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Pole-Assignment Descriptor Steady-State Kalman Estimators[1)]

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Abstract Using the modern time-series analysis method in the time domain, based on the autoregressive moving average (ARMA) innovation model and white noise estimators, and in terms of the pole-assignment principle in control theory, the pole-assignment descriptor steady-state Kalman estimators are presented for linear discrete-time descriptor stochastic systems. They have globally asymptotic stability, and can forget the effect of the initial state estimates at an exponentially decaying rate by assigning the poles of the estimators. They can handle the filtering, smoothing and prediction problems in a unified framework. They avoid the Riccati equation and the computation of the optimal initial state estimates so that reduce the computational burden. A simulation example shows their effectiveness.

Key words Descriptor systems, pole-assignment, steady-state Kalman estimator, modern time-series analysis method

1 Introduction

Descriptor systems often occur in many fields including circuit, economics, robotics, *etc*, and have attracted considerable attention in recent years. The disagvantages of classical descriptor Kalman filter given by Nikoukhah *et al.*[1] are that the solution of descriptor Riccati equation is required, which yields a larger computational burden, and it does not solve the smoothing problem. The descriptor Kalman estimators[2~6] proposed by the modern time-series analysis method can handle the filtering, smoothing and prediction problems in a unified framework, and avoid the Riccati equation. But their disadvantages are that they require the optimal initial state estimates[2~4], thus increase the computational burden. In this paper, using the modern time-series analysis method[6] and the pole assignment principle in control theory, based on the ARMA innovation model and white noise estimators, the pole assignment descriptor Kalman estimators are presented. They avoid the Riccati equation and computation of optimal initial state estimates. Not only they have the globally asymptotic stability, but also the effect of the initial state estimates can be rapidly forgotten by assigning the poles of the estimators.

Consider the discrete time descriptor stochastic system

$$M\boldsymbol{x}(t+1) = \Phi\boldsymbol{x}(t) + \Gamma\boldsymbol{w}(t) \tag{1}$$

$$\boldsymbol{y}(t) = H\boldsymbol{x}(t-k) + \boldsymbol{v}(t) \tag{2}$$

where the state $\boldsymbol{x}(t)\in R^n$, the measurement $\boldsymbol{y}(t)\in R^m$, $\boldsymbol{w}(t)\in R^r$, $k\geqslant 0$ is the measurement delay, $\boldsymbol{v}(t)\in R^m$, M, Φ, Γ and H are the constant matrices.

Assumption 1. M is a singular square matrix, that is, $\det(M)=0$.

Assumption 2. The system is regular, that is, $\det(zM-\Phi)\neq 0$, $z\in C$, where C is the complex field.

Assumption 3. The system is completely observable, that is,

$$\operatorname{rank}\begin{bmatrix} zM-\Phi \\ H \end{bmatrix} = n, \quad \forall z\in C \text{ and } \operatorname{rank}\begin{bmatrix} M \\ H \end{bmatrix} = n \tag{3}$$

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Assumption 4. $w(t)$ and $v(t)$ are correlated white noises with zero mean and

$$E\left\{\begin{bmatrix} w(t) \\ v(t) \end{bmatrix}\begin{bmatrix} w^{\mathrm{T}}(j) & v^{\mathrm{T}}(j) \end{bmatrix}\right\} = \begin{bmatrix} Q_w & S \\ S^{\mathrm{T}} & Q_v \end{bmatrix}\delta_{tj} \tag{4}$$

where E is the expectation operator, T denotes the transpose, $\delta_{tt}=1$, $\delta_{tj}=0$ $(t\neq j)$.

The descriptor Kalman filtering problem is to find the steady state Kalman estimators $\hat{x}(t|t+N)$ of the state $x(t)$ based on measurements $(y(t+N), y(t+N-1), \cdots)$. For $N=0$, $N>0$ or $N<0$, they are called the descriptor Kalman filter, smoother or predictor, respectively.

2 ARMA innovation model

From (1) and (2) we have

$$y(t) = H(M-q^{-1}\Phi)^{-1}\Gamma q^{-1-k}w(t) + v(t) \tag{5}$$

where q^{-1} is the backward shift operator. Introducing the left coprime factorization

$$H(M-q^{-1}\Phi)^{-1}\Gamma q^{-1-k} = A^{-1}Bq^{\tau} \tag{6}$$

where A and B are polynomial matrices having the form $X=X(q^{-1})=X_0+X_1q^{-1}+\cdots+X_{n_x}q^{-n_x}$, X_i are the coefficient matrices and n_x is the degree of X. We define $X_i=0$ $(i>n_x)$, and $A_0=I_m$, $B_0\neq 0$, τ is an integer, that is, $\tau=0$, $\tau>0$, or $\tau<0$.

Substituting (6) into (5) yields the ARMA innovation model

$$Ay(t) = D\varepsilon(t) \tag{7}$$

$$D\varepsilon(t) = Bq^{\tau}w(t) + Av(t) \tag{8}$$

where D is a stable polynomial matrix, $D_0=I_m$, and the innovation process $\varepsilon(t)\in R^m$ is the white noise with zero mean and variance matrix Q_ε. D and Q_ε can be obtained by using the Gevers-Wouters algorithm[6]. According to (7), the innovations $\varepsilon(t)$ can be computed recursively as

$$\varepsilon(t) = Ay(t) - D_1\varepsilon(t-1) - \cdots - D_{n_d}\varepsilon(t-n_d),\ t = n_d, n_d+1, \cdots \tag{9}$$

with the initial values $(\varepsilon(0), \cdots, \varepsilon(n_d-1))$.

3 Lemmas

Lemma 1[3,6]. For arbitrary integers i, j, t we have that

$$E[w(t)\varepsilon^{\mathrm{T}}(j)] = \Pi^w_{j-t},\quad E[v(t)\varepsilon^{\mathrm{T}}(j)] = \Pi^v_{j-t},\quad E[y(t)\varepsilon^{\mathrm{T}}(j)] = M_{t-j}Q_\varepsilon,$$

$$\Pi^w_i = Q_wF^{\mathrm{T}}_{i+(\tau\vee 0)} + SG^{\mathrm{T}}_{i+(\tau\vee 0)},\quad \Pi^v_i = Q_vG^{\mathrm{T}}_{i+(\tau\vee 0)} + S^{\mathrm{T}}F^{\mathrm{T}}_{i+(\tau\vee 0)} \tag{10}$$

where we define that $(a\vee b)=\max(a,b)$, $(a\wedge b)=\min(a,b)$. F_i, G_i and M_i can be computed recursively as

$$\begin{aligned} F_i &= -D_1F_{i-1} - \cdots - D_{n_d}F_{i-n_d} + \bar{B}_i,\quad F_i = 0\ (i<0),\quad \bar{B}_i = 0\ (i>n_{\bar{b}}), \\ G_i &= -D_1G_{i-1} - \cdots - D_{n_d}G_{i-n_d} + \bar{A}_i,\quad G_i = 0\ (i<0),\quad \bar{A}_i = 0\ (i>n_{\bar{a}}), \\ M_i &= -A_1M_{i-1} - \cdots - A_{n_a}M_{i-n_a} + D_i,\quad M_i = 0\ (i<0),\quad D_i = 0\ (i>n_d) \end{aligned} \tag{11}$$

where we define $\bar{B}=Bq^{(\tau\wedge 0)}$, $\bar{A}=Aq^{(-\tau\wedge 0)}$.

Lemma 2[3,6]. The optimal white noise estimators are given as

$$\hat{w}(t\mid t+N) = L^w_N\varepsilon(t+N)\ \text{and}\ \hat{v}(t\mid t+N) = L^v_N\varepsilon(t+N) \tag{12}$$

where $L^w_N=0(N<-(\tau\vee 0))$, $L^v_N=0(N<-(\tau\vee 0))$, and for $N\geqslant-(\tau\vee 0)$, we define

$$L^w_N = \sum_{i=-(\tau\vee 0)}^{N}\Pi^w_iQ_\varepsilon^{-1}q^{i-N}\ \text{and}\ L^v_N = \sum_{i=-(\tau\vee 0)}^{N}\Pi^v_iQ_\varepsilon^{-1}q^{i-N} \tag{13}$$

The optimal predictors $\hat{y}(t+i|t)$ can be computed recursively as

$$A(\tilde{q}^{-1})\hat{y}(t+j\mid t) = D\varepsilon(t+j),\quad j = 1, \cdots, i \tag{14}$$

where $\widetilde{A}(q^{-1})$ denotes the polynomial matrix $A(q^{-1})$ which operates only for time $(t+j)$, and $\varepsilon(t+j)=0$ $(j>0)$, $\hat{y}(i|j)=y(i)$ $(i\leqslant j)$.

Lemma 3[7]. Under Assumptions 2 and 3, there exists an $n\times m$ matrix K such that

$$\det[M-q^{-1}(\Phi+KH)] = \gamma q^{-n} \tag{15}$$

where $\gamma \neq 0$ is a constant.

4 Pole assignment descriptor steady state Kalman estimators

Premultiplying (2) by K, and setting $t=t+k$, and combining it with (1) yield

$$Mx(t+1) = (\Phi + KH)x(t) + \Gamma w(t) - Ky(t+k) + Kv(t+k) \tag{16}$$

which yields that $x(t)=[M-q^{-1}(\Phi+KH)]^{-1}[\Gamma w(t-1)-Ky(t+k-1)+Kv(t+k-1)]$. By Lemma 3, we have the nonrecursive representation of $x(t)$ as

$$x(t) = Rw(t+n-1) - Py(t+k+n-1) + Pv(t+k+n-1) \tag{17}$$

where we define the polynomial matrices R and P as

$$R = \mathrm{adj}[M - q^{-1}(\Phi + KH)]\Gamma/\gamma, \quad P = \mathrm{adj}[M - q^{-1}(\Phi + KH)]K/\gamma \tag{18}$$

where $\mathrm{adj}A$ denotes the adjoint of matrix A.

According to Assumption 3, $\mathrm{rank}[M^{\mathrm{T}} \quad H^{\mathrm{T}}]^{\mathrm{T}}=n$. Then there exists an $n\times m$ matrix T_0 such that $(M+T_0H)$ is nonsingular[6]. Premultiplying (2) by T_0, and setting $t=t+k$, and combining it with (1) yield the conventional system

$$x(t) = \Psi x(t-1) + \Psi_1 y(t+k) - \Psi_1 v(t+k) + \Psi_2 w(t-1) \tag{19}$$

$$y(t) = Hx(t-k) + v(t) \tag{20}$$

where $\Psi=(M+T_0H)^{-1}\Phi$, $\Psi_1=(M+T_0H)^{-1}T_0$ and $\Psi_2=(M+T_0H)^{-1}\Gamma$. And it can be proved[6] that (Ψ, H) is a completely observable pair.

Theorem 1. For the descriptor system (1)and (2), under Assumptions 1~4, we have the descriptor steady state Kalman estimators as

$$\hat{x}(t|t+N)= \Psi\hat{x}(t-1|t-1+N)+\Psi_1\hat{y}(t+k \mid t-1+N) - \Psi_1\hat{v}(t+k \mid t-1+N) + \Psi_2\hat{w}(t-1 \mid t-1+N) + K_N\varepsilon(t+N) \tag{21}$$

where the gain matrices K_N are given as

$$K_N = \left(\sum_{i=0}^{n_r} R_i \Pi^{w}_{N-n+1+i} - \sum_{i=0}^{n_p} P_i M_{k+n-1-i-N} Q_\varepsilon + \sum_{i=0}^{n_p} P_i \Pi^{v}_{N-k-n+1+i}\right) Q_\varepsilon^{-1} \tag{22}$$

where $\hat{w}(i|j)$, $\hat{v}(i|j)$ and $\hat{y}(i|j)$ can be computed by Lemma 2.

Proof. According to the projection property , we have

$$\hat{x}(t \mid t+N) = \hat{x}(t \mid t-1+N) + K_N\varepsilon(t+N), \quad K_N = E[x(t)\varepsilon^{\mathrm{T}}(t+N)]Q_\varepsilon^{-1} \tag{23}$$

Taking the projection operation for (19) yields

$$\hat{x}(t \mid t-1+N) = \Psi\hat{x}(t-1 \mid t-1+N) + \Psi_1\hat{y}(t+k \mid t-1+N) - \Psi_1\hat{v}(t+k \mid t-1+N) + \Psi_2\hat{w}(t-1 \mid t-1+N) \tag{24}$$

Substituting (24) into (23) yields (21). Substituting (17) into (23) and applying (10) yield (22). □

Theorem 2. If Ψ is a stable matrix, then the descriptor steady state Kalman estimators (21) are globally asymptotically stable, that is, $\hat{x}(t|t+N)$ are asymptotically independent of both the initial estimates $\hat{x}(t_0|t_0+N)$ and the innovation initial values $(\varepsilon(0), \cdots, \varepsilon(n_d-1))$.

Proof. The proof is similar to that as in [6], which is omitted. □

Theorem 3. If Ψ is an unstable matrix, then for the system (1) and (2) under Assumptions 1~4, we can suitably select an $n\times m$ matrix $\overline{T}_0$ such that $\overline{\Psi}=\Psi+\overline{T}_0H$ is stable, and the eigenvalues of $\overline{\Psi}$ can arbitrarily be assigned, so that the pole assignment descriptor steady state Kalman estimators have globally asymptotic stability as

$$\hat{x}(t|t+N)=\overline{\Psi}\hat{x}(t-1|t-1+N)+\Psi_1\hat{y}(t+k|t-1+N)-\overline{T}_0\hat{y}(t+k-1|t-1+N)- \Psi_1\hat{v}(t+k|t-1+N) + \overline{T}_0\hat{v}(t+k-1|t-1+N) + \Psi_2\hat{w}(t-1|t-1+N) + K_N\varepsilon(t+N) \tag{25}$$

Proof. Since (Ψ, H) is a completely observable pair, we can select $\overline{T}_0$ such that $\overline{\Psi}=\Psi+\overline{T}_0H$ is stable, and its eigenvalues can be arbitrarily assigned[6]. Premultiplying (20) by

$\overline{T}_0$, and setting $t=t+k$, and taking the projection operation on it and combining it with (21) yield (25). The globally asymptotic stability can be proved by Theorem 2. □

Remark 1. Given the innovation initial values, the poles of the estimators (25) are determined by the zeros of $\det(I_n - q^{-1}\overline{\Psi})=0$, i. e., the poles of the estimators (25) are determined by the eigenvalues of $\overline{\Psi}$.

Remark 2. In order to ensure that the effect of the initial values $\hat{x}(t_0 \mid t_0+N)$ is rapidly forgotten, we usually assign the eigenvalues of $\overline{\Psi}$ values close to the origin. If we assign the eigenvalues of $\overline{\Psi}$ values close to the bound of the unit circle, then although we can make $\overline{\Psi}$ a stable matrix, but the effect of the initial values $\hat{x}(t_0 \mid t_0+N)$ will be forgotten in a longer decaying process.

Remark 3. Theorem 2 is the special case of Theorem 3, that is, when Ψ is stable, we select $\overline{T}_0=0$. For the case that Ψ is a stable matrix but its eigenvalues are approximate to the unit circle, we can assign new eigenvalues of $\overline{\Psi}$ by using Theorem 3, in order that the effect of the initial values $\hat{x}(t_0 \mid t_0+N)$ can be rapidly forgotten.

Theorem 4. Assuming that the spectral radius of $\overline{\Psi}$ is ρ, $0<\rho<1$, that is, $\rho=\max(|\lambda_1|, \cdots, |\lambda_n|)$ where $\lambda_1, \cdots, \lambda_n$ are different real eigenvalues of $\overline{\Psi}$ and are inside the unit circle. Then the effect of the initial values $\hat{x}(t_0 \mid t_0+N)$ of the pole assignment Kalman estimators (25) decays to zero at the exponentially decaying rate $O(\rho^t)$, that is, arbitrarily taking two sets of the initial values of (25) as $(\hat{x}^{(i)}(t_0 \mid t_0+N); \varepsilon(0), \cdots, \varepsilon(n_d-1)), i=1,2$, which have the same innovation initial values, denoting the corresponding Kalman estimators (25) as $\hat{x}^{(i)}(t \mid t+N)$ and introducing the error

$$\boldsymbol{\delta}(t) = \hat{x}^{(1)}(t \mid t+N) - \hat{x}^{(2)}(t \mid t+N), \quad \boldsymbol{\delta}(t) = [\delta_1(t), \cdots, \delta_n(t)]^{\mathrm{T}} \tag{26}$$

we have

$$\delta_i(t) = O(\rho^t), \quad i = 1,2,\cdots,n \tag{27}$$

that is, $|\delta_i(t)| \leqslant \beta_i \rho^t$, $i=1,2,\cdots,n$.

Proof. Because the computation of measurement predictors and white noise estimators only dependent on the innovation initial values, and because there are the same innovation initial values in the two sets of initial values of (25), from (25) we have the difference equation

$$\boldsymbol{\delta}(t) = \overline{\Psi}\boldsymbol{\delta}(t-1) \tag{28}$$

Denoting the n linearly independent $n\times 1$ eigenvectors corresponding to the real eigenvalues $\lambda_1, \cdots, \lambda_n$ of $\overline{\Psi}$ as $\boldsymbol{\alpha}_1, \cdots, \boldsymbol{\alpha}_n$, where $\boldsymbol{\alpha}_i = [\alpha_{i1}, \cdots, \alpha_{in}]^{\mathrm{T}}, i=1,\cdots,n$, the difference equation (28) has the general solution[8]

$$\boldsymbol{\delta}(t) = c_1\lambda_1^t\boldsymbol{\alpha}_1 + \cdots + c_n\lambda_n^t\boldsymbol{\alpha}_n \tag{29}$$

where constants c_i can be computed from initial values $\boldsymbol{\delta}(t_0) = \hat{x}^{(1)}(t_0 \mid t_0+N) - \hat{x}^{(2)}(t_0 \mid t_0+N)$ by (29). In fact, taking $t=t_0$ in (29) yields linear equations

$$[\lambda_1^{t_0}\boldsymbol{\alpha}_1, \cdots, \lambda_n^{t_0}\boldsymbol{\alpha}_n][c_1, \cdots, c_n]^{\mathrm{T}} = \boldsymbol{\delta}(t_0) \tag{30}$$

Since $\boldsymbol{\alpha}_1, \cdots, \boldsymbol{\alpha}_n$ are linearly independent, the matrix $[\lambda_1^{t_0}\boldsymbol{\alpha}_1, \cdots, \lambda_n^{t_0}\boldsymbol{\alpha}_n]$ is nonsingular. Therefore, from (30) we have

$$[c_1, \cdots, c_n]^{\mathrm{T}} = [\lambda_1^{t_0}\boldsymbol{\alpha}_1, \cdots, \lambda_n^{t_0}\boldsymbol{\alpha}_n]^{-1}\boldsymbol{\delta}(t_0) \tag{31}$$

Equation (29) yields $|\delta_i(t)| \leqslant \beta_i\rho^t$, $\beta_i = \max(|c_1\alpha_{1i}|, \cdots, |c_n\alpha_{ni}|), i=1,2,\cdots,n$, that is, (27) holds. □

Remark 4. Theorem 4 gives the quantitative analysis that the effect of the initial values $\hat{x}(t_0 \mid t_0+N)$ exponentially decays to zero, and the decaying rate can be controlled by the spectral radius ρ of $\overline{\Psi}$.

5 Simulation example

Consider the descriptor stochastic system

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{x}(t+1) = \begin{bmatrix} 0.9 & 0 & 0 \\ -0.5 & 0.5 & 0 \\ -0.25 & 0 & 0.25 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \boldsymbol{w}(t) \tag{32}$$

$$\boldsymbol{y}(t) = [0 \quad 1 \quad 0] \boldsymbol{x}(t) + \boldsymbol{v}(t) \tag{33}$$

where $\boldsymbol{x}(t) = [x_1(t) \quad x_2(t) \quad x_3(t)]^{\mathrm{T}}$, $\boldsymbol{v}(t)$ is correlated with $\boldsymbol{w}(t)$, $\boldsymbol{v}(t) = 0.5\boldsymbol{w}(t) + \boldsymbol{\xi}(t)$. $\boldsymbol{w}(t)$ and $\boldsymbol{\xi}(t)$ are uncorrelated Gaussian white noises with zero mean and variances $\sigma_w^2 = 1$ and $\sigma_\xi^2 = 1$, respectively. The problem is to find $\hat{\boldsymbol{x}}(t|t)$.

Using Gevers-Wouters algorithm[6], the ARMA innovation model is easily obtained as

$$(1 - 0.9q^{-1})\boldsymbol{y}(t) = (1 - 0.5405q^{-1})\boldsymbol{\varepsilon}(t) \tag{34}$$

where $\sigma_\varepsilon^2 = 7.4471$, and

$$(1 - 0.5405q^{-1})\boldsymbol{\varepsilon}(t) = (2 - 0.8q^{-1})\boldsymbol{w}(t) + (1 - 0.9q^{-1})\boldsymbol{v}(t) \tag{35}$$

Applying Lemma 3 yields $K = [1 \quad -0.5 \quad 0]^{\mathrm{T}}$ and $\gamma = -0.125$. According to Theorem 1, taking $T_0 = [0.5 \quad 0.1 \quad 0.1]^{\mathrm{T}}$ we obtain

$$\boldsymbol{\Psi} = \begin{bmatrix} 2.15 & 0 & -1.25 \\ -2.5 & 0 & 2.5 \\ 1.9 & 0.5 & -1.5 \end{bmatrix}, \quad \boldsymbol{\Psi}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{\Psi}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad K_0 = \begin{bmatrix} 0.0265 \\ 0.6979 \\ 0.0265 \end{bmatrix} \tag{36}$$

Since $\boldsymbol{\Psi}$ has the eigenvalues $\lambda_1 = 1.5394$, $\lambda_2 = -1.4083$, and $\lambda_3 = 0.5189$, $\boldsymbol{\Psi}$ is an unstable matrix. Here we assign poles for two cases:

Case 1. Taking $\lambda_1 = 0.1$, $\lambda_2 = 0.2$ and $\lambda_3 = 0.4$, there exists $\overline{T}_0 = [0.649 \quad 0.05 \quad -0.234]^{\mathrm{T}}$ such that $\overline{\boldsymbol{\Psi}}$ has the above eigenvalues. Then we obtain the pole assignment descriptor steady state Kalman filter by Theorem 3 as

$$\begin{aligned} \hat{\boldsymbol{x}}(t \mid t) = {} & \overline{\boldsymbol{\Psi}}\hat{\boldsymbol{x}}(t-1 \mid t-1) + \boldsymbol{\Psi}_1\hat{\boldsymbol{y}}(t \mid t-1) - \overline{T}_0\boldsymbol{y}(t-1) + \\ & \overline{T}_0\hat{\boldsymbol{v}}(t-1 \mid t-1) + \boldsymbol{\Psi}_2\hat{\boldsymbol{w}}(t-1 \mid t-1) + K_0\boldsymbol{\varepsilon}(t) \end{aligned} \tag{37}$$

Taking the initial values $\hat{\boldsymbol{x}}(1|1) = [1.5 \quad 1.5 \quad 1.5]^{\mathrm{T}}$, $\boldsymbol{\varepsilon}(0) = 0$, the simulation results of the estimators (37) are shown in Fig. 1～Fig. 3, where the solid lines denote $x_i(t)$, and the dashed lines denote $\hat{x}_i(t|t)$, $i = 1, 2, 3$.

Case 2: Taking $\lambda_1 = 0.85$, $\lambda_2 = 0.95$ and $\lambda_3 = 0.9$, there exists $\overline{T}_0 = [-0.575 \quad 2.05 \quad -1.853]^{\mathrm{T}}$ such that $\overline{\boldsymbol{\Psi}}$ has the above eigenvalues. Then according to (37), taking the initial values $\hat{\boldsymbol{x}}(1|1) = [1.5 \quad 1.5 \quad 1.5]^{\mathrm{T}}$, $\boldsymbol{\varepsilon}(0) = 0$, the simulation results of the estimators (37) are shown in Fig. 4～Fig. 6, where the solid lines denote $x_i(t)$, and the dashed lines denote $\hat{x}_i(t|t)$, $i = 1, 2, 3$.

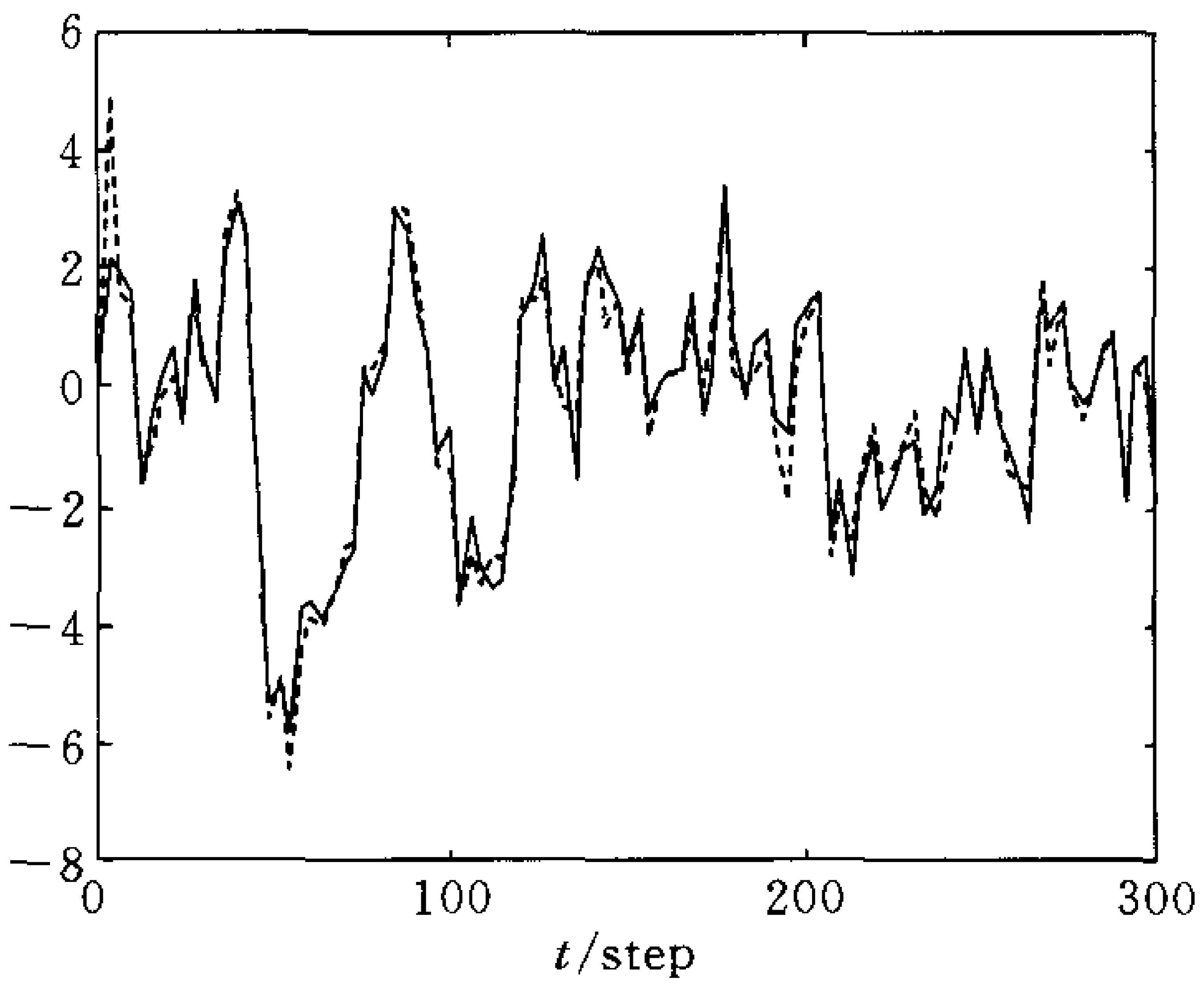

Fig. 1 $x_1(t)$ and descriptor Kalman filter $\hat{x}_1(t|t)$ (case 1)

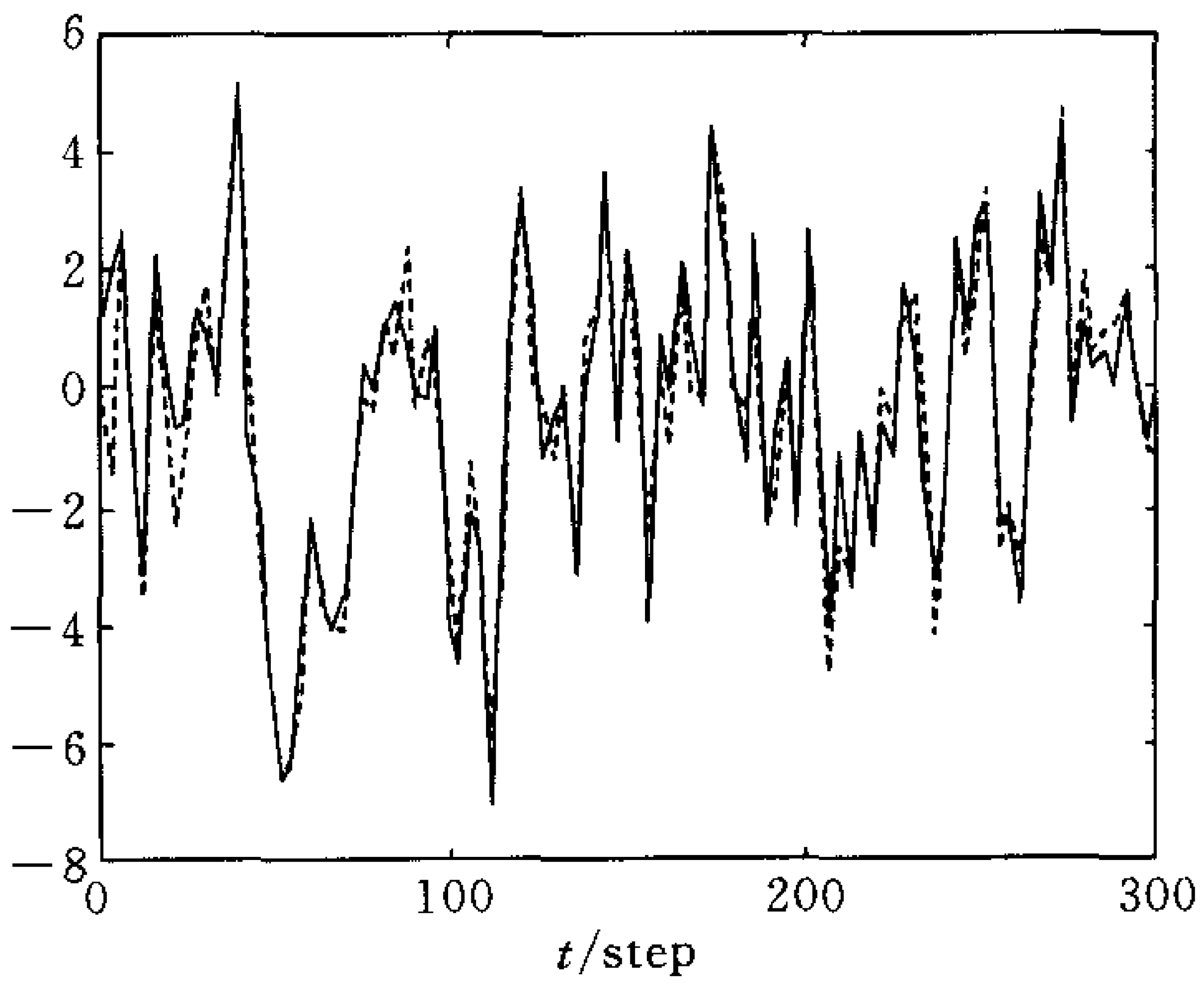

Fig. 2 $x_2(t)$ and descriptor Kalman filter $\hat{x}_2(t|t)$ (case 1)

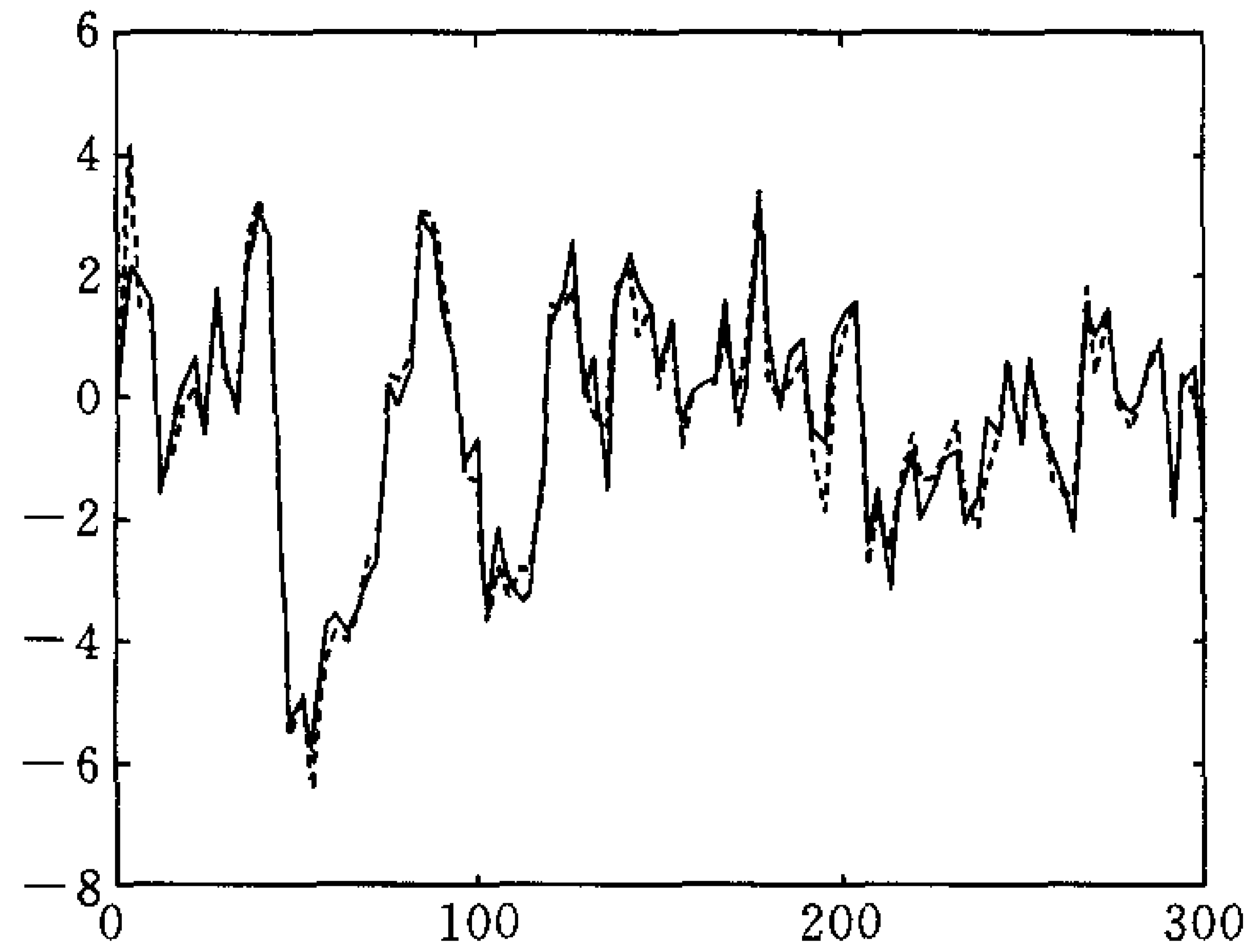

Fig. 3 $x_3(t)$ and descriptor Kalman filter $\hat{x}_3(t|t)$ (case 1)

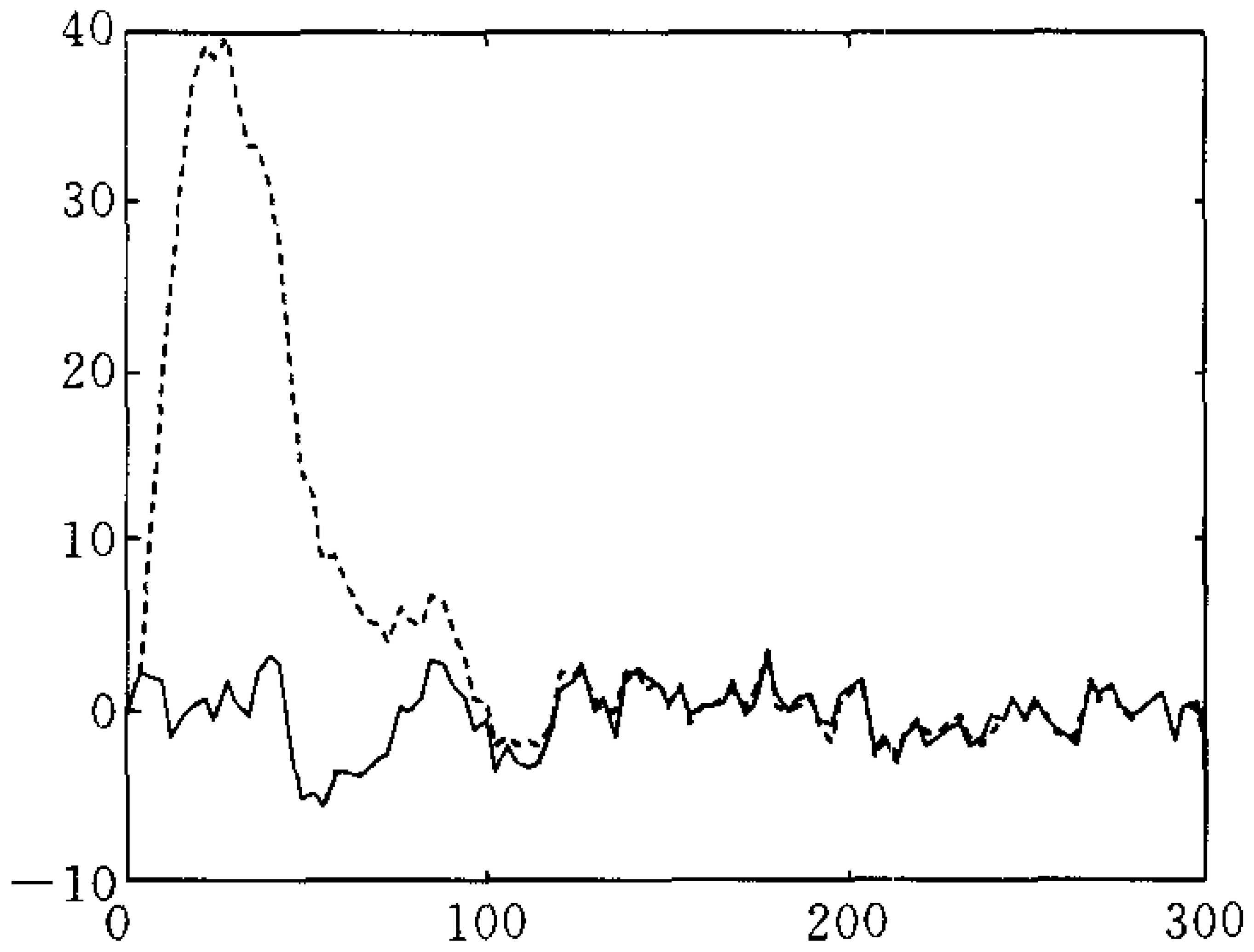

Fig. 4 $x_1(t)$ and descriptor Kalman filter $\hat{x}_1(t|t)$ (case 2)

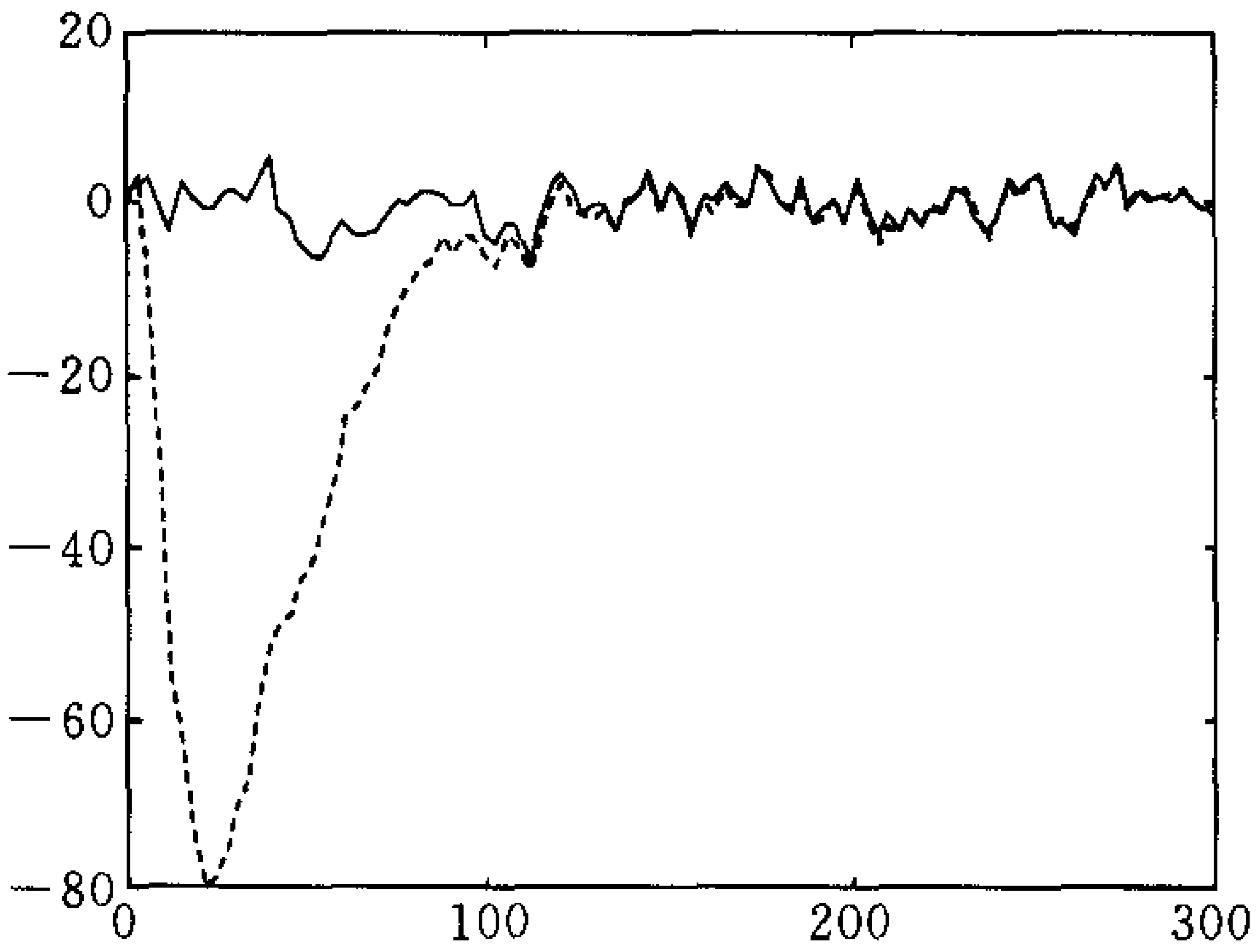

Fig. 5 $x_2(t)$ and descriptor Kalman filter $\hat{x}_2(t|t)$ (case 2)

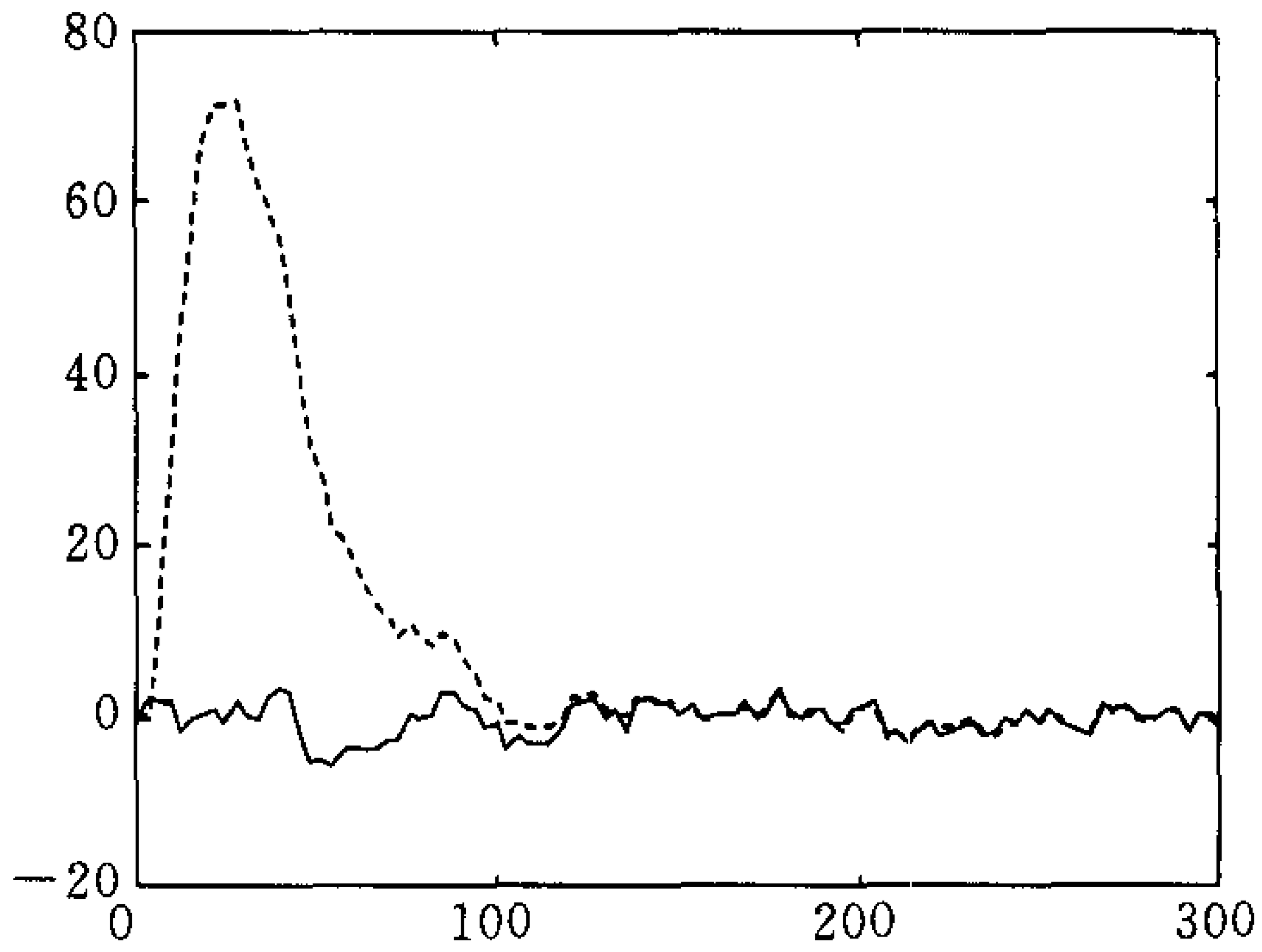

Fig. 6 $x_3(t)$ and descriptor Kalman filter $\hat{x}_3(t|t)$ (case 2)

From the above simulation results we see that if the absolute values of eigenvalues of $\bar{\Psi}$ are approximate to 0, then the effect of the initial values will decay rapidly. In Fig. 1~Fig. 3 we see that the state estimates satisfactorily track the true states after only a few steps. Contrarily, if the absolute values of eigenvalues of $\bar{\Psi}$ are approximate to 1, then the effect of the initial values will decay slowly with a longer decaying process. In Fig. 4~Fig. 6 we see that the estimate precision is not good for the first 100 steps and after 100 steps the estimates satisfactorily track the true states.

6 Conclusion

The unified pole assignment descriptor Kalman estimators have been presented by using the modern time-series analysis method[6]. Compared to the descriptor Kalman filter[1], they avoid the Riccati equation, and can handle the filtering, smoothing and prediction problems in a unified framework. Compared to the descriptor Kalman estimators[2~4], they avoid computing the optimal initial state estimates. Therefore, they can reduce the computational burden. Compared to Zhang *et al*'s results[5], they can handle the descriptor systems with correlated noises and with measurement delay, and the effect of initial state estimates can rapidly exponentially decay to zero. Compared to Dai's state observers[7], the state observers for determinate descriptor systems have been extended to the stochastic descriptor systems, and they can be considered as stochastic descriptor state observers.

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极点配置广义稳态 Kalman 估值器

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摘 要 应用时域上的现代时间序列分析方法,基于自回归滑动平均(ARMA)新息模型和白噪声估值器,应用控制理论中的极点配置原理,对线性离散时间广义随机系统提出了极点配置广义稳态 Kalman 估值器.它们具有全局渐近稳定性,且通过配置估值器的极点可按指数衰减速率使初始状态估值的影响快速消失.它们可在统一框架下处理滤波、平滑和预报问题.它们避免了 Riccati 方程和最优初始状态估值的计算,因而可减小计算负担.一个仿真例子说明了它们的有效性.

关键词 广义系统,极点配置,稳态 Kalman 估值器,现代时间序列分析方法

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