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A PD Type Fuzzy Logic Learning Control Approach for Repeatable Tracking Control Tasks

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Abstract In this paper, we consider repeatable tracking control tasks using a new control approach - PD type Fuzzy Logic Learning Control (FLLC). FLLC integrates two main control strategies: Fuzzy Logic Control as the basic control part and Learning Control as the refinement part. The new FLLC is constructed by simply adding an iterative learning mechanism to a fuzzy PD controller. The incorporation of the learning function into fuzzy PD controllers ensures exact tracking because it completely nullifies the effects of reference signal and periodic disturbances on the tracking error. Through rigorous proof based on energy function and functional analysis, we show that the proposed FLLC system achieves the following novel properties: (1) the tracking error sequence converges uniformly to zero; (2) learning control sequence converges to the desired control profile almost everywhere. Simulation is presented to show the validity of the proposed control method.

Key words Fuzzy logic control, learning control, PD control, repetitive tracking tasks.

1 Introduction

Fuzzy Logic Control (FLC) was originally advocated by Zadeh[1] and Mamdani[2] as a means of collecting human knowledge and experience to deal with uncertainties in the control process. In recent years, Fuzzy Logic Controllers have been widely used for industrial processes owing to their heuristic nature associated with simplicity and effectiveness especially for nonlinear uncertain systems. When a control task is given, a FLC is customized suitable for the task by experienced experts or stilled operators who "learn" to develop the FLC wherever the control task repeats.

The effectiveness of a FLC is mainly because of its structured nonlinearity. Many FLCs are essentially the fuzzy PD, fuzzy PI or fuzzy PID type controllers associated with nonlinear gains[3~6] Because of the nonlinear property of control gains, this kind of FLCs possesses the potential to improve and achieve better system performance. For instance, the farther the system error or change of error is off the equilibrium point, the higher the control gain is. Thus the closed-loop system will respond faster to the set-point change and recover faster from the load disturbance comparing to the conventional PID control.

Generally speaking, the nonlinear structure properties of a heuristically designed FLC cater well to the characteristics of the industrial process under control. However when a new control task is given, it is always imperative to re-adjust the FLC so as to produce reasonable responses. It will naturally take experienced experts or skilled operators long time and great efforts to re-adjust the FLC suitable for the new task through trial and error. A simple and feasible idea is to retain the well established FLC nonlinear structure and only tune the FLC parameters such as the input-output scaling coefficients. FLC auto-tuning methods[6,7] have been proposed

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which work effectively and can satisfy the specified gain margin and phase margin. The main limitation of FLC auto-tuning is that the auto-tuning schemes are only applicable to simple control tasks such as set-point control or step type load disturbance rejection. It would be a challenging work for a FLC to perform complicated tracking control tasks.

One way to partially address the trajectory tracking problem is to offer the FLC system a learning mechanism. Instead of letting experts learn to adjust, it is better to let FLC incorporate adaptive or learning functions to adjust itself to best meet the control task, which would be much more efficient and more accurate. Applying neural network into the FLC[8~11] is one such possible method. However, a neural controller tends to be over complicated due to its large number of nodes and weights. On the other hand, an over simplified neural network may not achieve sufficient tracking precision. As a kind of input-to-output mapping approaches, most neural controllers will reconstruct the whole control system, which is neither practical from control engineering point of view, nor advisable from both FLC point of view where the "good" nonlinear structure is to be retained.

In this paper we propose a new modular approach - Fuzzy Logic Learning Control (FLLC), which integrates two complement control methods, FLC and Iterative Learning Control (ILC), and improves the tracking performance through tasks repetitions.

In the configuration, FLLC consists of two control modules in an additive form: a simple fuzzy logic controller, and a learning mechanism which update the current control profile from the previous control sequence. Such a construction does not alter the existing FLC which is heuristic and proved effective from expert's experience. From the control point of view, FLC provides feedback and the learning mechanism realizes feedforward compensation. The necessity of incorporating learning function into FLC can be justified in terms of "internal model principle". The "internal model" of both reference signal and disturbance has to be incorporated into the feedforward loop because the feedback loop will not be able to provide the necessary control action when the tracking error approaches zero. In other words, when a FLC system is at the equilibrium point, the tracking error is zero and the control feedback part is also zero. However, to track a target trajectory and reject a persistent disturbance, a non-zero control profile will be demanded over the tracking period. Now if the control environment is repeatable or more or less repeatable over a finite duration, the proposed FLLC can provide a simple and effective way to possess such an internal model.

In this paper we limit our discussion to a simple PD type FLC. The proposed FLLC method based on the Fuzzy PD focuses on learning for the repeatable control tasks. The nonrepeatable factors such as random disturbance are assumed to be very small, consequently negligible. Through rigorous proof based on energy function, we show that FLLC system achieves the following novel properties: 1) the tracking error sequence converges uniformly to zero; 2) learning control sequence converges to the desired control profile almost everywhere.

The paper is organized as follows. In Section 2, problem formulation and control objective are introduced. In Section 3, the structure and properties of a PD type FLC are derived. In Section 4, FLLC with learning input updating is introduced with rigorous convergence analysis. In Section 5, simulation work is presented to demonstrate the effectiveness of the proposed scheme.

2 Problem Formulation

In this paper, we consider the second order nonlinear dynamical system described by

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = f(\boldsymbol{x}, t) + b(x_1, t)u, \end{cases} \tag{1}$$

where $x_1(t) = y(t)$, $\boldsymbol{x} = [x_1, x_2] \subseteq \mathcal{R}^2$ is the physically measurable state vector, and u is the control input. $f(\boldsymbol{x}, t)$ and $b(x_1, t)$ are nonlinear uncertain functions.

For this system we make the following assumptions:

A1) $f(\boldsymbol{x}, t)$ is bounded by a known function $f_{\max}(\boldsymbol{x}, t)$, and $0 < b_{\min} \leqslant b(x_1, t) \leqslant b_{\max}$ where $b_{\min}$ and $b_{\max}$ are known constants.

A2) $\forall h \in \{f, b\}$, $h(\boldsymbol{x}, t) \in \mathcal{C}(\mathcal{R}^2 \times [0, T_f])$ and $h(\boldsymbol{x}, t)$ satisfies the *Lipschitz* condition, $\|h(\boldsymbol{x}_1, t) - h(\boldsymbol{x}_2, t)\| \leqslant L_h \|\boldsymbol{x}_1 - \boldsymbol{x}_2\|$, $\forall t \in [0, T_f]$, $\forall \boldsymbol{x}_1, \boldsymbol{x}_2 \subseteq \mathcal{R}^2$ and for some positive constant $L_h < \infty$.

Given a finite initial state $\boldsymbol{x}_i(0)$ and a finite time interval $[0, T_f]$ where i denotes the iteration sequence, the control objective is to design a FLC combined with iterative learning such that, as $i \to \infty$, the system state $\boldsymbol{x}_i$ of the nonlinear uncertain system (1) tracks the desired trajectory $\boldsymbol{x}_d = [x_{d,1}, x_{d,2}] \subseteq \mathcal{R}^2$ which is generated by the following dynamics over $[0, T_f]$

$$\begin{cases} \dot{x}_{d,1} = x_{d,2}, \\ \dot{x}_{d,2} = \beta(\boldsymbol{x}_d, t) + r(t), \end{cases} \tag{2}$$

where $\beta(\boldsymbol{x}_d, t)$ is a known function and $r(t)$ is a reference input. As part of the repeatability condition, the initial states $\boldsymbol{x}_i(0) = \boldsymbol{x}_d(0)$ is available for all trials.

3 Properties of a Fuzzy PD Controller

For a large class of FLCs, fuzzy input variables are the error e and the change of error $\dot{e}$. The fuzzy rule table is then established on the phase plane $(e, \dot{e})$. In essence, these fuzzy controllers are the fuzzy PD type, fuzzy PI type or fuzzy PID type associated with nonlinear gains. Because of the nonlinear property of control gains, FLCs possess the potential to improve and achieve better system performance. Due to the existence of nonlinearity, it is usually difficult to conduct theoretical analysis and find out appropriate design methods.

Consider a typical class of fuzzy PD controllers[12] and the control system is shown in Fig.1 The inputs of the fuzzy rule base are the normalized error $(\omega_e e)$ and the normalized change of error $(\omega_{\dot{e}} \dot{e})$ where ω_e and $\omega_{\dot{e}}$ are weighting factors. The error and the change of error are defined as

$$\begin{cases} e(t) = y_{\text{ref}}(t) - y(t), \\ \dot{e}(t) = \dfrac{\mathrm{d}y_{\text{ref}}(t)}{\mathrm{d}t} - \dfrac{\mathrm{d}y(t)}{\mathrm{d}t}. \end{cases}$$

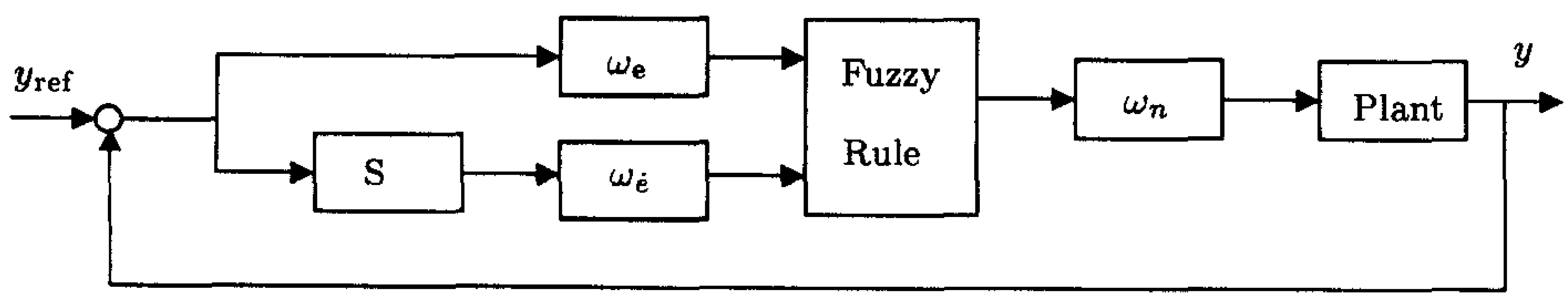

Fig.1 Overall structure of the FLC closed-loop system

The membership functions used to fuzzify the inputs are triangular in shape shown in Fig.2 and, consequently, there are four simple fuzzy control rules (Table 1) used in the FLC. The reasons to choose this type of FLC are 1) theoretical analysis is possible owing to the known structural knowledge; 2) the nonlinearity of the simplest fuzzy PD controller is the strongest in the case of linear distributed rules[13] ; 3) it is highly desirable to make the FLCs as simple as possible and leave the performance refining task to learning control, i.e., maximize the automated learning and minimize the heuristic learning efforts in deriving FLC rules.

The fuzzy output variables have trapezoidal shape membership functions and the lengths of their upper and lower bases are $2A$ and $2H$ (Fig.2), respectively. Zadeh's AND (MIN) and Lukasiewicz's OR are used in the fuzzy inference and the most general inference method, the Mamdamni's minimum inference method[6], is considered in the discussion. By using the center of gravity (COG) defuzzification method, Ying[12] has discussed the control property when $A \leqslant 0.5H$, and the overall control output can be obtained (inside the unsaturated region of the universe of discourse)

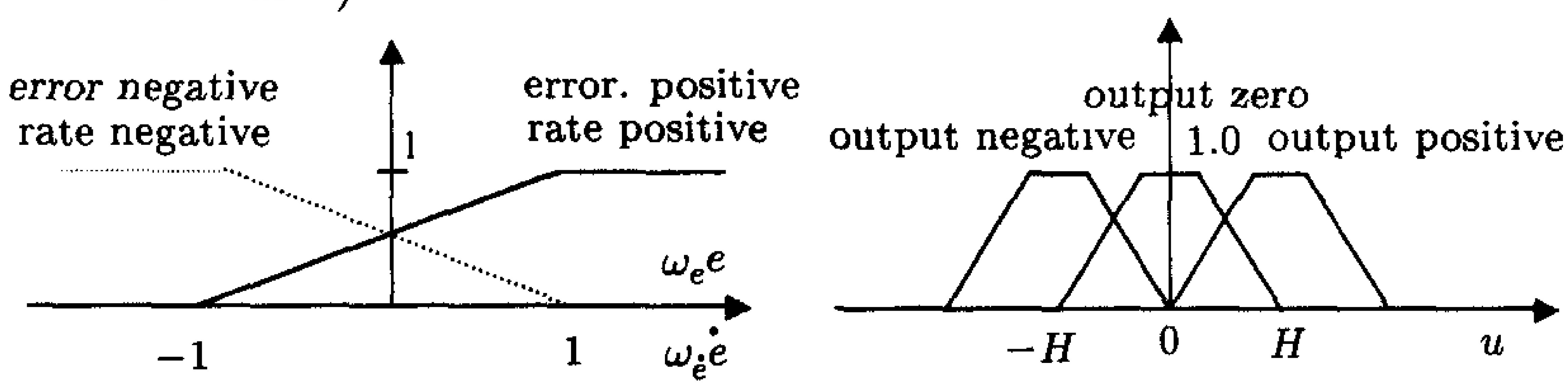

Fig.2 The membership functions of inputs $(\omega_e e, \omega_{\dot e}\dot e)$ and outputs

$$\begin{aligned} u_f =& k(e,\dot e)(\omega_e e + \omega_{\dot e}\dot e), \qquad (3)\\ k(e,\dot e) =& \{0.5H\omega_u[(1+\theta)+0.5(1-\theta)|\omega_e e - \omega_{\dot e}\dot e|]\}\backslash \\ & \{(3+\theta) - [(1+\theta)\max(\omega_e|e|, \omega_{\dot e}|\dot e|) + 0.5(1-\theta)((\omega_e e)^2 + (\omega_{\dot e}\dot e)^2)]\}, \end{aligned}$$

where $\theta = \dfrac{A}{H}$ and $k(e,\dot e)$ is the nonlinear part of the FLC output.

Table 1 Fuzzy control rules. N: Negative; P: Positive; Z:Zero

Rule 1	If error is N and change of error is N, control action is N
Rule 2	If error is N and change of error is P, control action is Z
Rule 3	If error is P and change of error is N, control action is Z
Rule 4	If error is P and change of error is P, control action is P

Let $H = 1$, $\theta = 0.5$ and $\omega_e = \omega_{\dot e} = \omega_u = 1$, the control surface of the FLC and the surface of $k(e,\dot e)$ of the unsaturated region are shown in Fig.3.

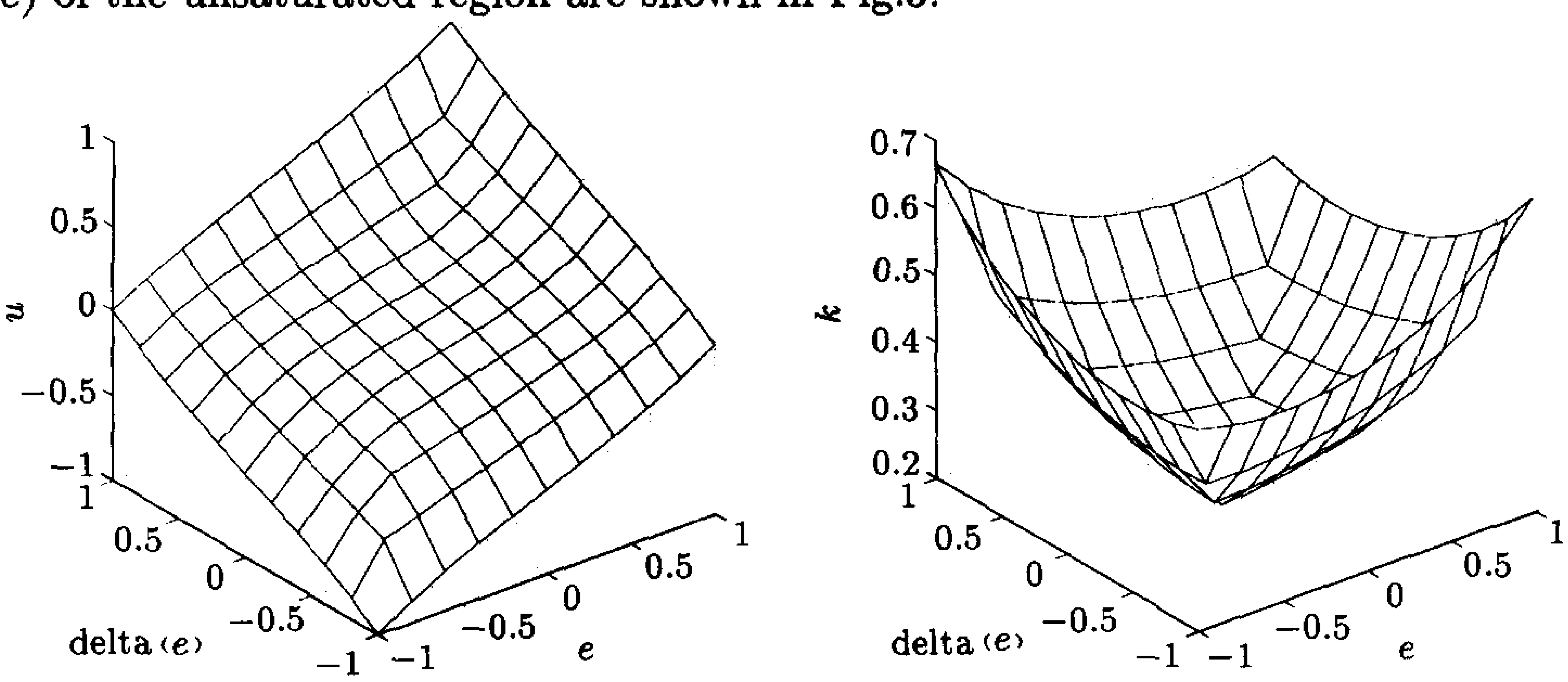

Fig.3 Control surface u (left) and nonlinear control gain k (right) produced by FLC (Unsaturated region)

In fact, from physical meanings, FLCs like this kind are most frequently used. In most cases, we find that the two-dimensional rule table has the skew-symmetry property[14]. The unsaturated phase plane is divided into two semi-planes by means of a switching line. Within the semi-planes positive and negative control outputs are produced respectively. While outside the unsaturated region, the output of FLCs will be partially or fully saturated. In general we can choose ω_e and $\omega_{\dot{e}}$ or use some simple design methods to ensure that the FLC works in the unsaturated region. Based on this, ILC technique is applied to improve the system performance. From (3), the FLC can be expressed as the following control law

$$u_f = k(e,\dot{e})\sigma,$$

where $\sigma = \omega_e e + \omega_{\dot{e}}\dot{e}$, $0 < k_{\min} < k(e,\dot{e}) \leqslant k_{\max}$ and u_f is bounded.

4 Fuzzy Logic Learning Control

The proposed FLLC is given below

$$u_i = u_{f,i} + sat(u_{i-1}), \tag{4}$$

$$u_{f,i} = k_i(e_i,\dot{e}_i)\sigma_i, \tag{5}$$

$$\sigma_i = \omega_e e_i + \omega_{\dot{e}}\dot{e}_i, \tag{6}$$

$$sat(u_{i-1}) = \begin{cases} u_{i-1} & |u_{i-1}| \leqslant u_M, \\ u_M \cdot \mathrm{sign}(u_{i-1}) & |u_{i-1}| > u_M. \end{cases}$$

where i denotes the iteration sequence, u_i is the system input, $u_0 \triangleq 0$ and $0 < k_{\min} < k(e_i,\dot{e}_i) \leqslant k_{\max}$. The saturation bound u_M is sufficiently large such that $u_M \geqslant \sup_{t\in[0,T_f]}|u_d(t)|$ to ensure the learnability. u_M is either a physical process limitation or a virtual saturation bound which can be arbitrarily large but finite. Moreover, the original FLC based on heuristic knowledge should ensure the system stability.

Remark 1. In the proposed FLLC, learning part is an "add-on" function to the existing FLC. In order to retain any advantages of the existing FLC, we only introduce the simplest learning mechanism that is the memory-based. Thus the system stability and the learning convergent rate will depend on the gain property of FLC.

To evaluate the learning performance, the following time-weighted $\mathcal{L}_2$ norm of $u_i - u_d$ is used

$$J_i(t) = \int_0^t e^{-\lambda\tau}[u_i(\tau) - u_d(\tau)]^2 \mathrm{d}\tau. \tag{7}$$

The difference of $J_i(t)$ between two successive trials for $i \geqslant 2$ can be derived as

$$\begin{aligned} \Delta J_i(t) = & J_i(t) - J_{i-1}(t) = \\ & \int_0^t e^{-\lambda\tau}(u_i - u_d)^2\mathrm{d}\tau - \int_0^t e^{-\lambda\tau}(u_{i-1} - u_d)^2\mathrm{d}\tau \leqslant \\ & \int_0^t e^{-\lambda\tau}(u_i - u_d)^2\mathrm{d}\tau - \int_0^t e^{-\lambda\tau}(sat(u_{i-1}) - u_d)^2\mathrm{d}\tau = \\ & \int_0^t e^{-\lambda\tau}[u_i - sat(u_{i-1})][u_i + sat(u_{i-1}) - 2u_d]\mathrm{d}\tau = \\ & \int_0^t e^{-\lambda\tau}\{u_{f,i}^2 + 2u_{f,i}[sat(u_{i-1}) - u_d]\}\mathrm{d}\tau. \end{aligned} \tag{8}$$

First we derive the expressions of $sat(u_{i-1}) - u_d$ and $u_i - u_d$. From (6) we can obtain

$$\sigma_i = \omega_e e_i + \omega_{\dot{e}}\dot{e}_i = \omega_e(x_{d,1} - x_{i,1}) + \omega_{\dot{e}}(x_{d,2} - x_{i,2}). \tag{9}$$

Differentiating (9) with respect to t yields

$$\dot{\sigma}_i = \omega_e(\dot{x}_{d,1} - \dot{x}_{i,1}) + \omega_{\dot{e}}(\dot{x}_{d,2} - \dot{x}_{i,2}). \tag{10}$$

Substituting (1) and (2) into (10) gives

$$\dot{\sigma}_i = \omega_e(x_{d,2} - x_{i,2}) + \omega_{\dot{e}}(\beta(\boldsymbol{x}_d, t) + r(t)) - h_i - l_i u_i = g_i - h_i - l_i u_i$$

where $h_i \triangleq \omega_{\dot{e}} f(\boldsymbol{x}_i, t)$, $l_i \triangleq \omega_{\dot{e}} b(x_{i,1}, t)$, $g_i \triangleq \omega_e(x_{d,2} - x_{i,2}) + g_d$, $g_d \triangleq \omega_{\dot{e}}\beta(\boldsymbol{x}_d, t) + \omega_{\dot{e}} r(t)$.
Then

$$u_i = -l_i^{-1}\dot{\sigma}_i + l_i^{-1} g_i - l_i^{-1} h_i. \tag{11}$$

We can derive the desired control, which makes $\dot{\sigma}_i = 0$, consequently $\sigma_i(t) = \sigma_i(0) = 0$, is

$$u_d = l_d^{-1} g_d - l_d^{-1} h_d, \tag{12}$$

where $h_d \triangleq \omega_{\dot{e}} f(\boldsymbol{x}_d, t)$, $l_d \triangleq \omega_{\dot{e}} b(x_{d,1}, t)$.
It can be derived that

$$u_i - u_d = -\, l_i^{-1}\dot{\sigma}_i - \gamma_i, \tag{13}$$

$$sat(u_{i-1}) - u_d = u_i - u_{f,i} - u_d = -u_{f,i} - l_i^{-1}\dot{\sigma}_i - \gamma_i, \tag{14}$$

where

$$\gamma_i = (l_d^{-1} g_d - l_i^{-1} g_i) - (l_d^{-1} h_d - l_i^{-1} h_i). \tag{15}$$

Here γ_i is the equivalent system uncertainties. We can derive that (See Appendix)

$$|\gamma_i| \leqslant c\|\boldsymbol{x}_d - \boldsymbol{x}_i\|, \tag{16}$$

where c is a positive constant.

To facilitate FLLC analysis, here we give three propositions which reveal the bound relationship among the quantities σ_i, $\boldsymbol{x}_i$, and γ_i.

Proposition 1. For system (1) given the desired trajectory in (2) and fuzzy logic controller (5), the following stands

$$\dot{\boldsymbol{x}}_d - \dot{\boldsymbol{x}}_i = A(\boldsymbol{x}_d - \boldsymbol{x}_i) + \boldsymbol{b}\dot{\sigma}_i, \tag{17}$$

$$\|\boldsymbol{x}_d - \boldsymbol{x}_i\| \leqslant \omega_{\dot{e}}^{-1}\|A\| \int_0^t |\sigma_i(\tau)| e^{\|A\|(t-\tau)} \mathrm{d}\tau + \omega_{\dot{e}}^{-1}|\sigma_i| \tag{18}$$

where $A = \begin{bmatrix} 0 & 1 \\ 0 & -\omega_{\dot{e}}^{-1}\omega_e \end{bmatrix}$, $\boldsymbol{b} = [0 \quad \omega_{\dot{e}}^{-1}]^{\mathrm{T}}$.

Proposition 2. For system (1) given the desired trajectory in (2) and fuzzy controller (5), the following stands

$$\int_0^t e^{-\lambda\tau} |\sigma_i(\tau)| \cdot |\gamma_i(\tau)| \mathrm{d}\tau \leqslant \left(c\omega_{\dot{e}}^{-1} + c\omega_{\dot{e}}^{-1}\|A\| T_f e^{\|A\| T_f}\right) \int_0^t e^{-\lambda\tau}\sigma_i^2(\tau)\mathrm{d}\tau. \tag{19}$$

Proposition 3. For system (1) given the desired trajectory in (2), under the control laws (4) and (5), the following stands

$$\|\boldsymbol{x}_i - \boldsymbol{x}_d\| \leqslant b_{\max} e^{lT_f} T_f^{\frac{1}{2}} J_i^{\frac{1}{2}}(T_f), \tag{20}$$

$$|\sigma_i| \leqslant b_{\max}(\omega_e^2 + \omega_{\dot{e}}^2)^{\frac{1}{2}} e^{lT_f} T_f^{\frac{1}{2}} J_i^{\frac{1}{2}}(T_f). \tag{21}$$

where $l \triangleq \max(\lambda, \|A\| + b_{\max} c)$.

Proof of all the propositions is shown in appendix.

Theorem 1. Consider the nonlinear system (1) satisfying assumptions A_1, A_2 and giving a desired trajectory $\boldsymbol{x}_d$ defined by (2). Under the control and learning laws (4) and (5), as $i \to \infty$, u_i converges to u_d almost everywhere, σ_i converges uniformly to 0 and $\boldsymbol{x}_i$ converges uniformly to $\boldsymbol{x}_d$.

Proof. Substituting (14) into (8) gives

$$\Delta J_i(t) \leqslant \int_0^t e^{-\lambda\tau}(-u_{f,i}^2 - 2u_{f,i}l_i^{-1}\dot{\sigma}_i - 2u_{f,i}\gamma_i)\mathrm{d}\tau.$$

Then $\Delta J_i(t) \leqslant \int_0^t e^{-\lambda\tau}(-2k_i l_i^{-1}\sigma_i\dot{\sigma}_i - 2k_i\sigma_i\gamma_i)\mathrm{d}\tau = -\int_0^t 2e^{-\lambda\tau}k_i l_i^{-1}\sigma_i\dot{\sigma}_i\mathrm{d}\tau - \int_0^t 2e^{-\lambda\tau}k_i\sigma_i\gamma_i\mathrm{d}\tau$. Since $b(x_1,t)$ is bounded, $l_i \in [\omega_{\dot{e}}b_{\min}, \omega_{\dot{e}}b_{\max}]$,

$$\Delta J_i(t) \leqslant -k_{\min}(\omega_{\dot{e}}b_{\max})^{-1}\int_0^{\sigma_i^2(t)} e^{-\lambda t}\mathrm{d}\sigma_i^2 + 2k_{\max}\int_0^t e^{-\lambda\tau}|\sigma_i||\gamma_i|\mathrm{d}\tau \leqslant$$

$$-k_{\min}(\omega_{\dot{e}}b_{\max})^{-1}e^{-\lambda t}\sigma_i^2 - \lambda k_{\min}(\omega_{\dot{e}}b_{\max})^{-1}\int_0^t e^{-\lambda\tau}\sigma_i^2\mathrm{d}\tau + 2k_{\max}\int_0^t e^{-\lambda\tau}|\sigma_i||\gamma_i|\mathrm{d}\tau.$$

Using proposition 2, we can derive

$$\begin{aligned}\Delta J_i(t) \leqslant & -k_{\min}(\omega_{\dot{e}}b_{\max})^{-1}e^{-\lambda t}\sigma_i^2 - \lambda k_{\min}(\omega_{\dot{e}}b_{\max})^{-1}\int_0^t e^{-\lambda\tau}\sigma_i^2\mathrm{d}\tau + \\ & 2k_{\max}(c\omega_{\dot{e}}^{-1} + c\omega_{\dot{e}}^{-1}\|A\|T_f e^{\|A\|T_f})\int_0^t e^{-\lambda\tau}\sigma_i^2(\tau)\mathrm{d}\tau = -k_{\min}(\omega_{\dot{e}}b_{\max})^{-1}e^{-\lambda t}\sigma_i^2 - \\ & k_{\min}(\omega_{\dot{e}}b_{\max})^{-1}\int_0^t [\lambda - 2k_{\max}k_{\min}^{-1}b_{\max}(c + c\|A\|T_f e^{\|A\|T_f})]e^{-\lambda\tau}\sigma_i^2(\tau)\mathrm{d}\tau.\end{aligned}$$

Since $2k_{\max}k_{\min}^{-1}b_{\max}(c + c\|A\|T_f e^{\|A\|T_f})$ is a finite positive constant, there exists a sufficiently large λ such that $\lambda \geqslant 2k_{\max}k_{\min}^{-1}b_{\max}(c + c\|A\|T_f e^{\|A\|T_f}) + k_{\min}^{-1}(\omega_{\dot{e}}b_{\max})$ to ensure

$$\Delta J_i(t) \leqslant -k_{\min}(\omega_{\dot{e}}b_{\max})^{-1}e^{-\lambda t}\sigma_i^2 - \int_0^t e^{-\lambda\tau}\sigma_i^2\mathrm{d}\tau. \tag{22}$$

According to (7), $J_i(t) \geqslant 0$, then from (22) we have

$$0 \leqslant J_i(t) \leqslant J_{i-1}(t) \leqslant \ldots \leqslant J_1(t).$$

From (22), taking the summation over $j = 1$ to i obtains

$$J_i(t) - J_1(t) \leqslant -k_{\min}(\omega_{\dot{e}}b_{\max})^{-1}e^{-\lambda t}\sum_{j=1}^{i}\sigma_j^2(t).$$

As $J_i \geqslant 0$, we have from the above that

$$\lim_{i\to\infty}\sum_{j=1}^{i}\sigma_j^2(t) \leqslant k_{\min}{}^{-1}(\omega_{\dot{e}}b_{\max})e^{\lambda t}J_1(t),$$

which concludes that

$$\lim_{i\to\infty}\sigma_i(t) = 0, \quad \forall t \in [0, T_f].$$

As $\lim_{i\to\infty}\sigma_i(t) = 0$, from (5) and (19), $\lim_{i\to\infty}u_{f,i} = 0$ and $\lim_{i\to\infty}\boldsymbol{x}_i = \boldsymbol{x}_d$. According to (16), $\lim_{i\to\infty}\gamma_i = 0$.

From (13) and (7), it can be obtained

$$\begin{aligned}\lim_{i\to\infty}J_i(t) = \lim_{i\to\infty}\int_0^t e^{-\lambda\tau}[u_i(\tau) - u_d(\tau)]^2\mathrm{d}\tau = \lim_{i\to\infty}\int_0^t e^{-\lambda\tau}(\omega_{\dot{e}}b_i)^{-2}\dot{\sigma}_i^2\mathrm{d}\tau = \\ \lim_{i\to\infty}\int_0^{\sigma(t)} e^{-\lambda\tau}(\omega_{\dot{e}}b_i)^{-2}\dot{\sigma}_i\mathrm{d}\sigma_i.\end{aligned} \tag{23}$$

From (5) we can obtain

$$\dot{\sigma}_i = \omega_e \dot{e}_i + \omega_{\dot{e}} \ddot{e}_i = \omega_e \dot{x}_{i,1} + \omega_{\dot{e}} \dot{x}_{i,2} = \omega_e x_{i,2} + \omega_{\dot{e}}(f + bu_i). \tag{24}$$

As u_i is bounded, $\dot{\sigma}_i$ is bounded. Since $e^{-\lambda\tau}(\omega_{\dot{e}} b_i)^{-2}\dot{\sigma}_i$ is bounded and $\lim_{i\to\infty} \sigma_i(t) = 0$, we can obtain

$$\lim_{i\to\infty} J_i(t) = 0, \qquad \forall t \in [0, T_f] \tag{25}$$

and u_i converges to u_d almost everywhere.

From Proposition 3 and (25), both $\boldsymbol{x}_i$ and σ_i are bounded. We have

$$\lim_{i\to\infty} \sup_{t\in[0,\ T_f]} |\sigma_i| = 0, \quad \lim_{i\to\infty} \sup_{t\in[0,\ T_f]} \|\boldsymbol{x}_d - \boldsymbol{x}_i\| = 0,$$

σ_i and $\boldsymbol{x}_i$ are *uniformly* convergent. From (5) and $e_i(0) = 0$, by solving the differential equation (1) with the FLLC, we can reach that e_i and $\dot{e}_i$ *uniformly* converge to zero as $i \to \infty$. □

5 Illustration Example

In this section, the FLLC will be applied to a simple nonlinear mass-spring-damper mechanical system[15] as shown in Fig.4 . The behavior of this system can be described by

$$\begin{aligned} &M\ddot{x} + g(x, \dot{x}) + f(x) = \phi(\dot{x})u, \\ &g(x, \dot{x}) = D(c_1 x + c_2 \dot{x} + c_3 \dot{x}^3), \\ &f(x) = c_4 x + c_5 x^3, \\ &\phi(\dot{x}) = 1 + c_6 \dot{x} + c_7 \dot{x}^3 + c_8 \sin \dot{x}, \end{aligned} \tag{26}$$

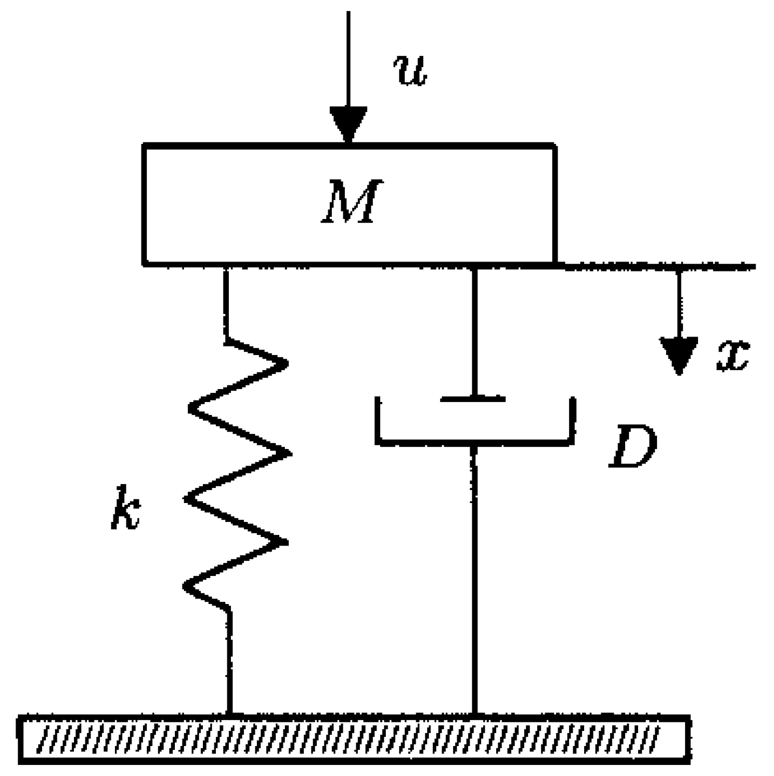

Fig.4 Mass-spring-damper system

where M is the mass and u is the force. $f(x)$, $g(x, \dot{x})$ and $\phi(\dot{x})$ describe the spring, the damper and the input nonlinearity and uncertainty respectively. The control task is to track the desired trajectory

$$x_d = 1.728 \times \sin^3(0.7t) \quad t \in [0, 9].$$

Case 1. The system parameters are set to be: $M = 1.0$, $D = 1.0$, $c_1 = 0.01$, $c_2 = 0.1$, $c_3 = 0$, $c_4 = 0.01$, $c_5 = 0$, $c_6 = 0.01$, $c_7 = 0$, $c_8 = -0.01$. The plant (26) can be rewritten as

$$\ddot{x} = -0.1\dot{x} - 0.02x + (1 - 0.01 \sin \dot{x} + 0.01\dot{x})u. \tag{27}$$

Considering the FLC described in Section 3, thus six sets of parameters are chosen without much elaborations, since there is no systematic way to fine tune the three FLC parameters (ω_e, $\omega_{\dot{e}}$, ω_u). ILC is further added to the FLC to improve the tracking performance. To demonstrate the effectiveness of the proposed FLLC, the maximum tracking error of each iteration ($e_{\max}$) is recorded and shown in Table 2.

Table 2 Comparison of FLLC with Different FLC Parameters

ω_e	$\omega_{\dot{e}}$	ω_u	FLC Error	$e_{\max}(i=1)$	$e_{\max}(i=2)$	$e_{\max}$ $(i=3)$	$e_{\max}$ $(i=4)$	Iter. Times ($e_{\max} < 10^{-3}$)
5	5	4	0.1721	0.0413	0.0166	0.0067	0.0039	17
6	6	5	0.1101	0.0191	0.0040	0.0018	0.0008	4
7	7	5	0.0923	0.0149	0.0028	0.0010	0.0005	4
8	8	6	0.0713	0.0076	0.0011	0.0003	0.0002	3
4	4	6	0.1440	0.0313	0.0089	0.0041	0.0017	6
8	8	4	0.2350	0.0593	0.0261	0.0082	0.0073	19

We can see that ILC can dramatically reduce the tracking error even if only one iteration is performed. After a number of learning iterations, the maximum tracking error can be reduced to less than 0.001 for all six sets of FLC parameters.

Case 2. The system parameters are chosen to be: $M = 1.0$, $D = 1.0$, $c_1 = 0.02$, $c_2 = 0.1$, $c_3 = 0.15$, $c_4 = 0.01$, $c_5 = 0.86$, $c_6 = 0.01$, $c_7 = 0$, $c_8 = 0.02$. The plant (28) can be rewritten as

$$\ddot{x} = -0.1\dot{x} - 0.03x - 0.15\dot{x}^3 - 0.86x^3 + (1 - 0.02\sin\dot{x} + 0.01\dot{x})u.$$

Applying FLLC with the same parameters as in Case 1, the tracking control results are summarized in Table 3.

Table 3 Comparison of FLLC with different FLC parameters

ω_e	$\omega_{\dot{e}}$	ω_u	FLC Error	$e_{\max}(i=1)$	$e_{\max}(i=2)$	$e_{\max}$ $(i=3)$	$e_{\max}$ $(i=4)$	Iter. Times ($e_{\max} < 10^{-3}$)
5	5	4	0.1373	0.0410	0.1379	0.0068	0.0035	10
6	6	5	0.0984	0.0234	0.0062	0.0020	0.0009	4
7	7	5	0.0844	0.0182	0.0044	0.0012	0.0006	4
8	8	6	0.0647	0.0112	0.0021	0.0004	0.0002	3
4	4	6	0.1286	0.0349	0.0104	0.0043	0.0020	6
8	8	4	0.0863	0.0197	0.0051	0.0015	0.0008	4

The FLLC can work equally well in the presence of stronger nonlinearities.

Case 3. From Table 1 and Table 2 , we can observe that the larger the (ω_e, $\omega_{\dot{e}}$, ω_u), the smaller the FLC tracking error. However, it is not advisable to reduce the tracking error only through increasing the FLC gains. Due to the discrete-time control nature, the FLC gains are limited by the system sampling period. Again consider the plant given in Case 1, but with a larger sampling period of 10ms.Choosing $\omega_e = 7$, $\omega_{\dot{e}} = 7$, $\omega_u = 5$ and applying FLLC, Fig.5 shows the control signal and the tracking error after six iterations. For comparison purpose, choosing higher FLC gains $\omega_e = 15$, $\omega_{\dot{e}} = 15$, $\omega_u = 20$ and only applying FLC, Fig.6 shows the control signal and the tracking error.

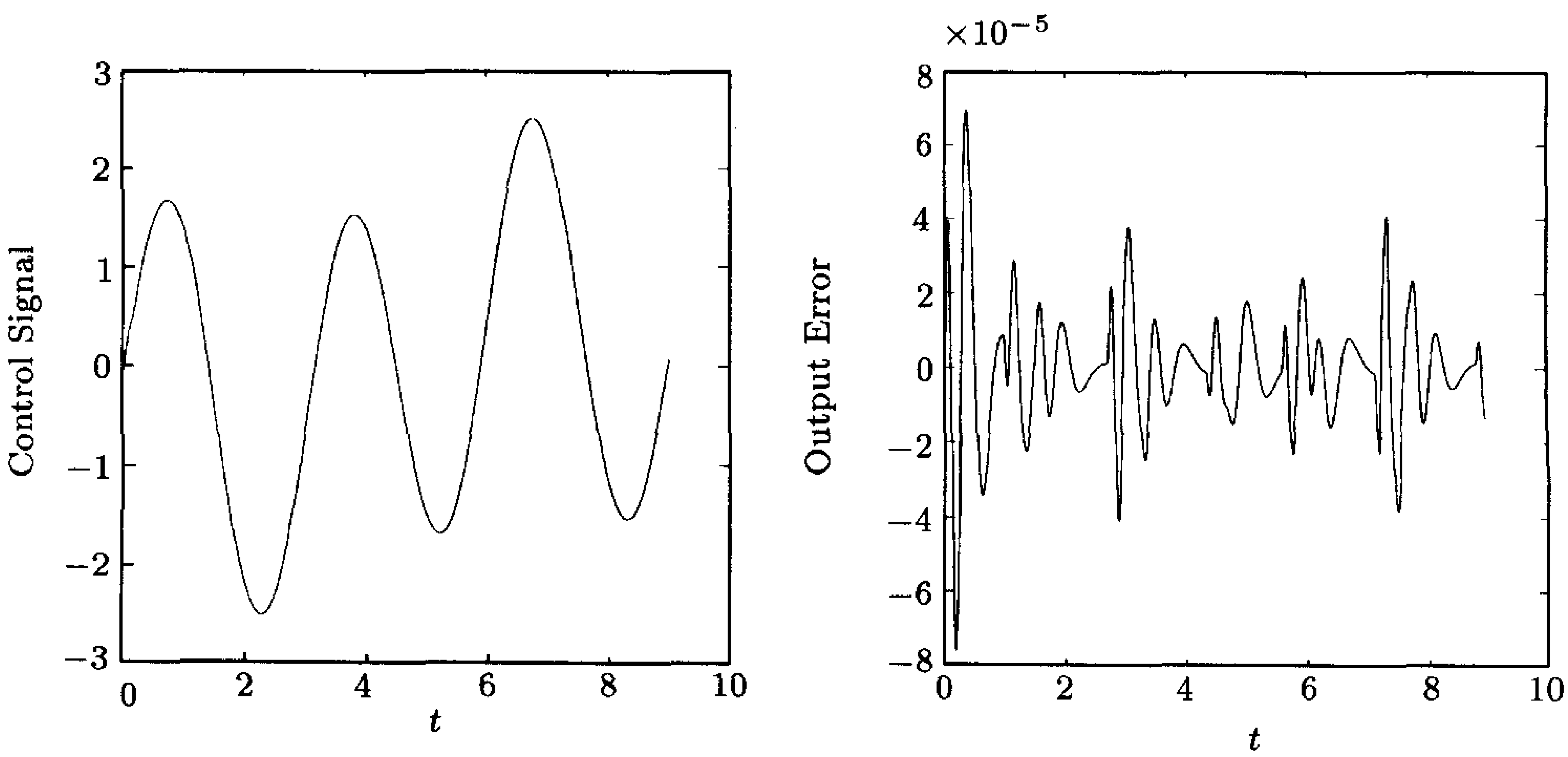

Fig.5 Control signal and output error of FLLC with low gain

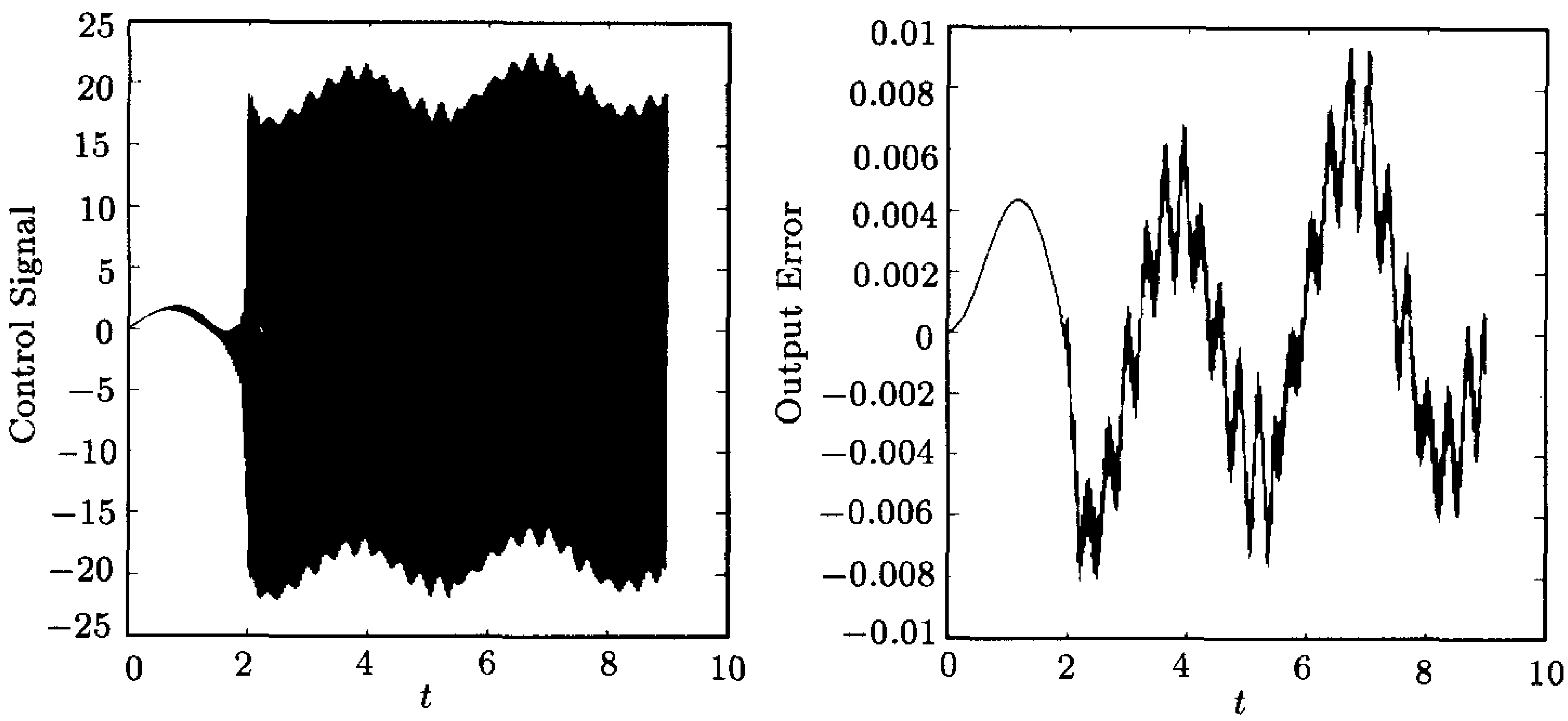

Fig.6 Control signal and output error of FLC with high gain

Due to large (ω_e, $\omega_{\dot{e}}$, ω_u), control chattering phenomenon occurs (Fig.6), yet the tracking error is about 100 times larger than that of FLLC. Obviously, in practice it is difficult for such a simple FLC to obtain accurate tracking performance. FLLC, on the other hand, can obtain much better tracking performance and much smoother control profiles with only a few iterations.

6 Conclusion

In this paper, a novel control scheme - Fuzzy Logic Learning Control (FLLC) is proposed for repeatable tracking control tasks. The new FLLC is constructed by simply adding an iterative learning mechanism to FLC without changing the original structure. By rigorous proof it shows that the FLLC method possesses the capability of improving control performance through learning iterations. Using FLLC, tracking error uniformly converges to zero, system states converge to the desired trajectory and learning control profile converges to the desired one almost everywhere.

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Appendix: Proof of propositions

Proof of Proposition 1. Combining(1) and (2) yields

$$\dot{x}_{d,1} - \dot{x}_{i,1} = x_{d,2} - x_{i,2}. \tag{28}$$

Rearranging (10) gives

$$\dot{x}_{d,2} - \dot{x}_{i,2} = -\omega_{\dot{e}}^{-1}\omega_e(x_{d,2} - x_{i,2}) + \omega_{\dot{e}}^{-1}\dot{\sigma}_i. \tag{29}$$

Combining (28) and (29) gives (17). Integrating both sides of (17) and noticing $\sigma_i(0) = 0$ and $\boldsymbol{x}_i(0) = \boldsymbol{x}_d(0)$ obtain

$$\boldsymbol{x}_d - \boldsymbol{x}_i = A\int_0^t (\boldsymbol{x}_d - \boldsymbol{x}_i)\,\mathrm{d}\tau + \boldsymbol{b}\sigma_i.$$

Taking the norm of the above and since $\|\boldsymbol{b}\| = \omega_{\dot{e}}^{-1}$, the following stands

$$\|\boldsymbol{x}_d - \boldsymbol{x}_i\| \leqslant \|A\|\int_0^t \|\boldsymbol{x}_d - \boldsymbol{x}_i\|\mathrm{d}\tau + \omega_{\dot{e}}^{-1}|\sigma_i|.$$

Applying *Bellman-Gronwell Lemma 1*[16], we can obtain (18). □

Derivations for the bound of γ_i From (15) we know

$$\gamma_i = (l_d^{-1}g_d - l_i^{-1}g_i) - (l_d^{-1}h_d - l_i^{-1}h_i).$$

It can be derived that

$$\begin{aligned}|\gamma_i| \leqslant & |l_d^{-1}g_d - l_i^{-1}g_d + l_i^{-1}g_d - l_i^{-1}g_i| + |l_d^{-1}h_d - l_i^{-1}h_d + l_i^{-1}h_d - l_i^{-1}h_i| \leqslant \\ & l_i^{-1}l_d^{-1}|l_d - l_i|\cdot|g_d| + l_i^{-1}|g_d - g_i| + l_i^{-1}l_d^{-1}|l_d - l_i|\cdot|h_d| + l_i^{-1}|h_d - h_i|.\end{aligned}$$

Since $g_d - g_i = \omega_e(x_{d,2} - x_{i,2})$, we have $|g_d - g_i| \leqslant \omega_e\|\boldsymbol{x}_d - \boldsymbol{x}_i\|$.

Under assumption A1), b_i^{-1} is bounded by $b_{\min}^{-1}$, so l_i^{-1} and l_d^{-1} are also bounded by $(\omega_{\dot{e}}b_{\min})^{-1}$. Since h_d and g_d are both bounded, we denote that $\bar{h}_d = \sup_{t\in[0,\ T_f]} h_d(t)$ and $\bar{g}_d = \sup_{t\in[0,\ T_f]} g_d(t)$. Using the *Lipschitz*-condition described in A2) we can obtain

$$|\gamma_i| \leqslant c\|\boldsymbol{x}_d - \boldsymbol{x}_i\|,$$

where $c = \omega_{\dot{e}}b_{\min}^{-1}(L_l\omega_{\dot{e}}b_{\min}^{-1}\bar{g}_d + \omega_{\dot{e}} + \omega_{\dot{e}}b_{\min}^{-1}L_l\bar{h}_d + L_h)$ which is a finite positive constant. □

Proof of Proposition 2. It can be obtained from (16) and(18) that

$$|\gamma_i| \leqslant c\omega_{\dot{e}}^{-1}\|A\|\int_0^t |\sigma_i(\tau)|\mathrm{e}^{\|A\|(t-\tau)}\mathrm{d}\tau + c\omega_{\dot{e}}^{-1}|\sigma_i(t)|. \tag{30}$$

Since $0 \leqslant \nu \leqslant \tau \leqslant t \leqslant T_f$, then $0 \leqslant \tau - \nu \leqslant \tau \leqslant T_f$ and $-\frac{\lambda}{2}\tau \leqslant -\frac{\lambda}{2}\nu$. Using *Hölder inequality*[16], it can be obtained from (30) that

$$
\begin{aligned}
&\int_0^t e^{-\lambda\tau}|\sigma_i(\tau)|\cdot|\gamma_i(\tau)|\mathrm{d}\tau \leqslant \\
&\int_0^t \left[\int_0^\tau c\omega_{\dot{e}}^{-1}\|A\|e^{\|A\|(\tau-\nu)}e^{-\lambda\tau}|\sigma_i(\tau)|\cdot|\sigma_i(\nu)|\mathrm{d}\nu\right]\mathrm{d}\tau + \int_0^t c\omega_{\dot{e}}^{-1}e^{-\lambda\tau}\sigma_i^2(\tau)\mathrm{d}\tau \leqslant \\
&c\omega_{\dot{e}}^{-1}\|A\|e^{\|A\|T_f}\int_0^t\left[\int_0^\tau e^{-\lambda\tau}|\sigma_i(\tau)|\cdot|\sigma_i(\nu)|\mathrm{d}\nu\right]\mathrm{d}\tau + \int_0^t c\omega_{\dot{e}}^{-1}e^{-\lambda\tau}\sigma_i^2(\tau)\mathrm{d}\tau = \\
&c\omega_{\dot{e}}^{-1}\|A\|e^{\|A\|T_f}\int_0^t e^{-\frac{\lambda}{2}\tau}|\sigma_i(\tau)|\left[\int_0^t e^{-\frac{\lambda}{2}\tau}|\sigma_i(\nu)|\mathrm{d}\nu\right]\mathrm{d}\tau + \int_0^t c\omega_{\dot{e}}^{-1}e^{-\lambda\tau}\sigma_i^2(\tau)\mathrm{d}\tau \leqslant \\
&c\omega_{\dot{e}}^{-1}\|A\|e^{\|A\|T_f}\int_0^t e^{-\frac{\lambda}{2}\tau}|\sigma_i(\tau)|\left[\int_0^t e^{-\frac{\lambda}{2}\nu}|\sigma_i(\nu)|\mathrm{d}\nu\right]\mathrm{d}\tau + \int_0^t c\omega_{\dot{e}}^{-1}e^{-\lambda\tau}\sigma_i^2(\tau)\mathrm{d}\tau = \\
&c\omega_{\dot{e}}^{-1}\|A\|e^{\|A\|T_f}\left[\int_0^t e^{-\frac{\lambda}{2}\tau}|\sigma_i(\tau)|\mathrm{d}\tau\right]^2 + \int_0^t c\omega_{\dot{e}}^{-1}e^{-\lambda\tau}\sigma_i^2(\tau)\mathrm{d}\tau \leqslant \\
&c\omega_{\dot{e}}^{-1}\|A\|e^{\|A\|T_f}\left[\int_0^t e^{-\lambda\tau}\sigma_i^2(\tau)\mathrm{d}\tau\right]\left[\int_0^t 1^2\mathrm{d}\tau\right] + \int_0^t c\omega_{\dot{e}}^{-1}e^{-\lambda\tau}\sigma_i^2(\tau)\mathrm{d}\tau \leqslant \\
&\left(c\omega_{\dot{e}}^{-1} + c\omega_{\dot{e}}^{-1}\|A\|T_f e^{\|A\|T_f}\right)\int_0^t e^{-\lambda\tau}\sigma_i^2(\tau)\mathrm{d}\tau. \qquad \square
\end{aligned}
$$

Proof of Proposition 3: From (13), it can be obtained that

$$\dot{\sigma}_i = l_i u_d - l_i u_i - l_i\gamma_i.$$

Substituting the above into (17) yields

$$\dot{\boldsymbol{x}}_d - \dot{\boldsymbol{x}}_i = A(\boldsymbol{x}_d - \boldsymbol{x}_i) + \boldsymbol{b}(l_i u_d - l_i u_i - l_i\gamma_i).$$

Since $\boldsymbol{x}_i(0) = \boldsymbol{x}_d(0)$, $\|\boldsymbol{b}\| = \omega_{\dot{e}}^{-1}$ and from assumption A1), $l_i \leqslant \omega_{\dot{e}} b_{\max}$, it can be obtained from the above that

$$\|\boldsymbol{x}_d - \boldsymbol{x}_i\| \leqslant \|A\|\int_0^t \|\boldsymbol{x}_d - \boldsymbol{x}_i\|\mathrm{d}\tau + b_{\max}\int_0^t |u_i - u_d|\mathrm{d}\tau + b_{\max}\int_0^t |\gamma_i|\mathrm{d}\tau$$

Substituting (16) into (31) yields

$$\|\boldsymbol{x}_d - \boldsymbol{x}_i\| \leqslant (\|A\| + b_{\max}c)\int_0^t \|\boldsymbol{x}_d - \boldsymbol{x}_i\|\mathrm{d}\tau + b_{\max}\int_0^t |u_{l,i} - u_d|\mathrm{d}\tau.$$

It can be obtained by the *Hölder inequality* and *Bellman-Gronwall Lemma 2*[16] that

$$
\begin{aligned}
\|\boldsymbol{x}_d - \boldsymbol{x}_i\| \leqslant & l_1\int_0^t \|\boldsymbol{x}_d - \boldsymbol{x}_i\|\mathrm{d}\tau + b_{\max}\int_0^t |u_{l,i} - u_d|\mathrm{d}\tau \leqslant \\
&\int_0^t b_{\max}e^{l(t-\tau)}|u_{l,i} - u_d|\mathrm{d}\tau \leqslant b_{\max}e^{lT_f}\int_0^{T_f} e^{-l\tau}|u_{l,i} - u_d|\mathrm{d}\tau \leqslant \\
&b_{\max}e^{lT_f}\left[\int_0^{T_f} e^{-2l\tau}(u_{l,i} - u_d)^2\mathrm{d}\tau\right]^{\frac{1}{2}}\left[\int_0^{T_f} 1^2\mathrm{d}\tau\right]^{\frac{1}{2}} \leqslant \\
&b_{\max}e^{lT_f}T_f^{\frac{1}{2}}\left[\int_0^{T_f} e^{-\lambda\tau}(u_{l,i} - u_d)^2\mathrm{d}\tau\right]^{\frac{1}{2}} = b_{\max}e^{lT_f}T_f^{\frac{1}{2}}J_i^{\frac{1}{2}}(T_f)
\end{aligned}
$$

where $l_1 = \|A\| + b_{\max}c$ and $l \triangleq \max(\lambda, l_1)$. From (5) we have

$$\sigma_i = [\omega_e \quad \omega_{\dot{e}}]^T(\boldsymbol{x}_d - \boldsymbol{x}_i),$$

$$|\sigma_i| \leqslant (\omega_e^2 + \omega_{\dot{e}}^2)^{\frac{1}{2}}\|\boldsymbol{x}_d - \boldsymbol{x}_i\|.$$

Hence from (32) and the above, we can obtain (21) which completes the proof. $\square$

PD型模糊学习控制及其在可重复轨迹跟踪问题中的应用

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摘 要 针对可重复轨迹跟踪问题，提出了一种 PD 型模糊学习算法. 该算法集成两种控制：作为基础的 PD 型模糊逻辑算法和改善系统性能的学习算法. 模糊学习控制在模糊控制基础上引入迭代学习算法，使得模糊 PD 控制器可以精确地跟踪可重复轨迹以及消除周期性扰动. 本文在能量函数和泛函分析的基础上，通过严格的推导表明 PD 型模糊学习算法可达到：1) 系统跟踪误差一致收敛到零；2) 学习控制序列几乎处处收敛到理想的控制信号.

关键词 模糊逻辑控制，学习控制，PD 控制，重复追踪任务.

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