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Robust H_∞ Control for Multiple Time-delay Uncertain Nonlinear System Based on Fuzzy Model and Neural Network[1)]

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Abstract A mixed control method combining fuzzy model-based control and neural network control is presented for a class of uncertain nonlinear system with multiple time delays. Firstly, fuzzy-model-based H_∞ control law is designed by means of LMI method for time-delay nonlinear multi-input systems modeled by the fuzzy T-S model with multiple time delays. Secondly, full adaptive RBF neural network control method is used to improve the scheme of the fuzzy H_∞ control. The effect of the unknown uncertainties and the error caused by fuzzy modeling is overcome by adaptive tuning of the weights, centers and widths of the RBF neural network on line, and no matching conditions or constraint conditions are required. The stability of the designed closed loop system is proved. The effectiveness of the proposed method is finally demonstrated through simulation on the multiple time-delay nonlinear chaos system.

Key words Multiple time-delay, fuzzy T-S model, neural network

1 Introduction

T-S fuzzy-model-based methodologies have emerged in recent years as a promising way to approach modeling and control of complex nonlinear system[1,2]. There have been some significant research efforts on the design of the fuzzy-model-based controller. Since uncertainties and time delay are frequently a source of instability in various engineering systems, the issue of robust fuzzy-model-based controller design for uncertain nonlinear system with time delay received considerable interest[3,4]. However, one limitation of these works is that uncertain nonlinearities are often required to satisfy certain types of matching conditions or constraint conditions. In fact, it is very difficult, if not impossible, to satisfy these conditions in many cases, especially in case that the uncertainties of the system are unknown. In many reported results[5~7], most of the stability analysis and design methods are based on fuzzy T-S model, the effect of approximation error due to fuzzy modeling is not considered. Despite of some robust stabilization technique is introduced to against modeling error in [8], an upper bound of the approximation error is used in analysis, which is difficult to find in application.

In this paper, fuzzy H_∞ control scheme is designed for a class of uncertain nonlinear systems with multiple time-delay. An adaptive RBF NN is used to eliminate the influence of unknown uncertainties and the fuzzy modeling error by tuning all parameters in the RBF expansion, thereby improving the robust h-infinity control performance. In this paper we extend the results in [9] to nonlinear multivariable system. The proposed adaptive RBF NN can be adopted to approximate nonlinear vector functions.

2 Problem formulation

Consider a class of uncertain nonlinear system with multiple time delay

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$$\dot{x}_1 = x_2 + \phi_1(t)$$
$$\dot{x}_2 = f_2(x_1, x_2, x_3, u) + \tilde{f}_2(x_1, x_2, x_3, u, t) + \sum_{k=1}^{h} g_{2k}(x(t-\tau_k)) + \sum_{k=1}^{h} \tilde{g}_{2k}(x(t-\tau_k)) + \phi_2(t)$$
$$\dot{x}_3 = f_3(x_1, x_2, x_3, u) + \tilde{f}_3(x_1, x_2, x_3, u, t) + \sum_{k=1}^{h} g_{3k}(x(t-\tau_k)) + \sum_{k=1}^{h} \tilde{g}_{3k}(x(t-\tau_k)) + \phi_3(t) \tag{1}$$

where $x_1, x_2 \in R^{n_1}, x_3 \in R^{n_2}$ are state vectors, $u \in R^m$ is the control input vector, f_i, g_{ik} are known smooth nonlinear functions, $\tilde{f}_i$, $\tilde{g}_{ik}$ are unknown uncertain nonliearties of the system and modeling error. $(i=2,3, k=1,\cdots,h)$, $\tau_k \geqslant 0 (k=1,\cdots,h)$ are time delays. $x(t)=0$, $\forall t<0$. $\phi(t)=[\phi_1(t)^T \ \phi_2(t)^T \ \phi_3(t)^T]^T$ denotes external disturbance. For convenience, we use the following notation: $x=[x_1^T, x_2^T, x_3^T]^T \in R^n (n=n1+n1+n2)$. $f=[f_2^T, f_3^T]^T$, $\tilde{f}=[\tilde{f}_2^T, \tilde{f}_3^T]^T$, $g=\sum_{k=1}^{h}[g_{2k}^T, g_{3k}^T]^T$, $\tilde{g}=\sum_{k=1}^{h}[\tilde{g}_{2k}^T, \tilde{g}_{3k}^T]^T$.

A fuzzy dynamic model described by fuzzy If-Then rules is used to approximate the known nonlinearities of the system in(1). The ith rule of the fuzzy model is of the following form:

Plant Rule i: If $z_1(t)$ is F_1^i and $\cdots z_p(t)$ is F_p^i Then

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + \sum_{k=1}^{h} A_{ik} x(t-\tau_k) + Bu(t) + \phi(t) \\ x(t) &= 0 \quad t \leqslant 0 \quad i = 1,2,\cdots,q \end{aligned} \tag{2}$$

where $z(t) \in R^p$ is the premise vector, F_j^i is the fuzzy set, q is the number of rules. A_i, A_{ik} are constant matrices with appropriate dimensions, $B=\begin{bmatrix} 0 & 0 \\ I_{n1} & 0 \\ 0 & I_{n2} \end{bmatrix} \in R^{n\times m}$. The final output of the fuzzy system is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(z(t))[A_i x(t) + \sum_{i=1}^{h} A_{ik} x(t-\tau_k)] + Bu(t) + \phi(t) \tag{3}$$

where $\omega_i(z(t)) = \prod_{j=1}^{p} F_j^i(z_j(t))$ is the overall truth value of the ith rule, $\mu_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{q}\omega_i(z(t))}$.

In this paper, we assume $\omega_i(z(t)) \geqslant 0$ and $\sum_{i=1}^{q}\omega_i(z(t)) > 0$ for $i=1,2,\cdots,q$. Therefore, we get $\mu_i(z(t)) \geqslant 0$, and $\sum_{i=1}^{q}\mu_i(z(t)) = 1 (i=1,2,\cdots,q)$. System (1) can be rearranged as the following equivalent system:

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(z(t))[A_i x(t) + \sum_{k=1}^{h} A_{ik} x(t-\tau_k)] + B[u(t) + \Delta(x(t), x(t-\tau), u, t)] + \phi(t) \tag{4}$$

where $x(t-\tau)=[x(t-\tau_1),\cdots,x(t-\tau_h)]^T$, $\Delta(x(t), x(t-\tau), u(t), t) = \tilde{f} + \tilde{g} + \Delta f + \Delta g$, $\tilde{f}$, $\tilde{g}$ are unknown uncertain nonliearties of the system, Δf and Δg denote errors caused by fuzzy modeling.

3 H_∞ controller based on fuzzy model

Suppose the following fuzzy state feedback controller based on T-S fuzzy model is employed to control the nonlinear system (3)

$$u_f(t) = \sum_{i=1}^{q}\mu_i(z(t))K_i\boldsymbol{x}(t) \tag{5}$$

Substituting (5) into (3) yields the corresponding fuzzy closed-loop system as follows

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{q}\mu_i(z(t))[(A_i + BK_i)\boldsymbol{x}(t) + \sum_{k=1}^{h}A_{ik}\boldsymbol{x}(t-\tau_k)] + \boldsymbol{\phi}(t) \tag{6}$$

Lemma 1[6]. For any $\boldsymbol{x}_1 \in R^m$, $\boldsymbol{x}_2 \in R^n$, $X \in R^{m\times n}$, and for any positive definite matrix $Y \in R^{n\times n}$ we have

$$\boldsymbol{x}_1^{\mathrm{T}}X\boldsymbol{x}_2 + \boldsymbol{x}_2^{\mathrm{T}}X^{\mathrm{T}}\boldsymbol{x}_1 \leqslant \boldsymbol{x}_1^{\mathrm{T}}XY^{-1}X^{\mathrm{T}}\boldsymbol{x}_1 + \boldsymbol{x}_2^{\mathrm{T}}Y\boldsymbol{x}_2 \tag{7}$$

Let us consider the following H_∞ control performance

$$\int_0^{t_f}\boldsymbol{x}(t)^{\mathrm{T}}R\boldsymbol{x}(t)\mathrm{d}t \leqslant \gamma^2\int_0^{t_f}\|\boldsymbol{\varphi}(t)\|^2\mathrm{d}t \tag{8}$$

where $\varphi(t) \in L_2[0\ \ t_f]$, γ is a prescribed positive constant, $R=R^{\mathrm{T}}$ is a positive definite weighting matrix, t_f is a terminal time of control. The inequality (8) also can be written as follows:

$$J = \frac{1}{\gamma}\int_0^{t_f}\boldsymbol{x}(t)^{\mathrm{T}}R\boldsymbol{x}(t)\mathrm{d}t - \gamma\int_0^{t_f}\boldsymbol{\varphi}(t)^{\mathrm{T}}\boldsymbol{\varphi}(t)\mathrm{d}t < 0 \tag{9}$$

Theorem 1. There exists fuzzy H_∞ state feedback controller (5) such that the close-loop fuzzy system (6) is stabilizable with prescribed H_∞ norm bound, if there exist a symmetric and positive definite matrix $P \in R^{n\times n}$, some matrices $K_i \in R^{m\times n}$ $(i=1,\cdots,q)$ and some scalars $\sigma_k>0(k=1,\cdots,h)$ satisfying the following LMI:

$$\begin{bmatrix} \Gamma_i & * & * \\ A_{di}^{\mathrm{T}} & -\sigma & * \\ I^{\mathrm{T}}Q & 0 & -\sigma^{-1} \end{bmatrix} < 0 \tag{10}$$

where $\Gamma_i = A_iQ + BW_i + QA_i^{\mathrm{T}} + W_i^{\mathrm{T}}B^{\mathrm{T}} + \frac{1}{\gamma}I_{n\times n} + \frac{1}{\gamma}QRQ$, $A_{di} = [A_{i1},\cdots,A_{ih}]_{n\times hn}$,

$\sigma = \mathrm{diag}(\sigma_1 I_{n\times n},\cdots,\sigma_h I_{n\times n})_{hn\times hn}$,

$I = [I_{n\times n},\cdots,I_{n\times n}]_{n\times hn}$, $I_{n\times n}$ is unity matrix,

$Q = P^{-1}$, $W_i = K_iP^{-1}$, $*$ denotes the transposed elements in the symmetric positions.

Proof. Select a Lyapunov function as

$$V(\boldsymbol{x}) = \boldsymbol{x}(t)^{\mathrm{T}}P\boldsymbol{x}(t) + \sum_{k=1}^{h}\int_{t-\tau_k}^{t}\sigma_k\boldsymbol{x}(s)^{\mathrm{T}}\boldsymbol{x}(s)\mathrm{d}s \tag{11}$$

where P is a symmetric and positive definite matrix, $\sigma_k>0$, $k=1,\cdots,h$are some constants. Clearly, $V(\boldsymbol{x})>0$, $\forall\, \boldsymbol{x}\neq 0$. The time derivative of $V(x)$ is

$$\dot{V}(x) = \sum_{i=1}^{q}\mu_i(z(t))\Big\{\boldsymbol{x}(t)^{\mathrm{T}}(H_i^{\mathrm{T}}P + PH_i)\boldsymbol{x}(t) + \sum_{k=1}^{h}\sigma_k[\boldsymbol{x}(t-\tau_k)^{\mathrm{T}}A_{ik}^{\mathrm{T}}P\boldsymbol{x}(t) + \boldsymbol{x}(t)^{\mathrm{T}}PA_{ik}\boldsymbol{x}(t-\tau_k)]\Big\} +$$

$$\boldsymbol{x}(t)^{\mathrm{T}}P\boldsymbol{\phi}(t) + \boldsymbol{\phi}^{\mathrm{T}}(t)P\boldsymbol{x}(t) + \sum_{k=1}^{h}\sigma_k\boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{x}(t) - \sum_{k=1}^{h}\sigma_k\boldsymbol{x}(t-\tau_k)^{\mathrm{T}}\boldsymbol{x}(t-\tau_k) \tag{12}$$

where $H_i = A_i + BK_i$. Applying Lemma 1 to (12), we get

$$\dot{V}(x) \leqslant \sum_{i=1}^{q}\mu_i(z(t))\left\{\boldsymbol{x}(t)^{\mathrm{T}}\left(S + \frac{1}{\gamma}PP\right)\boldsymbol{x}(t)\right\} + \gamma\boldsymbol{\phi}^{\mathrm{T}}(t)\boldsymbol{\phi}(t) \tag{13}$$

where $S = H_i^{\mathrm{T}}P + PH_i + PA_{di}\sigma^{-1}A_{di}^{\mathrm{T}}P + I\sigma I^{\mathrm{T}}$.

From (9) and (13), we have

$$J = \int_0^{t_f}\left(\frac{1}{\gamma}\boldsymbol{x}(t)^{\mathrm{T}}R\boldsymbol{x}(t) - \gamma\phi(t)^{\mathrm{T}}\phi(t)\right)\mathrm{d}t <$$

$$\int_0^{t_f}\left(\frac{1}{\gamma}\boldsymbol{x}(t)^{\mathrm{T}}R\boldsymbol{x}(t) - \gamma\phi(t)^{\mathrm{T}}\phi(t) + \dot{V}\right)\mathrm{d}t = \sum_{i=1}^{q}\mu_i(z(t))\int_0^{t_f}\boldsymbol{x}(t)^{\mathrm{T}}\left(S + \frac{1}{\gamma}PP + \frac{1}{\gamma}R\right)\boldsymbol{x}(t))\mathrm{d}t \tag{14}$$

If $S+\frac{1}{\gamma}PP+\frac{1}{\gamma}R<0$, we can get $J<0, \dot{V}(x)<0$. Let $Q=P^{-1}$. Pre-multiply and post-multiply Q on the both sides of (14), then apply Schur complement to the result. After some manipulation, we will get (10). This means the controlled fuzzy closed-loop system (6) is asymptotically stable, and the H_∞ control performance is achieved . This completes the proof of the theorem.

4 RBF neural network adaptive control

4.1 error dynamic model of system

Due to the effect of fuzzy modeling error and uncertain nonlinearities, it is very difficult to obtain desired control performance by only using fuzzy model-based H_∞ controller proposed in Theorem 1. In this situation, an adaptive RBF NN control design is proposed to ensure system stability and satisfactory control performance.

We define robust control by combining fuzzy H_∞ control with adaptive NN as follows:

$$\boldsymbol{u}(t) = \boldsymbol{u}_f(t) - \boldsymbol{u}_{nn}(t) \tag{15}$$

Substituting (15) into (4), we get

$$\begin{aligned}\dot{\boldsymbol{x}}(t) = &\sum_{i=1}^{q}\mu_i(z(t))[(A_i + BK_i)\boldsymbol{x}(t) + \sum_{k=1}^{h}A_{ik}\boldsymbol{x}(t-\tau_k)] + \\ &\boldsymbol{\phi}(t) + B(\Delta(\boldsymbol{x}(t), \boldsymbol{x}(t-\tau_k), \boldsymbol{u}_f, t) - \boldsymbol{u}_{nn})\end{aligned} \tag{16}$$

Consider fuzzy closed-loop system

$$\dot{\boldsymbol{x}}_f(t) = \sum_{i=1}^{q}\mu_i(\boldsymbol{z}(t))[(A_i + BK_i)x_f(t) + \sum_{k=1}^{h}A_{ik}x_f(t-\tau_k)] + \boldsymbol{\phi}(t) \tag{17}$$

Let $\boldsymbol{e}(t)=\boldsymbol{x}(t)-\boldsymbol{x}_f(t)$ be fuzzy modeling error, and the error dynamic equation can be expressed as

$$\begin{aligned}\dot{\boldsymbol{e}}(t) = &\sum_{i=1}^{q}\mu_i(z(t))[(A_i + BK_i)\boldsymbol{e}(t) + \sum_{k=1}^{h}A_{ik}\boldsymbol{e}(t-\tau_k)] + \\ &B(\Delta(\boldsymbol{x}(t), \boldsymbol{x}(t-\tau_k), \boldsymbol{u}_f(t), t) - \boldsymbol{u}_{nn})\end{aligned} \tag{18}$$

It is clear that if the function $\Delta(\boldsymbol{x}(t), \boldsymbol{x}(t-\tau_k), \boldsymbol{u}_f(t), t)$ in (18) can be canceled by the output of the RBF neural network, then from Theorem 1, we known that the resulting closed-loop system can be stabilize and achieve desired performance.

4.2 RBF neural network weight, centers and widths updating laws

Let $X=[x(t)^{\mathrm{T}}, x(t-\tau)^{\mathrm{T}}, u_f(t)^{\mathrm{T}}, t]^{\mathrm{T}} \in R^N (N=n+nh+m+1)$ be the input vector. When $X\in A_d, A_d$ is a compact set, RBF neural network can be used to approximate uncertain term $\Delta(X)$. It can be expressed as

$$\boldsymbol{u} = W^{\mathrm{T}}G(X, \xi, \eta) \tag{19}$$

where $W=(w_{ij})_{l\times m}$ is the weight matrix, $l>1$ is the number of neural network nodes, and $G=(G_i)_{l\times 1}$ is a Gaussian function.

$$G_i(X) = \exp\left[\frac{-(X-\xi_i)^{\mathrm{T}}(X-\xi_i)}{\eta_i^2}\right], \quad i = 1, 2, \cdots, l \tag{20}$$

where ξ_i is the center of the Gaussian function which has the same dimensions as X, η_i is the width of the Gaussion function, and $\boldsymbol{\xi}=[\xi_1, \cdots, \xi_l]^{\mathrm{T}}, \boldsymbol{\eta}=[\eta_1, \cdots, \eta_l]^{\mathrm{T}}$. It has been proven that RBF NN (20) is capable of uniformly approximating any real continuous nonlinear function on the compact set A_d with arbitrary any accuracy as

$$\Delta(X) = W^{*\mathrm{T}}G(X, \xi^*, \eta^*) + \varepsilon_f(X) \tag{21}$$

where W^*, ξ^*, η^* are ideal constant weights, center and width, respectively, $\varepsilon_f(X)$ is the approximation error.

Assumption 1[9]. There exist ideal weight W^*, center ξ^* and width η^*, such that

$\|\varepsilon_f(X)\| < \varepsilon^*$ with constant $\varepsilon^* > 0$ for all $X \in A_d$, and there exist constants $\bar{W}, \bar{\xi}, \bar{\eta}$ satisfying $\|W^*\|_F \leqslant \bar{W}$, $\|\xi^*\| \leqslant \bar{\xi}$, $\|\eta^*\| \leqslant \bar{\eta}$, respectively.

Define the output of the adaptive neural network as

$$u_{nn} = \hat{W}^{\mathrm{T}} G(X, \hat{\xi}, \hat{\eta}) \tag{22}$$

where $\hat{W}, \hat{\xi}, \hat{\eta}$ are the estimate values of the W^*, ξ^*, η^*. Let $\tilde{W} = W^* - \hat{W}$, $\tilde{\xi} = \xi^* - \hat{\xi}$, $\tilde{\eta} = \eta^* - \hat{\eta}$, $\tilde{Z} = \text{block\_diag}\{\tilde{W}, \tilde{\xi}, \tilde{\eta}\}$. The function approximation error can be expressed as

$$\Delta(X) - u_{nn} = W^{*\mathrm{T}} G^* - \hat{W}^{\mathrm{T}} \hat{G} + \varepsilon_f(X) = \tilde{W}^{\mathrm{T}} \tilde{G} + \tilde{W}^{\mathrm{T}} \hat{G} + \hat{W}^{\mathrm{T}} \tilde{G} + \varepsilon_f(X) \tag{23}$$

In order to deal with $\tilde{G}$, the Taylor's series of $G(X, \xi^*, \eta^*)$ is taken at $\xi^* = \hat{\xi}$ and $\eta^* = \hat{\eta}$. This produces

$$G(X, \xi^*, \eta^*) = G(X, \hat{\xi}, \hat{\eta}) + G'_\xi(\xi^* - \hat{\xi}) + G'_\eta(\eta^* - \hat{\eta}) + O(X, \tilde{\xi}, \tilde{\eta}) \tag{24}$$

where $G'_\xi = \text{diag}(g_{\xi i}) \in R^{l \times Nl}$, $G'_\eta = \text{diag}(g_{\eta i}) \in R^{l \times l}$. $O(X, \tilde{\xi}, \tilde{\eta})$ is the high-order term, and

$$g_{\xi i} = 2\frac{(X - \xi_i)^{\mathrm{T}}}{\eta_i^2} \exp\left[\frac{-(X - \xi_i)^{\mathrm{T}}(X - \xi_i)}{\eta_i^2}\right],$$

$$g_{\eta i} = 2\frac{\|X - \xi_i\|^2}{\eta_i^3} \exp\left[\frac{-(X - \xi_i)^{\mathrm{T}}(X - \xi_i)}{\eta_i^2}\right],\quad i = 1, 2, \cdots, l$$

Using (24), the high-order term $O(X, \tilde{\xi}, \tilde{\eta})$ is bounded by

$$\|O(X, \tilde{\xi}, \tilde{\eta})\| = \|\tilde{G} - G'_\xi \tilde{\xi} - G'_\eta \tilde{\eta}\| \leqslant \|\tilde{G}\| + \|G'_\xi\| \|\tilde{\xi}\| + \|G'_\eta\| \|\tilde{\eta}\| \leqslant c_1 + c_2\|\tilde{\xi}\| + c_3\|\tilde{\eta}\| \tag{25}$$

where c_1, c_2, c_3 are some bounded constants due to the fact that RBF and its derivative are always bounded by constants.

By substituting (24) into (23), we obtain

$$\Delta(X) - u_{nn} = \tilde{W}^{\mathrm{T}}(\hat{G} - G'_\xi \hat{\xi} - G'_\eta \hat{\eta}) + \hat{W}^{\mathrm{T}}(G'_\xi \tilde{\xi} + G'_\eta \tilde{\eta}) + d_f \tag{26}$$

where $d_f = \tilde{W}^{\mathrm{T}}(G'_\xi \xi^* + G'_\eta \eta^*) + W^* O(X, \tilde{\xi}, \tilde{\eta}) + \varepsilon_f(X)$. From (25), we have

$$\|d_f\| \leqslant \|\tilde{W}\|_F c_2 \bar{\xi} + \|\tilde{W}\|_F c_3 \bar{\eta} + \bar{W}(c_1 + c_2\|\tilde{\xi}\| + c_3\|\tilde{\eta}\|) + \varepsilon^* \tag{27}$$

It is obvious that d_f is bounded. Substituting (26) into (18), we get

$$\begin{aligned}\dot{e}(t) = \sum_{i=1}^{q} \mu_i(z(t))[(A_i + BK_i)e(t) + \sum_{k=1}^{h} A_{ik} e(t - \tau_k)] + \\ B[\tilde{W}^{\mathrm{T}}(\hat{G} - G'_\xi \hat{\xi} - G'_\eta \hat{\eta}) + \hat{W}^{\mathrm{T}}(G'_\xi \tilde{\xi} + G'_\eta \tilde{\eta}) + d_f]\end{aligned} \tag{28}$$

Theorem 2. Consider the closed-loop system consisting of the nonlinear system (1), the fuzzy state feedback control law (5) and adaptive NN control law (22). Assume there exists sufficiently large compact set $A_d \in R^N$ such that $X = [x(t)^{\mathrm{T}}, x(t-\tau)^{\mathrm{T}}, u_f(t)^{\mathrm{T}}, t]^{\mathrm{T}} \in A_d$. If the adaptation laws of the RBF neural network are designed as (29)～(31), then the closed-loop system with control law (15) are uniformly ultimately bounded stable.

$$\dot{\hat{W}} = L_1(\hat{G} - G'_\xi \hat{\xi} - G'_\eta \hat{\eta}) \boldsymbol{e}^{\mathrm{T}} PB - \lambda_w \|\boldsymbol{e}^{\mathrm{T}} PB\| L_1 \hat{W} \tag{29}$$

$$\dot{\hat{\xi}} = L_2(\boldsymbol{e}^{\mathrm{T}} PB \hat{W}^{\mathrm{T}} G'_\xi)^{\mathrm{T}} - \lambda_\xi \|\boldsymbol{e}^{\mathrm{T}} PB\| L_2 \hat{\xi} \tag{30}$$

$$\dot{\hat{\eta}} = L_3(\boldsymbol{e}^{\mathrm{T}} PB \hat{W}^{\mathrm{T}} G'_\eta)^{\mathrm{T}} - \lambda_\xi \|\boldsymbol{e}^{\mathrm{T}} PB\| L_3 \hat{\eta} \tag{31}$$

where L_1, L_2, L_3 are positive definite matrices with appropriate dimensions, constants λ_w, $\lambda_\xi, \lambda_\eta$ are positive, e is the difference between states of fuzzy close-loop system (20) and states of nonlinear system (1). P is symmetric positive definite matrix which is defined as in Theorem 1. $e(t)$, $\tilde{W}$, $\tilde{\xi}$, $\tilde{\eta}$ are uniformly ultimately bounded (UUB) if $\|\boldsymbol{e}(t)\| > \Omega_1$ or $\|\tilde{Z}\|_F > \Omega_2$, where Ω_1, Ω_2 are constants.

Proof. Consider the error dynamic equation (28), and we select a Lyapunov function as

$$V = \boldsymbol{e}(t)^{\mathrm{T}} P\boldsymbol{e}(t) + \sum_{k=1}^{h} \int_{t-\tau_k}^{t} \sigma_k \boldsymbol{e}(s)^{\mathrm{T}} \boldsymbol{e}(s) \mathrm{d}s + tr(\tilde{W}^{\mathrm{T}} L_1^{-1} \tilde{W}) + \tilde{\xi}^{\mathrm{T}} L_2^{-1} \tilde{\xi} + \tilde{\eta}^{\mathrm{T}} L_3^{-1} \tilde{\eta}$$

The time derivative of V is

$$\dot{V} = \dot{e}(t)^{\mathrm{T}} P e(t) + e(t)^{\mathrm{T}} P \dot{e}(t) + \sum_{k=1}^{h} \sigma_k e(t)^{\mathrm{T}} e(t) - \sum_{k=1}^{h} \sigma_k e(t-\tau_k)^{\mathrm{T}} e(t-\tau_k) + 2tr(\widetilde{W}^{\mathrm{T}} L_1^{-1} \dot{\hat{W}}) + 2\tilde{\xi}^{\mathrm{T}} L_2^{-1} \dot{\hat{\xi}} + 2\tilde{\eta}^{\mathrm{T}} L_3^{-1} \dot{\hat{\eta}} \tag{32}$$

From Theorem 1 we know Eq. (14) is negative. Let $\lambda = -\lambda_{\min}(S)$; we get

$$\dot{V} < -\lambda e(t)^{\mathrm{T}} e(t) + 2e(t)^{\mathrm{T}} PB[\widetilde{W}^{\mathrm{T}}(\hat{G} - G'_{\xi}\hat{\xi} - G'_{\eta}\hat{\eta}) + \hat{W}^{\mathrm{T}}(G'_{\xi}\tilde{\xi} + G'_{\eta}\tilde{\eta}) + d_f] - 2tr(\widetilde{W}^{\mathrm{T}} L_1^{-1} \dot{\hat{W}}) - 2\tilde{\xi}^{\mathrm{T}} L_2^{-1} \dot{\hat{\xi}} - 2\tilde{\eta}^{\mathrm{T}} L_3^{-1} \dot{\hat{\eta}} \tag{33}$$

By substituting (29)~(31) into (33), we obtain

$$\dot{V} < -\lambda e(t)^{\mathrm{T}} e(t) + 2\|e^{\mathrm{T}} PB\| \{ \|\widetilde{W}\|_F c_2 \bar{\xi} + \|\widetilde{W}\|_F c_3 \bar{\eta} + \overline{W}(c_1 + c_2\|\tilde{\xi}\| + c_3\|\tilde{\eta}\|) + \varepsilon^* + \lambda_w(\|\widetilde{W}\|_F \overline{W} - \|\widetilde{W}\|_F^2) + \lambda_\xi(\|\tilde{\xi}\|\bar{\xi} - \|\tilde{\xi}\|^2) + \lambda_\eta(\|\tilde{\eta}\|\bar{\eta} - \|\tilde{\eta}\|^2)\} = -\lambda e(t)^{\mathrm{T}} e(t) + 2\|e^{\mathrm{T}} PB\| \{ (C + C_1\|\widetilde{W}\|_F + C_2\|\tilde{\xi}\| + C_3\|\tilde{\eta}\|) - \lambda_w\|\widetilde{W}\|_F^2 - \lambda_\xi\|\tilde{\xi}\|^2 - \lambda_\eta\|\tilde{\eta}\|^2 \} \tag{34}$$

where $C = \overline{W}c_1 + \varepsilon^*$, $C_1 = c_2\bar{\xi} + c_3\bar{\eta} + \lambda_w\overline{W}$, $C_2 = \overline{W}c_2 + \lambda_\xi\bar{\xi}$, $C_3 = \overline{W}c_3 + \lambda_\eta\bar{\eta}$. C, C_1, C_2, C_3 are constants and bounded. Let $\lambda_m = \min\{\lambda_w, \lambda_\xi, \lambda_\eta\}$, $\widetilde{Z} = \mathrm{diag}(\widetilde{W}, \tilde{\xi}, \tilde{\eta})$, $Y = (C_1, C_2, C_3)^{\mathrm{T}}$, $\lambda_M = \|Y\|$. It is obvious that $C_1\|\widetilde{W}\|_F + C_2\|\tilde{\xi}\| + C_3\|\tilde{\eta}\| \leqslant \lambda_M\|\widetilde{Z}\|_F$. So we can conclude that

$$\dot{V} < -\lambda e(t)^{\mathrm{T}} e(t) - 2\|e^{\mathrm{T}} PB\|\lambda_m\left(\|\widetilde{Z}\|_F - \frac{\lambda_M}{2\lambda_m}\right)^2 + 2\|e^{\mathrm{T}} PB\|\left(C + \frac{\lambda_M^2}{4\lambda_m}\right) \tag{35}$$

Let $\Omega_1 = 2\|e^{\mathrm{T}} PB\|\left(C + \frac{\lambda_M^2}{4\lambda_m}\right)/\lambda$ and $\Omega_2 = \frac{\lambda_M}{2\lambda_m} + \sqrt{\left(C + \frac{\lambda_M^2}{4\lambda_m}\right)/\lambda}$. If $\|e(t)\| > \Omega_1$ or $\|\widetilde{Z}\|_F > \Omega_2$, then $\dot{V} < 0$. This implies $e(t), \widetilde{W}, \tilde{\xi}, \tilde{\eta}$ are uniformly ultimately bounded. This completes the proof. □

5 Simulation studies

Consider nonlinear chaotic system with multiple time delays

$$\begin{aligned} x_1(t) &= 2.5x_2(t) \\ x_2(t) &= -\left(\frac{1}{2.5}x_1(t)\right)^3 - \frac{1}{2.5}x_1(t) - 0.1x_2(t) + 0.01x_1(t-0.015) + \\ &\quad 0.01x_1^2(t-0.015) + 0.01x_2(t-0.015) + 0.01x_1(t-0.02) + \\ &\quad 0.01x_1^2(t-0.02) + 0.01x_2(t-0.02) + 25\cos(1.29t) + 4.5u(t) + \phi_1(t) \end{aligned} \tag{36}$$

where $\phi_1(t)$ is a white noise with zero mean and variance 0.1. The control objective is to design robust H_∞ control law such that the chaotic system is stable. Define $\gamma = 1$, $R = \mathrm{diag}\{0.1, 0.1\}$.

This system is chaotic without control. The trajectory of the system with $u(t) = 0$ is shown in Figs. 1 and 2. Define two fuzzy sets for state x_1 with labels F_1^1 (positive large, M_1) and F_1^2 (negative large, M_2) which are characterized by the following membership functions respectively $\mu_{F_1^1}(x_1(t)) = \frac{x_1(t) - M_2}{M_1 - M_2}$, $\mu_{F_1^2}(x_1(t)) = \frac{M_1 - x_1(t)}{M_1 - M_2}$ ($M_1 = 10$, $M_2 = -8$). So we can get fuzzy model parameters as

$$A_1 = \begin{bmatrix} 0 & 2.5 \\ -6.8 & 0.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & -2.5 \\ -4.496 & 0.1 \end{bmatrix},$$

$$A_{11} = A_{12} = A_{21} = A_{22} = \begin{bmatrix} 0 & 0 \\ 0.01 & 0.01 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

According to Theorem 1, we can obtain the following feedback gains: $K_1 = [1.6011 \quad -4.7417]$, $K_2 = [-0.3975 \quad -4.2111]$. According to Theorem 2, an adaptive RBF NN control is introduced, the weights, center and width of the RBF are updated online according to the update laws (29)~(31). For comparison purpose, we apply the robust H_∞ controller proposed in [4] to the system, which is designed based on T-S fuzzy model and without considering fuzzy modeling error. The result is illustrated in Fig. 4. It is clear that a quite satisfactory control result is obtained by using the proposed controller.

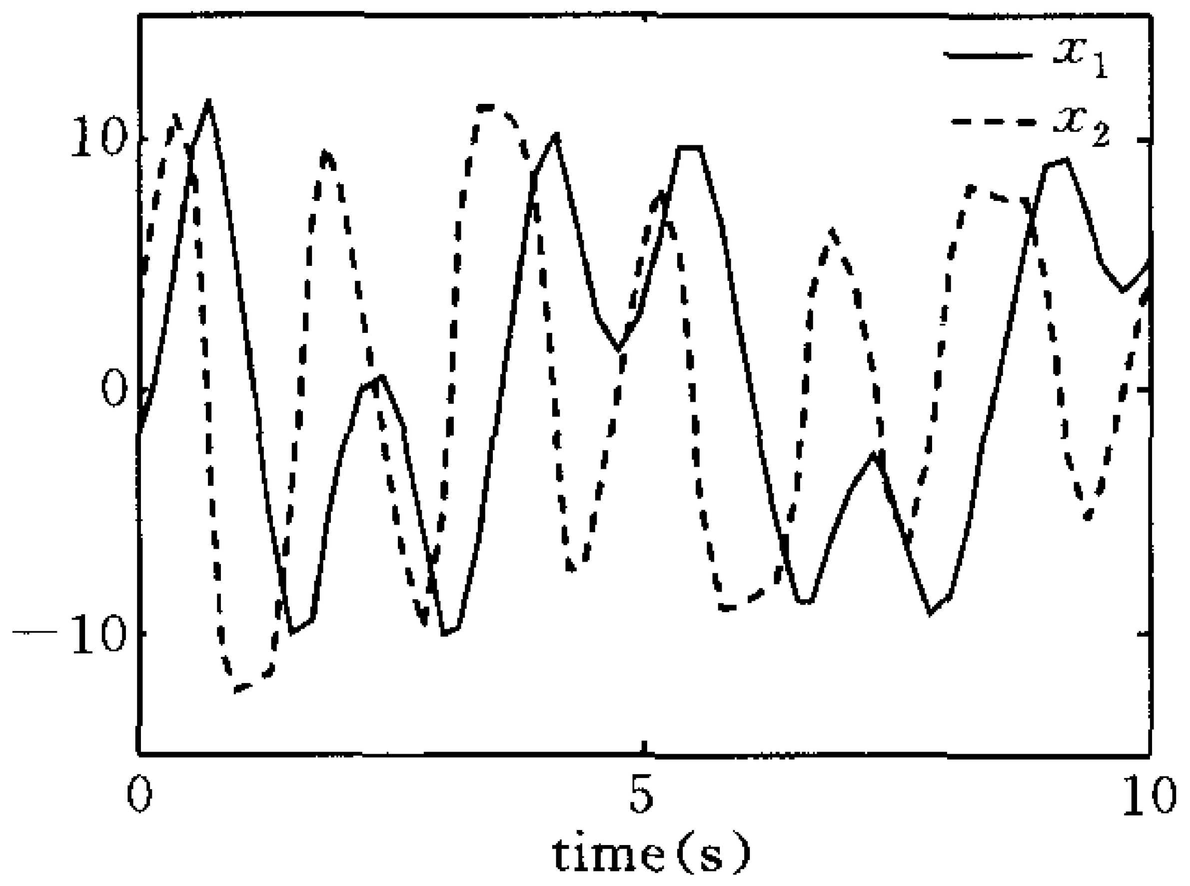

Fig. 1 Chaotic behavior of the system with no control force

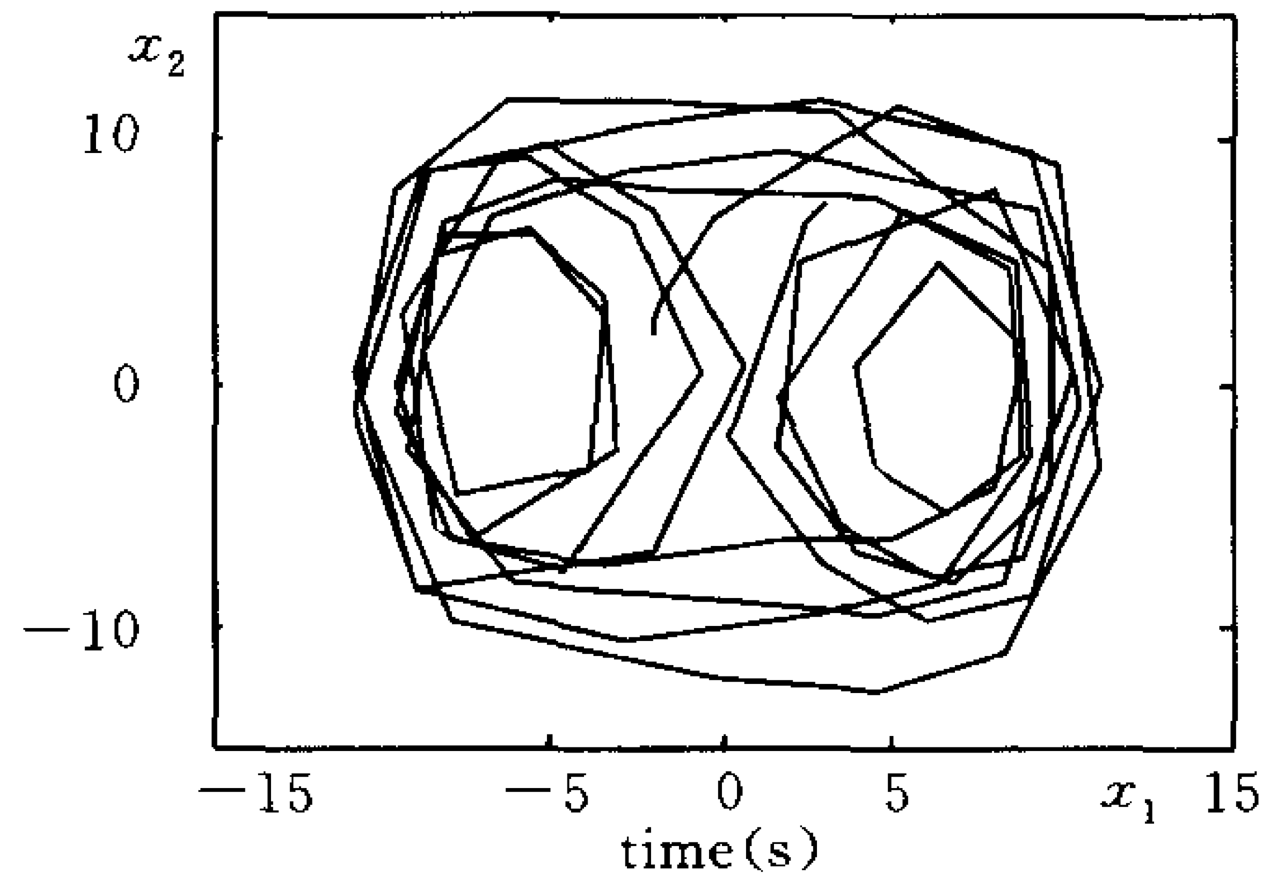

Fig. 2 Phase-plane trajectory of chaotic system

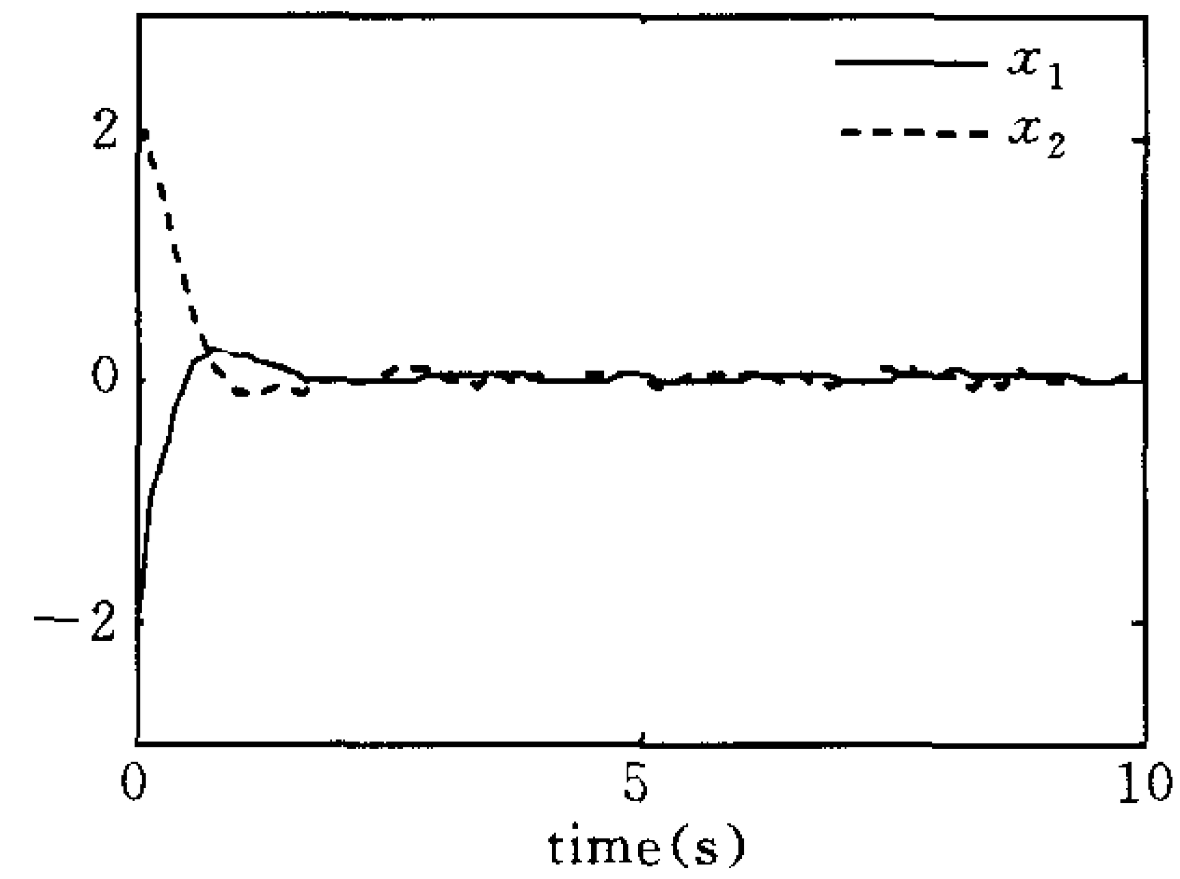

Fig. 3 Closed-loop controlled system trajectory using the proposed controller

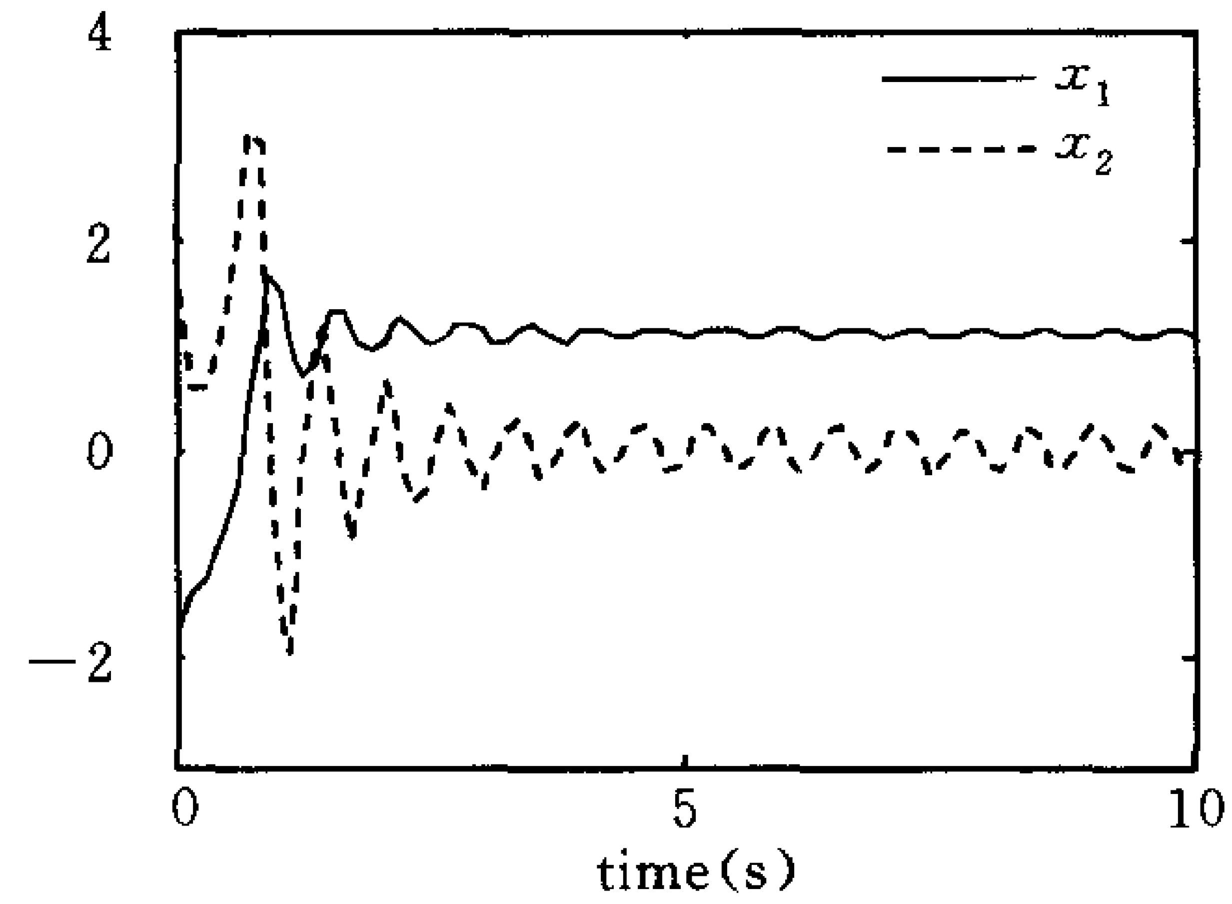

Fig. 4 The response of the system controlled by fuzzy H_∞ controller

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基于模糊模型和神经网络的多时滞不确定非线性系统的鲁棒 H_∞ 控制

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摘 要 针对一类具有多时滞的不确定非线性系统，提出了一种基于模糊模型和神经网络的组合控制方法．利用具有多时滞的模糊 T-S 模型对系统进行近似建模并给出基于线性矩阵不等式(LMI)的模糊 H_∞ 控制律．提出完全自适应 RBF 神经网络控制方法，通过在线自适应调整 RBF 神经网络的权重、函数中心和宽度，来对消系统的未知不确定性和模糊建模误差的影响，不要求系统的不确定项和模糊建模误差满足任何匹配条件或约束，并证明了闭环系统的稳定性．最后，将所提出的方法应用到一具有多时滞的非线性混沌系统，仿真结果表明了该方法的有效性．

关键词 多时滞，模糊 T-S 模型，神经网络
中图分类号 TP18

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