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Robust Control of Robotic Manipulators Using Fuzzy Inverse Model[1)]

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Abstract In this paper, a novel fuzzy inverse model control using fuzzy clustering and sliding mode control (SMC) is proposed for the trajectory tracking of robotic manipulators having unknown dynamics. The fuzzy model of a two-link manipulator is built based on the derivation of TS models using fuzzy C-means clustering algorithm and the robot fuzzy inverse model is obtained. Then, in the proposed control framework of fuzzy inverse model, sliding mode control and the time-delay control approach are used to compensate for the fuzzy modeling errors and disturbances, thus the system stability is guaranteed and its tracking performance is improved. The system stability and convergence of tracking errors are proven by stability theory. Finally, an example for the trajectory tracking of a two-link manipulator illustrates the performance improvement of the proposed control approach.

Key words Robotic manipulators, fuzzy clustering, sliding mode control, tracking control.

1 Introduction

Fuzzy logic controllers have found successful applications in robotic manipulators for control problems, which could be difficult to deal with by other conventional approach[1]. Essentially, the SMC theory usually is used to drive state trajectory toward a specified sliding surface and maintain its motion along the sliding surface in the state space. Conventionally, it is difficult for human experts to examine all the input-output data from a complex system to find a number of proper rules for the fuzzy system. To cope with this difficulty, several approaches to generate fuzzy if-then rules from numerical data have been proposed. In the early 1990's, the idea of tuning the parameters of a fuzzy model using I/O data becomes a focus of interest in research. In 1991, Wang and Mendel proposed a method for generating fuzzy rules by learning from example[2]. Jang proposed a fuzzy-neural networks (ANFIS) method for parameter adjustment of an TSK fuzzy model[3] in 1993. Also in the same year, Sugeno and Yasukawa published a paper[4] on a FL-based approach to qualitative modeling, where structure identification could be achieved simultaneously using a Mamdani position gradient-type fuzzy model by a combination of fuzzy c-means clustering and group method of data handling. However more and more authors have demonstrated how to improve and enhance the ability of fuzzy controller[5~8], especially integrating fuzzy theory and SMC into fuzzy controller design to acquire stability and consistent performance is a vigorous area of fuzzy control. The best of properties of the SMC is its robustness to parameter changes or external disturbance[9]. Hwang and Lin[10] incorporated fuzzy set theory to construct control rules according to the concepts of SMC for attenuating the chattering phenomena, but it can not be guaranteed with the stability

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of fuzzy control system. Ting et al. [11] gave a set of fuzzy control rules and applied adaptive manner to adjust the hitting control, but the problems of how to find a suitable equivalent control is still not solved. A self-organizing and the adaptive fuzzy sliding mode controllers were proposed by Yoo[12,13], however, as the system dimension increases, the rule number will become so large that the implementation will be very difficult. Sun et al.[14] proposed an adaptive fuzzy controller based on sliding mode for robot manipulators. Though a good control performance can be obtained with less fuzzy control rules, the control performance is still sensitive to the choice of the neighborhood size around the sliding surface. There is no systematic approach to determine the neighborhood size yet.

In this paper, a new method is developed to generate fuzzy rules from numerical data. Unlike traditional fuzzy modeling methods, the optimal number of rules (clusters) in the paper is determined by input-output data pairs rather than by only output data or input data singly. The fuzzy system constructed is used as an approximator to the robot inverse dynamics, by which the equivalent control can be constructed for SMC systems. Under the assumption that the system disturbance and dynamics variations are slow with respect to sampling interval, introduction of the $u(k-1)$ in the control law $u(k)$ results in the partial cancellation of the system disturbance and dynamics variations. Moreover, the reduced disturbance can be further compensated by the sliding mode control. The control gain in MIMO sliding mode control can be obtained through Lyapunov synthesis approach. Unlike the sliding model control in continuous time systems, the control gain obtained here should be subject to an inequality interval, i.e. an upper bound and lower bound. Then a new fuzzy control scheme is presented for robot tracking control, which can alleviate the chattering inherent in sliding mode control without the sacrifice of robustness against model uncertainties and external disturbances. The simulations of a two-link manipulator demonstrate properties of the proposed control approach.

The rest of the paper is organized as follows. In Section 2, some basics for a fuzzy system, robot model and its properties are reviewed. Derivation of fuzzy clustering is given and a new fuzzy control approach based on sliding mode is presented in Section 3. An illustrative example is given in Section 4. Finally, Section 5 concludes the paper.

2 Preliminaries

2.1 Definitions

Definition. A fuzzy rule base, $R=\bigcup_{l=1}^{M} R^l$, is a union of fuzzy rules, in which each rule R^l is of the form

$$R^l:\ \text{IF } \boldsymbol{z}(k) \text{ is } F^l \quad \text{THEN } \boldsymbol{z}(k+1)=A_l\boldsymbol{z}(k)+B_l\boldsymbol{u}(k)+\boldsymbol{w}_l, \quad l=1,2,\cdots,m. \tag{1}$$

Where R^l denotes the l-th approximation inference rule, F^l is fuzzy set, $(A_l, B_l, \boldsymbol{w}_l)$ is the l-th local model of the fuzzy system, m is the number of approximation inference rules, $\boldsymbol{u}(k)\in R^g$ are control input variables of the system, $\boldsymbol{z}(k)\in R^p$ are the state variables of the system.

The final output of the system is inferred by taking the weighted average of all local models. Because the model in (1) only represents the properties of the system in a local region, it is termed as a fuzzy dynamic model.

2.2 Mathematical Model for an n -link Robot Manipulator

Consider the discrete dynamic model of an n-link rigid robot manipulator

$$D(\boldsymbol{q}(k+1))\dot{\boldsymbol{q}}(k+1)-D(\boldsymbol{q}(k))\dot{\boldsymbol{q}}(k)-f(\boldsymbol{q}(k),\dot{\boldsymbol{q}}(k))T=T\boldsymbol{u}(k), \tag{2}$$

where T is the sampling interval, $D(\boldsymbol{q}(k))=D^{\mathrm{T}}(\boldsymbol{q}(k))(>0)\in R^n$ is the inertia matrix, $f(\boldsymbol{q}(k),\dot{\boldsymbol{q}}(k))$ represents centrifugal, Coriolis and gravitational torques, and $\boldsymbol{u}(k)$ is the piecewise constant genaralized force input:

$$u(t) = u(k) \quad \text{for} \quad kT \leqslant t < (k+1)T. \tag{3}$$

The discrete dynamic model for an n-link robot represented in (2) shows a more accurate performance compared with models obtained through discretization of Euler-Lagrange dynamic equation. (2) can be written in an explicit form by setting

$$q(k+1) \cong \bar{q}(k) = q(k) + a(k)T\dot{q}(k) \tag{4}$$

and

$$D(q(k+1)) \cong D[q(k) + a(k)T\dot{q}(k)] = D(\bar{q}(k)), \tag{5}$$

where $a(k)$ represents the change of the slope of the robot joint trajectories at any discrete time instant. With (4), (2) can be written through some mathematical operations as

$$\dot{q}(k+1)-\dot{q}(k)+D^{-1}(\bar{q}(k))((D(\bar{q}(k))-D(q(k)))\dot{q}(k)-Tf(q(k),\dot{q}(k))) = D^{-1}(\bar{q}(k))Tu(k). \tag{6}$$

It should be noted that $D(q)$ is a positive symmetric matrix defined by $D_m \leqslant \|D(q)\| \leqslant D_M$ with $D_m, D_M > 0$ being known constants.

3 Fuzzy Controller Design

Consider the following robot discrete time state equation,

$$z(k+1) = Az(k) + Bu(k) + w(k). \tag{7}$$

Where $z(k) = (q^{\mathrm{T}}(k), \dot{q}^{\mathrm{T}}(k))^{\mathrm{T}}$ is the state and $u(k)$ is the control input, $w(k)$ is the lumped system parameter and disturbance uncertainty.

A sliding surface for the discrete state space is defined as

$$s(k) = C(z(k) - z_d(k)) = 0. \tag{8}$$

Where $z_d(k) = (q_d^{\mathrm{T}}(k), \dot{q}_d^{\mathrm{T}}(k))^{\mathrm{T}}$ is the desired trajectory to be tracked, and C is the matrix.

In this section, a fuzzy controller design approach based on fuzzy clustering and sliding mode techniques will be developed for robot control. The goal is to design a control law $u(k)$ which ensures that the robot joint disaplacement $q(k)$ follows the desired trajectory $q_d(k)$. We first use fuzzy clustering to get the dynamics model of the robot, then a composition controller is developed.

3.1 Derivation of TS models via Fuzzy Clustering

Through using fuzzy input/output space clustering identification algorithm and let $z(k) = (q^{\mathrm{T}}(k), \dot{q}^{\mathrm{T}}(k))^{\mathrm{T}}$ to get the state equation of the robot articulator. The clustering Identification algorithm can be divided into two stages. The first stage is the identification of the membership functions, including the determination of the number of fuzzy rules, and the estimation of the parameters in the membership functions. The second one is the identification of the local rule maps and the global rule interpolation.

$$R^l: \text{ IF } (z^{\mathrm{T}}u_e(k)) \text{ is } F^l \qquad \text{THEN } z(k+1) = A_l z(k) + B_l u_e(k) + w_l. \tag{9}$$

1) Identification of membership functions

In the following discussion all the membership functions are chosen as the BSMF (Bell shaped membership function). First, we define the following criterion function

$$J(\mu, z) = \sum_{t=1}^{N}\sum_{l=1}^{m} \mu_l(t)^{\omega}\|z(k) - \bar{z}_l\|^2, \tag{10}$$

$$\sum_{l=1}^{m}\mu_l(t)=1,$$

where m is the number of the rules , $\bar{z}=[\bar{z}_1,\bar{z}_2,\cdots,\bar{z}_m]$ is mean prototypes , $\|z(k)-\bar{z}_l\|$ is the distance of the feature point to the mean prototype , ω, a shape factor, is used to control the shape of the membership function.

With the known weighting factors, the necessary conditions for minimizing $J(\mu,\bar{z},\alpha)$ are

$$\bar{z}_l=\frac{\sum_{t=1}^{N}\mu_l(k)^{\omega}z(k)}{\sum_{t=1}^{N}\mu_l(k)^{\omega}},\quad l=1,2,\cdots,m, \tag{11a}$$

$$\alpha_l=[\Phi^{\mathrm{T}}D_l\Phi]^{-1}\Phi^{\mathrm{T}}D_lY,\quad l=1,2,\cdots,m, \tag{11b}$$

$$\Phi=[\varphi(1),\varphi(2),\cdots,\varphi(N)]^{\mathrm{T}},\ D_l=\mathrm{diag}[\mu_l(k)]_{N\times N},$$

$$\mu_l(k)=\left\{\sum_{j=1}^{m}\frac{(\|z(k)-\bar{z}_l\|^2)^{\sigma}}{(\|z(k)-\bar{z}_j\|^2)^{\sigma}}\right\}^{-1},\ \sigma=1/(\omega-1), \tag{11c}$$

$$l=1,2,\cdots,m.$$

Fuzzy input output space clustering algorithm can be divided into the following several steps.

Step 1. Choose the shape factors ω, and pick an termination threshold $\varepsilon>0$ and an initial membership function $\mu_l^{(0)},\sum_{l=1}^{m}\mu_l^{(0)}=1$.

Step 2. Update $\mu^{(k)}\to\mu^{(k+1)}$ according to (11c) if $I_t=0$, otherwise,

$$\mu_i^{k+1}(t)=0,\forall i\in\tilde{I}_t \text{ and } \sum_{i\in I_t}\mu_i^{(k+1)}(t)=1.$$

Where

$$I_t\cong\left\{i|1\leqslant i\leqslant m;\|z(t)-\bar{z}_i\|^2=0\right\},\quad \tilde{I}_t\cong\{1,2,\cdots,m\}-I_t.$$

Step 3. If $\|\mu^{(k+1)}-\mu^{(k)}\|\leqslant\varepsilon$, then stop; otherwise go to step 2.

2) Identification of local linear models

In order to retain the local behavior of the system which is represented by the local linear models, the BSMF are fixed during the identification of the local models. If the BSMF and the parameters in the local models are adjusted together such that the global fitting error is minimized, then the resulting model is a global nonlinear function model rather than the fuzzy model discussed in this paper. For this, we can get a good global approximation to the given system, the local fuzzy rules no longer represent the local behavior of the robotic manipulator. Since the multi-input multi-output fuzzy model can be represented by m multiple inputs and single output systems, we only consider the multiple inputs/single output system and still use $z(k+1)$ to represent one of the output components. Thus, one of the output components can be rewritten as

$$z(k+1)=\sum_{i=1}^{M}\mu_i(k)f_i(z(k)). \tag{12}$$

Where $\mu_i(k)$ is the membership function of the i-th local linear model and recall (9),

$$f_i(z(k))=A_iz(k)+B_iu_e(k)+w_i(k).$$

The goal of the structure identification (12) is to select the most significant terms from a set of candidates to describe $\boldsymbol{z}(k+1)$. If a certain term is not selected during the process, it indicates the term has little effect on the fuzzy model (12).

Suppose that the following model has been selected in the above structure algorithm.

$$\hat{\boldsymbol{z}}(k+1) = v^{\mathrm{T}}(k, \boldsymbol{\mu})\hat{\alpha}. \tag{13}$$

Where $\hat{\boldsymbol{z}}(k+1)$ and $\hat{\alpha}$ denote the estimates of $\boldsymbol{z}(k+1)$ and α respectively, $v(k, \boldsymbol{\mu}) = (\boldsymbol{z}(k), \boldsymbol{u}_e(k))$, $\alpha = (A_i, B_i, w_i)$. The parameter estimation is fulfilled by minimizing the following criterion,

$$J(\hat{\theta}) = \|U - \Phi(\boldsymbol{\mu})\hat{\alpha}\|^2. \tag{14}$$

Where,

$$U = (\boldsymbol{z}(1), \boldsymbol{z}(2), \cdots, \boldsymbol{z}(N))^{\mathrm{T}}, \tag{15}$$

$$\Phi(\boldsymbol{\mu}) = \begin{bmatrix} z_0(1) & z_1(1) & \cdots & z_M(1) & u_0(1) & u_1(1) & \cdots & u_M(1) \\ z_0(2) & z_1(2) & \cdots & z_M(2) & u_0(2) & u_1(2) & \cdots & u_M(2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ z_0(N) & z_1(N) & \cdots & z_M(N) & u_0(N) & u_1(N) & \cdots & u_M(N) \end{bmatrix}. \tag{16}$$

The minimization of the performance loss (14) results in

$$\hat{\alpha} = [\Phi(\boldsymbol{\mu})^{\mathrm{T}} \Phi(\boldsymbol{\mu})]^{-1} \Phi(\boldsymbol{\mu})^{\mathrm{T}} U. \tag{17}$$

The convergence of the identification algorithm can be referred to the conventional least-squares algorithms for more detail.

3.2 Fuzzy Controller Design

The preceding development is to get the fuzzy model of robotic manipulators using fuzzy cluster algorithm. Now, consider the following control law

$$\boldsymbol{u}(k) = \boldsymbol{u}_e(k) + \boldsymbol{u}_n(k), \tag{18}$$

where

$$\boldsymbol{u}_e(k) = -(CB)^{-1}\{CA(\boldsymbol{z}(k) - \boldsymbol{z}(k-1)) - (CB)\boldsymbol{u}(k-1) + C(\boldsymbol{z}_d(k) - \boldsymbol{z}_d(k+1))\} \tag{19}$$

and

$$\boldsymbol{u}_n(k) = -K_m \mathrm{sgn}(\boldsymbol{s}) - K\boldsymbol{s}, \tag{20}$$

with

$$A(\mu) = \sum_{l=1}^{m} \mu_l A_l, \quad B(\boldsymbol{\mu}) = \sum_{l=1}^{m} \mu_l B_l,$$

represent the equivalent control component and nonlinear control component, respectively. Besides, $K = K^{\mathrm{T}} > 0, K_m = \mathrm{diag}(k_{m11}, k_{m22}, \cdots, k_{mnn})$ is the gain coefficient, $\boldsymbol{u}_n(k)$ was used to compensate for the modeling errors and disturbance uncertainty.

Substituting (7) into (8) and together with (18), we have

$$\boldsymbol{s}(k+1) = \boldsymbol{s}(k) + C(\boldsymbol{w}(k) - \boldsymbol{w}(k-1)) - K_m \mathrm{sgn}(\boldsymbol{s}) = \boldsymbol{s}(k) + \boldsymbol{e}(k) - K_m \mathrm{sgn}(\boldsymbol{s}). \tag{21}$$

where

$$\boldsymbol{e}(k) = C(\boldsymbol{w}(k) - \boldsymbol{w}(k-1)).$$

The following theorem gives a stable learning law, and guarantees the asymptotic stability of the system.

Theorem 1. For system (7), the control law (18) can ensure the system switching function converge to the sliding boundary layer exponentially fast. Where the control gain K_m can be determined as

$$d_i(k) \leqslant k_{mii} \leqslant 2s_i(k) + d_i(k), \tag{22}$$

where

$$d_i(k) = |e_i(k)|.$$

Proof. Let the Lyapunov function is defined by

$$V(k) = \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{s}(k) \tag{23}$$

Thus, the forward difference $\Delta V(k)$ of the Lyapunov function is obtained as

$$\begin{aligned}\Delta V(k) =& V(k+1) - V(k) = \frac{1}{2}\left[\boldsymbol{s}^{\mathrm{T}}(k+1)\boldsymbol{s}(k+1) - \boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{s}(k)\right] = \\ & \boldsymbol{s}^{\mathrm{T}}(k)\Delta\boldsymbol{s}(k+1) + \frac{1}{2}\Delta\boldsymbol{s}^{\mathrm{T}}(k+1)\Delta\boldsymbol{s}(k+1) \leqslant \\ & \boldsymbol{s}^{\mathrm{T}}(k)(\boldsymbol{e}(k) - K_m\mathrm{sgn}(\boldsymbol{s})) + \frac{1}{2}(\boldsymbol{e}(k) - K_m\mathrm{sgn}(\boldsymbol{s}))^{\mathrm{T}}(\boldsymbol{e}(k) - K_m\mathrm{sgn}(\boldsymbol{s})) = \\ & \boldsymbol{s}^{\mathrm{T}}\boldsymbol{e}(k) - \boldsymbol{s}^{\mathrm{T}}\mathrm{diag}(k_{m11}, \cdots, k_{mii}, \cdots, k_{mnn})\mathrm{sgn}(\boldsymbol{s}) + \\ & \frac{1}{2}(K_m^2 + \mathbf{e}^{\mathrm{T}}(k)\boldsymbol{e}(k) - \mathbf{e}^{\mathrm{T}}(k)K_m\mathrm{sgn}(\boldsymbol{s}) - K_m\mathrm{sgn}(\boldsymbol{s})\boldsymbol{e}(k)) = \\ & \sum_{i=1}^{n}[s_i(k)d_i(k) - |s_i(k)|k_{mii} + \frac{1}{2}k_{mii}^2 + \frac{1}{2}d_i^2(k) - d_i(k)k_{mii}].\end{aligned} \tag{24}$$

It is easy to verify that if (22) is satisfied, $\Delta V(k+1) < 0$. Then, it comes to a conclusion that the system has a stable sliding motion. □

4 Application

In this section, the above developed control approach is employed in the position control of a 2-link manipulator[14]. The dynamical equaion of a 2-link manipulator is

$$\begin{bmatrix} D_{11}(\phi) & D_{12}(\phi) \\ D_{12}(\phi) & D_{22}(\phi) \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} F_{12}(\phi)\dot{\phi}^2 + 2F_{12}(\phi)\dot{\theta}\dot{\phi} \\ -F_{12}(\phi)\dot{\theta}^2 \end{bmatrix} + \begin{bmatrix} q_1(\theta,\phi)g \\ q_2(\theta,\phi)g \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \tag{25}$$

where

$$\begin{aligned} & D_{11}(\phi) = (m_1 + m_2)(r_1')^2 + m_2(r_2')^2 + 2m_2r_1'r_2'\cos(\phi) + J_1, \\ & D_{12}(\phi) = m_2(r_2')^2 + m_2r_1'r_2'\cos(\phi), \\ & D_{22}(\phi) = m_2(r_2')^2 + J_2, \\ & q_1(\theta,\phi) = -(m_1+m_2)r_1'\cos(\theta) - m_2r_2'\cos(\phi+\theta), \\ & q_2(\theta,\phi) = -m_2r_2'\cos(\theta+\phi), F_{12}(\phi) = m_2r_1'r_2'\sin(\phi), \\ & |u_1| \leqslant 360(\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}), \quad |u_2| \leqslant 182(\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}). \end{aligned}$$

The parameters of the robot are

$$\begin{aligned} & r_1' = 1\mathrm{m}, \quad r_2' = 0.8\mathrm{m}, \quad J_1 = J_2 = 5\mathrm{kg}\cdot\mathrm{m}^2, \\ & m_1 = 0.5\mathrm{kg}, \quad m_2 = 6.25\mathrm{kg}. \end{aligned}$$

The desired joint angle trejectory for the robot to follow is

$$\begin{aligned} & \theta_d(t) = 0.5(\sin t + \sin 2t), \\ & \phi_d(t) = 0.5(\cos 3t + \cos 4t), \end{aligned} \tag{26}$$

where $t \in [0, 2\pi]$, the sampling interval is $\pi/50$, the fuzzy I/O space clustering algorithm is used to get the following fuzzy state model with only three rules,

$$\boldsymbol{v}(k+1) = \mu \boldsymbol{f}(\boldsymbol{z}(k)), \tag{27}$$

where $\boldsymbol{v}$ is $[\theta \quad \phi]^{\mathrm{T}}, \mu \in R^{2\times 6}, \boldsymbol{z}(k)$ is $[(\theta(k)\dot{\theta}(k)u_1(k-1); \phi(k) \quad \dot{\phi}(k) \quad u_2(k-1)]^{\mathrm{T}}$.
The membership function and the clustering center for the first joint of the manipulator are obtained as,

$$\begin{aligned}
&\mu_l(\boldsymbol{z}) = \left(\sum_{j=1}^{3} \frac{\|\boldsymbol{z} - \bar{\boldsymbol{z}}_l\|^\sigma}{\|\boldsymbol{z} - \bar{\boldsymbol{z}}_j\|^\sigma}\right)^{-1}, \\
&l = 1, 2, 3, \sigma = 1/(\omega - 1) = 1/(1.2 - 1), \\
&\bar{\boldsymbol{z}}_1 = (0.1998, 0.0426, 47.2887)^{\mathrm{T}}, \\
&\bar{\boldsymbol{z}}_2 = (-0.2151, -0.1543, 128.4728)^{\mathrm{T}}, \\
&\bar{\boldsymbol{z}}_3 = (-0.2795, 0.0107, 191.5544)^{\mathrm{T}}.
\end{aligned} \tag{28}$$

The local linear models of the first robot joint are,

$$\begin{aligned}
&A_{11} = \begin{bmatrix} 0.9958 & 0.0627 \\ 0 & 0 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 0.0078 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{w}_{11} = \begin{bmatrix} 0.1029 \\ 0 \end{bmatrix}, \\
&A_{12} = \begin{bmatrix} 0.9842 & 0.0724 \\ 0 & 0 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 0.0046 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{w}_{12} = \begin{bmatrix} -0.0472 \\ 0 \end{bmatrix}, \\
&A_{13} = \begin{bmatrix} -0.9903 & -0.0596 \\ 0 & 0 \end{bmatrix}, \quad B_{13} = \begin{bmatrix} 0.00021 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{w}_{13} = \begin{bmatrix} -0.2771 \\ 0 \end{bmatrix}.
\end{aligned} \tag{29}$$

The angle for the first robot joint can be rewritten as,

$$\begin{aligned}
\theta(k+1) = &\sum_{l=1}^{3} \mu_l f_l(\boldsymbol{z}(k)) = \mu_1 f_1 + \mu_2 f_2 + \mu_3 f_3 = \\
&0.9958\mu_1\theta(k) + 0.0627\mu_1\dot{\theta}(k) + 0.0078\mu_1 u_1(k-1) + 0.1029\mu_1 + \\
&0.9842\mu_2\theta(k) - 0.0724\mu_2\dot{\theta}(k) + 0.0046\mu_2 u_1(k-1) - 0.0472\mu_2 - \\
&0.9903\mu_3\theta(k) - 0.0596\mu_3\dot{\theta}(k) + 0.0021\mu_3 u_1(k-1) - 0.2771\mu_3.
\end{aligned} \tag{30}$$

Similarly, the membership function and the clustering center of the second robot joint are obtained as

$$\mu_l(\boldsymbol{z}) = \left(\sum_{j=1}^{3} \frac{\|\boldsymbol{z} - \bar{\boldsymbol{z}}_l\|^\sigma}{\|\boldsymbol{z} - \bar{\boldsymbol{z}}_j\|^\sigma}\right)^{-1}, \quad l = 1, 2, 3, \quad \sigma = 1/(\omega - 1) = 1/(1.2 - 1).$$

$$\begin{aligned}
&\bar{\boldsymbol{z}}_1 = (-0.2711 \quad 0.5268 \quad 103.3799)^{\mathrm{T}}, \\
&\bar{\boldsymbol{z}}_2 = (-0.1093 \quad 0.0140 \quad 40.9023)^{\mathrm{T}}, \\
&\bar{\boldsymbol{z}}_3 = (0.3311 \quad -0.5142 \quad -21.9805)^{\mathrm{T}}.
\end{aligned}$$

The local linear models of the second robot joint are,

$$\begin{aligned}
&A_{21} = \begin{bmatrix} -0.9791 & -0.0616 \\ 0 & 0 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 0.0037 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{w}_{11} = \begin{bmatrix} -0.0050 \\ 0 \end{bmatrix}, \\
&A_{22} = \begin{bmatrix} 0.9785 & 0.0618 \\ 0 & 0 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0.0054 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{w}_{12} = \begin{bmatrix} 0.0097 \\ 0 \end{bmatrix}, \\
&A_{23} = \begin{bmatrix} 0.9804 & -0.0611 \\ 0 & 0 \end{bmatrix}, \quad B_{23} = \begin{bmatrix} 0.0014 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{w}_{13} = \begin{bmatrix} 0.0001 \\ 0 \end{bmatrix}.
\end{aligned} \tag{31}$$

Then the angle of the second robot joint is,

$$\phi(k+1)=\sum_{l=1}^{3}\mu_l f_l(z(k))=\mu_1 f_1+\mu_2 f_2+\mu_3 f_3=$$
$$-0.9791\mu_1\phi(k)-0.0616\mu_1\dot{\phi}(k)+0.0037\mu_1 u_2(k-1)-0.005\mu_1+$$
$$0.9785\mu_2\phi(k)+0.0618\mu_2\dot{\phi}(k)+0.0054\mu_2 u_2(k-1)+0.0097\mu_2+$$
$$0.9804\mu_3\phi(k)-0.0611\mu_3\dot{\phi}(k)+0.0014\mu_3 u_2(k-1)+0.0001\mu_3. \tag{32}$$

To demonstrate the modeling precision by fuzzy clustering shown above, Fig.1 denotes the approximation errors for two joint angles of the robotic manipulator. It can be seen that the model precision is good enough for the fuzzy inverse model control.

Simulation is done using a fourth-order Runge-Kutta algorithm with an integral step of 0.01s, controller-sampling integral are also selected as 0.01s. The initial simulation conditions are: $(\theta(0),\dot{\theta}(0),u_1(0))=(1,-0.5,0),(\phi(0),\dot{\phi}(0),u_2(0))=(1,-2,0)$. Substituting (27) and (31) into (18), and choosing $0.0062\leqslant k_{111}\leqslant 0.1798, 0\leqslant k_{122}\leqslant 0, 0.0023\leqslant k_{211}\leqslant 0.00153, 0\leqslant k_{222}\leqslant 0$, the system responses of tracking errors are shown as Fig.2.

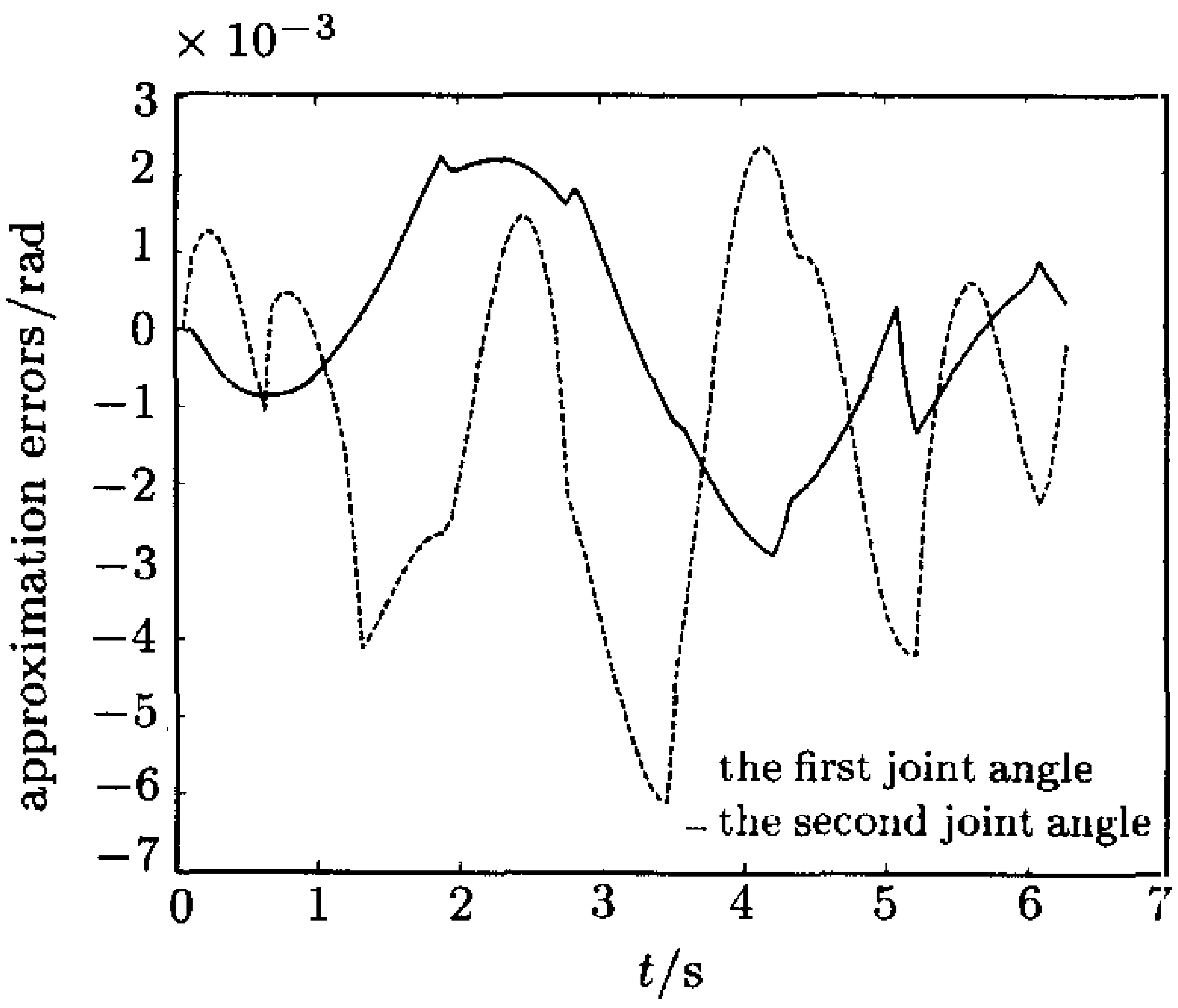

Fig.1 Approximation errors of the first and second manipulator

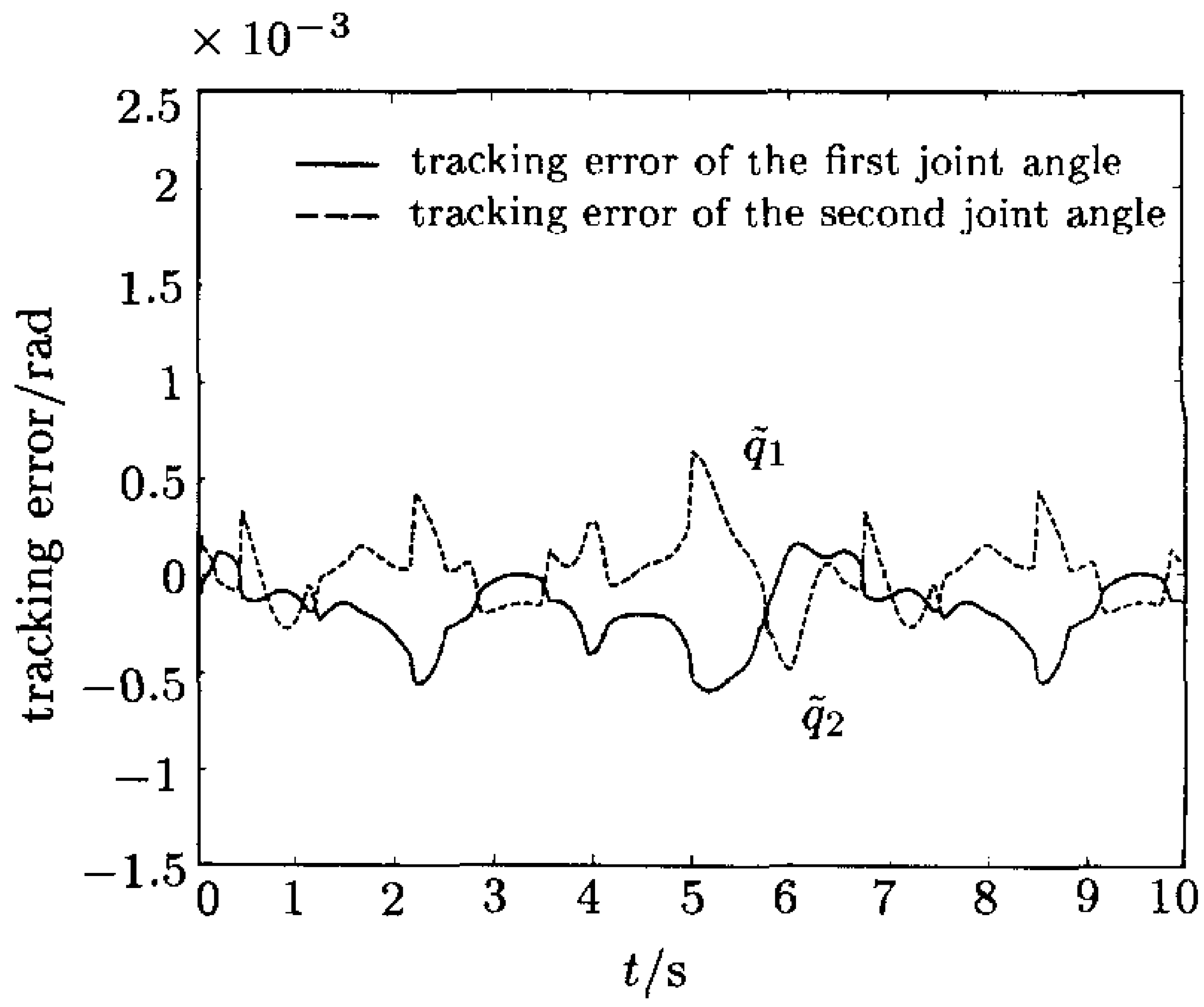

Fig.2 Robot tracking error responses using fuzzy controller

It has been shown that the number of fuzzy rules can be reduced by fuzzy clustering, thus a simplified fuzzy control structure is resulted. In this example, the satisfied control performance is obtained with only three fuzzy rules, the modeling error is compensated by discrete SMC control.

To study the contribution of fuzzy controller based on the derivation of TS models using fuzzy C-means clustering algorithm, Fig.3 shows the tracking error responses with $\boldsymbol{u}=-KS$ only. It is shown that the addition of this fuzzy controller makes a significant improvement in the tracking performance (See Fig.2).

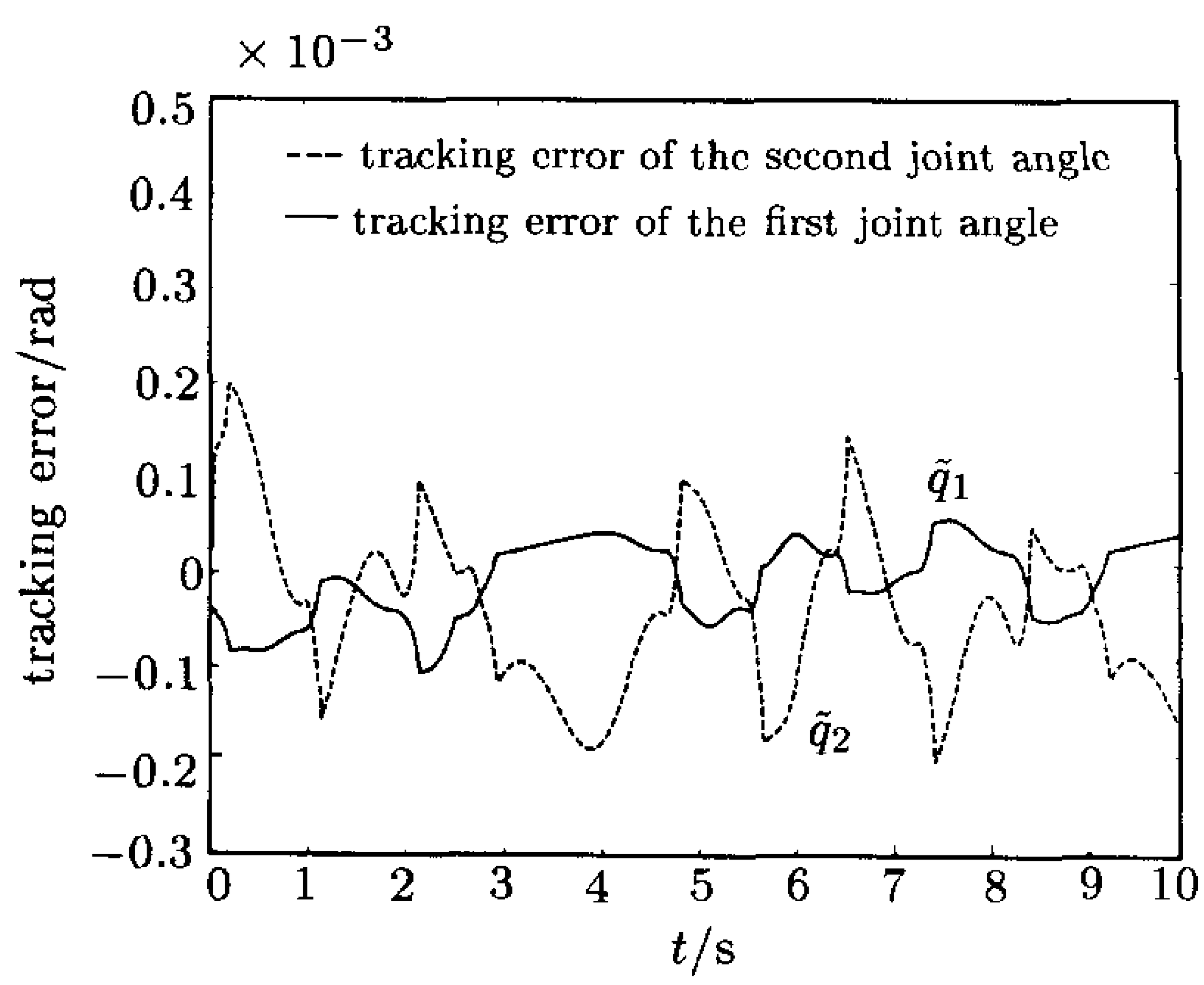

Fig.3 Robot tracking error responses without fuzzy controller

5 Conclusions

A new fuzzy controller based on TS fuzzy clustering model is proposed in this paper for the trajectory tracking of robotic manipulators with unknown nonlinear dynamics. Main idea is the integration of fuzzy inverse model approach and the discrete sliding mode control, where fuzzy control is used to approximate the robot dynamics nonlinearities while discrete sliding mode control is used to compensate the system remaining perturbations for improving the system performance and guaranteeing the system stability. Main contributions of the paper include:

1) As the controller contains an estimate of the perturbations, the remaining part of the system perturbations will be the difference of perturbations in two adjacent sampling instants. Under the assumption that the system perturbations vary slowly with respect to sampling interval, the remaining perturbations are very small in magnitude. Moreover, the remaining perturbations can be further compensated by the sliding mode control such that a good tracking performance can be guaranteed.

2) A similar result is derived that the control gains in MIMO discrete sliding mode control are also subject to an inequality interval, which is consistent to that for SISO discrete sliding mode control. Since the system disturbances are reduced significantly by the perturbation estimations in the controller, the estimation conservation is overcome.

The simulation results show that the fuzzy approximator constructed through I/O data provides a new alternative for further simplifying the design of fuzzy controllers. Besides, after incorporating the linguistic information into the fuzzy controller, the adaptation speed became much faster, and a better transient performance and learning convergence are obtained.

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机械手的模糊逆模型鲁棒控制

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摘 要 提出一种基于模糊聚类和滑动模控制的模糊逆模型控制方法，并将其应用于动力学方程未知的机械手轨迹控制. 首先，采用 C 均值聚类算法构造两关节机械手的高木 - 关野 (T-S) 模糊模型，并由此构造模糊系统的逆模型. 然后，在提出的模糊逆模型控制结构中，离散时间滑动模控制和时延控制 (TDC) 用于补偿模糊建模误差和外扰动，保证系统的全局稳定性并改进其动态和稳态性能. 系统的稳定性和轨迹误差的收敛性可以通过稳定性定理来证明. 最后，以两关节机械手的轨迹跟随控制为例，揭示了该设计方法的控制性能.

关键词 机械手，模糊聚类，离散滑动模控制，跟随控制.

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