MODELING AND ANALYSIS OF CAPACITATED TRANSFER LINES WITH UNRELIABLE MACHINES AND DETERMINISTIC PROCESSING TIMES*

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Abstract This paper presents an efficient method of equivalient workstations for modeling and analysis of multistage transfer lines with unreliable machines and finites buffers. The deterministic processing times for discrete parts and random failuse and repair times for machines are assumed. These buffers lead to blockage and starvation in operation due to limited storage capacities and make the problem of modelling and analysis very difficult to treat because they have large state spaces and cannot be decomposed exactly. A single buffer between two reliable workstations is analysed first. Then an equivalent workstation without starvation and blockage is constructed. Thereafter connecting all the equivalent workstations in series we get the equivalent transfer line. A set of preformance measures such as the production rates, efficiencies and average inventory levels are derived in explicit analytical expressions. Finally two numerical examples are given for comparing these calculated results with those of S. B. Gershwin (1987)[3] and C. R. Glasseye & Y. Hong (1993)[5] and illustrating the application of the method in engineering design directly.

Key words Intelligent manufacturing system (IMS), Computer integrated manufacturing System (CIMS), Inventory levels and throughput in transfer lines, Reliability of transfer lines.

1 INTRODUCTION

The unreliable multistage transfer line with finite buffers is a special large system, which is subject to interruptions in operation due to finite buffers and random failure and repair times. The performance characteristic of such a system is very difficult to solve analytically because it is with large state spaces and can not decompose exactly.

The literature shows that a lot of good work about this problems has been done by some researchers with different conditions in different ways. As the character of processing times is concerned the related literature can be divided into two major classes. The first consists of pepers on manufacturing system with deterministic processing times, but allow random failures. Such as [1 to 5], in which [2,3] were solved by discrete approach and [1,4,5] by continuous approach. The second class of papers includes those with random processing times. Moreover the failure and repair times are also random. Ref. [6 to 10] belong to this class, in which only [6] was solved by discrete approach and others by continuous method.

As far as the authors are aware the exact results have been obtained only in a few papers for a very limited stage lines, such as [1] for two stage production line and [2] for three stage line. For more than three stages only the approximate approaches and simulation methods have been studied.

The research work in this paper falls into the first class, an equivalent workstation method is proposed for analysizing the n-stage transfer line with deterministic processing times. A finite intermediate buffer between two reliable workstations is solved first by Markov process into $K+1$ states, but only two states (unfull and unempty) are used for constructing the equialent workstation. Then connent all the equivalent workstations in series to form an equivalent transfer line. This new model can be treated as a continuous operating production system, thus to remove the interrupting operations from the production system. Finally, two examples are given for illustrating the application of this new method in engineering analysis and design and compare the results with Ref. [3,4,5].

2 MODEL DESCRIPTION AND ASSUMPTIONS

Consider a n-stage transfer line as shown in Fig.1.

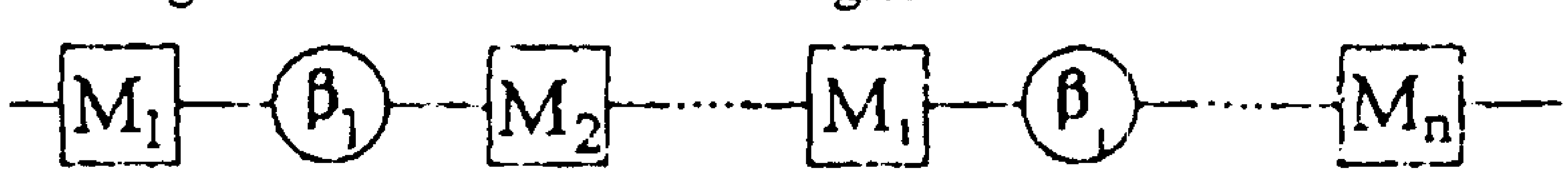

Fig.1 n-stage transfer line

Here M_i indicates the ith machine and β_i indicate ith buffer, $i=1,2,\cdots,n$.

Most assumptions listed below for formulating the mathematical model are standard in literatures and realistic in some practical problems.

1) The processing times for a discrete part at the individual workstations are deterministic. Let ω_i denote the rated production rate of ith machine in pieces per time unit. Transportation takes negligible time compared to machining times.

2) Any failured machines are repaired without delay. The life times and repair times are distributed exponentially with parameters λ_i and μ_i respectively.

3) If the buffer β_{i-1} is empty and M_{i-1} (or another before it) in failure condition (under repair), then M_i is starved; if M_{i-1} is running but $\omega_i > \omega_{i-1}$, then M_i has to slow down to rate ω_{i-1}. If the buffer β_i is full and M_{i+1} (or another after it) in failure condition (under repair), then M_i is blocked, if M_{i+1} is running but $\omega_{i+1} < \omega_i$, then M_i has to slow down to rate ω_{i+1}.

4) The first workstation is never starved and the last workstation is never blocked.

5) Machines fail only while processing parts. That means the machine in starvation or blockage state, it can not fall.

3 ANALYSIS OF INTERMEDIATE BUFFERS

Consider an intermediate buffer as an isolated system between two reliable workstations as shown in Fig.2

For random processing times, failure times and repair times, the buffer states have been analyzed by Markov processes [9,10] and the steady state results obtained.

The probability for the jth state of buffer

$$P_j = \frac{\rho_i(1-\rho_i)}{1-\rho_{k_i+1}} \qquad (\sum_{j=1}^{k_i} \rho_j = 1) \tag{1}$$

where $\rho_i = \omega_i / \omega_{i+1}, k_i$ = capacity of the buffer storage (including one unit is workstation) obviously, when $\rho_i = 1$,

$$P_i = \frac{1}{k_i + 1} \tag{1a}$$

For constructing the equivalent workstation in the next section, we need only the following two states

$$\begin{cases} \overline{P}_{ki} = 1 - P_{ki} = \dfrac{1-\rho_1^{ki}}{1-\rho_1^{ki+1}} & \text{(unfull)} \\ \overline{P}_{oi} = 1 - P_{oi} = \dfrac{\rho_i(1-\rho_1^{ki})}{1-\rho_1^{ki+1}} & \text{(unempty)} \end{cases} \tag{2}$$

Then the average inventory level in buffer.

$$M_{ki} = \sum_{j=1}^{k_i} jP_j = \frac{\rho_i - (k_i+1)\rho_i^{ki+1} + k_i\rho_i^{ki+2}}{1-\rho_i-\rho_i^{ki+1}+\rho_1^{ki+2}} \tag{3}$$

Although the above three equations are based upon the random processing rate ω_i but they are still good for deterministic processing rate ω_i, at $\rho_i = \omega_i / \omega_{i+1} = 1$, where $\overline{P}_{0i} = \dfrac{k_i}{k_i+1} = \overline{P}_{ki}$, and

$$M_{ki} = \frac{k_1}{2}$$

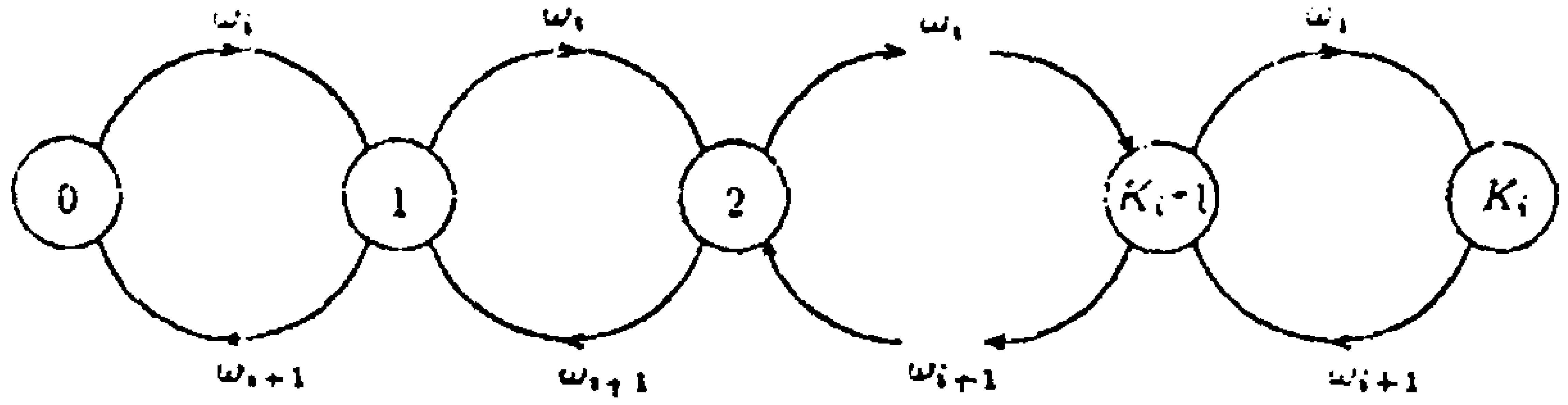

Fig.2 Diagram of buffer state transition

However, in the case of deterministic processing rate, if $\omega_i < \omega_{i+1}$, then $P_{oi} = 1(\overline{P}_{oi} = 0)$, $P_{ki} = 0(\overline{P}_{ki} = 1)$ and $M_{ki} = 0$. If $\omega_i > \omega_{i+1}$, then $P_{oi} = 0(\overline{P}_{oi} = 1)$, $P_{ki} = 1(\overline{P}_{ki} = 0)$, and $M_{ki} = k_i$.

4 EQUIVALENT WORKSTATION

The equivalent workstation is defined as an isolated machine without either starvation nor blockage and free from failures.

Let P_{ji} represent the probability of the jth state for ith workstation in the transffer line, where $j = 1$ to 5 represent. The state probability of normal working blockage starvation both blockage and starvation and repair respectively. Then probability flow diagram of states for ith workstation is shown as Fig.3.

According to definitions of blockage (Pb) and starvation (Ps) as given in assumption (3), we have

$$\begin{cases} P_{bi} = P(\overline{P}_{o(i-1)}, P_{ki}, P_{a(i+1)}) = \overline{P}_{o(i-1)} P_{ki} \overline{P}_{a(i+1)} \\ P_{Si} = P(P_{o(i-1)}, \overline{P}_{ki}, \overline{P}_{a(i-1)}) = P_{o(i-1)} \overline{P}_{ki} \overline{P}_{a(i-1)} \end{cases} \tag{4}$$

$$\begin{cases} \text{where } P_{ai} = \dfrac{\mu_i}{\mu_i + \lambda_i} = A_i, \text{availability of isolated } M_i \\ \text{and define } B_i = \overline{P}_{bi} \overline{P}_{ai}, \text{availability of buffers related to } M_i \end{cases} \tag{4a}$$

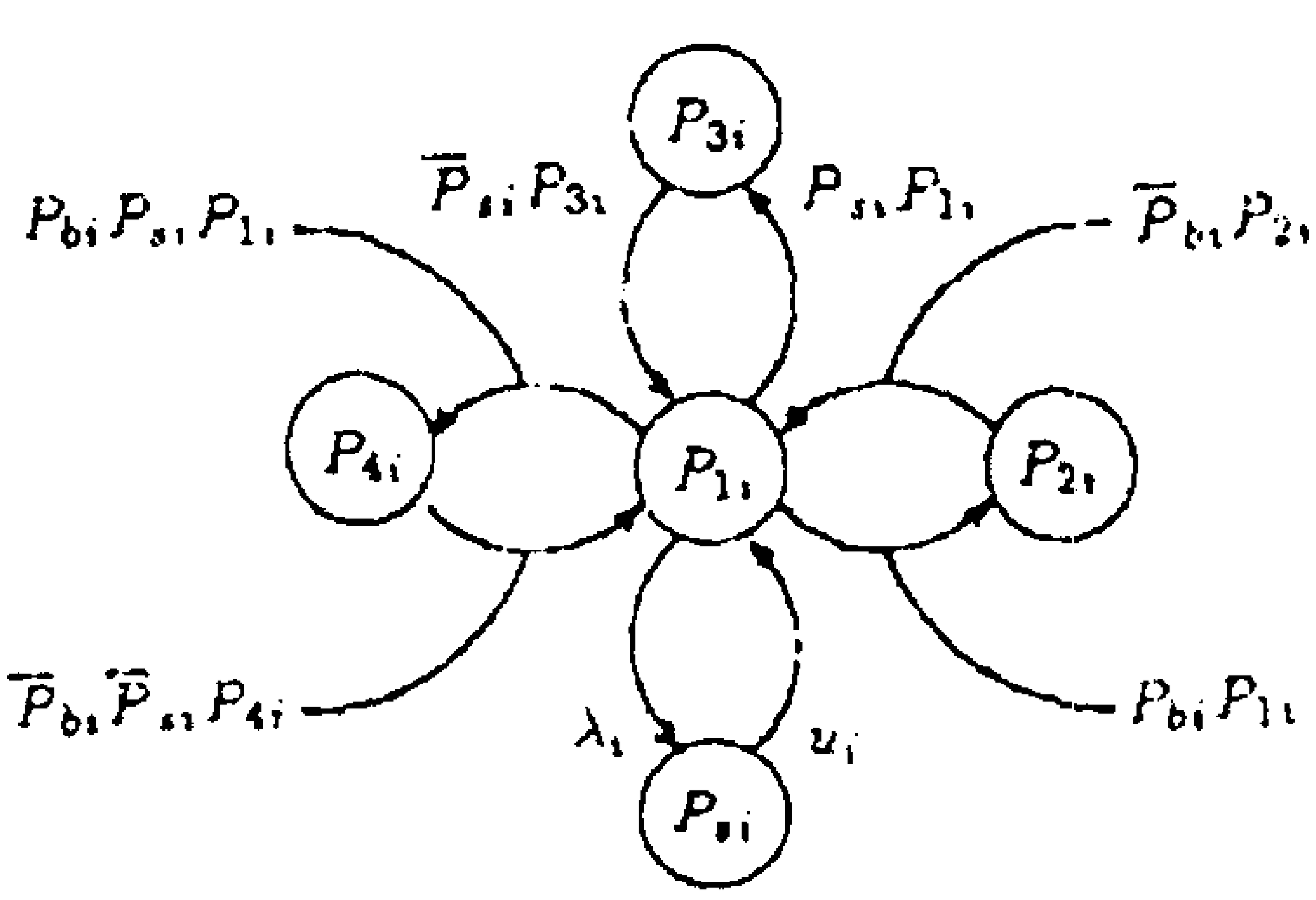

Fig.3 State transtion diagram of machine M_i

Then

$$\begin{cases} \dot{P}_{1i} = \overline{P}_{bi} P_{2i} + \overline{P}_{Si} P_{ai} + \overline{P}_{bi} \overline{P}_{Si} P_{4i} + \mu_i P_{5i} (P_{bi} + P_{Si} + P_{bi} P_{Si} + a_i) P_{1i} \\ \dot{P}_{2i} = P_{bi} P_{1i} - \overline{P}_{bi} P_{2i} \\ \overline{P}_{3i} = P_{Si} P_{1i} - \overline{P}_{Si} P_{2i} \\ \overline{P}_{4i} = P_{bi} P_{Si} P_{1i} - \overline{P}_{bi} \overline{P}_{Si} P_{4i} \\ \overline{P}_{5i} = \lambda_i P_{1i} - \mu_i P_{si} \end{cases} \tag{5}$$

For the steady state. the left sides of (5) should be equal to zero. Then solve the equations simultaneously, we get

$$P_{2i}=\frac{P_{bi}}{P_{bi}}P_{1i},\quad P_{3i}=\frac{P_{Si}}{P_{Si}}P_{1i},\quad P_{4i}=\frac{P_{bi}P_{Si}}{B_i}P_{1i},\quad P_{5i}=\frac{\lambda_i}{\mu_i}P_{1i}$$

$$\sum_{j=1}^{5}P_{ji}=1=(1+\frac{P_{bi}}{\overline{P}_{bi}}+\frac{P_{Si}}{\overline{P}_{Si}}+\frac{P_{b1}P_{Si}}{B_i}+\frac{\lambda_i}{\mu_i})P_{1i}=(\frac{1}{B_i}+\frac{\lambda_i}{\mu_i})P_{1i}$$

$$\begin{cases} P_{1i}=\dfrac{1}{1/B_i+\lambda_i/\mu_i}=\dfrac{\mu_iB_i}{\mu_i+\lambda_iB_i}=A_i'=E_i \text{ probability of normal working,} \\ \text{avaiability (efficiency) of equivalent workstation} \\ P_{2i}=\dfrac{P_{bi}}{\overline{P}_{bi}}A_i' \text{ (probability of blockage for } M_i) \\ P_{3i}=\dfrac{P_{si}}{\overline{P}_{si}}A_i' \text{ (probability of starvation for } M_i) \\ P_{4i}=\dfrac{P_{bi}P_{Si}}{B_i}A_i' \text{ (probability of both blockage and starvation for } M_i) \\ P_{5i}=\dfrac{\lambda_i}{\mu_i}A_i'=\dfrac{\lambda_iB_i}{\mu_i+\lambda_iB_i} \text{ (probability of repair for } M_i) \end{cases} \tag{6}$$

The above five expressions in (6) are also the time probortions of respective steady state. The idle time proportion due to blockage and starvation of M_i is

$$P_{bSi}=P_{2i}+P_{3i}+P_{4i}=\frac{\mu_i(1-B_i)}{\mu_i+\lambda_iB_i} \tag{6a}$$

The steady state production rate (throughput) of M_i,

$$W_i=P_{1i}\omega_i=\frac{\omega_i\mu_iB_i}{\mu_i+\lambda_iB_i},\quad i=1,2,\cdots,n \tag{7}$$

5 EQUIVALENT TRANSFER LINE

Connecting all the equivalent workstation in series we get an equivalent multistage transfer line as shown in Fig.4.

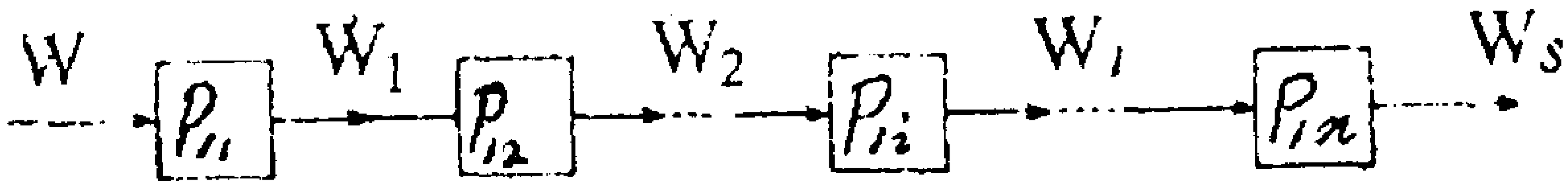

Fig.4 Equivalent transfer line

From the principle of conservation of processing pieces flow in the transfer line, the manufacturing rate for each workstation (7) should be equal to system production rate, namely

$$\begin{cases} W_s=W_i=\dfrac{\omega_i\mu_iB_i}{\mu_i+\lambda_iB_i}=\omega_i/(\dfrac{1}{B_i}+\dfrac{\lambda_i}{\mu_i})=A_i'\omega_i \\ \text{or } W_s=P_{1i}\omega_i=A_i'\omega=E_i\omega_i,\quad i=1,2,\cdots,n \end{cases} \tag{8}$$

The above equation is also the necessary and sufficient condition for multistage repairable transfer line without any creation or missing of processing piece along the line.

The efficiency of the system is defined as the probability of production at full rate, or the ratio of actual processing time at all the individual workstations to average system production time.

$$E_s = \sum_{i=1}^{n} \frac{1}{\omega_i} \Big/ \sum_{i=1}^{n} \frac{1}{W_i} = \frac{W_s}{n} \sum \frac{1}{\omega_i} = \frac{1}{n} \sum_{i=1}^{n} E_i \tag{9}$$

The above equation may be used as the objective function for optimal design.

If equation (8) can not hold, then the system production rate should be the bottleneck throughput,

$$W_s = \min W_i = \min_i \frac{\omega_i u_i B_i}{u_i + \lambda_i B_i} \tag{10}$$

In the homogeneous transfer line (e.i: $\rho = \rho_i = \omega_i / \omega_{i+1} = 1$, and let $k_i = k_{i+1} = k$, $\lambda_i = \lambda_{i+1} = \lambda$, $\mu_i = \mu_{i+1} = \mu$), we have

$$\begin{cases} P_{bi} = \overline{P}_{0(i-1)} P_{ki} \overline{P}_{a(i+1)} = \dfrac{k}{k+1} \times \dfrac{1}{k+1} \times \dfrac{\lambda}{\mu+\lambda} = \dfrac{K}{(k+1)^2} \times \dfrac{\lambda}{\mu+\lambda} \\ P_{Si} = P_{0(i-1)} \overline{P}_{ki} \overline{P}_{a(i-1)} = \dfrac{1}{k+1} \times \dfrac{k}{k+1} \times \dfrac{\lambda}{\mu+\lambda} = \dfrac{K}{(k+1)^2} \times \dfrac{\lambda}{\mu+\lambda} \\ B_i = \overline{P}_{Si} \overline{P}_{Si} = (1 - P_{bi})(1 - P_{Si}) = [1 - \dfrac{K}{(k+1)^2} \times \dfrac{\lambda}{\mu+\lambda}]^2 \end{cases} \tag{4b}$$

Then substituting B_i in (8) and (9) we get W_S and E_S.

6 AVERAGE INVENTORY LEVELS IN THE INTERMEDIATE BUFFERS OF MULTISTAGE TRANSFER LINE

Eqn.(3) represents the average inventory level of a buffer between two reliable workstations with random processing times. In the multistage transfer lines, we can determine the average steady state inventory levels for each buffer as follows:

Table 1 Procedeure for calculating the average inventory levels

weight	Condition	Inventory levels	Remarks
A_i'	no failure	$A_i' \times k_i / 2, i = 1,2,\cdots,n-1$	Eqn.(3)
A_1'	1st failue	all buffers empty	
A_2'	2nd failure	$\overline{A}_2' k_1$, all others empty	
A_3'	3rd failure	$\overline{A}_3' k_1, \overline{A}_3' k_2$	
⋮	⋮	⋮	
A_{n-1}'	(n-1)th failure	$\overline{A}_{n-1}' k_1, \overline{A}_{n-1}' k_2, \cdots, \overline{A}_{n-1}' k_{n-2}, 0$	
$A = A_1' + \sum_1^n \overline{A}_i'$		$Mk_i = \dfrac{\frac{1}{n}\sum_1^n A_i' k_i / 2 + k_i \sum_2^n \overline{A}'}{\frac{1}{n}\sum_1^n A_i' + \sum_2^n \overline{A}_i'}$	

7 NUMERICAL RESULTS

Example 1 Take a 3-stage homogeneous transfer line of case 1 from Table 2 Ref. [3] with parameters: $(\lambda_i, \mu_i, \omega_i)$, k_i as (0.05, 0.1, 1). $i=1,2,3$ $k_1=k_2=5$.

Find. 1) System production rate, W_s 2) System efficiency, E_s 3) Buffer inventory levels, Mk_1, Mk_2.

Solution. 1) Calculations results as listed in the following table.

i	P_{bi}	P_{si}	B_i	W_i	
1	1/18	0	17/18	0.6415094	By eqn.(4) and (4b)
2	5/108	5/108	0.9095508	0.6252173	also eqn.(8)
3	0	1/18	17/18	0.6415094	
W_s=min W_i=W_2=0.6252 (0.4680, 0.4542)					

2) By eqn.(9). $E_S=\frac{1}{3}(0.6415+0.6252+0.6415)=0.6361$

3) From Table 1 of section 6. we have

$$Mk_1=\frac{\frac{1}{3}\sum_1^3 A_i'\times\frac{k_1}{2}+k_1\sum_2^3 \overline{A}_i'}{\frac{1}{3}\sum_1^3 A_i'+\sum_1^3 A_i'}=\frac{5.2566165}{1.7278667}=3.0923(3.1723,3.088)$$

$$Mk_2=\frac{\frac{1}{3}\sum_1^3 A_i'\times\frac{k_2}{2}+k_2\sum \overline{A}_3'}{\frac{1}{3}\sum_1^3 A_i'+\sum_1^3 \overline{A}_i'}=\frac{3.38275}{1.7278667}=1.9578(1.8277,1.912)$$

Nate. The answer in brackets were given by Ref. [4] and [3]. Our answers for average inventory levels are very close to theirs. But the system production rate. Ws is rather large than theirs, this is due to that we use the new model of equivalent workstations as expressed by eqn.(4) and (10) (based upon assumptions 3 and 5 instead of the approximate equation (3) of Ref. [4] or eqn.(7) of Ref. [3]. If we substitute our P_{ki} and P_{oi} in their equation, we get $W_S=0.4444$. Moreover, if we use P_{ki} and P_{oi} instead of P_{bi} and P_{Si}, we have $W_S=0.4630$.

Example 2 Take a 4-stage homogeneous transfer line from Table 1 of Ref. [5] with parameters as follows:

i	1	2	3	4
λ_i	0.04	0.02	0.03	0.02
μ_i	0.08	0.04	0.06	0.04
k_i	20	0	20	

Find the same answers as example 1.

Solution: Doing the same work as before, we obtain the answers and compare them with those of Ref. [5] as listed in the following table:

Mk_1	Mk_2	Mk_3	W_S	Remarks
15.0	0.0	5 0	0.431	Simul
				Ref. [5]
14.9	0.0	5.1	0.430	Approx.
14 8	0.0	5.5	0.494	This paper

The efficiency, E_S=0.5768

8 CONCLUSION

The main contributions of the major findings are : (1) reducing the total state spaces from $2^n\prod_1^{n-1}(k_i+1)$ to $2(n-1)$ for the $(n-1)$ buffers in the line; (2) Converting the interrupting operations into a coutinuous production line by the method of equivalent workstation; (3) Using P_{bi} and P_{Si} instead of P_{ki} and P_{oi} in Ref. [3, 6-10] to improve the production rate in the workstations and transfer line; (4) All the important performance measure such as production rates efficiencies (effectualities) and average inventory levels have been derived in explicit expressions; (5) The numerical results of this paper are compared with those of Ref. [3, 4, 5] as shown in two examples; (6) This proposed method can be used directly in engineering design.

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