MODELING AND ANALYSIS OF INTELLIGENT MANUFACTURING SYSTEM CONSISTING OF TWO REPAIRABLE MACHINES AND FINITE INTELLIGENT BUFFER IN SERIES*

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Abstract A new model is proposed for analyzing the intelligent manufacturing systems (IMS) consisting of repairable machine and finite buffers in series. The unreliable machines stop during times to repair and the finite storage buffers lead to blockage and starvation due to the random machine operation times of failure and repair times. Then the machines operate in interrupting state and make the problem of modeling and analysis for the system very difficult to treat. An equivalent workstation without starvation and blockage is constructed by means of quening theory of the discrete Markovian process. Connecting all the equivalent workstations in series, we get the new model of the whole system. Thus the interrupting discrete system has been converted to a continuous production line which can be solved by ordinary differential equations on the normal way. Then from the law of conservation of working piece flow in the transfer line, some explicit expressions for the measures of performance for the systems in steady state have been derived (such as system production rate, efficiency, expected in —process inventory, etc.).

Key words. IMS; CIMS; DEDS; reliability

1 INTRODUCTION

The unreliable multistage manufacturing system with random processing times and finite buffers is a special kind of large scale systems. It is subject to interruptions in operation due to blockage and starvation in the transfer line and very difficult to treat because of its large state spares and undercomposability by nature.

So far there are some researchers have done a lot of good works about this problem with

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different given conditions in different ways. But most of these works belong to two—stage systems, such as : (1) for transfer lines with deterministic processing times [1,2]; (2) for continuous materials flow through the lines [3, 4] and (3) with random processing times [5, 6]. As for three—stage transfer lines only a few papers have been published [7, 8, 9]. For more than three stages, the problem of modeling and analysis can be solved only in very strict constrained cases by approximate methods, such as decomposition approaches [10, 11] or aggregation approaches [12, 13]. However almost all these methods are rather complicate in computations and not convenient to practical uses.

A complete new method is proposed in this paper on the base of "equivalent workstations" [14, 15] for solving an unreliable intelligent manufacturing system (IMS) with random processing times of discrete workpieces in stations. For the purpose of simplification and explicitness, we limit the problem to two—stage transfer lines and compare the numercial results with those of Gershwin and Berman [6]. Obviously this new method is more simple in calculation and very easy extends to multistage transfer lines.

2 ASSUMPTIONS AND MODEL DESCRIPTION

Consider an intelligent manufacturing system of two workstations M_1 with intermediate buffer β_1 connected in series as shown in Fig. 1.

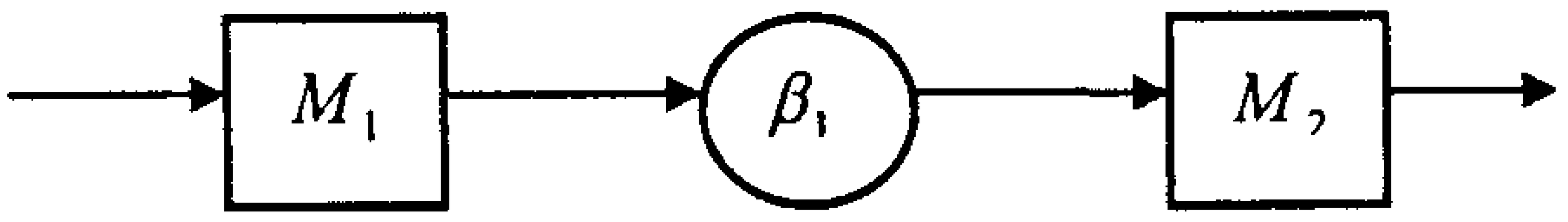

Fig. 1. Two-machine IMS

The following assumptions are given for analysis in this paper:

1) Any failues occured in the workstations can be repaired without delay. The processing time, the time to fail, and the time to repair are distributed exponentially with parameters ω_i (production rate), λ_i (failure rate) and μ_i (repaire rate).

2) Buffers are always reliable (the failures of buffers may be included in the adjacent workstation if any). Transformation of working pieces takes negligible time compared to the processing time.

3) The first workstation is never starved, and the last workstation is never blocked.

4) The operating workstation is controlled to stop automatically while either starvation or blockage occurs.

5) The stopped workstations are not vulnerable to fail, because they are not operating.

3 ANALYSIS OF THE INTERMEDIATE BUFFERS[14,15]

Consider an intermediate storage buffer as an isolated system between two reliable stations as shown in Fig.2.

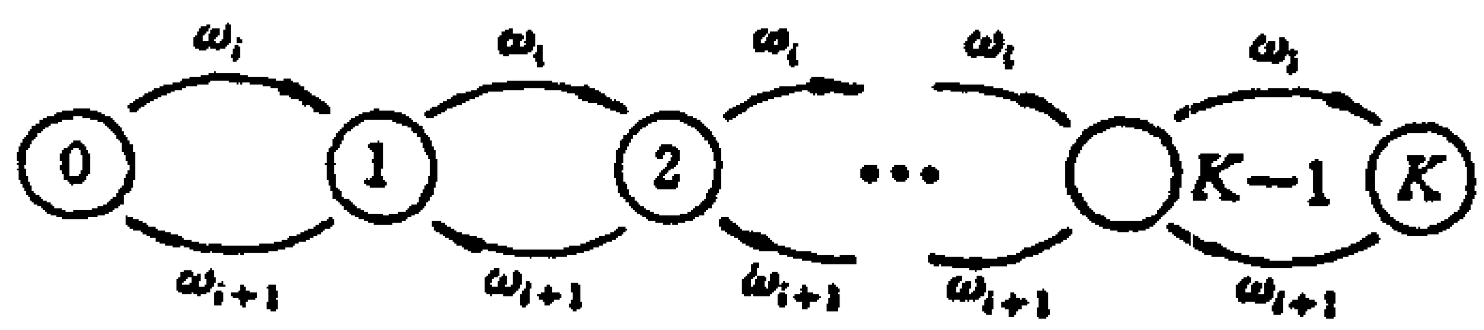

Fig.2. Transitive state diagram of buffer

Where K indicates the capacity of buffer. Here K includes one unit in the workstation.

Let P_j present the steady state probabilities of the jth state of buffer, and $\rho = \omega_1/\omega_2$, then

$$\sum_{j=0}^{K} P_j = 1 \tag{1}$$

and

$$P_j = \frac{\rho_j(1-\rho)}{1-\rho^{K+1}} \tag{2}$$

Obviously, when $\rho = 1$,

$$P_j = \frac{1}{K+1} \tag{2a}$$

For constructing the equivalent workstation in the next section, we need only the following two states:

$$P_K^- = 1 - P_K = \frac{1-\rho^K}{1-\rho^{K+1}} \text{(unfull)}$$

$$P_0^- = 1 - P_0 = \frac{\rho(1-\rho^K)}{1-\rho^{K+1}} \text{(unempty)} \tag{3}$$

Then the average in — process inventory in the buffer can be calculated as following:

$$M_K = 1P_1 + 2P_2 + \cdots + KP_K = \frac{\rho-(1+K)\rho^{K+1}+K\rho^{K+2}}{1-\rho-\rho^{K+1}+\rho^{K+2}} \tag{4}$$

4 CONSTRUCTION AND ANALYSIS OF THE EQUIVALENT WORKSTATION

For generalization, considering the effects of two adjacent buffers on the ith workstation, we have five independent working states probabilities as following:

1) M_i in the normal working states with probability:

$$P'_{ai} = P_{\overline{0(i-1)}} P^-_{K_i} P_{ai} = B_i P_{ai}$$

where P_{ai} is the reliable state probability of M_i, $B_i = P_{\overline{0(i-1)}} P^-_{Ki}$ is the availability of the adjacent buffers relation to ith workstation.

2) M_i stopped due to blockage on downstream buffer with probability:

$$P_{\overline{0(i-1)}} P_{Ki} P_{ai}$$

3) M_i stopped due to starvation on upstream buffer with probability:

$$P_{0(i-1)} P^-_{Ki} P_{ai}$$

4) M_i stopped due to both starvation and blockage simultaneously with probability:

$$P_{0(l-1)}P_{K1}P_{ai}$$

5) M_i stopped on repair with probability: P_{bi}

Obviously, sum of the above five state probabilities is the total probability, i.e.

$$[P_{\overline{0(i-1)}}P_{Ki}^{-} + P_{\overline{0(i-1)}}P_{Ki} + P_{0(i-1)}P_{Ki}^{-} + P_{0(i-1)}P_{Ki}]P_{ai} + P_{bi} = 1 \tag{5}$$

It is easy to prove that the sum of the 4 terms in bracket of (5) is equal to one, then, we have

$$P_{ai} + P_{bi} = 1 \tag{6}$$

The equivalent workstation is defined as a reliable machine which can process pieces without any starvation from emptiness of unstream buffers and blockage from fullness of downstream buffers.

Then from view point of equivalent workstation, only the first term of eq(5) is necessary for consideration. Since only the steady state solution is required for IMS problem, so we can obtain the equivalent workstation availability which is defined as (see above)

$$A = P'_{ai} = P_{\underline{\overline{0(i-1)}}}P_{Ki}^{-}P_{ai} = B_i P_{ai} \tag{7}$$

in which, P_{ai} should be determined.

Based upon assumption (5), the failure occurs only in working machine, so the processing state and repair state of machine i will transfer between P'_{ai} and P_{bi} (not P_{ai} and P_{bi}) as shown in Fig. 3.

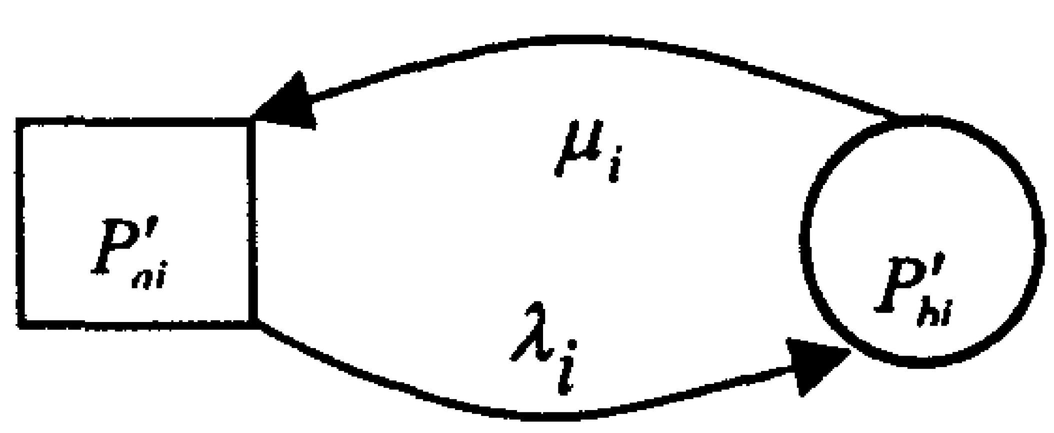

Fig.3. Equivalent repairable workstation

From Fig.3, we have

$$P'_{ai} = \lambda_i P'_{ai} - \mu_i P_{bi} \tag{8}$$

For the steady state solution, let $P'_{ai} = 0$, and take care of eq(8),

$$P_{bi} = \frac{\lambda_i}{\mu_i} P'_{ai} = \frac{\lambda_i B_i P_{ai}}{\mu_i} = \frac{B_i \lambda_i (1 - P_{bi})}{\mu_i}$$

Solving the above equation, we get the probability to repair, and the reliable state probability of M_i,

$$P_{bi} = \frac{\lambda_i B_i}{\mu_i + \lambda_i B_i}$$

$$P_{ai} = \frac{\mu_i}{\mu_i + \lambda_i B_i} \tag{9}$$

The efficiency (effectuality) of ith equivalent workstation is equal to its availability, i.e.

$$E_i = A'_i = P'_{ai} = \frac{\mu_i B_i}{\mu_i + \lambda_i B_i} \tag{10}$$

The manufacturing rate of the ith equivalent workstation,

$$W_i = \omega_i + A'_i = \frac{\omega_i \mu_i B_i}{\mu_i + \lambda_i B_i} \tag{11}$$

The average manu facturing time for each piece in the ith equivalent workstation,

$$T_i = 1 / W_i \tag{12}$$

5 MODELING AND ANALYSIS OF TWO - STAGE REPAIRABLE INTELLIGENT MANUFACTURING SYSTEM

Although the equivalent workstation method can be used for multistage transfer lines, now we take two - stage IMS as an example for explanation. Connecting two equivalent workstations in series as shown in Fig. 4.

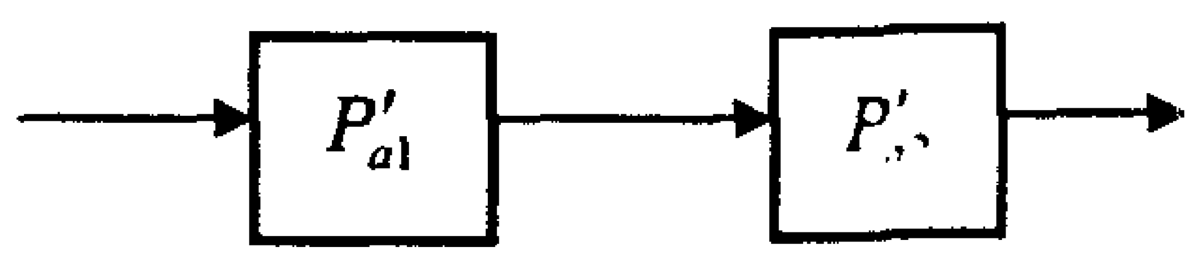

Fig. 4. Equivalent two-stage IMS

From the law of conservation of workpiece flow in the transfer line and eq(11), we have

$$W_s = W_i = \frac{\omega_1 \mu_1 B_1}{\mu_1 + \lambda_1 B_1} = \frac{\omega_2 \mu_2 B_2}{\mu_2 + \lambda_2 B_2} \qquad (i = 1,2) \tag{13}$$

Solve the above equation, we get

$$\omega_i = W_s \left(1/B_i + \lambda_i / \mu_i \right) \tag{13a}$$

and

$$\rho = \omega_1 / \omega_2 = \left(\lambda_1 / \mu_1 \right) / \left(\lambda_2 / \mu_2 \right) \tag{13b}$$

(13b) means that ρ is independent of K_1 (capacity of the buffer). It is also the necessary and sufficient condition for a two-stage repairable IMS with conservation of material flow.

The average processing time for each workpiece in the two workstations:

$$T_p = 1/\omega_1 + 1/\omega_2 \tag{14}$$

The average stayed time (including blockage, stacvation and repain) for each workpiece in two workstations.

$$T_q = 1/W_1 + 1/W_2 = 2/W_s \tag{15}$$

The system efficiency (effectuality)

$$E_s = T_p / T_q = W_s / 2 \left(1/\omega_1 + 1/\omega_2 \right) \tag{16}$$

6 NUMERICAL EXAMPLES

EXAMPLE 1

Take the standard parameter values from computational experience by S.B. Gershwin and O. Berman Ref[6] as an example and check the results with those of [6].

Table 1 Standard parameter values

i	1	2
ω_i	1	2
λ_i	3	4
μ_i	5	6

Find the manufacturing rates of the workstations, the system production rate, system efficiency (effectuality) and the average in-process inventory.

Solution:

$B_1 = P_{41}^- = (1-\rho_4)/(1-\rho_5) = 0.9688$

$B_2 = \rho B_1 = 0.4844$

$W_1 = (\omega_1\mu_1\lambda_1)/(\mu_1 + \lambda_1 B_1) = 0.6126$

$W_2 = (\omega_2\mu_2\lambda_2)/(\mu_2 + \lambda_2 B_2) = 0.7323$

which satisfied with Fig[2, 3, 4] of Ref[6].

$W_s = \min W_1 = W_2 = 0.6126$

$M_K = (\rho - (K+1)\rho^{K+1} + K\rho^{K+2}) / (1 - \rho - \rho^{K+1} + \rho^{K+2}) = 0.9677$

$E_s = W_s / 2(1/\omega_1 + 1/\omega_2) = 0.4595$ which satisfied with Fig.5 of [6]

EXAMPLE 2

Design a two-stage IMS with given parameters: $\lambda_1 = 3$, $\lambda_2 = 4$, $\mu_1 = 5$, $\mu_2 = 6$, and the average system production rate $W_s = 10$ pieces/hr. (required).

Find the workstation manufacturing rates ω_1, ω_2, the system efficiency E_s, and the average in-process in-ventory MK also choose the reasonable storage capacity K of the buffer.

Solution

$$\rho = (\lambda_1/\mu_1)*(\mu_2/\lambda_2) = 0.9$$

Table 2 Calculating table

K	2	3	4	K	8	∞
B_1	0.2710	0.7880	0.8398	B_1	0.9297	1
B_2	0.2439	0.7092	0.7558	B_2	0.8368	0.9
ω_1	42.9004	18.6900	17.9078	ω_1	16.7558	16.0
ω_2	47.6671	20.7667	19.8976	ω_2	18.6176	17.7778
E_s	0.2214	0.5083	0.5305	E_s	0.5670	0.5938
MK	0.9299	1.3687	1.7903	MK	0.3080	9.0

Form table 2, we see that E_s increase with K, but the rate is rapidly decreasing for $K > 3$. So $K = 3$ is the reasonable storage capacity.

7 CONCLUSION

The main contributions and findings are:

(1) The excellent achivement is to connect the interruption operations due to finite buffer and unreliable machines into continuous production line and make the very difficult problem being easy to solve by ordinary mathematical tools.

(2) Reducing the dimensions of the storage buffer from $(K+1)$ states to two (P_0^- and P_K^-) by means of Markov chain and reducing the dimensions of the workstations from five to one, thus simplify the calculation very much.

(3) By the law of conservation of workpieces flow, the necessary and sufficient conditions without any lost workpieces in the transfer line have been obtained as shown in eqn(13) and (13b),

which are very useful for economical and optimal engineering design.

(4) From the equivalent workstations and equivalent IMS, a set of performance measures have been derived in explicit analytical expressions, such as the station and system average production rates, the expected in-process inventory, efficiencies (effectualities), and time to process pieces and stay in the line. All these derived formulas can be used directly for theoretical analysis and practical engineering design.

(5) The results of numerical example can justify the proposed model by comparing them with those of Ref[6]. Example 2 is given for illustrating that this method is convinient to practical engineering design.

(6) This equivalent workstation method can be easily extended to multistage transfer line analysis and design.

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