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Robust H_∞ Control for a Class of Nonlinear Systems with Input Unmodeled Dynamics[1)]

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Abstract The problem of robust H_∞ control of a class of nonlinear systems with input dynamical uncertainty is dealt with. By the recursive design approach, a robust controller is constructed such that the closed-loop system has an arbitrarily small L_2 gain from disturbance to output and in the absence of disturbance, the closed-loop system is globally asymptotically stable.

Key words H_∞ control, input unmodeled dynamics, backstepping.

1 Introduction

The problem of H_∞ control occupies an important part of classical as well as modern control theory. Several important problems such as robust control, output regulation, model reference and tracking control can be recast as an H_∞ problem[1,2]. The robust H_∞ control of nonlinear systems with structural uncertainty has attracted considerable attention, as in [3], while, with the nonlinear systems having input unmodeled dynamics, such a problem has not received sufficient attentions. This paper deals with the robust H_∞ control problem of a class of nonlinear systems with input unmodeled dynamics and presents a method for the construction of the robust controllers.

2 Problem formulation

We consider a class of nonlinear systems of the form

$$\begin{aligned}
\dot{\boldsymbol{z}} &= f(\boldsymbol{z},x_1) + p(\boldsymbol{z},x_1)\boldsymbol{w} \\
\dot{x}_1 &= x_2 + f_1(\boldsymbol{z},\boldsymbol{x}) + p_1(\boldsymbol{z},\boldsymbol{x})\boldsymbol{w} \\
&\vdots \\
\dot{x}_{r-1} &= x_r + f_{r-1}(\boldsymbol{z},\boldsymbol{x}) + p_{r-1}(\boldsymbol{z},\boldsymbol{x})\boldsymbol{w} \\
\dot{x}_r &= v + f_r(\boldsymbol{z},\boldsymbol{x}) + p_r(\boldsymbol{z},\boldsymbol{x})\boldsymbol{w} \\
y &= x_1 \\
\dot{\boldsymbol{\xi}} &= A(\boldsymbol{\xi}) + \boldsymbol{b}u \\
v &= c(\boldsymbol{\xi}) + u
\end{aligned} \tag{1}$$

where $\boldsymbol{z}\in R^{n-r}$, $\boldsymbol{\xi}\in R^p$, $x_i\in R$, $i=1,\cdots,r$, $\boldsymbol{x}=(x_1,\cdots,x_r)$ are the system states, $u\in R$, $y\in R$, $\boldsymbol{w}\in R^q$ are the system input, output and disturbance, respectively, f and p are continuous functions with $f(\boldsymbol{0},0)=\boldsymbol{0}$. Throughout this paper, we restrict continuous functions f_i, p_i, $i=1,\cdots,r$ by the following assumptions.

A_1) There exists a number $\mu>0$ such that for each $i=1,\cdots,r$,

$$|f_i(\boldsymbol{z},\boldsymbol{x})| \leqslant \mu(\|\boldsymbol{z}\| + |x_1| + \cdots + |x_i|)$$

A_2) There exists a number $M>0$ such that for each $i=1,\cdots,r$, $\|p_i(\boldsymbol{z},\boldsymbol{x})\|\leqslant M$.

The ξ-subsystem

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$$\dot{\boldsymbol{\xi}} = A(\boldsymbol{\xi}) + \boldsymbol{b}u, \quad v = c(\boldsymbol{\xi}) + u \tag{2}$$

with input $u \in R$ and output $v \in R$ represents the input unmodeled dynamics, in which $\boldsymbol{b} \in R^p$ is an unknown vector and $A(\boldsymbol{\xi}), c(\boldsymbol{\xi})$ are unknown continuous functions vanishing at zero. Clearly, system (2) has relative degree of zero. To restrict the admissible unmodeled dynamics, we make the following assumptions.

B_1) There exists a constant $K > 0$ such that $\|\boldsymbol{b}\| \leqslant K$.

B_2) The zero dynamics of $\boldsymbol{\xi}$-subsystem

$$\dot{\boldsymbol{\xi}} = A(\boldsymbol{\xi}) - \boldsymbol{b}c(\boldsymbol{\xi}) \triangleq A_0(\boldsymbol{\xi})$$

is such that, when it is disturbed by $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$

$$\dot{\boldsymbol{\zeta}} = A_0(\boldsymbol{\zeta} + \boldsymbol{d}_1) + \boldsymbol{d}_2$$

there exist positive numbers α_1, α_2 and a C^1, positive definite, radially unbounded function $V(\boldsymbol{\zeta})$ such that

$$\frac{\partial V}{\partial \boldsymbol{\zeta}}(A_0(\boldsymbol{\zeta} + \boldsymbol{d}_1) + \boldsymbol{d}_2) \leqslant \alpha_1 \|(\boldsymbol{d}_1^{\mathrm{T}}, \boldsymbol{d}_2^{\mathrm{T}})^{\mathrm{T}}\|^2 - \alpha_2 \|\boldsymbol{\zeta}\|^2 \tag{3}$$

B_3) There exists a constant $\bar{c} > 0$ such that

$$|c(\boldsymbol{\xi})| \leqslant \bar{c} \|\boldsymbol{\xi}\|$$

The admissible conditions B_1), B_2) and B_3) were used in [4, 5], under which the robust stabilization problem of nonlinear systems like (1) were investigated. The objective of this paper is, under the assumptions A_1)$\sim$$\mathrm{A}_2$) and B_1)$\sim$$\mathrm{B}_3$), to construct a state feedback controller for systems (1) to attain global disturbance attenuation with internal stability. The precise statement of this control problem is that for any given $\gamma > 0$, find a state feedback controller

$$u = u(\boldsymbol{x}) \tag{4}$$

with $u(\mathbf{0}) = 0$ such that

1) for any $\boldsymbol{w}(t) \in L_2[0, \infty)$, the response of the closed-loop system (1)$\sim$(4) starting from the origin satisfies

$$\int_0^T |y(t)|^2 \mathrm{d}t \leqslant \gamma^2 \int_0^T \|\boldsymbol{w}(t)\|^2 \mathrm{d}t, \text{for all } T \geqslant 0$$

2) when $\boldsymbol{w}(t) = \mathbf{0}$, the closed-loop system is globally asymptotically stable at the equilibrium $(\boldsymbol{z}, \boldsymbol{x}, \boldsymbol{\xi}) = \mathbf{0}$.

In terms of [2, 6], a dissipation inequality plays a fundamental role in the problem of H_∞ control, on the basis of which the problem of H_∞ control becomes how to construct a controller and a storage function that make the dissipation inequality true.

Lemma 1. Consider the system

$$\begin{aligned} \dot{\boldsymbol{x}} &= f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{w} \\ y &= h(\boldsymbol{x}) \end{aligned} \tag{5}$$

where $\boldsymbol{x} \in R^n, \boldsymbol{w} \in R^q, y \in R$, $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are continuous functions with $f(\mathbf{0}) = \mathbf{0}, h(\mathbf{0}) = 0$. $\gamma > 0$ is a given number. If there exist a C^1, positive definite, radially unbounded function $V(\boldsymbol{x})$ and a class K_∞ function $\alpha(r)$, such that

$$\frac{\partial V}{\partial \boldsymbol{x}}[f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{w}] + [h(\boldsymbol{x})]^2 \leqslant \gamma^2 \|\boldsymbol{w}\|^2 - \alpha(\|\boldsymbol{x}\|) \tag{6}$$

then for any $\boldsymbol{w}(t) \in L_2[0, \infty)$, the output of system (5) starting from the origin satisfies

$$\int_0^T |y(t)|^2 \mathrm{d}t \leqslant \gamma^2 \int_0^T \|\boldsymbol{w}(t)\|^2 \mathrm{d}t, \text{ for any } T > 0$$

and when $\boldsymbol{w}(t) = \mathbf{0}$, system (5) is globally asymptotically stable at the equilibrium $\boldsymbol{x} = \mathbf{0}$.

Remark 1. According to [7, 8], the significance of the dissipation inequality (6) is that system (5) is ISS with respect to $\boldsymbol{w}$, which means that the effect of $\boldsymbol{w}$ on the system states is attenuated as well as the effect of $\boldsymbol{w}$ on the output[7,9].

3 Main Conclusions

Lemma 2. Consider nonlinear systems described by equations of the form

$$\begin{aligned} \dot{x}_1 &= x_2 + f_1(\boldsymbol{z},\boldsymbol{x}) + p_1(\boldsymbol{z},\boldsymbol{x})\boldsymbol{w} \\ &\vdots \\ \dot{x}_{n-1} &= x_n + f_{n-1}(\boldsymbol{z},\boldsymbol{x}) + p_{n-1}(\boldsymbol{z},\boldsymbol{x})\boldsymbol{w} \\ \dot{x}_n &= u + f_n(\boldsymbol{z},\boldsymbol{x}) + p_n(\boldsymbol{z},\boldsymbol{x})\boldsymbol{w} \end{aligned} \tag{7}$$

where $\boldsymbol{z}\in R^m$ and $\boldsymbol{w}\in R^q$ are viewed as the external signals, $x_i\in R$, $i=1,\cdots,n$ are the system states and $\boldsymbol{x}=(x_1,\cdots,x_n)$, $u\in R$ is the system input. Functions $f_i(\boldsymbol{z},\boldsymbol{x})$, $p_i(\boldsymbol{z},\boldsymbol{x})$, $i=1,\cdots,n$ are continuous and satisfy assumptions A_1) and A_2). Then, for any given numbers γ_1, γ_2, $\gamma_3>0$, there exist a positive definite quadratic function $V(x_1,\cdots,x_n)$, a positive number λ and a linear feedback controller

$$u = a_1x_1 + \cdots + a_nx_n \tag{8}$$

such that the time derivative of $V(x_1,\cdots,x_n)$ along the trajectories of the closed-loop system (7),(8) satisfies

$$\dot{V}(x_1,\cdots,x_n) \leqslant \gamma_1\|\boldsymbol{w}\|^2 + \gamma_2\|\boldsymbol{z}\|^2 - \lambda\cdot(|x_1|^2+\cdots+|x_n|^2) - \gamma_3|x_1|^2 \tag{9}$$

for all $\boldsymbol{z}\in R^m$, $\boldsymbol{w}\in R^q$ and $x_i\in R$, $i=1,\cdots,n$.

Proof. We give the proof using backstepping. Let $\delta_1=n^{-1}\gamma_1$ and $\delta_2=n^{-1}\gamma_2$.

With x_2 viewed as a virtual input, construct the Lyapunov function

$$V_1(x_1) = \frac{1}{2}x_1^2$$

By using of A_1) and A_2) and computing the time derivative of $V_1(x)$along (7), we obtain that

$$\begin{aligned} \dot{V}_1(x_1) \leqslant{}& x_1x_2 + \mu x_1^2 + \mu|x_1|\,\|\boldsymbol{z}\| + \delta_1\|\boldsymbol{w}\|^2 + \frac{1}{4}M^2\delta_1^{-1}x_1^2 \leqslant \\ & \delta_1\|\boldsymbol{w}\|^2 + \delta_2\|\boldsymbol{z}\|^2 - |x_1|^2 - \gamma_3|x_1|^2 + \\ & x_1\left(x_2 + \mu x_1 + \frac{1}{4}M^2\delta_1^{-1}x_1 + \frac{1}{4}\mu^2\delta_2^{-1}x_1 + x_1 + \gamma_3x_1\right) \end{aligned}$$

Choosing the virtual control law

$$x_2 = -\left(\mu + \frac{1}{4}M^2\delta_1^{-1} + \frac{1}{4}\mu^2\delta_2^{-1} + 1 + \gamma_3\right)x_1$$

we obtain that

$$\dot{V}_1(x_1) \leqslant \delta_1\|\boldsymbol{w}\|^2 + \delta_2\|\boldsymbol{z}\|^2 - |x_1|^2 - \gamma_3|x_1|^2$$

Suppose that at step i, there exist a virtual controller

$$x_{i+1} = \mu_1x_1 + \cdots + \mu_ix_i \tag{10}$$

a positive definite quadratic form $V_i(x_1,\cdots,x_i)$ and a positive number λ_i such that

$$\dot{V}_i(x_1,\cdots,x_i) \leqslant i\cdot\delta_1\|\boldsymbol{w}\|^2 + i\cdot\delta_2\|\boldsymbol{z}\|^2 - \lambda_i(|x_1|^2+\cdots+|x_i|^2) - \gamma_3|x_1|^2 \tag{11}$$

With x_{i+2} viewed as a virtual input, construct the Lyapunov function

$$V_{i+1}(x_1,\cdots,x_{i+1}) = V_i(x_1,\cdots,x_i) + \frac{1}{2}(x_{i+1} - (\mu_1x_1 + \cdots + \mu_ix_i))^2$$

For notional convenience, denote $f_k=f_k(\boldsymbol{z},\boldsymbol{x})$, $p_k=p_k(\boldsymbol{z},\boldsymbol{x})$, $i=1,\cdots,r$. Letting

$$v_{i+1} = x_{i+1} - (\mu_1x_1 + \cdots + \mu_ix_i)$$

the time derivative of V_{i+1} along system (7) is

$$\dot{V}_{i+1} = \frac{\partial V_i}{\partial x_1}(x_2 + f_1 + p_1\boldsymbol{w}) + \cdots + \frac{\partial V_i}{\partial x_i}((\mu_1x_1 + \cdots + \mu_ix_i) + f_i + p_i\boldsymbol{w}) +$$

$$v_{i+1}(x_{i+2} - \mu_1 x_2 - \cdots - \mu_i x_{i+1} + \partial V_i/\partial x_i) +$$
$$v_{i+1}(f_{i+1} - \mu_1 f_1 - \cdots - \mu_i f_i) + v_{i+1}(p_{i+1} - \mu_1 p_1 - \cdots - \mu_i p_i)\boldsymbol{w}$$

Thanks to assumption A_2), there exists a constant $M_{i+1} > 0$ such that

$$\| p_{i+1} - \mu_1 p_1 - \cdots - \mu_i p_i \| \leqslant M_{i+1}$$

Using the completion of square, it is straightforward to show that

$$| v_{i+1}(p_{i+1} - \mu_1 p_1 - \cdots - \mu_i p_i)\boldsymbol{w} | \leqslant \delta_1 \| \boldsymbol{w} \|^2 + \frac{1}{4}\delta_1^{-1} M_{i+1}^2 v_{i+1}^2 \tag{12}$$

From the fact that quadratic function $x_1^2 + \cdots + x_i^2 + v_{i+1}^2$ is positive definite, there exists a positive number λ_{i+1} such that

$$\lambda_i(x_1^2 + \cdots + x_i^2 + v_{i+1}^2) \geqslant 2\lambda_{i+1}(x_1^2 + \cdots + x_{i+1}^2) \tag{13}$$

Owing to assumption A_1), there exists a positive number κ_{i+1}, such that

$$| f_{i+1} - \mu_1 f_1 - \cdots - \mu_i f_i | \leqslant \kappa_{i+1}(\| \boldsymbol{z} \| + | x_1 | + \cdots + | x_i | + | x_{i+1} |)$$

It follows by the completion of square that

$$| v_{i+1}(f_{i+1} - \mu_1 f_1 - \cdots - \mu_i f_i) | \leqslant$$
$$\frac{1}{4}(i+1)\lambda_{i+1}^{-1}\kappa_{i+1}^2 v_{i+1}^2 + \lambda_{i+1}(| x_1 |^2 + \cdots + | x_{i+1} |^2) + \frac{1}{4}\kappa_{i+1}^2 \delta_2^{-1} v_{i+1}^2 + \delta_2 \| \boldsymbol{z} \|^2 \tag{14}$$

Using (11), (12) and (14), we have

$$\begin{aligned}\dot{V}_{i+1} \leqslant\ & i \cdot \delta_1 \| \boldsymbol{w} \|^2 + i \cdot \delta_2 \| \boldsymbol{z} \|^2 - \lambda_i(x_1^2 + \cdots + x_i^2 + v_{i+1}^2) - \gamma_3 | x_1 |^2 + \\ & v_{i+1}(x_{i+2} - (\mu_1 x_2 + \cdots + \mu_i x_{i+1}) + \partial V_i/\partial x_i + \lambda_i v_{i+1}) + \\ & | v_{i+1}(f_{i+1} - \mu_1 f_1 - \cdots - \mu_i f_i) | + | v_{i+1}(p_{i+1} - \mu_1 p_1 - \cdots - \mu_i p_i)\boldsymbol{w} | \leqslant \\ & (i+1)\cdot \delta_1 \| \boldsymbol{w} \|^2 + (i+1)\cdot \delta_2 \| \boldsymbol{z} \|^2 - \lambda_{i+1}(x_1^2 + \cdots + x_i^2 + x_{i+1}^2) - \gamma_3 | x_1 |^2 + \\ & v_{i+1}\Big(x_{i+2} - (\mu_1 x_2 + \cdots + \mu_i x_{i+1}) + \partial V_i/\partial x_i + \lambda_i v_{i+1} + \\ & \frac{1}{4}(i+1)\lambda_{i+1}^{-1}\kappa_{i+1}^2 v_{i+1} + \frac{1}{4}\delta_1^{-1} M_{i+1}^2 v_{i+1} + \frac{1}{4}\delta_2^{-1}\kappa_{i+1}^2 v_{i+1} \Big)\end{aligned}$$

Since V_i is a quadratic function, $\partial V_i/\partial x_i$ is a linear function. Choosing the virtual controller

$$\begin{aligned}x_{i+2} =\ & \mu_1 x_2 + \cdots + \mu_i x_{i+1} - \partial V_i/\partial x_i - \\ & \Big(\frac{1}{4}(i+1)\kappa_{i+1}^2 \lambda_{i+1}^{-1} + \frac{1}{4}\delta_1^{-1} M_{i+1}^2 + \frac{1}{4}\delta_2^{-1}\kappa_{i+1}^2 + \lambda_i \Big) v_{i+1}\end{aligned} \tag{15}$$

which is linear, we obtain that

$$\dot{V}_{i+1} \leqslant (i+1)\cdot \delta_1 \| \boldsymbol{w} \|^2 + (i+1)\cdot \delta_2 \| \boldsymbol{z} \|^2 - \lambda_{i+1}(x_1^2 + \cdots + x_{i+1}^2) - \gamma_3 | x_1 |^2 \tag{16}$$

Repeating the above inductive argument n times, we complete the proof of Lemma 2. □

Theorem 3. Consider system (1). Suppose that it satisfies assumptions A_1) and A_2), and its input unmodeled dynamics satisfies the admissible conditions B_1), B_2) and B_3). If there exist a C^1, positive definite, radially unbounded function $V_z(\boldsymbol{z})$, positive numbers β_1 and β_2 such that

$$\frac{\partial V_z}{\partial \boldsymbol{z}}[f(\boldsymbol{z}, x_1) + p(\boldsymbol{z}, x_1)w] \leqslant \beta_2 \| (\boldsymbol{w}, x_1)^{\mathrm{T}} \|^2 - \beta_1 \| \boldsymbol{z} \|^2 \tag{17}$$

for all $\boldsymbol{z} \in R^{n-r}, \boldsymbol{w} \in R^q, x_1 \in R$, then for any given $\gamma > 0$, there exists a linear feedback controller

$$u = u(x_1, \cdots, x_r) \tag{18}$$

such that

1) for every disturbance $\boldsymbol{w}(t) \in L_2[0, \infty)$, the output of the closed-loop system (1)～(18) starting from the origin satisfies

$$\int_0^T |y(t)|^2 dt \leqslant \gamma^2 \int_0^T \|w(t)\|^2 dt, \text{ for all } T \geqslant 0$$

2) when $w(t)=\mathbf{0}$, the closed-loop system is globally asymptotically stable at the equilibrium origin.

Proof. We first give the proof for $r>1$.

Without loss of generality, we suppose that $\beta_2=1$.

Choose $\gamma_1=1, \gamma_2=\beta_1/2, \gamma_3=3\gamma^{-2}+1$. In terms of Lemma 2 and the construction of the Lyapunov function in the proof, there exist a linear controller

$$u_r = a_1 x_1 + \cdots + a_r x_r$$

a positive number λ and a positive definite quadratic function with the form

$$V_r(x_1, \cdots x_r) = V_{r-1}(x_1, \cdots, x_{r-1}) + \frac{1}{2}(x_r - (\mu_1 x_1 + \cdots + \mu_{r-1} x_{r-1}))^2 \tag{19}$$

such that

$$\begin{gathered}\frac{\partial V_r}{\partial x_1}(x_2 + f_1 + p_1 w) + \cdots + \frac{\partial V_r}{\partial x_r}(a_1 x_1 + \cdots + a_r x_r + f_r + p_r w) \leqslant \\ \|w\|^2 + \frac{1}{2}\beta_1 \|z\|^2 - \lambda(x_1^2 + \cdots + x_r^2) - (3\gamma^{-2}+1)|x_1|^2\end{gathered} \tag{20}$$

Denote $v_r = x_r - \mu_1 x_1 - \cdots - \mu_{r-1} x_{r-1}$. By introducing a transformation $\boldsymbol{\eta} = \boldsymbol{\xi} - \boldsymbol{b} v_r$, system (1) becomes

$$\begin{aligned}\dot{\boldsymbol{\eta}} &= A_0(\boldsymbol{\eta} + \boldsymbol{b} v_r) + \boldsymbol{\phi}(\boldsymbol{z}, \boldsymbol{x}) + \psi(\boldsymbol{z}, \boldsymbol{x}) \boldsymbol{w} \\ \dot{\boldsymbol{z}} &= f(\boldsymbol{z}, \boldsymbol{x}) + p(\boldsymbol{z}, \boldsymbol{x}) \boldsymbol{w} \\ \dot{x}_1 &= x_2 + f_1(\boldsymbol{z}, \boldsymbol{x}) + p_1(\boldsymbol{z}, \boldsymbol{x}) \boldsymbol{w} \\ &\vdots \\ \dot{x}_{r-1} &= x_r + f_{r-1}(\boldsymbol{z}, \boldsymbol{x}) + p_{r-1}(\boldsymbol{z}, \boldsymbol{x}) \boldsymbol{w} \\ \dot{x}_r &= c(\boldsymbol{\eta} + \boldsymbol{b} v_r) + u + f_r(\boldsymbol{z}, \boldsymbol{x}) + p_r(\boldsymbol{z}, \boldsymbol{x}) \boldsymbol{w} \\ y &= x_1\end{aligned} \tag{21}$$

where

$$\begin{aligned}\psi(\boldsymbol{z}, \boldsymbol{x}) &= \boldsymbol{b}(\mu_1 p_1 + \cdots + \mu_{r-1} p_{r-1} - p_r) \\ \boldsymbol{\phi}(\boldsymbol{z}, \boldsymbol{x}) &= \boldsymbol{b}(\mu_1(x_2 + f_1) + \cdots + \mu_{r-1}(x_r + f_{r-1}) - f_r)\end{aligned}$$

Owing to assumptions B_1) and B_2), there exists a positive definite and proper function $V_\eta(\boldsymbol{\eta})$ such that

$$\begin{aligned}\dot{V}_\eta(\boldsymbol{\eta}) \leqslant & \alpha_1 \|\boldsymbol{b} v_r\|^2 + \alpha_1 \|\boldsymbol{\phi}(\boldsymbol{z}, \boldsymbol{x}) + \psi(\boldsymbol{z}, \boldsymbol{x}) \boldsymbol{w}\|^2 - \alpha_2 \|\boldsymbol{\eta}\|^2 \leqslant \\ & \alpha_1 K^2 v_r^2 + 2\alpha_1 (\|\boldsymbol{\phi}(\boldsymbol{z}, \boldsymbol{x})\|^2 + \|\psi(\boldsymbol{z}, \boldsymbol{x})\|^2 \|\boldsymbol{w}\|^2) - \alpha_2 \|\boldsymbol{\eta}\|^2 \leqslant \\ & \alpha_1 (K^2 v_r^2 + 2\|\boldsymbol{\phi}(\boldsymbol{z}, \boldsymbol{x})\|^2) + 2\alpha_1 \|\psi(\boldsymbol{z}, \boldsymbol{x})\|^2 \|\boldsymbol{w}\|^2 - \alpha_2 \|\boldsymbol{\eta}\|^2\end{aligned}$$

Due to A_2), $\psi(\boldsymbol{z}, \boldsymbol{x})$ is a bounded function, so there exists a positive number $\overline{M}$ such that $\|\psi(\boldsymbol{z}, \boldsymbol{x})\|^2 \leqslant \overline{M}$. Due to A_1), there exists a number $\bar{\kappa} > 0$ such that

$$\|\boldsymbol{\phi}(\boldsymbol{z}, \boldsymbol{x})\| \leqslant \bar{\kappa}(\|\boldsymbol{z}\| + |x_1| + \cdots + |x_r|)$$

Furthermore, there exists a number $\overline{L} > 0$ such that

$$\alpha_1 (K^2 v_r^2 + 2\|\boldsymbol{\phi}(\boldsymbol{z}, \boldsymbol{x})\|^2) \leqslant \overline{L}(\|\boldsymbol{z}\|^2 + x_1^2 + \cdots + x_r^2)$$

Let $\kappa = \min\left\{\frac{1}{2}\lambda \overline{L}^{-1}, \frac{1}{4}\beta_1 \overline{L}^{-1}, (2\alpha_1 \overline{M})^{-1}\right\}$ and $\overline{V}_\eta(\boldsymbol{\eta}) = \kappa V_\eta(\boldsymbol{\eta})$. We have

$$\dot{\overline{V}}_\eta(\boldsymbol{\eta}) \leqslant \frac{1}{4}\beta_1 \|\boldsymbol{z}\|^2 + \frac{1}{2}\lambda(x_1^2 + \cdots + x_r^2) + \|\boldsymbol{w}\|^2 - \kappa \alpha_2 \|\boldsymbol{\eta}\|^2 \tag{22}$$

Construct the Lyapunov function

$$\overline{W}(\boldsymbol{z}, x_1, \cdots, x_r, \boldsymbol{\eta}) = V_r(x_1, \cdots, x_r) + \overline{V}_\eta(\boldsymbol{\eta}) + V_z(\boldsymbol{z}) \tag{23}$$

and compute its time derivative along the solutions of system (21) with noting that $\partial V_r / \partial x_r = v_r$. Using (17), (20) and (22) and rearranging the terms, we obtain

$$\dot{\bar{W}} \leqslant \frac{\partial V_r}{\partial x_1}(x_2 + f_1 + p_1 w) + \cdots + \frac{\partial V_r}{\partial x_r}(u_r + f_r + p_r w) + v_r(c(\boldsymbol{\eta} + \boldsymbol{b} v_r) + u - u_r) + \frac{1}{4}\beta_1 \| \boldsymbol{z} \|^2 + \frac{1}{2}\lambda(x_1^2 + \cdots + x_r^2) + \| \boldsymbol{w} \|^2 - \kappa\alpha_2 \| \boldsymbol{\eta} \|^2 + \| \boldsymbol{w} \|^2 + x_1^2 - \beta_1 \| \boldsymbol{z} \|^2 \leqslant 3 \| \boldsymbol{w} \|^2 - 3\gamma^{-2} x_1^2 - \frac{1}{2}\lambda(x_1^2 + \cdots + x_r^2) - \kappa\alpha_2 \| \boldsymbol{\eta} \|^2 - \frac{1}{4}\beta_1 \| \boldsymbol{z} \|^2 + v_r(u - u_r + c(\boldsymbol{\eta} + \boldsymbol{b} v_r)) \tag{24}$$

With the help of B_1) and B_3), the last term in (24) is

$$v_r(u + c(\boldsymbol{\eta} + \boldsymbol{b} v_r) - u_r) \leqslant v_r(u - u_r) + | v_r | \bar{c} \cdot (\| \boldsymbol{\eta} \| + \| \boldsymbol{b} v_r \|) \leqslant v_r(u - u_r) + \bar{c} K v_r^2 + \frac{1}{2}\bar{c}^2(\kappa\alpha_2)^{-1} v_r^2 + \frac{1}{2}(\kappa\alpha_2) \| \boldsymbol{\eta} \|^2 \tag{25}$$

Substituting (25) in (24) gives

$$\dot{\bar{W}} \leqslant 3 \| \boldsymbol{w} \|^2 - 3\gamma^{-2} x_1^2 - \frac{1}{2}\lambda(x_1^2 + \cdots + x_r^2) - \frac{1}{2}\kappa\alpha_2 \| \boldsymbol{\eta} \|^2 - \frac{1}{4}\beta_1 \| \boldsymbol{z} \|^2 + v_r\left(u - u_r + \bar{c} K v_r + \frac{1}{2}\bar{c}^2(\kappa\alpha_2)^{-1} v_r\right) \tag{26}$$

Let

$$W(\boldsymbol{z}, x_1, \cdots, x_r, \boldsymbol{\eta}) = \frac{1}{3}\gamma^2 \bar{W}(\boldsymbol{z}, x_1, \cdots, x_r, \boldsymbol{\eta})$$

Then, it is easy to show that linear feedback controller

$$u = u_r - \bar{c} K v_r - \frac{1}{2}\bar{c}^2(\kappa\alpha_2)^{-1} v_r \tag{27}$$

leads to

$$\dot{W} \leqslant \gamma^2 \| \boldsymbol{w} \|^2 - y^2 - \frac{1}{6}\gamma^2\left(\lambda(x_1^2 + \cdots + x_r^2) + \kappa\alpha_2 \| \boldsymbol{\eta} \|^2 + \frac{1}{2}\beta_1 \| \boldsymbol{z} \|^2\right) \tag{28}$$

From Lemma 1, we complete the proof for $r>1$.

When $r=1$, the proof is analogous to the above by using a directly transformation

$$\boldsymbol{\eta} = \boldsymbol{\xi} - \boldsymbol{b} x_1 \tag{29}$$

□

4 Conclusion

We have addressed the H_∞ control problem of a class of nonlinear systems with input unmodeled dynamics. State feedback controllers, which are robust to the unmodeled dynamics, are constructed to attain global disturbance attenuation with internal stability to an arbitrary degree of accuracy in the L_2 gain sense.

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一类带不确定输入动态非线性系统的鲁棒 H_∞ 控制

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摘　要　研究一类带不确定输入动态非线性系统的 H_∞ 控制问题. 在输入动态存在的条件下，利用反传设计方法构造了鲁棒状态反馈控制器，使得闭环系统在零初始状态下从干扰到输出的 L_2 增益任意小，同时在干扰输入恒为零时闭环系统是全局渐近稳定的.

关键词　H_∞ 控制，不确定输入动态，反传设计

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