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非线性广义系统的右可逆性[1)]

吴热冰 李春文 刘艳红
(清华大学自动化系 北京 100084)
(E-mail: wurebing98@mails.tsinghua.edu.cn)

摘 要 研究了广义非线性系统的右可逆性,给出构造性的求逆算法以克服以往结果中需求解非线性方程组的困难,从而使得求逆算法对任意足够光滑的非线性广义系统皆为可行.

关键词 广义系统,非线性系统,逆系统
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Right Invertibility of Nonlinear Singular Systems

WU Re-Bing LI Chun-Wen LIU Yan-Hong
(*Department of Automation, Tsinghua University, Beijing* 100084)
(E-mail: wurebing98@mails.tsinghua.edu.cn)

Abstract In this paper the right invertibility of nonlinear singular system is studied. Constructive algorithm is presented to solve the problem of solving implicit nonlinear equations in the existing results, so that the algorithm becomes feasible for any nonlinear singular systems that are smooth enough.

Key words Singular systems, nonlinear systems, inverse system

1 引言

广义系统模型自 20 世纪 70 年代提出以来,由于其能够描述一些常微分方程不能精确描述的系统,引起了广泛关注,研究已初步形成对广义线性系统的包括定性分析、结构特性、反馈控制等在内的理论体系[1],但对广义非线性系统的研究目前还远不完善.在此我们讨论广义非线性系统的右可逆性,这对分析控制系统的结构性质以及进一步的线性化、跟踪、解耦等问题,有着重要的基础意义.这方面的研究始于 20 世纪 80 年代末对连续和离散广义线性系统逆问题[2,3]的讨论.此后,文献[5]将求逆算法推广到广义仿射非线性系统.我们知

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道,可逆性研究的是系统输入输出之间的映射关系,求逆的思路通常是从系统的状态空间表述导出一组包含状态参量的 I/O 准微分方程,再从这组方程出发研究系统的可逆性. 但以往得到这组方程的过程[4,5]都涉及到对某些非线性方程的求解和代入,这使得导出的方程事实上成为隐式而无法应用. 本文针对该问题提出一种新的求逆算法,算法能够给出这些代数方程以及逆系统的显式表达,从而大大提高了算法的可应用范围.

2 递归算法

考虑如下广义非线性系统

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x},\boldsymbol{u},z), \quad 0 = \boldsymbol{g}(\boldsymbol{x},\boldsymbol{u},z), \quad \boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x},\boldsymbol{u},z) \tag{1}$$

其中 $\boldsymbol{x}\in M_0\subset R^n$, $\boldsymbol{z}\in J_0\subset R^l$, $\boldsymbol{u}\in L_0\subset R^m$, $\boldsymbol{y}\in N_0\subset R^r$分别为系统的状态、扩展状态、输入和输出,$\boldsymbol{u}(t)$是时间的光滑函数. 假设式(1)是指数 1 的,即函数 $\boldsymbol{g}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{z})$满足

$$\operatorname{rank}\left[\frac{\partial \boldsymbol{g}(\boldsymbol{x},\boldsymbol{u},z)}{\partial z}\right] = l, \forall\,(\boldsymbol{x},\boldsymbol{u},\boldsymbol{z},\boldsymbol{y}) \in M_0 \times L_0 \times J_0 \times N_0 \tag{2}$$

下面递归地定义一组系统 S_k^g

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},z),\ \boldsymbol{x} \in M_k, \boldsymbol{u} \in L_k \\ 0 = \boldsymbol{g}(\boldsymbol{x},\boldsymbol{u},z),\ \boldsymbol{z} \in J_k \\ \boldsymbol{h}_k(\boldsymbol{x},\boldsymbol{u},z,\boldsymbol{y},\cdots,\boldsymbol{y}^{(k)}) = 0,\ (\boldsymbol{y},\cdots,\boldsymbol{y}^{(k)}) \in N_k \end{cases} \tag{3}$$

其中 $h_0(\boldsymbol{x},\boldsymbol{u},z,\boldsymbol{y})=\boldsymbol{y}-\boldsymbol{h}(\boldsymbol{x},\boldsymbol{u},z)$, $\Omega_k^g=[L_k,M_k,J_k,N_k]$的定义如后所示.

在式(3)中将两个代数方程分别对时间求导,并消去第二个方程中的$\dot{z}$项,则有

$$\left[\frac{\partial \boldsymbol{h}_k}{\partial \boldsymbol{x}} - \frac{\partial \boldsymbol{h}_k}{\partial z}\left(\frac{\partial \boldsymbol{g}}{\partial z}\right)^{-1}\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}}\right]\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},z) + \left[\frac{\partial \boldsymbol{h}_k}{\partial \boldsymbol{u}} - \frac{\partial \boldsymbol{h}_k}{\partial z}\left(\frac{\partial \boldsymbol{g}}{\partial z}\right)^{-1}\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{u}}\right]\dot{\boldsymbol{u}} + \sum_{i=0}^{k}\frac{\partial \boldsymbol{h}_k}{\partial \boldsymbol{y}^{(i)}}\boldsymbol{y}^{(i+1)} = 0 \tag{4}$$

为方便起见,在此引入一个记号. 一般地,对任意函数 $F(\boldsymbol{\mu},\boldsymbol{\varepsilon},\cdot)$,记 $F(\boldsymbol{\mu},\boldsymbol{\varepsilon},\cdot)$对其部分变元 $\boldsymbol{\mu}$ 在对 $\boldsymbol{\varepsilon}$ 的约束 $H(\boldsymbol{\varepsilon},\cdot)=0$ 下的偏导数为

$$\left(\frac{\partial F}{\partial \boldsymbol{\mu}}\right)_H = \left.\frac{\partial F}{\partial \boldsymbol{\mu}}\right|_{H(\cdot)=0} = \frac{\partial F}{\partial \boldsymbol{\mu}} - \frac{\partial F}{\partial \boldsymbol{\varepsilon}}\left(\frac{\partial H}{\partial \boldsymbol{\varepsilon}}\right)^{-1}\frac{\partial H}{\partial \boldsymbol{\mu}} \tag{5}$$

应用这个记号,式(4)可以记为

$$\left(\frac{\partial \boldsymbol{h}_k}{\partial \boldsymbol{x}}\right)_g \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},z) + \left(\frac{\partial \boldsymbol{h}_k}{\partial \boldsymbol{u}}\right)_g \dot{\boldsymbol{u}} + \sum_{i=0}^{k}\frac{\partial \boldsymbol{h}_k}{\partial \boldsymbol{y}^{(i)}}\boldsymbol{y}^{(i+1)} = 0 \tag{6}$$

假设方程(6)在初始点 x_0的某个邻域内有定常秩,即 x_0是一个正则点. 记 $\mu_k=\operatorname{rank}(\partial \boldsymbol{h}_k/\partial \boldsymbol{u})_g$, $\bar{\Omega}_{k+1}^g=[L_{k+1},M_{k+1},J_{k+1},\bar{N}_{k+1}]\subseteq\Omega_k^g$,其中

$$\begin{aligned} M_{k+1} &= \{\boldsymbol{x} \mid \boldsymbol{x} \in M_k, \operatorname{rank}(\partial \boldsymbol{h}_k/\partial \boldsymbol{u})_g = \mu_k\}, \\ L_{k+1} &= \{\boldsymbol{u} \mid \boldsymbol{u} \in L_k, \operatorname{rank}(\partial \boldsymbol{h}_k/\partial \boldsymbol{u})_g = \mu_k\}, \\ J_{k+1} &= \{\boldsymbol{z} \mid \boldsymbol{z} \in J_k, \operatorname{rank}(\partial \boldsymbol{h}_k/\partial \boldsymbol{u})_g = \mu_k\}, \\ \bar{N}_{k+1} &= \{(\boldsymbol{y},\cdots,\boldsymbol{y}^{(k)}) \mid (\boldsymbol{y},\cdots,\boldsymbol{y}^{(k)}) \in N_k, \operatorname{rank}(\partial \boldsymbol{h}_k/\partial \boldsymbol{u})_g = \mu_k\}. \end{aligned}$$

容易证明,联立代数方程组

$$\begin{cases} \boldsymbol{g}(\boldsymbol{x},\boldsymbol{u},z) = 0 \\ \boldsymbol{h}_k(\boldsymbol{x},\boldsymbol{u},z,\boldsymbol{y},\boldsymbol{y}^{(1)},\cdots,\boldsymbol{y}^{(k)}) = 0 \end{cases} \tag{7}$$

中必有包括代数约束 $\boldsymbol{g}(\boldsymbol{x},\boldsymbol{u},z)=0$ 在内的$(l+\mu_k)$个方程,它们对变量$[\boldsymbol{z}^{\mathrm{T}},\boldsymbol{u}^{\mathrm{T}}]^{\mathrm{T}}$互相独立. 记 $\bar{\boldsymbol{h}}_k(\cdot)$(其选取显然不唯一)为这些方程中包含在 $\boldsymbol{h}_k(\cdot)$中的 μ_k个函数,并记 $\boldsymbol{h}_k(\cdot)$中剩

余$(r-\mu_k)$个函数为$\hat{\boldsymbol{h}}_k(\cdot)$，则一定存在$(r-\mu_k)\times\mu_k$维的变换矩阵$E_k^g(\boldsymbol{x},\boldsymbol{u},\boldsymbol{z},\boldsymbol{y},\boldsymbol{y}^{(1)},\cdots,\boldsymbol{y}^{(k)})$，使得在$\bar{\Omega}_{k+1}^g$中有$(\partial\hat{\boldsymbol{h}}_k/\partial\boldsymbol{u})_g=E_k^g(\cdot)(\partial\bar{\boldsymbol{h}}_k/\partial\boldsymbol{u})_g$. 这样，就可以根据以上这些关系将方程(6)中对应于$\hat{\boldsymbol{h}}_k(\cdot)$中的输入导数项消去，得到

$$\hat{\boldsymbol{h}}_k(\boldsymbol{x},\boldsymbol{u},\boldsymbol{z},\boldsymbol{y},\cdots,\boldsymbol{y}^{(k+1)})=\left[\left(\frac{\partial\hat{\boldsymbol{h}}_k}{\partial\boldsymbol{x}}\right)_g-E_k^g(\cdot)\left(\frac{\partial\bar{\boldsymbol{h}}_k}{\partial\boldsymbol{x}}\right)_g\right]\boldsymbol{f}(\cdot)+\sum_{i=0}^{k}\left[\frac{\partial\hat{\boldsymbol{h}}_k}{\partial\boldsymbol{y}^{(i)}}-E_k^g(\cdot)\frac{\partial\bar{\boldsymbol{h}}_k}{\partial\boldsymbol{y}^{(i)}}\right]\boldsymbol{y}^{(i+1)} \tag{8}$$

令$\boldsymbol{h}_{k+1}(\cdot)=[\bar{\boldsymbol{h}}_k^{\mathrm{T}}(\cdot),\hat{\boldsymbol{h}}_k^{\mathrm{T}}(\cdot)]^{\mathrm{T}}$，定义第$(k+1)$个递归系统$S_{k+1}^g$为

$$\begin{cases}\dot{\boldsymbol{x}}=\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{z}),\ \boldsymbol{x}\in M_{k+1},\ \boldsymbol{u}\in L_{k+1}\\ 0=\boldsymbol{g}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{z}),\ \boldsymbol{z}\in J_{k+1}\\ \boldsymbol{h}_{k+1}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{z},\boldsymbol{y},\cdots,\boldsymbol{y}^{(k+1)})=0,\ (\boldsymbol{y},\cdots,\boldsymbol{y}^{(k+1)})\in N_{k+1}\end{cases} \tag{9}$$

其中$N_{k+1}=\{(\boldsymbol{y},\cdots,\boldsymbol{y}^{(k+1)})\,|\,(\boldsymbol{x},\boldsymbol{u},\boldsymbol{y},\cdots,\boldsymbol{y}^{(k)})\in\bar{\Omega}_{k+1}^g\}$.

由于上述过程只涉及代数运算，方程(8)显然是可以显式写出的，所以在得到形如(9)的系统S_{k+1}^g后，递归算法可以继续进行下去. 如果在有限步骤内有$\mu_k=r$，则记满足这个条件的最小整数α为系统的右相对阶数. 至此，可以得到r个对$\boldsymbol{u}$相互独立的方程

$$\boldsymbol{h}_\alpha(\boldsymbol{x},\boldsymbol{u},\boldsymbol{z},\boldsymbol{y},\boldsymbol{y}^{(1)},\cdots,\boldsymbol{y}^{(\alpha)})=0 \tag{10}$$

可以证明，上述方程实际上就是文[5]中得到的相应方程. 这种处理方法还可以推广到指数不为 1 的广义非线性系统，本文限于篇幅不再赘述.

3　求逆算法

定义 1. 称广义系统

$$\dot{\hat{\boldsymbol{x}}}=\boldsymbol{f}(\hat{\boldsymbol{x}},\boldsymbol{v},\hat{\boldsymbol{z}}),\quad 0=\boldsymbol{g}(\hat{\boldsymbol{x}},\boldsymbol{v},\hat{\boldsymbol{z}}),\quad \boldsymbol{w}=\hat{\boldsymbol{h}}(\hat{\boldsymbol{x}},\boldsymbol{v},\hat{\boldsymbol{z}}) \tag{11}$$

为广义系统(1)的一个右逆系统. 若系统(11)是可解的，且对满足一定初始条件的函数$\bar{\boldsymbol{y}}(t)$，当$\boldsymbol{v}(t)=\bar{\boldsymbol{y}}(t)$，$\boldsymbol{w}(t)=\boldsymbol{u}(t)$时，有$\boldsymbol{y}(t)=\bar{\boldsymbol{y}}(t)$.

根据隐函数定理，式(10)必存在关于$\boldsymbol{u}$的解，记为$\boldsymbol{u}=h_\alpha^{-1}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{y},\boldsymbol{y}^{(1)},\cdots,\boldsymbol{y}^{(\alpha)})$. 构造系统$\hat{\Sigma}_0$如下：

$$\begin{cases}\dot{\boldsymbol{x}}_I=f(\boldsymbol{x}_I,\boldsymbol{h}_\alpha^{-1}(\boldsymbol{x}_I,\boldsymbol{z},\boldsymbol{v},\boldsymbol{v}^{(1)},\cdots,\boldsymbol{v}^{(\alpha)}),\boldsymbol{z})\\ 0=\boldsymbol{g}(\boldsymbol{x}_I,\boldsymbol{h}_\alpha^{-1}(\boldsymbol{x}_I,\boldsymbol{z},\boldsymbol{v},\boldsymbol{v}^{(1)},\cdots,\boldsymbol{v}^{(\alpha)}),\boldsymbol{z})\\ \boldsymbol{w}=\boldsymbol{h}_\alpha^{-1}(\boldsymbol{x}_I,\boldsymbol{z},\boldsymbol{v},\boldsymbol{v}^{(1)},\cdots,\boldsymbol{v}^{(\alpha)})\end{cases} \tag{12}$$

其中$\boldsymbol{x}_I\in R^n$，$\boldsymbol{v}\in R^r$和$\boldsymbol{w}\in R^m$分别为系统$\hat{\Sigma}_0$的状态、输入和输出. 由文[5]，系统$\hat{\Sigma}_0$是系统(1)的一个右逆系统. 但由于非线性方程一般不存在解析解，故$\hat{\Sigma}_0$一般不可能精确实现. 为此，进一步将式(10)对t求导

$$\left(\frac{\partial\boldsymbol{h}_\alpha}{\partial\boldsymbol{x}}\right)_g\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{z})+\left(\frac{\partial\boldsymbol{h}_\alpha}{\partial\boldsymbol{u}}\right)_g\dot{\boldsymbol{u}}+\sum_{i=0}^{\alpha}\frac{\partial\boldsymbol{h}_\alpha}{\partial\boldsymbol{y}^{(i)}}\boldsymbol{y}^{(i+1)}=0 \tag{13}$$

则矩阵$(\partial\boldsymbol{h}_\alpha/\partial\boldsymbol{u})_g$在开集$\Omega_\alpha^g$上行满秩，从而有

$$\dot{\boldsymbol{u}}=U_g(\boldsymbol{x},\boldsymbol{u},\boldsymbol{z},\boldsymbol{y},\boldsymbol{y}^{(1)},\cdots,\boldsymbol{y}^{(\alpha+1)}) \tag{14}$$

根据式(14)对系统$\hat{\Sigma}_0$引入m阶动态补偿，即可构造系统$\hat{\Sigma}_1$如下：

$$
\left\{\begin{aligned}
\dot{x}_I &= f(x_I, p, z) \\
\dot{p} &= U_g(x_I, p, z, v, v^{(1)}, \cdots, v^{(\alpha+1)}) \\
0 &= g(x_I, p, z) \\
w &= p
\end{aligned}\right. \tag{15}
$$

其中$(x_I, p) \in R^{n+m}$，$v \in R^r$和$w \in R^m$分别为系统$\hat{\Sigma}_1$的状态、输入和输出.

后面将证明$\hat{\Sigma}_1$也是式(1)的一个逆系统.进一步还有可能降低补偿动态的阶数.具体地，若方程(10)对于输入是部分可解的，即输入可分解为两部分，记为$\bar{u}$和$\hat{u}$，其中$\bar{u}$的个数为m_1，使得从式(10)中可以选择m_1个可显式解出$\bar{u}$的对$\bar{u}$独立的方程组$\bar{h}_\alpha(\cdot)=0$.具体地，记$h_\alpha(\cdot)=[\bar{h}_\alpha^{\mathrm{T}}(\cdot), \hat{h}_\alpha^{\mathrm{T}}(\cdot)]^{\mathrm{T}}$，$u=[\bar{u}^{\mathrm{T}}, \hat{u}^{\mathrm{T}}]^{\mathrm{T}}$，并设方程$\bar{h}_\alpha(\cdot)=0$的显式解为

$$
\bar{u} = \bar{h}_\alpha^{-1}(x, \hat{u}, z, y, y^{(1)}, \cdots, y^{(\alpha)}) \tag{16}
$$

将$[g^{\mathrm{T}}(\cdot), \bar{h}_\alpha^{\mathrm{T}}(\cdot)]^{\mathrm{T}}=0$看作关于$[z^{\mathrm{T}}, \bar{u}^{\mathrm{T}}]^{\mathrm{T}}$的约束，则对方程$\hat{h}_\alpha(\cdot)=0$对$t$求导得

$$
\left(\frac{\partial \hat{h}_\alpha}{\partial \hat{u}}\right)_{g\oplus\bar{h}_\alpha}\dot{\hat{u}} + \left(\frac{\partial \hat{h}_\alpha}{\partial x}\right)_{g\oplus\bar{h}_\alpha} f(x, \bar{h}_\alpha^{-1}(\cdot), \hat{u}, z) + \sum_{i=0}^{\alpha}\left(\frac{\partial \hat{h}_\alpha}{\partial y^{(i)}}\right)_{g\oplus\bar{h}_\alpha} y^{(i+1)} = 0 \tag{17}
$$

可以证明式(17)中$\dot{\hat{u}}$的系数矩阵$(\partial\hat{h}_\alpha/\partial\hat{u})_{g\oplus\bar{h}_\alpha}$是行满秩的，这一点由矩阵$(\partial h_\alpha/\partial u)_g$和$(\partial\bar{h}_\alpha/\partial\bar{u})_g$的行满秩性质及下面线性变换的行秩不变性容易得到

$$
\begin{bmatrix} I_{m_1} & 0 \\ -\left(\frac{\partial \hat{h}_\alpha}{\partial \bar{u}}\right)_g \left(\frac{\partial \bar{h}_\alpha}{\partial \bar{u}}\right)_g^{-1} & I_{r-m_1} \end{bmatrix}
\begin{bmatrix} \left(\frac{\partial \bar{h}_\alpha}{\partial \bar{u}}\right)_g & \left(\frac{\partial \bar{h}_\alpha}{\partial \hat{u}}\right)_g \\ \left(\frac{\partial \hat{h}_\alpha}{\partial \bar{u}}\right)_g & \left(\frac{\partial \hat{h}_\alpha}{\partial \hat{u}}\right)_g \end{bmatrix}
=
\begin{bmatrix} \left(\frac{\partial \bar{h}_\alpha}{\partial \bar{u}}\right)_g & \left(\frac{\partial \bar{h}_\alpha}{\partial \hat{u}}\right)_g \\ 0 & \left(\frac{\partial \hat{h}_\alpha}{\partial \hat{u}}\right)_{g\oplus\bar{h}_\alpha} \end{bmatrix} \tag{18}
$$

因此，方程(17)存在关于$\dot{\hat{u}}$的解

$$
\dot{\hat{u}} = \hat{U}_g(x, \hat{u}, z, y, y^{(1)}, \cdots, y^{(\alpha+1)}) \tag{19}
$$

从而可得具有m_2阶动态补偿的逆系统$\hat{\Sigma}_2$

$$
\left\{\begin{aligned}
\dot{x}_I &= f(x_I, \bar{h}_\alpha^{-1}(x_I, \hat{p}, z, v, v^{(1)}, \cdots, v^{(\alpha+1)}), \hat{p}, z) \\
0 &= g(x_I, \bar{h}_\alpha^{-1}(x_I, \hat{p}, z, v, v^{(1)}, \cdots, v^{(\alpha+1)}), \hat{p}, z) \\
\dot{\hat{p}} &= \hat{U}_g(x_I, \hat{p}, z, v, v^{(1)}, \cdots, v^{(\alpha+1)}) \\
w &= [\bar{h}_\alpha^{-1}(x_I, \hat{p}, z, v, v^{(1)}, \cdots, v^{(\alpha+1)}), \hat{p}]^{\mathrm{T}}
\end{aligned}\right. \tag{20}
$$

其中$(x_I, \hat{p}) \in R^{n+m_2}$，$v \in R^r$和$w \in R^m$分别为系统$\hat{\Sigma}_2$的状态、输入和输出.

定理 1. 系统$\hat{\Sigma}_1$和$\hat{\Sigma}_2$是广义系统(1)的逆系统.

证明. 易见$\hat{\Sigma}_1$是$\hat{\Sigma}_2$的一种特殊形式，因此只对$\hat{\Sigma}_2$给出证明即可.设$v(t)$为满足如下性质的函数：存在$u(t) \in L_0$和$y(t) \in Y = \{y(t) \mid y(t) = y[t; x_0; z_0; u(t)],\ u(t) \in L_0\}$，使得$v^{(i)}(t_0) = y^{(i)}(t_0) = y^{(i)}(t_0; x_0; z_0; u(t_0))$，$0 \leqslant i \leqslant \alpha$.则由前面的讨论易得

$$
\begin{aligned}
\hat{h}_\alpha(x_I, \bar{h}_\alpha^{-1}(x_I, \hat{p}, z, v, v^{(1)}, \cdots, v^{(\alpha)}), \hat{p}, z, v, v^{(1)}, \cdots, v^{(\alpha)}) &= \mathrm{const} \\
\bar{h}_\alpha(x_I, \bar{h}_\alpha^{-1}(x_I, \hat{p}, z, v, v^{(1)}, \cdots, v^{(\alpha)}), \hat{p}, z, v, v^{(1)}, \cdots, v^{(\alpha)}) &= 0
\end{aligned} \tag{21}
$$

其中$\bar{p} = \bar{h}_\alpha^{-1}(z, \hat{p}, v, v^{(1)}, \cdots, v^{(\alpha)})$.取$\hat{\Sigma}_2$的初值为$\hat{p}(t_0) = \hat{u}(t_0)$，$x_I(t_0) = x(t_0)$，则

$$\widehat{h}_\alpha(x_I,\bar{h}_\alpha^{-1}(x_I,\widehat{p},z,v,v^{(1)},\cdots,v^{(\alpha)}),\widehat{p},z,v,v^{(1)},\cdots,v^{(\alpha)})(t_0)=$$
$$\widehat{h}_\alpha(x,\bar{h}_\alpha^{-1}(x,\widehat{u},z,y,y^{(1)},\cdots,y^{(\alpha)}),\widehat{u},z,y,y^{(1)},\cdots,y^{(\alpha)})(t_0)=$$
$$\widehat{h}_\alpha(x,\bar{u},\widehat{u},z,y,y^{(1)},\cdots,y^{(\alpha)})(t_0)=0 \tag{22}$$

由式(20)可得 $\widehat{h}_\alpha(x_I,\bar{p},\widehat{p},z,v,v^{(1)},\cdots,v^{(\alpha)})=0$，故方程 $h_\alpha(z,w,v,v^{(1)},\cdots,v^{(\alpha)})=0$ 成立，并有 $w=h_\alpha^{-1}(x_I,z,v,v^{(1)},\cdots,v^{(\alpha)})$. 故对系统(20)必存在形如式(12)的系统，它们对同一输入产生相同输出. 由于式(12)是逆系统，故 $v(t)$ 可由式(12)重构，从而也可由系统(20)重构. 又系统(20)是指数为 1 的，故可解. 由定义 1，系统(20)是系统(1)的逆系统. 证毕.

4 结束语

本文提出的构造性右逆求逆算法，通过显式递归算法和动态补偿，克服了文[3,6]中算法的形式化缺陷，适用于一般非线性广义系统. 最后指出的是，虽然本文只讨论了指数为 1 的情形，但是仿照文[5]，算法可以容易地推广到一般广义非线性系统.

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吴热冰 1998 年于清华大学获得学士学位，现为清华大学博士研究生，主要研究兴趣为自动控制理论及应用、量子力学控制理论.

(**WU Re-Bing** Received his bachelor degree in 1998 from Tsinghua University. Now he is a Ph. D. candidate at Tsinghua University. His research interests include control theory and its applications, and quantum mechanical control theory.)

李春文 1982 年和 1989 年于清华大学获得学士和博士学位，现为清华大学教授、博士生导师. 主要研究兴趣为自动控制理论及应用、非线性控制的逆系统方法、稳定性的正则判别函数法等.

(**LI Chun-Wen** Received his bachelor degree in 1982 and the Ph. D. degree in 1989, both from Tsinghua University. Now he is the professor of Tsinghua University. His research interests include control theory and its applications, and inverse system method for nonlinear control theory.)

刘艳红 现为清华大学博士研究生，主要研究兴趣为自动控制理论及应用.

(**LIU Yan-Hong** She is now a Ph. D. candidate at Tsinghua University. Her research interests include control theory and its applications, and singular control systems.)