h}(x_1, \alpha(x_1, v, w), w) + v, w) \equiv 0 \tag{14}$$

From (13), it is easy to see

$$\left.\frac{\partial \alpha(x_1, v, w)}{\partial(x_1, v, w)}\right|_{x_1=0, v=0, w=0} = -\left[\bar{A}_{22}^{-1}\bar{A}_{21} \quad \bar{A}_{22}^{-1}B_2 \quad \bar{A}_{22}^{-1}\bar{P}_2\right] \tag{15}$$

Substituting (12) into (9) (11) leads to a new system

$$\dot{x}_1 = \hat{f}_1(x_1, v, w) = \tilde{A}_{11} x_1 + \tilde{B}_1 v + \tilde{P}_1 w + \tilde{f}_1(x_1, v, w) \tag{16}$$

$$e = \hat{h}_1(x_1, v, w) = \tilde{C}_1 x_1 + \tilde{D} v + \tilde{Q} w + \tilde{h}_1(x_1, v, w) \tag{17}$$

where $\tilde{A}_{11} = \bar{A}_{11} - \bar{A}_{12}\bar{A}_{22}^{-1}\bar{A}_{21}$, $\tilde{B}_1 = B_1 - \bar{A}_{12}\bar{A}_{22}^{-1}B_2$, $\tilde{P} = \bar{P}_1 - \bar{A}_{12}\bar{A}_{22}^{-1}\bar{P}_2$, $\tilde{C}_1 = C_1 - C_2\bar{A}_{22}^{-1}\bar{A}_{21}$, $\tilde{D} = -C_2\bar{A}_{22}^{-1}B_2$, $\tilde{Q} = Q - C_2\bar{A}_{22}^{-1}\bar{P}_2$, and

$$\tilde{f}_1(x_1, v, w) = \bar{f}_1(x_1, \alpha(x_1, v, w), v, w) + \bar{A}_{12}\alpha(x_1, v, w) + \bar{A}_{12}\bar{A}_{22}^{-1}\bar{A}_{21}x_1 + \bar{A}_{12}\bar{A}_{22}^{-1}B_2 v + \bar{A}_{12}\bar{A}_{22}^{-1}\bar{P}_2 w \tag{18}$$

$$\tilde{h}_1(x_1, v, w) = h_1(x_1, \alpha(x_1, v, w), v, w) + C_2\alpha(x_1, v, w) + C_2\bar{A}_{22}^{-1}\bar{A}_{21}x_1 + C_2\bar{A}_{22}^{-1}B_2 v + C_2\bar{A}_{22}^{-1}\bar{P}_2 w \tag{19}$$

$$\hat{f}_1(x_1, v, w) = \breve{f}_1(x_1, \alpha(x_1, v, w), K\breve{h}(x_1, \alpha(x_1, v, w), w) + v, w) \tag{20}$$

$$\hat{h}_1(x_1, v, w) = \breve{h}(x_1, \alpha(x_1, v, w), w) \tag{21}$$

We can see that system (16,17) is a standard state space system. Now, we design output feedback controller to regulate system (16,17). From [5], we know that if $(\tilde{A}_{11}, \tilde{B}_1)$ is stabilizable and $\left(\begin{bmatrix}\tilde{A}_{11} & \tilde{P}_1 \\ 0 & S\end{bmatrix}, \ [\tilde{C}_1 \quad \tilde{Q}]\right)$ is detectable, then the output regulation problem is solvable by output feedback if and only if there exist sufficiently smooth functions $\pi(w)$, $\pi(0)=0$, and $\upsilon(w), \upsilon(0)=0$, both defined in a neighbourhood of the origin of R^q, such that

$$\frac{\partial \pi(w)}{\partial w}s(w) = \hat{f}_1(\pi(w), \upsilon(w), w) \tag{22}$$

$$0 = \hat{h}_1(\pi(w), \upsilon(w), w) \tag{23}$$

Before designing the controller for (16,17), we give two lemmas:

Lemma 2. $(\tilde{A}_{11}, \quad \tilde{B}_1)$ is stabilizable if and only if (E, A, B) is R-stabilizable.

Lemma 3. $\left(\begin{bmatrix}\tilde{A}_{11} & \tilde{P}_1 \\ 0 & S\end{bmatrix}, \ [\tilde{C}_1 \quad \tilde{Q}]\right)$ is detectable if and only if

$$\left(\begin{bmatrix}E & 0 \\ 0 & I_q\end{bmatrix}, \ \begin{bmatrix}A & P \\ 0 & S\end{bmatrix}, \ [C \quad Q]\right)$$

is R-detectable.

By H2), H3) and Lemmas 2, 3, there exist $H \in R^{m\times r}$, $G_1 \in R^{r\times p}$, $G_2 \in R^{q\times p}$ such that

$$\sigma(\tilde{A}_{11} + \tilde{B}_1 H) \subset C^-, \ \sigma\left(\begin{bmatrix}\tilde{A}_{11} & \tilde{P}_1 \\ 0 & S\end{bmatrix} - \begin{bmatrix}G_1 \\ G_2\end{bmatrix}[\tilde{C}_1 \quad \tilde{Q}]\right) \subset C^- \tag{24}$$

By [5], the output feedback controller for system (16,17) is

$$\dot{z}_1 = \hat{f}_1(z_1, \upsilon(z_2) + H(z_1 - \pi(z_2)), z_2) - G_1[\hat{h}_1(z_1, \upsilon(z_2) + H(z_1 - \pi(z_2)), z_2) - e] \tag{25}$$

$$\dot{z}_2 = s(z_2) - G_2[\hat{h}_1(z_1, \upsilon(z_2) + H(z_1 - \pi(z_2)), z_2) - e] \tag{26}$$

$$v = \upsilon(z_2) + H(z_1 - \pi(z_2)) \tag{27}$$

In the following, we will prove that the output dynamic feedback controller $u = \upsilon(z_2) + H(z_1 - \pi(z_2)) + Ke$ can solve the output regulation problem for system (1). Since system (1) is r. s. e. to (6)～(8), without loss of generality, we consider the closed-loop system composed of (6)～(8)

$$\dot{x}_1 = \breve{f}_1(x_1, x_2, v(z_2) + H(z_1 - \pi(z_2)) + K\breve{h}(x_1, x_2, w), w) \tag{28}$$

$$0 = \breve{f}_2(x_1, x_2, v(z_2) + H(z_1 - \pi(z_2)) + K\breve{h}(x_1, x_2, w), w) \tag{29}$$

$$\dot{z}_1 = \hat{f}_1(z_1, v(z_2) + H(z_1 - \pi(z_2)), z_2) - G_1[\hat{h}_1(z_1, v(z_2) + H(z_1 - \pi(z_2)), z_2) - \breve{h}(x_1, x_2, w)] \tag{30}$$

$$\dot{z}_2 = s(z_2) - G_2[\hat{h}_1(z_1, v(z_2) + H(z_1 - \pi(z_2)), z_2) - \breve{h}(x_1, x_2, w)] \tag{31}$$

$$e = \breve{h}(x_1, x_2, w) \tag{32}$$

Let the linearization of the above closed-loop be $E_c\dot{x}_c = A_c x_c$, where $E_c = \text{blockdiag}\{I_r, 0, I_r, I_q\}$,

$$A_c = \begin{bmatrix} A_{11} + B_1KC_1 & A_{12} + B_1KC_2 & B_1H & B_1F \\ A_{21} + B_2KC_1 & A_{22} + B_2KC_2 & B_2H & B_2F \\ G_1C_1 & G_1C_2 & \hat{A}_{11} & \hat{P}_1 \\ G_2C_1 & G_2C_2 & -G_2(\tilde{C}_1 + \tilde{D}H) & S - G_2(\tilde{Q} + \tilde{D}F) \end{bmatrix}$$

$$F = \left.\frac{\partial v(z_2)}{\partial z_2}\right|_{z_2=0} - H\left.\frac{\partial \pi(z_2)}{\partial z_2}\right|_{z_2=0}$$

$$\hat{A}_{11} = \tilde{A}_{11} + \tilde{B}_1H - G_1(\tilde{C}_1 + \tilde{D}H),\ \hat{P}_1 = \tilde{P}_1 + \tilde{B}_1F - G_1(\tilde{Q} + \tilde{D}F)$$

To show (E_c, A_c) satisfies R1), we give the following lemma.

Lemma 4. (E_c, A_c) is regular, impulse-free and satisfies

$$\sigma(E_c, A_c) = \sigma(\tilde{A}_{11} + \tilde{B}_1H) \cup \sigma\left(\begin{bmatrix} \tilde{A}_{11} & \tilde{P}_1 \\ 0 & S \end{bmatrix} - \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}[\tilde{C}_1 \quad \tilde{Q}]\right) \tag{33}$$

Base on the above discussion, we give a main result.

Theorem 1. Assume H1)~H3) hold and suppose that there exist sufficiently smooth functions $\pi(w)$, $\pi(0)=0$, and $v(w)$, $v(0)=0$, both defined in a neighborhood of the origin of R^q, such that (22)(23) hold. Then the output regulation problem for system (1) is solvable and the controller is a reduced-order normal controller with $r+q$ dimensions.

Proof. Without loss of generality, we prove for system (6)~(8). From (24) and Lemma 4, we know (E_c, A_c) is strongly stable, i. e., closed-loop (28)~(32) satisfies R1). By Lemma 1, (28)~(32) also satisfies R2) if and only if there exists a sufficiently smooth function $\chi_c(w)$, $\chi_c(0)=0$, defined in a neighbourhood of the origin of R^q, such that (4)(5) hold. Now, we construct $\chi_c(w)$ using $\pi(w)$ and $v(w)$. Let

$$\chi_c(w) = [\pi^T(w) \quad \alpha^T(\pi(w), v(w), w) \quad \pi^T(w) \quad w^T]^T$$

It is easy to check that $\chi_c(w)$ satisfies (4)(5). □

Lemma 5. There exist sufficiently smooth functions $\pi(w)$, $\pi(0)=0$, and $v(w)$, $v(0)=0$, both defined in a neighborhood of the origin of R^q satisfying (22)~(23) if and only if there exist sufficiently smooth functions $\chi(w)$, $\chi(0)=0$, and $\mu(w)$, $\mu(0)=0$, both defined in a neighborhood of the origin of R^q satisfying

$$E\frac{\partial \chi(w)}{\partial w}s(w) = f(\chi(w), \mu(w), w) \tag{34}$$

$$0 = h(\chi(w), w) \tag{35}$$

Remark 4. In [3], the authors proved it necessary for the solvability of the regulation problem that there exist sufficiently smooth functions $\chi(w)$, $\chi(0)=0$, and $\mu(w)$, $\mu(0)=0$, both defined in a neighbourhood of the origin of R^q satisfying (34)(35).

From Theorem 1 and Lemma 5, we get the following theorem.

Theorem 2. Under Hypotheses H1), H2) and H3), the output regulation problem via a reduced-order normal controller is solvable if and only if there exist sufficiently smooth functions $\chi(w)$, $\chi(0)=0$, and $\mu(w)$, $\mu(0)=0$, both defined in a neighbourhood of the origin of R^q, such that (34)(35) hold.

Remark 5. Theorem 2 improves Theorem 4 in [3]. We have removed the assumption of normalizability and the controller can be designed as reduced-order controller.

Applying Theorem 2 to the output regulation problem of singular linear systems, we can derive Theorem 4.6 in [6]. Furthermore, we can give a new reduced-order controller as the form

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix}=\begin{bmatrix}\tilde{A}_{11} & \tilde{B}_1(\Gamma-H\Pi)+\tilde{P}_1-G_1(\tilde{Q}+\tilde{D}(\Gamma-H\Pi))\\ -G_2(\tilde{C}_1+\tilde{D}H) & S-G_2(\tilde{Q}+\tilde{D}(\Gamma-H\Pi))\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix}+\begin{bmatrix}G_1\\ G_2\end{bmatrix}e$$

$$u=\begin{bmatrix}H & \Gamma-H\Pi\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix}+Ke$$

where matrices Π, Γ are the solutions of the regulator equations

$$\Pi S=\tilde{A}_{11}\Pi+\tilde{B}_1\Gamma+\tilde{P}_1$$
$$0=\tilde{C}_1\Pi+\tilde{D}\Gamma+\tilde{Q}$$

Remark 6. Without assumption of normalizability, a reduced-order controller is given by [6] with the help of an appropriate system decomposition. But the system decomposition used in [6] is dependent on the assumption of regularity. In our paper, we do not assume the singular linear system is regular. So even for singular linear systems, this paper is also meaningful.

4 Conclusion

In this paper, the output regulation problem of singular nonlinear systems via output dynamic feedback is researched. The reduced-order normal controller is given. The assumption of normalizability is removed and the necessary and sufficient conditions are given. By applying the result to singular linear systems, a new reduced-order controller is given.

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ZHU Jian-Dong Received his Master's and Ph. D. degrees from Shandong University in 1999 and 2002. His research interests include linear singular systems, nonlinear singular systems and robust control theory.

CHENG Zhao-Lin Professor of Shandong University. His research interests include singular systems, nonlinear control and robust control theory.

经输出动态反馈的非线性奇异系统输出调节问题

朱建栋　　程兆林

（山东大学数学与系统科学院　济南　250100）

（E-mail: zhu@math.sdu.edu.cn）

摘　要　讨论非线性奇异系统的输出调节问题，在没有可正常化的假设条件下，给出了降阶的正常状态空间的输出动态反馈控制器的设计，并导出了这种控制器存在的充分必要条件.

关键词　非线性奇异系统，输出调节，输出动态反馈

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