A STUDY OF OPTIMIZING DESIGN OF CIM SYSTEMS WITH UNRELIABLE MACHINES AND FINITE BUFFERS*

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Abstract On the basis of equivalent workstations, the optimizing design problem of CIMS has been studied and analyzed in different cases, such as with or without buffers before the first and after the last stations, and with or without constraints of resources. An example is given for illustrating the application of the proposed method.

Key words Computer integrated manufacturing system, discrete event dynamic system, intelligent manufacturing system, optimizing design of system, system reliability.

1 INTRODUCTION

There have been a lot of research papers on CIMS in literature up to now. Most of them only dealt with the problem of two stage systems, a few studied three stage transfer lines, and the n-stage $(n > 3)$ problems are rarely seen. Almost all methods used in these papers are based on approximate solution and the calculation amount of these methods is very large. Only several papers discussed the buffer optimizing problem [1,2]. Papers on the optimizing design problem of CIMS have appeared very seldom.

On the basis of two papers [3,4], we use the equivalent workstation model and the equilibrium principle of workpiece flow to study the optimizing design problem of CIMS. Under the condition that there is no workpieces to be destroyed or rejected in the middle of the production line optimizing design has been studied and accurate analytical solutions have been obtained, respectively, in the following cases: (1) there are and there are not identical buffers before the first station and after the last station, (2) buffers in the middle of the CIMS have different capacities, and (3) the total resource is constrained or not.

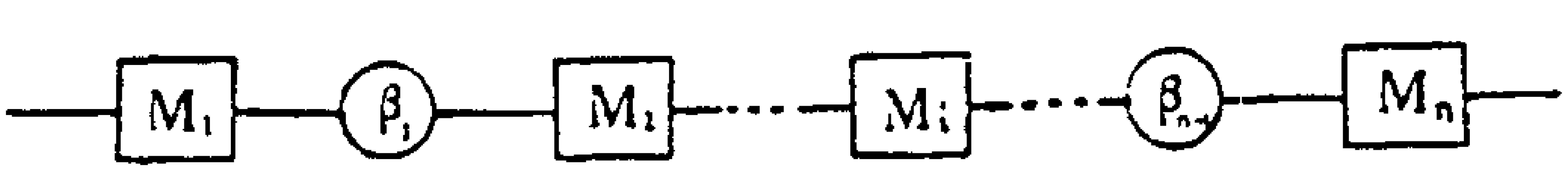

Fig.1 Serial CIMS structure

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2 MODELING AND ANALYSIS OF SERIAL REPAIRABLE CIMS

In order to get ready for optimizing design of the CIMS, we will use an equivalent workstation model on the basis of a published paper [4] to derive system effectuality (referred to as efficiency in some literature) and take it as the objective function of optimizing design.

Suppose that there is a serial processing system with n workstations as shown in Fig.1, where M_i represents the ith workstation and β_i represents the ith buffer.

2.1 Assumptions

In the following, we present six assumptions and take them as the basis for modeling, analysis, and design thereafter.

1) When workstations fail (including failures in buffers), they can be repaired immediately and after repair they can completely recover normal functioning. Processing time of the workpieces, failure time, and repair time of the workstations are of negative exponential distribution with parameters ω_i, λ_i and μ_i.

2) Transportation takes negligible time compared with processing time of a workpiece.

3) When a buffer is full of workpieces, the workstation before the buffer will be blocked and stop working automatically until this buffer has an empty place.

4) When a buffer is wholly empty, the workstation after it will be starved and stop working.

5) During the idle times of starvation and blockage, any workstation would not fail.

6) The buffer before the first station and the buffer after the last station may have equal capacities or their capacities are infinite (i.e., the first station is not starveing and the last station is not blocked).

2.2 The states of buffers

According to the analysis in paper [4], four states are picked from (K_i+1) states of a buffer for constructing and analyzing the equivalent workstation.

$$\left.\begin{aligned}&\text{Empty}\quad P_{0i}=\frac{1-\rho_i}{1-\rho_i^{K_i+1}};\quad \text{Full}\quad P_{K_i i}=\frac{\rho_i^{K_i}(1-\rho_i)}{1-\rho_i^{K_i+1}}\\&\text{Non-empty}\quad P_{\overline{0i}}=1-P_{0i}=\frac{\rho_i(1-\rho_i^{K_i})}{1-\rho_i^{K_i+1}};\quad \text{Non-full}\quad P_{\overline{K_i i}}=1-P_{Ki}=\frac{1-\rho_i^{K_i}}{1-\rho_i^{K_i+1}}\end{aligned}\right\}\tag{1}$$

where K_i represents the capacity of the ith buffer (including one storage unit in the workstation), and $\rho_i=\omega_i/\omega_{i+1}$. The average inventory level (level of storage) in the ith buffer is

$$M_{Ki}=\sum_{k_i=1}^{K_i}k_iP_{ki}=\frac{\rho_i(K_i+1)\rho_i^{K_i+1}+K_i\rho_i^{K_i+2}}{1-\rho_i-\rho_i^{K_i+1}+\rho_i^{K_i+2}},k_i=1,2,\cdots,K_i.\tag{2}$$

2.3 Modeling and analysis of the equivalent workstation

An equivalent workstation is a station where the raw materials before it are not lacking (i.e., the upstream buffer is not empty) and it should not be blocked by the workpieces after it (i.e., the downstream is not full). For a given workstation between buffers, the corresponding equivalent workstation includes not only the given workstation but also the influences of two adjacent buffers on it. According to analysis in paper [4], the availability of the ith equivalent workstation can be shown as

$$A_i = P'_{ai} = P_{\overline{0,(i-1)}} P_{\overline{Ki}} P_{ai} = B_i P_{ai} \qquad \text{(1) represents normal working probability)} \qquad (3)$$

where $B_i = P_{\overline{0(i-1)}} P_{\overline{K2}}$ represents the availability of the front buffer and the back buffer of the ith workstation, and P_{ai} represents non-failure probability of the ith workstation (including normal working, starving, and blocking).

Supposing that P_{bi} represents repair probability of the ith workstation, we have

$$P_{ai} + P_{bi} = 1. \qquad (4)$$

According to Assumption 5), we can derive [4]

$$\left.\begin{aligned} P_{ai} &= \frac{\mu_i}{\mu_i + \lambda_i B_i} \\ P_{bi} &= \frac{\lambda_i B_i}{\mu_i + \lambda_i B_i} \end{aligned}\right\} \qquad (5)$$

The steady-state productivity (production rate and throughput) of the ith equivalent workstation is

$$W_i = A_i \omega_i = P_{1i} \omega_i = \frac{\omega_i \mu_i B_i}{\mu_i + \lambda_i B_i} \qquad \text{(1) workpieces/2) hour)} \qquad (6)$$

The rated (design) productivity of the ith workstation is

$$\omega_i = \frac{W_i}{A_i} = \frac{W_i(\mu_i + \lambda_i B_i)}{\mu_i B_i} \qquad (6a)$$

The actual processing time per workpiece in the ith equivalent workstation is

$$T_{1i} = \frac{1}{\omega_i} = \frac{A_i}{W_i} = \frac{B_i P_{ai}}{W_i} \qquad (7)$$

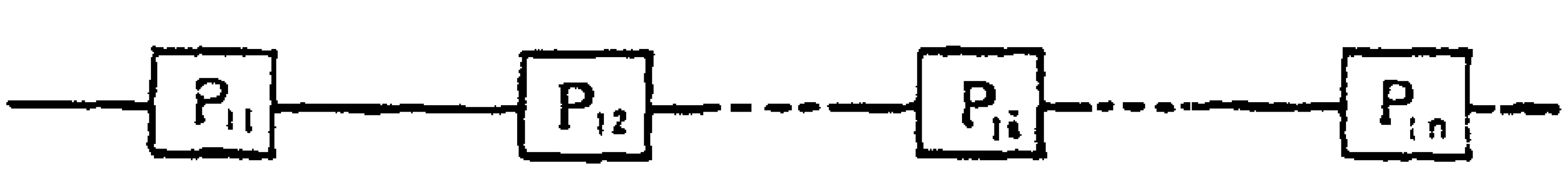

Fig.2 Equivalent production line

The average time that the ith equivalent workstation processes one workpiece is

$$T_{qi} = \frac{1}{W_i}. \qquad (8)$$

The effectuality of the ith equivalent workstation is

$$E_i = \frac{T_{1i}}{T_{qi}} = \frac{W_i}{\omega_i} = \frac{\mu_i B_i}{\mu_i + \lambda_i B_i} = A_i \qquad (9)$$

From the above formulas we can see that effectuality of a single equivalent workstation equals its availability.

2.4 Modeling and analysis of a series equivalent production line

Connecting n equivalent workstations in series, the serial production line in Fig.1 will change into one equivalent production line as shown in Fig.2, where P_{1i} is as shown in (6).

We define the effectuality of a system as the probability that it is in the states being equivalent to the full production rate at some moment under the provided conditions. In other words, the effectuality of a system equals the mean value of every equivalent workstation effectuality, that is,

$$E_s = \sum_{i=1}^{n} \frac{1}{\omega_i} \Big/ \sum_{i=1}^{n} \frac{1}{W_i} = \frac{W_s}{n} \sum_{i=1}^{n} \frac{1}{\omega_i} = \frac{1}{n} \sum_{i=1}^{n} E_i \quad . \tag{10}$$

This formula is a main objective function for the system optimizing design.

3 OPTIMIZING DESIGN PROBLEM OF THE SERIAL CIMS

On the basis of modeling and analysis indicated above, we can study the optimizing design of the system. First, according to the conservation (equilibrium) principle of workpiece flow we can do general optimizing design for the system under the different buffer conditions. Second, we will get the maximum value of the objective function (system effectuality), namely, generalized optimum solution under the constrained resources.

As far as the system control is concerned, when some buffer is full, the workstation before it will stop working automatically to avoid a workpiece being lost in the middle of the production line (Assumption 3 can be carried out).

Considering economy and optimal operation of the system, every workstation should aviod idle time due to starving and blocking as far as possible. Using the conservation of workpiece flow, from Eqn.(6) we can get

$$W_s = W_i = E_i\omega_i = \frac{\omega_i \mu_i B_i}{\mu_i + \lambda_i B_i} = \frac{\omega_i}{\frac{1}{B_i} + \frac{\lambda_i}{\mu_i}} \qquad (i = 1,2,\cdots,n) \ . \tag{11}$$

The above equation is the necessary and sufficient condition that the production line will work continuously as a homogeneous line does.

Comparing Eqn. (10) with Eqn. (11), we can see that the optimizing design problem of a system is to solve the maximum value of E_s or $\sum_{i=1}^{n} E_i$ or $\sum_{i=1}^{n} \frac{W_i}{\omega_i}$.

3.1 General optimizing design problem

Given the intermediate buffer's capacity (K_i), the optimizing design problems of the system are discussed according to the capacity of the buffers before the first and after the last workstations, respectively.

1) Suppose that the buffers before the first workstation and after the last one have the same capacity as the intermediate buffers, and raw material input and product output are stochastic. Eqn. (11) may be written as

$$\begin{aligned} W_s &= W_1 = W_2 = \cdots = W_n = E_1\omega_1 = \cdots = E_n\omega_n \\ &= \frac{\omega_1}{\frac{1}{B_1} + \frac{\lambda_1}{\mu_1}} = \frac{\omega_2}{\frac{1}{B_2} + \frac{\lambda_2}{\mu_2}} = \cdots = \frac{\omega_n}{\frac{1}{B_n} + \frac{\lambda_n}{\mu_n}} , \end{aligned} \tag{11a}$$

and

$$\rho_i = \frac{\omega_i}{\omega_{i+1}} = \frac{E_{i+1}}{E_i} = \left(\frac{1}{B_i} + \frac{\lambda_i}{\mu_i}\right) \Big/ \left(\frac{1}{B_{i+1}} + \frac{\lambda_{i+1}}{\mu_{i+1}}\right). \tag{12}$$

When K_i is given, we may get the following theorem.

Theorem 1. When the system productivity $W_s = W_i$ is given and every buffer's capacity K_i is the same, the system effectuality E_s reaches its maximum value (optimum solution) at the points of $\rho_i = \rho_{i+1} = 1$ and $\lambda_i / \mu_i = \lambda_{i+1} / \mu_{i+1}$.

Proof. From Eqn. (1), we can prove that $B_i = B_{i+1} = B = (K/(K+1))^2$ when $\rho_i = \rho_{i+1} = 1$, and at the same time B reaches its maximum value (when $\rho_i = \rho_{i+1} \neq 1$). From Eqn. (12), we can see that $\lambda_i / \mu_i = \lambda_{i+1} / \mu_{i+1}$ is also the necessary condition of the optimum solution when $\rho_i = \rho_{i+1} = 1$ and $B_i = B_{i+1}$. Again, from Eqn. (11), we can see that when ω_i and λ_i / μ_i are fixed and B_i reaches its maximum value, E_i reaches its maximum value and E_s in turn reaches its maximum value (see Eqn.(10)), which is the optimum solution. This completes the proof.

2) Suppose that the first workstation is never starved and the last workstation is never blocked. In this case, the optimizing design problem is more complicated.

Except for the first and the last workstations, we first solve the optimizing design problem of the other workstations in the middle according to the method in Theorem 1, and then revise the solution from the two ends.

Because of the first workstation not being starved, $B_1 = P_{\overline{K_1 i}}$ and $B_2 = P_{\overline{01}} P_{\overline{K_2 i}} = \rho_1 P_{\overline{K_1 i}} P_{\overline{K_2 i}} = \rho_1 P_{K_2 i} B_1$. In order to increase E_1, let $\rho_1 < 1$, that is, let $\omega_1 < \omega_2$. Because of $P_{K_2 i} < 1$, $B_1 > B_2$. Let $K_1 > K_2 = K$; then we can get

$$\frac{\lambda_i}{\mu_i} = \frac{\lambda_{i+1}}{\mu_{i+1}}. \tag{13}$$

$$B_1 = P_{\overline{K_1}} = \frac{1-\rho_1^{K_1}}{1-\rho_i^{K_{i+1}}} \text{ and } B_2 = P_{\overline{01}} P_{\overline{K_2 i}} = \rho_1 P_{\overline{K_1 i}} P_{\overline{K_2 i}},$$

$$E_1 = A_1 = \frac{1}{\dfrac{1}{B_1} + \dfrac{\lambda_1}{\mu_1}} = E_n = A_n \text{ and } \omega_1 = \frac{W_s}{E_1} = \omega_n. \tag{13a}$$

The first workstation and the last owrkstation are symmetric, and the ith workstation and the $(n-i+1)$ th workstation are also symmetric. Finally, we can get the suboptimum solution of the system by using Eqn. (10).

3.2 Generalized optimum solution

According to the models and analysis results above, by increasing the buffer's capacity K_i, we can increase system productivity W_s and system effectuality E_s. But K_i and W_s are restricted by the investment and cannot be increased unlimitedly. So this is a generalized optimizing problem.

Use Eqn. (10) to put the optimizing model of the system as follows

$$\max E_s = \frac{1}{n}\sum_{i=1}^{n} \frac{W_i}{\omega_i} = \frac{1}{n}\sum_{i=1}^{n} E_i, \tag{14-1}$$

$$\text{s.t.} \sum_{i=1}^{n}(a_i w_i)+\sum_{i=1}^{n\pm 1}(b_i K_i)\leqslant C. \tag{14-2}$$

where C is the upper limit of the total system investment (including workstation and buffer cost), a_i is the unit design productivity cost of the ith workstation, and b_i is the cost per storage unit in the ith buffer. The sign "+" in Eqn.(14-2) represents that there are identical buffers at both ends of the production line. The sign "—" represents that the first workstation is never starved and the last workstation is never blocked.

According to Theorem 1, when $\rho_i=1$ and $K_i=K$, we have [4]

$$B_i=B_1=B_2=\cdots=B_n=\left(\frac{K}{K+1}\right)^2. \tag{15-1}$$

When the resource is used up, Eqn. (14-2) becomes equality. When $a_i=a, b_i=b, \omega_i=\omega$ and $K_i=K$, we have

$$K=\frac{C-naw}{(n\pm 1)b}=\frac{1}{(n\pm 1)b}\left[C-naw\left(\frac{1}{B}+\frac{\lambda}{\mu}\right)\right]. \tag{15-2}$$

Solving Eqn.(15-2), we get

$$B=\frac{1}{\frac{(n\pm 1)b}{naw}\left(\frac{C}{(n\pm 1)b}-K\right)-\frac{\lambda}{\mu}}. \tag{15-2a}$$

Table 1

	1	2	3	4	5	Remarks
λ_i	0.001	0.002	0.002	0.002	0.001	n=5
ρ_i	0 02	0.04	0.04	0.04	0.02	W_S = 10 workpieces/hour
λ_i/ρ_i	0.05	0.05	0.05	0.05	0.05	

Table 2

Classification	Production Line	K_i	ω_i	E_s
General optimizing design	Homogeneous production line $\rho_1=1, B_i=\left(\frac{K_i}{K_i+1}\right)$	Let $K_i=4$	Eqn.(11a) 16.125001	Eqn.(10) 0.625155 (optimum solution)
	The first station is not starveling, the last station is not blocked	Let $K_2=K_3=4$ $K_1=K_4=6$	$\omega_1=11.63420=\omega_5$ $\omega_2=15.96417=\omega_4$ $\omega_3=16.12500$	0.718406 (suboptimum solution)
Generalized optimizing design	Homogeneous production line $a_i=6, b_i=3, c=505$	Let $K_i=5$	13.83333	0.722892 (optimum solution)
	The first station is not starveling, the last station is not blocked $a_i=6, b_i=3, c=505$	Let $K_2=K_3=4$ $K_1=K_4=6$	$\omega_1=11\ 01885=\omega_5$ $\omega_2=12.55285=\omega_4$ $\omega_3=12.60$	0.840397 (suboptimum solution)

Substituting Eqn.(15-2a) into Eqn.(15-1), we can get the solution for K. Here K has three roots, but K must be a positive integer. So, we can try some positive integers to find out solution K and need not solve the cubic equation.

Discussion

(1) If the system is absolutely reliable, let $\lambda / \mu = 0$. In this case we can also get the optimal solution so long as we put $\lambda \ \mu = 0$ into related equations.

(2) When the first workstation is never starved and the last workstation is never blocked, except for taking a negative sign of $(n \pm 1)$ in the equations above, we should deal with the two end workstations with the special method as before.

4 CALCULATION EXAMPLE (UNDER THE CONDITION OF CONSERVATION OF WORKPIECE FLOW)

Example: Suppose there is a 5-stage repairable CIMS and every buffer's capacity is the same (including the input buffer and the output beffer). The system productivity and its reliability data are given in Table 1. With K_i being fixed, for general optimizing design, generalized optimizing design, with or without buffers at both ends of the system, and the other cases, we solve:

(1) Every workstation rated (design) productivity ω_i;

(2) The maximum system effectuality E_s (i.e., optimum solution).

The solutions are listed in Table 2 (detailed calculation can be seen in Ref. [5]).

5 CONCLUSION

On the basis of the equivalent workstation and under the condition of conservation of workpiece flow, this paper presents detailed researches on the optimizing design problem of the system and gives some good results. These results can be applied to production practice directly. The results may be summarized in seven points as follows:

1) The necessary and sufficient conditions of the CIM system under the conservation of the workpiece flow for optimizing design are $\rho_i = \omega_i / \omega_{i+1} = 1$ and

$$\frac{\lambda_i}{\mu_i} = \frac{\lambda_{i+1}}{\mu_{i+1}} (i = 1, 2, \cdots, n-1);$$

2) The methods of the CIMS optimizing design is discussed and the definition of the system effectuality [Eqn.(10)] is given;

3) Optimum solution is obtained by taking the system effectuality (or productivity) as the objective function;

4) With K_i being fixed, the general optimum solution of the system is obtained;

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5) In the case of the resource being constrained, the generalized optimizing design model is derived and its optimum solution is given;

6) The application of this method is illustrated with an example;

7) The optimizing design method given by this paper can also be applied to other similar systems (such as IMS, DEDS, and CPMS).

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