Robust Trajectory Tracking Controller Design for Mobile Robots with Bounded Input¹⁾

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Abstract Designing of robust trajectory tracking controller for mobile robots with parametric uncertainties in the dynamic model and with bounded input is investigated. Based on the established full dynamic error model, a robust tracking controller is designed using receding horizon control (RHC) and linear matrix inequalities (LMIs). Simultaneously asymptotically tracking position, heading angle and velocities of mobile robots is realized in accordance with nonholonomic and input constraints. The sufficient condition of the system stability is given in terms of linear matrix inequalities. Simulation results verify the feasibility and effectiveness of the proposed method.

Key words Robust tracking, full dynamic error model, receding horizon control, linear matrix inequalities

1 Introduction

Motion control of mobile robots with nonholonomic constraint (mobile robot for short in the rest) has received considerable attention over the past few years. Depending on whether the nonholonomic system is represented by a kinematic or dynamic model, the tracking problem can be classified as either a kinematic tracking problem or a dynamic tracking problem. Most of the existing results are obtained based on kinematic model, that is, the system velocities are treated as control inputs and the dynamics is ignored. The kinematic model of mobile robots is of limited use, as pointed out in [1], "this simplified representation does not correspond to the reality of moving vehicle which has unknown mass, friction, drive train compliance and backlash effects". Therefore, much effort has been paid to the use of dynamic model where uncertainties in the robot physical parameters can be explicitly taken into considerations in designing the controller. The controller designing schemes based on dynamic model can be basically classified into two kinds; one is the nominal controller is designed based on the assumption that each parameter in the dynamic model is exactly known; while the other takes the uncertainties of the dynamic model into account and the robust controller designing is performed. The main trajectory tracking approaches based on the dynamic model include input/output feedback linearization method^[2,3]; sliding mode control method^[4~7] and backstepping method^[8~10]. However, few results have been obtained on control design of the presence of uncertainties of the dynamic model^[11~13].

In this paper, a full dynamic error model of mobile robot is established and represented in the polytopic form. And then using RHC-LMI method, the asymptotic convergence to zero of the trajectory tracking error is realized with respect to the selected performance index in the presence of the system uncertainty, as well as the nonholonomic constraint.

2 Construction of full dynamic error model and problem statement

In this paper, a three-wheeled mobile robot moving on a horizontal plane is considered

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as shown in Fig. 1. The mobile robot consists of two driving rear wheels and a castor front

wheel. The radius of the wheels is denoted by r and the length of the rear wheel axis is 2l. The system inputs are two torques τ_1 and τ_2 , provided by two motors attached to the rear wheels.

The full dynamic model of the considered wheeled mobile robot is given by [7,8,14,15]

$$\ddot{x} = \frac{\lambda}{m} \sin\phi + \beta u_1 \cos\phi$$

$$\ddot{y} = -\frac{\lambda}{m} \cos\phi + \beta u_1 \sin\phi$$

$$\ddot{\phi} = \alpha u_2$$

$$\dot{x} \sin\phi - \dot{y} \cos\phi = 0$$
(1)

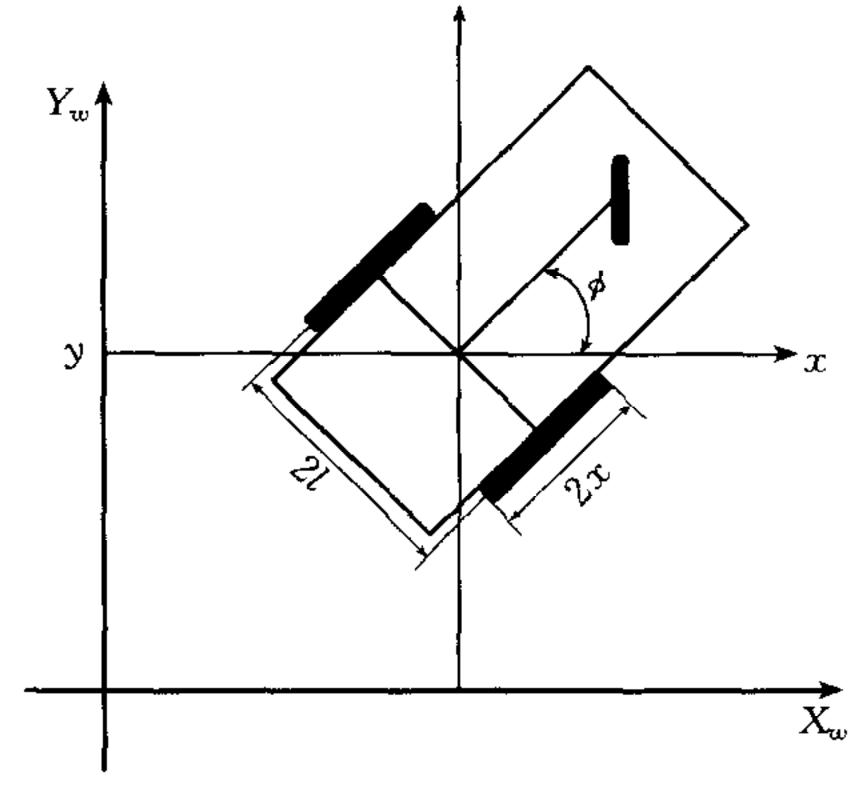


Fig. 1 The configuration of the mobile robot

where $\beta=1/(rm)$, $\alpha=l/(rI)$, and that m and I denote the mass and the moment of inertia of the mobile robot, respectively. $u_1=\tau_1+\tau_2$ and $u_2=\tau_1-\tau_2$ are the control inputs, and λ is the Lagrange multiplier, given by $\lambda=-m\dot{\phi}(\dot{x}\cos\phi+\dot{y}\sin\phi)$. The parameters r,m,I,l are supposed uncertain with bounded uncertainties:

$$r = \overline{r} + \Delta r$$
, $m = \overline{m} + \Delta m$, $I = \overline{I} + \Delta I$, $l = \overline{l} + \Delta l$

with \overline{r} , \overline{m} , \overline{l} , \overline{l} being the nominal values, and

$$|\Delta r| \leqslant \Delta r_{
m max}$$
, $|\Delta m| \leqslant \Delta m_{
m max}$, $|\Delta I| \leqslant \Delta I_{
m max}$, $|\Delta l| \leqslant \Delta l_{
m max}$

with Δr_{max} , Δm_{max} , Δl_{max} being known constants. Therefore, the above variation of the model parameters produces bounded variation intervals for α and β , that is,

$$lpha=ar{lpha}+\Deltalpha$$
 , $eta=ar{eta}+\Deltaeta$, $|\Deltalpha|\leqslant\Deltalpha_{
m max}$, $|\Deltaeta|\leqslant\Deltaeta_{
m max}$

Using Lagrangian formulism, the full dynamic model (1) can be rewritten as follows.

$$\dot{\mathbf{z}} = \begin{bmatrix} v \cos \phi \\ v \sin \phi \\ \omega \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \beta u_1 \\ \alpha u_2 \end{bmatrix}$$
(2)

where $z = [x \ y \ \phi \ v \ \omega]^T$ where v and ω are linear and angular velocities, respectively. Let $z_d = [x_d \ y_d \ \phi_d \ v_d \ \omega_d]^T$ and u_d be the desired position, heading angle, linear velocity, angular velocity and the input corresponding to the desired trajectory, respectively. Define:

$$e_1 = x - x_d$$
, $e_2 = y - y_d$, $e_3 = \phi - \phi_d$, $e_4 = v - v_d$, $e_5 = \omega - \omega_d$

Also define $e = [e_1 \ e_2 \ e_3 \ e_4 \ e_5]^T$. Then we obtain

$$\dot{e} = \begin{bmatrix}
0 & 0 & 0 & \cos(\phi_d + e_3) & 0 \\
0 & 0 & 0 & \sin(\phi_d + e_3) & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \beta & 0 \\ 0 & \alpha \end{bmatrix} \begin{bmatrix} u_{1e} \\ u_{2e} \end{bmatrix} + \begin{bmatrix} v_{1e} \\ u_{2e} \end{bmatrix} + \begin{bmatrix} v_{1e} \\ u_{2e} \end{bmatrix} + \begin{bmatrix} v_{1e} \\ u_{2e} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} v_{1e} \\ u_{2e} \\ u_{2e} \end{bmatrix} =$$

where $u_{1e} = u_1 - u_{d1}$, $u_{2e} = u_2 - u_{d2}$. Rewrite (3) in a simplified form as follows.

$$\dot{e} = Ae + Bu_e + \Delta \tag{4}$$

In order to facilitate the designing of controller, the last term in (4) is linearized about the equilibriums, i. e.,

Then system (3) becomes

$$\dot{e} = (A + L)e + Bu_e \tag{6}$$

Equation (6) is discretized using calculus of differences with a sampling period T, and we have

$$e(k+1) = \begin{bmatrix} 1 & 0 & -Tv_d \sin(\phi_d) & T\cos(\phi_d(k) + e_3(k)) & 0\\ 0 & 1 & Tv_d \cos(\phi_d) & T\sin(\phi_d(k) + e_3(k)) & 0\\ 0 & 0 & 1 & 0 & T\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} e(k) + \begin{bmatrix} 0 & 0\\ 0 & 0\\ 0 & 0\\ T\beta & 0\\ 0 & T\alpha \end{bmatrix} \begin{bmatrix} u_{1e}(k)\\ u_{2e}(k) \end{bmatrix}$$

$$(7)$$

Rewrite (7) in the polytopic form with the control input constraint, and we have $e(k+1) = \widetilde{A}(k)e(k) + \widetilde{B}(k)u_e(k)$

$$[\widetilde{A}(k) \ \widetilde{B}(k)] \in \Omega = \sum_{i=1}^{L} \delta_{i} (\widetilde{A}_{i} \ \widetilde{B}_{i})$$

$$\sum_{i=1}^{L} \delta_{i} = 1, \ \delta_{i} \geqslant 0$$

$$\|\mathbf{u}_{e} + \mathbf{u}_{d}\|_{2} \leqslant u_{\text{max}}$$

$$(8)$$

where u_{max} is a known constant. For system (8), we choose L=4, and

$$\widetilde{A}_{1} = \begin{bmatrix} 1 & 0 & -Tv_{d}\sin(\phi_{d}) & -T & 0 \\ 0 & 1 & Tv_{d}\cos(\phi_{d}) & -T & 0 \\ 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \widetilde{A}_{2} = \begin{bmatrix} 1 & 0 & -Tv_{d}\sin(\phi_{d}) & T & 0 \\ 0 & 1 & Tv_{d}\cos(\phi_{d}) & T & 0 \\ 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & Tv_{d}\cos(\phi_{d}) & -T & 0 \\ 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \widetilde{A}_{4} = \begin{bmatrix} 1 & 0 & -Tv_{d}\sin(\phi_{d}) & T & 0 \\ 0 & 1 & Tv_{d}\cos(\phi_{d}) & -T & 0 \\ 0 & 1 & Tv_{d}\cos(\phi_{d}) & T & 0 \\ 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\widetilde{B}_{1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & T\beta_{\min} & 0 \\ 0 & T\alpha_{\min} \end{bmatrix}, \quad \widetilde{B}_{2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ T\beta_{\max} & 0 \\ 0 & T\alpha_{\max} \end{bmatrix}, \quad \widetilde{B}_{3} = \widetilde{B}_{4} = 0$$

where α_{\min} and α_{\max} as well as β_{\min} and β_{\max} denote the minimum and maximum of the varying ranges of α and β , respectively.

In this paper, we design a state-feedback controller for the uncertain system (8) guaranteeing the robust asymptotic tracking of the given desired trajectory z_d in the presence of

system uncertainties while satisfying the nonholonomic and input constraints.

3 Robust controller design

RHC is an open-loop optimization design procedure where plant measurements are obtained at each sampling time k and a model of the process is used to predict future outputs of the system. The control input sequences are computed by minimizing a cost function J_p (k) over a prediction horizon. In the receding horizon framework, only the first computed control move is implemented. At the next sampling time, the above procedure is repeated using the new measurements from the plant. The following robust performance index is chosen for system (8).

$$\min_{\mathbf{u}_{e}(k+i|k), i=0,1,\dots,m} \max_{[\tilde{A}(k+i)] \in \Omega, i \geqslant 0} J_{\infty}(k),$$

$$J_{\infty}(k) = \sum_{i=0}^{\infty} (\mathbf{e}(k+i|k)^{\mathrm{T}} \mathbf{Q}_{1} \mathbf{e}(k+i|k) + \mathbf{u}_{e}(k+i|k)^{\mathrm{T}} \mathbf{R} \mathbf{u}_{e}(k+i|k))$$
(9)

where e(k+i|k) denotes the error at time k+i predicted based on the measurements at time k; $u_e(k+i|k)$ is the control move at time k+i computed by optimization problem (9) at time k; e(k|k) and $u_e(k|k)$ denote the error and the control to be implemented at time k, respectively; Q_1 , R are positive definite and symmetric weighting matrices. Select a quadratic function $V(e) = e^T P e$, P > 0 for state e(k|k) = e(k) of system (8) with V(0) = 0. At sampling time k, suppose Vsatisfies the following inequality for all e(k+i|k), $u_e(k+i|k)$, $i \ge 0$ satisfying (8), and for any $\lceil \widetilde{A}(k+i) \rceil \in \Omega$, $i \ge 0$,

$$V(e(k+i+1|k)) - V(e(k+i|k)) \le -(e(k+i|k)^{\mathrm{T}}Q_1e(k+i|k) + u_e(k+i|k)^{\mathrm{T}}Ru_e(k+i|k))$$
(10)

For the robust performance objective function to be finite, we must have $e(\infty|k) = 0$ where $e(\infty|k)$ is the error prediction of infinite horizon at sampling time k and hence, $V(e(\infty|k)) = 0$. Summing (10) from i=0 to $i=\infty$, we get

$$-V(e(k \mid k)) \leq -J_{\infty}(k)$$

Thus

$$\max_{\tilde{B}(k+i)} \int_{\tilde{B}(k+i)}^{\infty} \int_{\tilde{B}(k+i)}^{\infty} J_{\infty}(k) \leq V(e(k \mid k))$$

This gives an upper bound on the robust performance objective. Thus, the goal of the robust RHC algorithm is to synthesize, at each time step k, a constant state-feedback control law $\mathbf{u}_e(k+i|k) = F\mathbf{e}(k+i|k)$ to minimize this upper bound $V(\mathbf{e}(k|k))$ and only the first computed input $\mathbf{u}_e(k|k) = F\mathbf{e}(k|k)$ is implemented. At the next sampling time, the state $\mathbf{e}(k+1)$ is measured and the optimization is repeated to compute F again.

Minimizing $V(e(k|k)) = e(k|k)^T Pe(k|k)$, P > 0 is equivalent to

$$\min_{\gamma,Q} \gamma$$

$$s. t. \begin{bmatrix} 1 & e(k \mid k)^{\mathrm{T}} \\ e(k \mid k) & Q \end{bmatrix} \geqslant 0$$
(11)

where $Q = \gamma P^{-1} > 0$ (see [16]). Moreover, from inequality (10) it follows that $(\widetilde{A}(k+i)+\widetilde{B}(k+i)F)^{\mathrm{T}}P(\widetilde{A}(k+i)+\widetilde{B}(k+i)F)-P+F^{\mathrm{T}}RF+Q_1 \leq 0$

Substituting $P = \gamma Q^{-1}$, Q > 0, into the above inquality, multiplying both sides by Q, substituting Y = FQ and using Schur complements, we can draw the following conclusion^[16].

Conclusion 1. If and only if there exist Q > 0, Y = FQ and γ such that

$$\begin{bmatrix} Q & Q\widetilde{A}_{j}^{T} + Y^{T}\widetilde{B}_{j}^{T} & QQ_{1}^{\frac{1}{2}} & Y^{T}R^{\frac{1}{2}} \\ \widetilde{A}_{j}Q + \widetilde{B}_{j}Y & Q & 0 & 0 \\ Q_{1}^{\frac{1}{2}}Q & 0 & \gamma I & 0 \\ R^{\frac{1}{2}}Y & 0 & 0 & \gamma I \end{bmatrix} \geqslant 0, \quad [\widetilde{A}_{j} \quad \widetilde{B}_{j}] \in \Omega, \ j = 1, 2, 3, 4$$

(12)

the quadratic function V can be guaranteed to satisfy (10), that is, the uncertain system (8) has robust stability and the feedback matrix is given by $F = YQ^{-1}$.

In the following, the constraint imposed on the control input u is transformed into the constraint imposed on the control error u_e . Since $u = u_e + u_d$, we have

$$\|\boldsymbol{u}\|_2 = \|\boldsymbol{u}_e + \boldsymbol{u}_d\|_2 \leqslant u_{\text{max}}$$

Due to $\|u_e + u_d\|_2 \le \|u_e\|_2 + \|u_d\|_2$, we obtain

$$\|\boldsymbol{u}_e\|_2 \leqslant (u_{\text{max}} - \|\boldsymbol{u}_d\|_2) = \tilde{u}_{\text{max}}$$

Therefore, using Schur complement, $\|u_e\|_2 \leq \tilde{u}_{\max}$, if [16]

$$\begin{bmatrix} \tilde{u}_{\max}^2 I & Y \\ Y^T & Q \end{bmatrix} \geqslant 0 \tag{13}$$

From the above analysis, we can have the following theorem.

Theorem 1. The receding horizon state feedback control law $u_e(k) = Fe(k)$ robustly asymptotically stabilizes the uncertain system (8) in the presence of the control input constraint and the feedback matrix is given by

$$F = YQ^{-1} \tag{14}$$

where Q>0, Y are obtained from the solution (if it exists) to the following linear objective minimization problem:

$$\min_{\gamma,Q,Y} \gamma$$
 (15)

subject to

$$\begin{bmatrix} 1 & e(k \mid k)^{\mathrm{T}} \\ e(k \mid k) & Q \end{bmatrix} \geqslant 0$$
 (16)

and (12) as well as (13).

4 Simulation results

In this section, physical parameters of a LABMATE mobile robot [7] are adopted to perform simulation to test the proposed control scheme using MATLAB. The nominal values of the LABMATE are as follows: $m=80 \,\mathrm{kg}$, $I=2 \,\mathrm{kgm^2}$, $r=0.075 \,\mathrm{m}$, $l=0.325 \,\mathrm{m}$. The sampling time is chosen as $T=0.1 \,\mathrm{s}$. The given desired reference trajectory is $\mathbf{z}_d=[2 \sin(t) \ 2 \cos(t) \ -t \ 2 \ -1]^{\mathrm{T}}$, that is, $v_d=2 \,\mathrm{m/s}$, $\omega_d=-1 \,\mathrm{rad/s}$, $\mathbf{u}_d=0$. At time t=0, $\mathbf{z}_d(0)=[0\ 2\ 0\ 2\ -1]^{\mathrm{T}}$. The initial condition of the mobile robot is chosen at $\mathbf{z}(0)=[4\ 2\ -\pi/3\ 0\ 0]^{\mathrm{T}}$. The weighting matrices in the performance index are $Q_1=I$, $R=2\times 10^{-5}$ and $u_{\max}=4$. Here the effect of the main varying parameters is considered, namely the mass m and the moment of inertia I. Let the mass and the moment of inertia be $m=(\overline{m},1.25\overline{m},1.5\overline{m})$ and $I=(\overline{I},1.25\overline{I},1.5\overline{I})$, where \overline{m} and \overline{I} are the nominal mass and moment of inertia, respectively. The corresponding parameters are $(\bar{a},(4/5)\bar{a},(2/3)\bar{a})$ and $(\bar{\beta},(4/5)\bar{\beta},(2/3)\bar{\beta})$. The simulation time is 20s. Simulation results are shown in Fig. 2 to Fig. 4.

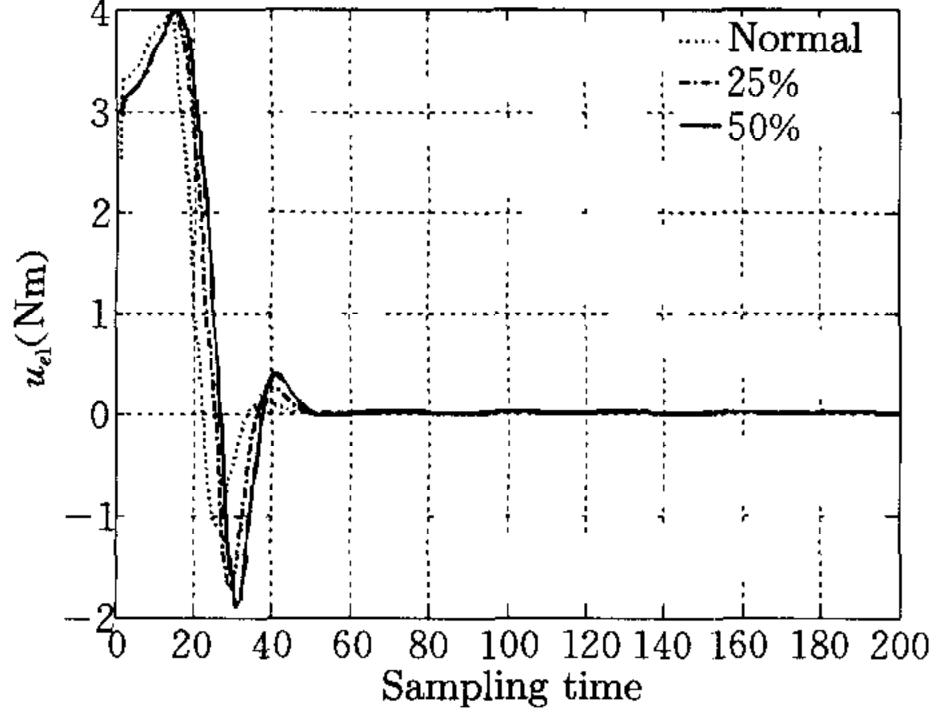


Fig. 2 Control input error u_{e1}

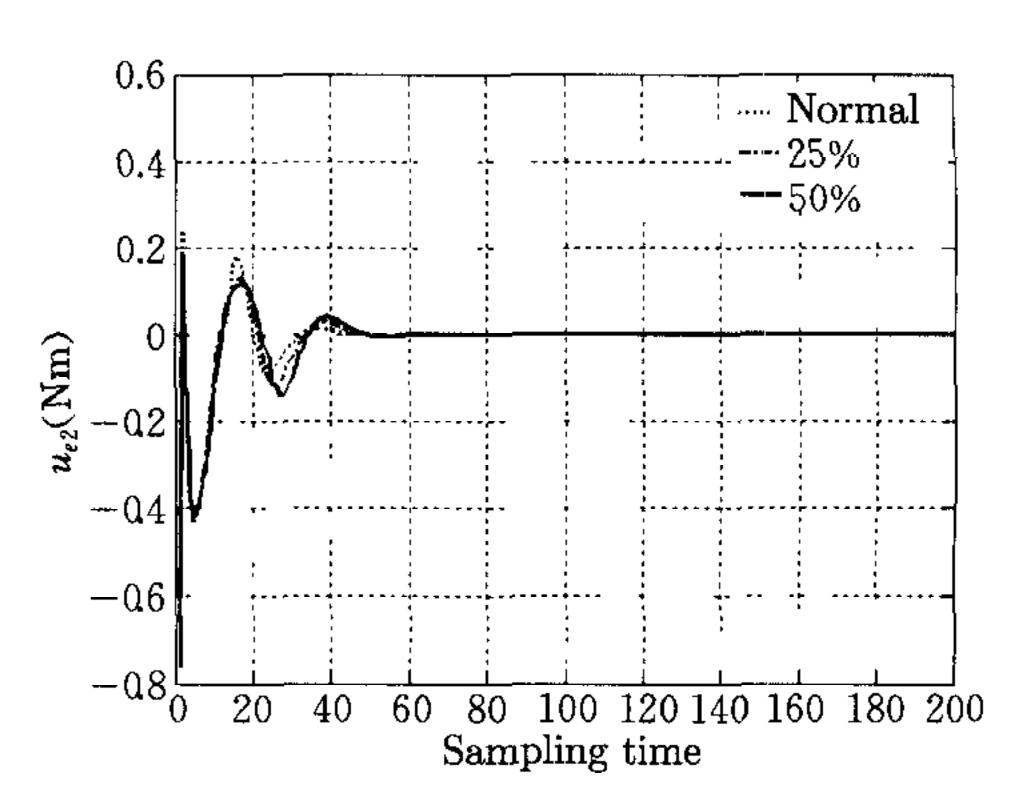


Fig. 3 Control input error u_{e2}

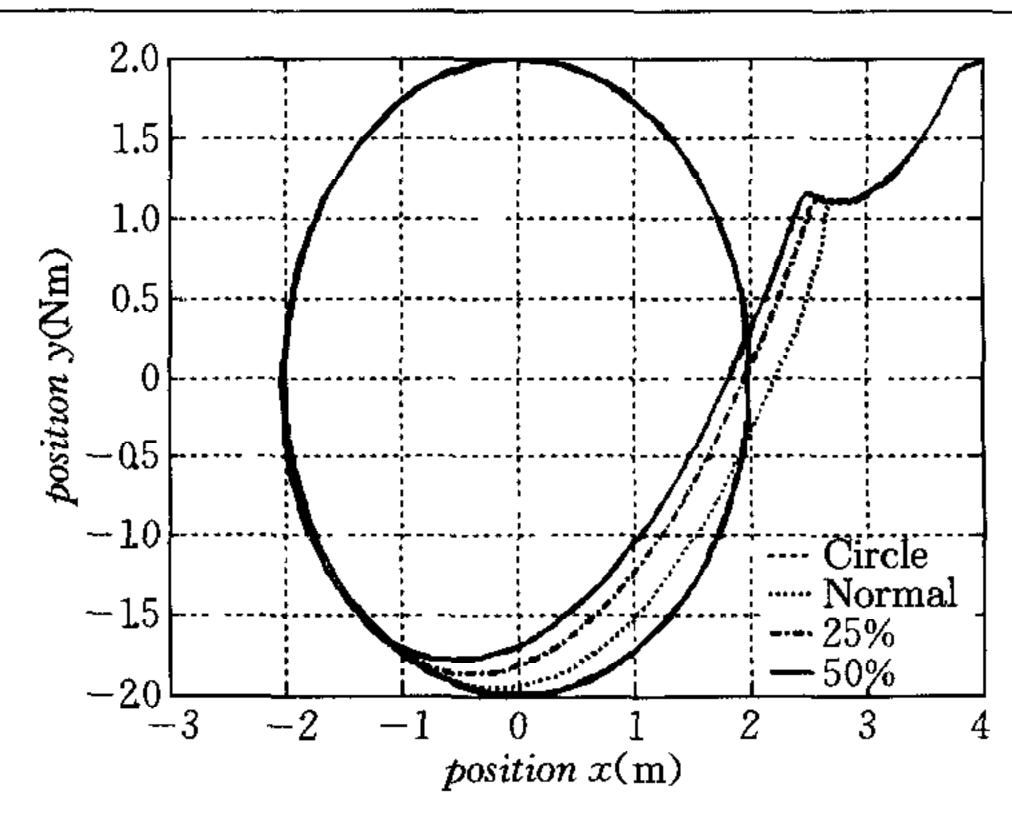


Fig. 4 Position trajectories

As seen from the simulation results, the proposed approach can realize the robust asymptotic tracking of the desired reference trajectory in the presence of parametric uncertainty in the dynamic model of the mobile robot and control input constraint.

5 Conclusions

First, the full dynamic error model is established and represented in the polytopic form. Secondly, a robust tracking controller is designed using RHC-LMI scheme and the simultaneously asymptotical tracking of position, heading angle and velocities of the mobile robot is realized in the presence of the parameter uncertainty in the dynamic model and control input constraint. Finally, simulation results verify the feasibility and effectiveness of the presented method.

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控制受限的移动机器人鲁棒跟踪控制器设计

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摘 要 研究了非完整移动机器人动力学模型中带有参数不确定和控制受限的鲁棒轨迹跟踪控制器的设计问题. 在建立移动机器人的全动态误差模型的基础上,应用滚动时域控制(RHC)和线性矩阵不等式(LMIs)方法,设计了鲁棒跟踪控制器,在满足非完整和控制约束的条件下,实现了机器人位置,导向角以及速度的同时渐近跟踪. 系统稳定性的充分条件以 LMI 的形式给出. 仿真结果验证了提出方法的可行性和有效性.

关键词 鲁棒跟踪,全动态误差模型,滚动时域控制,线性矩阵不等式中图分类号 TP24