AN UNIVERSAL HEURISTIC METHOD FOR SOLVING THE REDUNDANCY OPTIMIZATION PROBLEMS*

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1 INTRODUCTION

The system reliability can be increased by different means, in which the structural redundancy is the most effective method for improving the system reliability to any desired level. Although there are many algorithms[1] for solving the redundancy problems, but very few of them are quite successful in various respects.

In this paper we provide an universal heuristic method by choosing the largest sum of the purtial derivatives of the system reliability with respect to all the current constrained resources separately in a certain stage as the selection factor. This method is good for series parallel and complex system with either linear or non-linear constraiats and objective functions.

2 PROBLEM FORMULATION AND COMPUTATION PROCEDURE

Consider a reliability optimization problem for series parallel or complex systems subject to multiple constraints. Define the selection factor as:

$$C_i(m_i) \Rightarrow \sum_{l=1}^{k} \frac{\partial[R_{si}(m_i)]/R_{si}}{\partial[W_{li}(m_i)]/W_l} \tag{1}$$

where $i = 1,2,\cdots,n$ (no of stages)

k = No. of constratats

$R_{si}(m_i)$ denotes the current system reliability as a function of m_i (the current number of the components in paralell in the 1th stage).

$W_{li}(m_i)$ denotes the amount of the 1th resource comsumed in the lth stage.

For the series parallel system with multiple constrainsts, we have

$$R_{si}(m_i) = \prod_{i=1}^{n} \left(1 - q_i^{m_i}\right)$$

and

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* Proceedings of China-Japan Reliability Symposium, Shanghai, China, 1987.

$$W_{li}(m_i) = w_{li} m_i$$

w_{li} denotes the resource of each unit, then equation (1) becomes[3]

$$C_i(m_i) = \frac{\ln(q_i)}{\left(1 - q_i^{-m_i}\right)} \sum_{l=1}^{k} \frac{w_l}{w_{li}} \tag{1a}$$

For non-series-parallel (complex) system with multiple nonlinear constraints, equation(1) becomes

$$\begin{aligned} C_i(m_i) &= \sum_{l=1}^{k} \frac{\Delta[R_{si}(m_i)]/R_{si}}{\Delta[w_{li}(m_i)]/w_l} \\ &= \sum_{l=1}^{k} \frac{R_{si}[m_1, \cdots, (m_i+1), \cdots, m_n] - R_{si}(m_1, \cdots, m_i, \cdots, m_n)}{w_{li}[m_1, \cdots, (m_i+1), \cdots, m_n] - w_{li}(m_1, \cdots, m_i, \cdots, m_n)} \frac{w_l}{R_{si}} \end{aligned} \tag{1b}$$

The sequential steps involved in solving the optimization problems can be outlined as follows:

Step 1. Assign $m_i = 1$ for $i = 1, 2, \cdots, n$. Check to see that if the constraints are violated and make sure that the system as stated is possible.

Step 2. Compute the value of the selection factor $C_j(m_i)$ for all stages ($i = 1, 2, \cdots, n$) by formula (1a) or (1b) or (1c). Add a redundancy component to the stage which has the highest selection factor $C_j(m_i)$

Step 3. Check to see whether any constraints are violated.

1) If no constraints has been violated, go back to step 2.

2) If any constraint is violated, go to step 4.

3) If any constraint is exactly reached, stop. Here the set of (m_i) is the optimum allocation for the problem.

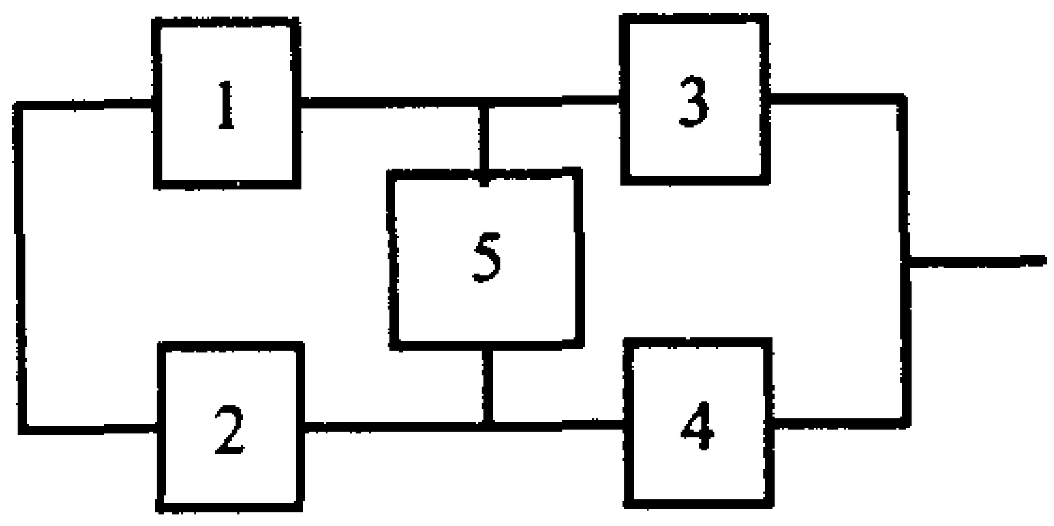
Step 4. Remove the redundancy component added in step 2 and add a redundancy compoment to the next stage with sub-highest selection factor.

Step 5. Repent step 3 to check the constraints until no component can be added to any stages, stop. The current set of (m_i) will be the optimum allocation for the problem, and the corresponding R_{si} is max, system reliability (R_s).

3 COMPARISON AMONG SEVERAL METHEDS

To compare the result, we use several popular heuristic methods to solve the same problem as following:

The system configuration and the data of various subsystems are



	1	2	3	4	5	
q_i	.3	.25	.15	.20	.10	
w_i	2.	2.	3.	3.	1.	$w \le 20$

The problem is

max
$$R_s = R_1R_3 + R_2R_4 + R_1R_5R_4 + R_2R_5R_3 - R_1R_2R_3R_4 - R_1R_3R_4R_5 - R_1R_2R_4R_5 - R_1R_2R_3R_5 - R_2R_3R_4R_5 + 2R_1R_2R_3R_4R_5$$

s.t.
$$\sum_{l=1}^{5} w_i m_i \le W$$

here
$$R_i = 1 - q_i^{m_i} \qquad i = 1,2,\cdots,n$$

The results of comparision is given in the following:

Method	M_i	R_s
by Sharma's [5]	(2,2,1,2,3)	0.9884
by Aggarwal's [6]	(3,2,1,2,1)	0.9914
by Naksgswaend's [7]	(3,2,1,2,1)	0.9914
by Wang's [8]	(2,2,1,2,1)	0.9921
by Shu's	(3,2,2,1,1)	0.9932

It is noted that the same problem solved by our approach yields a system reliability of 0.9932, which is the optimal solution. Therefore, the new selection factor proposed in this paper is a great improvement for the old.

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