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An Aggregation Based Robust Model Predictive Controller[1)]

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Abstract A new model predictive controller ABRMPC is presented for constrained linear system with bounded additive perturbation. On the one hand, by introducing input amplitude decaying aggregation strategy, the number of optimization variables at every time is greatly reduced so that the on-line computation is simplified. On the other hand, having taken into account the perturbation at each time, some new state constraints are obtained and thus the state of the real system is kept in the original constraint domain. The paper emphasizes on how to get the feasible solution at next time and gives the condition that the decaying coefficient should satisfy. When time goes to infinite, it is proved that the system input becomes zero and the controller drives the system state to approach the invariant set. It is concluded that the controlled system is robustly stable. The conclusions are finally verified through a simulation example.

Key words Predictive control, robust sbability, aggregation, perturbation

1 Introduction

MPC (Model Predictive Control) has attracted wide attention in recent years because of its efficiency in dealing with constraints[1]. A typical MPC involves on-line solving a constrained nonlinear optimization problem. The computation burden depends on the number of control variables and is often huge. In order to reduce the on-line computation burden, Du *et al.*[2] proposed an input amplitude decaying aggregation strategy where the number of on-line optimization variables is greatly reduced.

The closed-loop stability of model predictive control has been the focus of research for a long time. For nominal systems without uncertainty or additive perturbation, the problem has been well solved for a certain degree. Mayne *et al.*[3] had summarized four stability conditions for such systems. However, in real applicants, the controlled plant is often perturbed and the model/plant mismatch always exists so that the robustness problem is also interesting to the development of stability research. Scokaert *et al.*[4] firstly proposed the condition of exponential stability and then asymptotic stability for systems with decaying additive perturbation. By means of Tailor formula, Nicolao *et al.*[5] proposed the conditions of robust stability for systems with gain or additive perturbation and further provided more detailed conclusions for linear systems. But their conclusions could not be used because the expression of optimal cost function is hard to obtain. Scokaert *et al.*[6] proposed "min-max feedback model predictive controller" which optimizes the "worst-case" cost function and gets a control sequence when the system state is out of an invariant set. At the same time the stability property of the controller was discussed. But the computation burden for the feedback controller is huge. For constrained linear systems with additive bounded perturbation, Chisci *et al.*[7] proposed a robust MPC. They got the perturbation domain at every time and designed the controller to make the system state within a smaller domain after considering the domain of perturbation. The model used in the controller is a nominal one. They also proved that the controller can drive the system states into the invariant set from outside.

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Combining the concept in [7] and the amplitude decaying aggregation strategy in [2], for perturbed constrained linear systems we present an aggregation based robust model predictive controller named ABRMPC. The advantage of the controller lies in that the computation burden can be reduced and at the same time the state of the real system can be kept in the constrained domain. Furthermore, the controller can drive the real system states into the invariant set from outside if certain assumptions and conditions are satisfied, which makes the controlled system robustly stable.

2 Preparation

Firstly we give the following definitions and assumptions.

Definition 1[7]. For two sets S, T and $S \supseteq T$, define $S-T=\{z \mid z+y \in S, z \in S, \forall y \in T\}$, and $S+T=\{z \mid z=x+y, \forall x \in S, \forall y \in T\}$.

From Definition 1, we can see that for an element z in S, if $z+y \in S$ for any $y \in T$, then $z \in S-T$; $S+T$ is the set of elements which are the sum of $\forall x \in S$ and $\forall y \in T$. Assume S, T are the sections surrounded by the dot-dash lines in Fig. 1 below. Then $S-T$, $S+T$ are the sections surrounded by the real lines in (a) and (b).

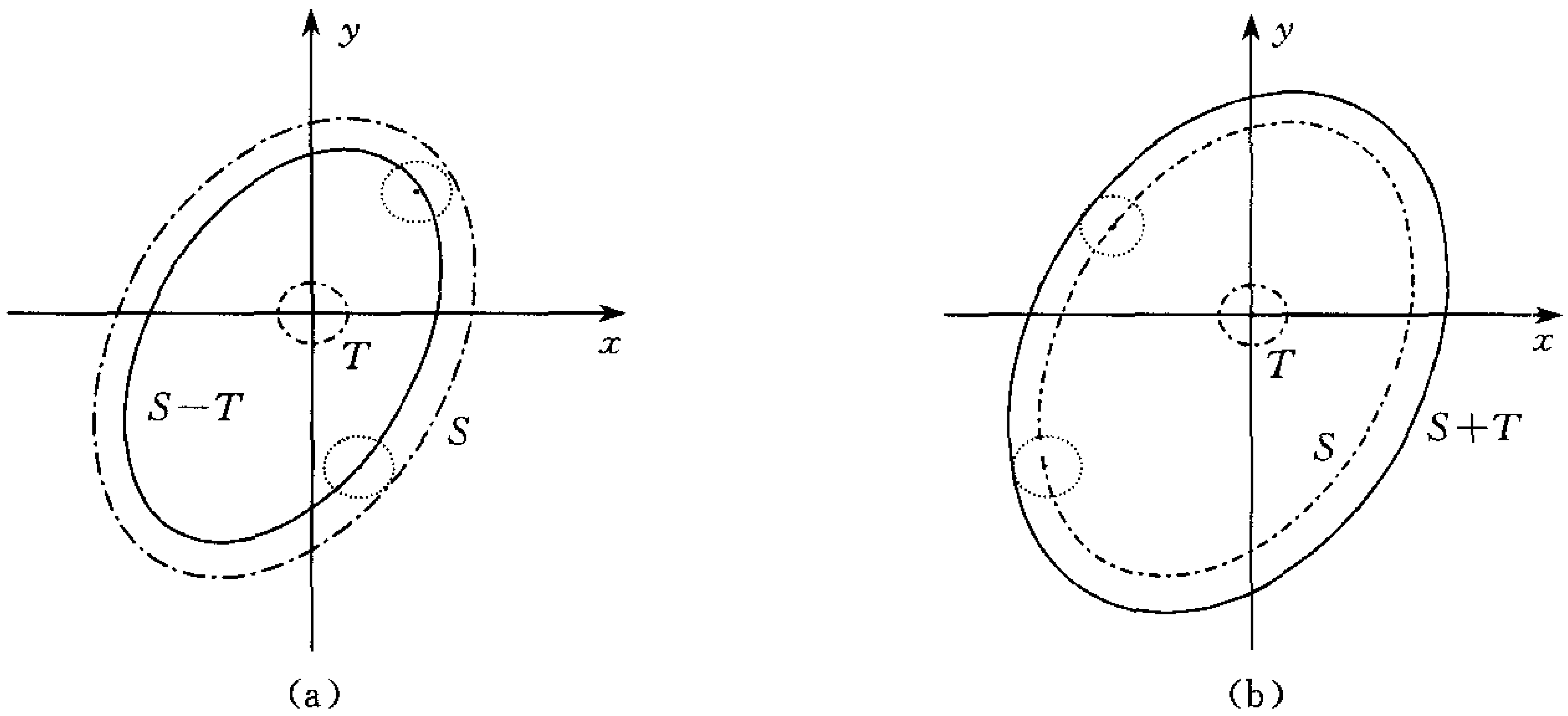

Fig. 1 $S-T$ and $S+T$

Definition 2. Assume Y is a closed compact set and $0 \in Y$, denote the boundary of Y by $\delta(Y)$ and define $\bar{r}(Y)=\max_{y \in \delta(Y)} \|y\|$.

$\bar{r}(Y)$ can be regarded as the radius of the minimum outer hyperball of Y.

Definition 3. For $\forall r>0$, denote B_r as a hyperball with the radius r centered at the origin.

Consider the system described by the following equation:

$$x(k+1) = Ax(k) + Bu(k) + \omega(k) \tag{1}$$

where $x(k) \in X \subseteq R^n$, $u(k) \in U \subseteq R^m$ and $\omega \in \Omega \subseteq R^n$ are the system state, system input and bounded additive perturbation respectively. X, U and Ω are all closed compact sets. $0 \in X$, $0 \in U$ and U is a convex set. (A, B) is controllable. For a matrix $F \in R^{m \times n}$, define $FX=\{Fx \mid x \in X\}$ and assume that $V=U-FX$ is not empty, $0 \in V$ and V is convex. F is the following feedback matrix which makes the eigenvalues of $(A+BF)$ small enough.

Assumption 1. $\max\{|\lambda(A)|\}<1$ and $\|A\| \triangleq \bar{\sigma}(A) \leqslant 1$ hold. Otherwise, since (A,B) is controllable the requests above can be achieved through feedback. We can have the system input as $u(k)=Fx(k)+v(k)(v(k) \in V)$ so that system (1) becomes $x(k+1)=\phi x(k)+Bv(k)+\omega(k)$ where $\phi=A+BF$. Since (A,B) is controllable, we can make the eigenvalues of ϕ approach the origin and differ with each other. For simplicity, we assume that Assumption 1 directly holds for (1).

Let $R_0=\varnothing$, $R_1=\{\boldsymbol{\omega}_1|\boldsymbol{\omega}_1\in\Omega\}$, $R_2=\{A\boldsymbol{\omega}_1+\boldsymbol{\omega}_2|\boldsymbol{\omega}_1,\boldsymbol{\omega}_2\in\Omega\}$, $\cdots$, $R_n=\left\{\sum_{i=1}^{n}A^{i-1}\boldsymbol{\omega}_{n-i+1}|\boldsymbol{\omega}_{n-i+1}\in\Omega,i=1,2,\cdots,n\right\}$, $\cdots$. According to Assumption 1, there exists a positive integer s such that $A^{s+i}\cong 0$ for $i=1,2,\cdots$. Then we have $R_{s+1}=R_{s+2}=\cdots$ and $R_{s+1}\supseteq R_s\supseteq R_{s-1}\supseteq\cdots\supseteq R_1\supseteq R_0$. At time k, for $\forall\,\boldsymbol{x}(k)\in R_{s+1}$, we have $\boldsymbol{x}(k)=A^s\boldsymbol{\omega}_1+A^{s-1}\boldsymbol{\omega}_2+\cdots+\boldsymbol{\omega}_{s+1}$ where $\boldsymbol{\omega}_1,\boldsymbol{\omega}_2,\cdots,\boldsymbol{\omega}_{s+1}\in\Omega$. If $\boldsymbol{u}(k)=0$ and the perturbation at that time is $\boldsymbol{\omega}(k)$, we have $\boldsymbol{x}(k+1)=\boldsymbol{\omega}(k)+A^{s+1}\boldsymbol{\omega}_1+A^s\boldsymbol{\omega}_2+\cdots+A\boldsymbol{\omega}_{s+1}$, $\boldsymbol{\omega}(k)\in\Omega$. Since $A^{s+1}=0$, $\boldsymbol{x}(k+1)\in R_{s+1}$ holds according to the definition of R_{s+1}.

As discussed above, we adopt R_{s+1} as the invariant set. The system state of (1) can be kept in the set if the input is $\boldsymbol{u}(k)=0$. If we denote $X_i=X-R_i$, then $X_0\supseteq X_1\supseteq X_2\supseteq\cdots\supseteq X_s\supseteq X_{s+1}=X_{s+2}=\cdots$. We assume $0\in X_{s+1}$ and select $r_1>0$, $r_2>0$, $\cdots$, as large as possible and $r_1\geqslant r_2\geqslant\cdots$ so that $B_{r_1}\subseteq X_1$, $B_{r_2}\subseteq X_2$, $\cdots$. We consider the following assumptions:

Assumption 2. For B_{r_i} defined above and the set Ω, we have $B_{r_{i+1}}+A^i\Omega\subseteq B_{r_i}$, $i=1,2,\cdots$.

The assumption means that, for $\forall\,\boldsymbol{x}\in B_{r_{i+1}}$ and $\forall\,\boldsymbol{\omega}\in\Omega$, we have $\boldsymbol{x}+A^i\boldsymbol{\omega}\in B_{r_i}$. The assumption is reasonable because according to Assumption 1, $\|A\|\leqslant 1$ and the set Ω is bounded so that $\|A\|^i\|\boldsymbol{\omega}\|$ can be small. If there exists an i which does not agree with Assumption 2, we can choose an appropriate r'_{i+1} satisfying $r'_{i+1}<r_{i+1}$ and $B_{r'_{i+1}}+A^i\Omega\subseteq B_{r_i}$. Then adjust r_{i+2}, r_{i+3}, $\cdots$ so that the assumption can be satisfied.

Having given the definitions and assumptions above, we now present the ABRMPC controller (aggregation based robust model predictive control) in the following.

3 ABRMPC controller

Similar to [7], the controller we propose will meet the following requirements: 1) with additive perturbation, the states of system (1) should remain in the constraint domain X; 2) $\boldsymbol{u}(k)\to 0$ when $k\to\infty$.

Firstly, consider the nominal model predictive controller when $\boldsymbol{\omega}(k)=0$:

$$\begin{aligned}\min_{U(k)}\quad & J(k)=\sum_{i=0}^{N-1}\|\boldsymbol{u}(k+i\mid k)\|^2 \\ \text{s.t.}\quad & \boldsymbol{x}(k+1)=A\boldsymbol{x}(k)+B\boldsymbol{u}(k)\quad \boldsymbol{x}(k\mid k)=\boldsymbol{x}(k)\\ & \boldsymbol{u}(k+i\mid k)\in U\\ & \boldsymbol{x}(k+i\mid k)\in X\quad i=0,1,\cdots,N-1\\ & \boldsymbol{x}(k+N\mid k)=0\\ & U(k)=[\boldsymbol{u}(k),\boldsymbol{u}(k+1\mid k),\cdots,\boldsymbol{u}(k+N-1\mid k)]\end{aligned}\tag{2}$$

where $\boldsymbol{x}(k)$ is the system state at time k.

If we regard $\|\boldsymbol{u}(k+i|k)\|^2$ as the energy, then the physical meaning of the performance index is to minimize the energy needed to drive the system state to the origin in every predictive horizon.

The nominal predictive controller has the following property.

Theorem 1. If the optimal solution of the nominal predictive controller (2) at time k is $U_0(k)$ and its first element $\boldsymbol{u}_0(k)$ is applied to the system at that time, then $\boldsymbol{u}_0(k)\to 0$ when $k\to\infty$.

Proof. Since $\boldsymbol{x}(k+N+1|k)=0$ when $\boldsymbol{x}(k+N|k)=0$ and $\boldsymbol{u}(k+N|k)=0$, $U(k+1)=[\widetilde{U}(k),0]$ is a feasible solution for the controller where $\widetilde{U}(k)$ are the 2nd to the Nth elements of $U_0(k)$. Denote the optimal cost function for $U_0(k)$ at time k by $J_0(k)$ and the cost function for $U(k+1)$ at time $k+1$ by $J(k+1)$. Then according to the optimality principle we have

$$J_0(k)-J_0(k+1)\geqslant J_0(k)-J(k+1)=\|\boldsymbol{u}_0(k)\|^2\tag{3}$$

We sum the right and left sides respectively of the above inequalities starting from time k and get $J_0(k)-J_0(\infty)\geqslant\sum_{i=0}^{\infty}\|u_0(k+i)\|^2$. Since $J_0(k)$ is finite and $J_0(\infty)>0$, $\sum_{i=0}^{\infty}\|u_0(k+i)\|^2$ is finite too. Then $\lim_{k\to\infty}u_0(k)=0$ is proved. □

We can see that at every time the optimization variables for controller (2) are $\boldsymbol{u}(k+i|k)$, $i=0,1,\cdots,N-1$. Since the constraints $\boldsymbol{x}(k+i|k)\in X$, $\boldsymbol{u}(k+i|k)\in U$ may be nonlinear, the optimization at each time is a nonlinear programming problem. The computation burden is thus huge and will increase greatly with N. In order to solve the problem, Du *et al.*[2] introduced aggregation transformation $U(k)=H\boldsymbol{u}(k)$ where the aggregation matrix $H=[I_m \quad \rho(k)I_m \quad \cdots \quad \rho^{N-1}(k)I_m]^{\mathrm{T}}$ where $0<\rho(k)<1$ means that the input amplitude is decaying. Apply this strategy to controller (2) the number of on-line optimization variables could be reduced to single $\boldsymbol{u}(k)$ and $\rho(k)$. If $\rho(k)$ is given, then only $\boldsymbol{u}(k)$ is the optimization variable.

The introduction of aggregation means additive constraints in problem (2):

$$\boldsymbol{u}(k+i\mid k)=\rho^i(k)\boldsymbol{u}(k) \quad i=0,1,\cdots,N-1 \tag{4}$$

From the proof of Theorem 1, we can see that $U(k+1)=[\widetilde{U}(k),0]$ is feasible at time $k+1$ is important to the stability of system. However, when additive constraint (4) is added to controller (2), $U(k+1)$ is not feasible any longer because in most cases $\rho^N(k)\boldsymbol{u}(k)=0$ does not hold. As a result, the system stability can not be ensured as in Theorem 1. Although the on-line computation burden has been reduced by means of aggregation, it is necessary to rediscuss the stability of the controller when aggregation constraint (4) has been introduced.

We note that it is a nominal case we have discussed above. When there exists additive perturbation or $\boldsymbol{\omega}(k)\neq 0$, the state constraints may be violated. Then for the real system (1), the controller should have the property of robustness. Since $\boldsymbol{x}(k+1)=A\boldsymbol{x}(k)+B\boldsymbol{u}(k)+\boldsymbol{\omega}(k)\in\{A\boldsymbol{x}(k)+B\boldsymbol{u}(k)+\boldsymbol{z}\mid\boldsymbol{z}\in R_1\}$, $\boldsymbol{x}(k+2)=A^2\boldsymbol{x}(k)+AB\boldsymbol{u}(k)+B\boldsymbol{u}(k+1)+A\boldsymbol{\omega}(k)+\boldsymbol{\omega}(k+1)\in\{A^2\boldsymbol{x}(k)+AB\boldsymbol{u}(k)+B\boldsymbol{u}(k+1)+\boldsymbol{z}\mid\boldsymbol{z}\in R_2\},\cdots$, by combining the definitions of R_i and X_i, we can see that if we denote the system state at every time as $\boldsymbol{x}(k+i)=A^i\boldsymbol{x}(k)+\sum_{j=0}^{i-1}A^jB\boldsymbol{u}(k+i-1-j)+\sum_{j=0}^{i-1}A^j\boldsymbol{\omega}(k+i-1-j)\triangleq\boldsymbol{x}_n(k+i)+\sum_{j=0}^{i-1}A^j\boldsymbol{\omega}(k+i-1-j)$, then $\boldsymbol{x}(k+i)\in X$ holds if the nominal state $\boldsymbol{x}_n(k+i)\in X_i$. This means that with the existence of perturbation, if the nominal system state is kept within a certain bounded domain, for the real system the constraint $\boldsymbol{x}(k+i|k)\in X$ can be satisfied. Then it is possible to get stability property.

Through the discussion above, we introduce the concept of aggregation and robustness into controller (2) and get the following controller ABRMPC (aggregation based robust model predictive control):

$$\min_{\rho(k),\boldsymbol{u}(k)} \quad J(k)=\sum_{i=0}^{N-1}\|\boldsymbol{u}(k+i\mid k)\|^2+\frac{M}{\rho(k)} \tag{5}$$

$$\begin{aligned}\text{s. t.}\quad & \boldsymbol{x}(k+1)=A\boldsymbol{x}(k)+B\boldsymbol{u}(k),\quad \boldsymbol{x}(k\mid k)=\boldsymbol{x}(k)\\ & \boldsymbol{u}(k+i\mid k)=\rho^i(k)\boldsymbol{u}(k),\quad i=1,2,\cdots,N-1\\ & \boldsymbol{u}(k+i\mid k)\in U\\ & \|\boldsymbol{x}(k+j\mid k)\|\leqslant r_j,\quad j=0,1,\cdots,N,\ \text{where } N\geqslant s+2\\ & 0<\rho(k)<\lambda\end{aligned}$$

where λ is a positive number less than 1 and M is a positive constant. Comparing with problem (2), we can see that the optimization variables of problem (5) are $\boldsymbol{u}(k)$ and $\rho(k)$

so that the number of optimization variables has been greatly reduced; moreover, $\frac{M}{\rho(k)}$ is added to the performance index which ensures that the decrease of $\boldsymbol{u}(k+i|k)$ is not too quick. Aggregation constraint (4) is added in and the constraints $\boldsymbol{x}(k+i|k)\in X$ and $\boldsymbol{x}(k+N|k)=0$ are replaced by $\|\boldsymbol{x}(k+j|k)\|\leqslant r_j$.

4 Property of the ABRMP controller

Theorem 2. Suppose Assumptions 1 and 2 hold for the ABRMPC controller (5) and the following inequality holds:

$$r_N - \|A\|^{N-1}r_1 - \|A^{N-1}B\|\bar{r}(U) - \|A^N\|\bar{r}(\Omega) > 0 \tag{6}$$

Then there exists $\varepsilon>0$ such that for $\lambda=\varepsilon$ if the optimal solution of the controller at time k is $U_0(k)=[\boldsymbol{u}_0(k),\rho_0(k)\boldsymbol{u}_0(k),\cdots,\rho_0^{N-1}(k)\boldsymbol{u}_0(k)]$, then $U(k+1)=\rho_0(k)U_0(k)$ is a feasible solution for the controller at time $k+1$.

Proof. As $U_0(k)$ is the optimal solution at time k, there exist a feasible solution $U(k)$ and $0<\rho(k)<1$ such that $\rho^i(k)\boldsymbol{u}(k)\in U(i=1,2,\cdots,N-1)$.

Applying $\boldsymbol{x}(k+1)=A\boldsymbol{x}(k)+B\boldsymbol{u}(k)$ we have

$$\begin{aligned}\boldsymbol{x}(k+i\mid k) &= A^{i-1}\boldsymbol{x}(k+1\mid k)+[\rho(k)A^{i-2}B+\\ &\cdots+\rho^{i-1}(k)B]\boldsymbol{u}(k),\quad i=1,2,\cdots,N\end{aligned} \tag{7}$$

Since $\|\boldsymbol{x}(k+1|k)\|\leqslant r_1$ and $\|\boldsymbol{u}(k)\|\leqslant\bar{r}(U)$ according to Definition 2, we get

$$\|\boldsymbol{x}(k+N\mid k)\|\leqslant \|A\|^{N-1}r_1+[\rho(k)\|A^{N-2}B\|+\cdots+\rho^{N-1}(k)\|B\|]\bar{r}(U) \tag{8}$$

Define $t_N(\rho)=\rho\|A^{N-2}B\|+\cdots+\rho^{N-1}\|B\|$ and it is easy to show

$$\frac{\mathrm{d}t_N(\rho)}{\mathrm{d}\rho}\geqslant 0 \text{ and } t_N(0)=0 \tag{9}$$

By combining it with inequality (6), it is clear that there exists a positive number $\varepsilon>0$ so that for $\forall\,\rho(k)<\lambda=\varepsilon$ the following inequality holds

$$\|A\|^{N-1}r_1+t_N(\rho(k))\bar{r}(U)\leqslant r_N-\|A^{N-1}B\|\bar{r}(U)-\|A^N\|\bar{r}(\Omega)\leqslant r_N \tag{10}$$

Assume that $\boldsymbol{x}_0(k+i|k)(i=1,2,\cdots,N)$ is the system state corresponding to the optimal solution $U_0(k)=[\boldsymbol{u}_0(k),\rho_0(k)\boldsymbol{u}_0(k),\cdots\rho_0^{N-1}(k)\boldsymbol{u}_0(k)]$ at time k, and that $\boldsymbol{x}(k+1+i|k+1)(i=1,2,\cdots,N+1)$ is the system state for $U(k+1)=\rho_0(k)U_0(k)$. For the real system $\boldsymbol{x}(k+1|k+1)=A\boldsymbol{x}(k|k)+B\boldsymbol{u}_0(k)+\boldsymbol{\omega}(k)$, then we get $\boldsymbol{x}(k+2|k+1)=\boldsymbol{x}_0(k+2|k)+A\boldsymbol{\omega}(k)$. Since $\boldsymbol{u}_0(k)$ is the optimal solution, $\|\boldsymbol{x}_0(k+2|k)\|\leqslant r_2$, i. e., $\boldsymbol{x}_0(k+2|k)\in B_{r_2}$. By Assumption 2 we get $\boldsymbol{x}(k+2|k+1)\in B_{r_1}$, i. e., $\|\boldsymbol{x}(k+2|k+1)\|\leqslant r_1$. Similarly, we have

$$\|\boldsymbol{x}(k+i+1\mid k+1)\|\leqslant r_i\quad i=2,3,\cdots,N-1 \tag{11}$$

From $\boldsymbol{x}(k+N+1|k+1)=A\boldsymbol{x}_0(k+N|k)+\rho_0^N(k)B\boldsymbol{u}_0(k)+A^N\boldsymbol{\omega}(k)=A^N\boldsymbol{x}(k+1|k)+[\rho_0(k)A^{N-1}B+\rho_0^2(k)A^{N-2}B+\cdots+\rho_0^N(k)B]\boldsymbol{u}_0(k)+A^N\boldsymbol{\omega}(k)$ and $\|\boldsymbol{x}_0(k+1|k)\|\leqslant r_1$, we have

$$\begin{aligned}\|\boldsymbol{x}(k+N+1\mid k+1)\|\leqslant &\|A\|\,\|A\|^{N-1}r_1+\rho_0(k)[\,\|A^{N-1}B\|+\\ &\rho_0(k)\|A^{N-2}B\|+\cdots+\rho_0^{N-1}(k)\|B\|\,]\bar{r}(U)+\|A^N\|\bar{r}(\Omega)\end{aligned} \tag{12}$$

Since $\rho_0(k)<1$ and according to Assumption 1, $\|A\|\leqslant 1$, from (12) and the definition of $t_N(\rho)$, it is easy to show

$$\begin{aligned}&\|\boldsymbol{x}(k+N+1\mid k+1)\|\leqslant\\ &\quad\|A\|^{N-1}r_1+t_N(\rho_0(k))\bar{r}(U)+\|A^{N-1}B\|\bar{r}(U)+\|A^N\|\bar{r}(\Omega)\leqslant r_N\end{aligned} \tag{13}$$

From the above, we know that $U(k+1)=\rho_0(k)U_0(k)$ is a feasible solution for ABRMPC controller at time $k+1$.

Theorem 3. For ABRMPC controller (5), the conditions of Theorem 2 and $\lambda=\min\{1-\eta,\varepsilon\}$ hold, where η is a positive number which is small enough. If the system initial state $\boldsymbol{x}(0)\notin$

R_{s+1}, where R_{s+1} is the invariant set defined before, then $\boldsymbol{x}(i)$ asymptotically approaches R_{s+1} when $i \to \infty$.

Proof. Assume that $U_0(k)=[\boldsymbol{u}_0(k), \rho_0(k)\boldsymbol{u}_0(k), \cdots, \rho_0^{N-1}(k)\boldsymbol{u}_0(k)]$ is the optimal solution for the ABRMPC controller at time k, and that the system input at the time is $\boldsymbol{u}_0(k)$. The following proving process will be divided into two parts: I) $\boldsymbol{u}_0(k) \to 0$ when $k \to \infty$; II) For system (1), $\boldsymbol{x}_0(k) \to R_{s+1}$ when $k \to \infty$.

I) Denote $J_0(k)$ as the cost function for the optimal solution $U_0(k)$ of ABRMPC controller at time k, and $J(k+1)$ as the cost function for the feasible solution $U(k+1)=\rho_0(k)U_0(k)$ at time $k+1$. Then according to the optimality principle we get the following inequality

$$J_0(k) - J_0(k+1) \geqslant J_0(k) - J(k+1) = [1-\rho_0^{2N}(k)]\|\boldsymbol{u}_0(k)\|^2 \tag{14}$$

Similarly, we have

$$\vdots$$

$$J_0(k+j) - J_0(k+1+j) \geqslant [1-\rho_0^{2N}(k+j)]\|\boldsymbol{u}_0(k+j)\|^2$$

$$\vdots$$

We sum up the right and left sides of the inequalities above from time k, respectively, and get

$$J_0(k) - J_0(\infty) \geqslant \sum_{i=0}^{\infty}\{[1-\rho_0^{2N}(k+i)]\|\boldsymbol{u}_0(k+i)\|^2\}$$

We note that $\rho(k) \leqslant 1-\eta$ leads to $1-\rho_0^{2N}(k+i) \geqslant 1-(1-\eta)^{2N}$ and if we define $\xi = 1-(1-\eta)^{2N}$, then we get $J_0(k)-J_0(\infty) \geqslant \xi\sum_{i=0}^{\infty}\|\boldsymbol{u}_0(k+i)\|^2$.

Since $J_0(\infty) \geqslant 0$, $\xi\sum_{i=0}^{\infty}\|\boldsymbol{u}_0(k+i)\|^2 > 0$ and ξ is a constant positive number, we have $\sum_{i=0}^{\infty}\|\boldsymbol{u}_0(k+i)\|^2 < \infty$ so that $\lim_{i\to\infty}\|\boldsymbol{u}_0(k+i)\| = 0$, which shows that $\lim_{k\to\infty}\|\boldsymbol{u}_0(k)\| = 0$.

II) Since $|\lambda(A)| < 1$, for the state equation $\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + B\boldsymbol{u}(k)$ used in the ABRMPC controller we have

$$\lim_{i\to\infty}\boldsymbol{x}(i) = \lim_{i\to\infty}\Big[A^i\boldsymbol{x}(0) + \sum_{j=0}^{i-1}A^jB\boldsymbol{u}_0(i-1-j)\Big] = 0 \tag{15}$$

For the real system (1), we have

$$\lim_{i\to\infty}\boldsymbol{x}(i) = \lim_{i\to\infty}\Big[A^i\boldsymbol{x}(0) + \sum_{j=0}^{i-1}A^jB\boldsymbol{u}_0(i-1-j) + \sum_{j=0}^{i-1}A^j\boldsymbol{\omega}(i-1-j)\Big] = \lim_{i\to\infty}\Big[\sum_{j=0}^{i-1}A^j\boldsymbol{\omega}(i-1-j)\Big] \tag{16}$$

Then $\lim_{i\to\infty}\boldsymbol{x}(i) \in R_{s+1}$ is proved. □

Remark. Combining the concepts of aggregation and robust stability, for the constrained linear system with bounded additive perturbation we present a new infinite horizon predictive controller ABRMPC. The work of the paper focuses on two points: a) how to get the feasible solution at time $k+1$ when the optimal solution at time k is available; b) how to drive the state of the real system to the invariant set when the optimal control law is applied to the plant. □

5 Simulation example

Consider the following ABRMPC problem:

$$\min_{\rho(k),u(k)} \quad J(k) = \sum_{i=0}^{60} \|u(k+i \mid k)\|^2 + \frac{500}{\rho(k)}$$

$$\text{s.t.} \quad x(k+1) = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix} x(k) + \begin{bmatrix} -2 \\ 1 \end{bmatrix} u(k) \quad x(0) = \begin{bmatrix} 54 \\ 36 \end{bmatrix}$$

$$u(k+i \mid k) = \rho^i(k) u(k) \quad i = 1,2,\cdots,60$$

$$u(k+i \mid k) \in [-3,3]$$

$$\|x(k+j \mid k)\| \leqslant r_j \quad j = 0,1,\cdots,61$$

$$0 < \rho(k) < \lambda$$

The constraints of system (1) are $X = \{x \mid \|x\| \leqslant 50\}$, $\Omega = \{(\omega_1, \omega_2)^T \mid \|(\omega_1, \omega_2)^T\| \leqslant 0.9\}$. We take $s = 55$ and $\eta = 0.0001$. According to the definitions of X_i and r_i we get the values of r_j: $r_1 = 49.1000$, $r_2 = 48.3800$, $\cdots$. We also get the invariant set $R_{s+1} = \{(x_1, x_2)^T \mid \|(x_1, x_2)^T\| \leqslant 4.5\}$. According to (10) we have $\varepsilon = 0.9729$. Fig. 2 is the state trajectory of the real system (with k from 0 to 50).

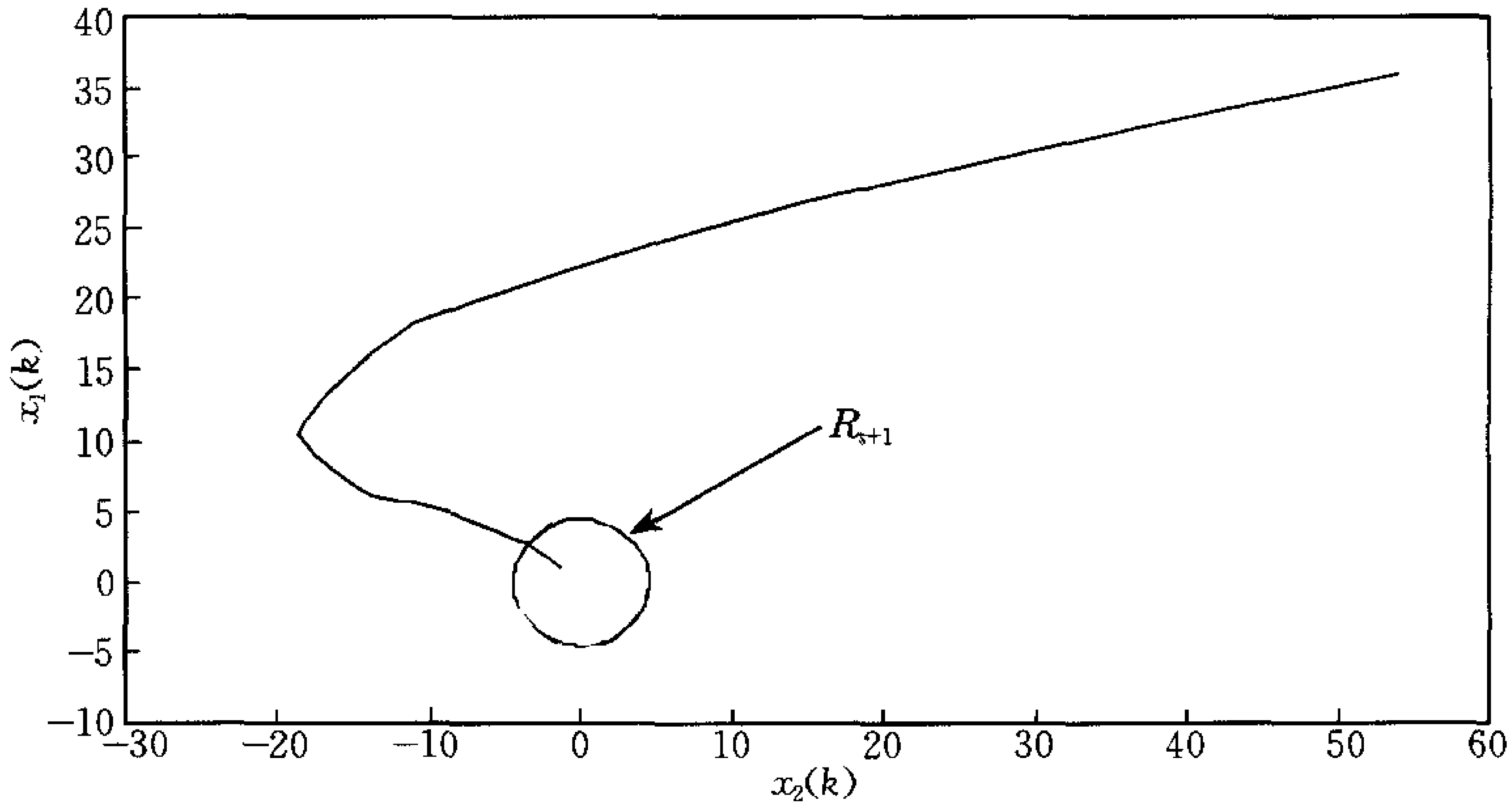

Fig. 2 The State trajectory

6 Conclusions

In this paper a new predictive controller ABRMPC is presented for the constrained linear system with additive bounded perturbation. Although in the controller the system state has been kept in a smaller domain at each time, the state of the real system is ensured to be in the feasible domain. On the other hand, from Theorem 3 we see that by using the controller the state of the real system can asymptotically approach the invariant set and can at last be kept in the set with $u(k) = 0$ so that the controller is robustly stable.

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一种基于集结的鲁棒 MPC 控制器

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摘 要 针对存在有界外界扰动的有约束线性系统,本文提出了一种新的 MPC 控制器 ABRMPC. 一方面,通过引入输入幅值衰减集结策略,使得各时刻优化的变量数大大减少,简化了在线计算;另一方面,在对各步的扰动进行了考虑之后,得到新的状态约束,可保证系统实际状态始终处于原约束域内. 本文重点对如何得到下一时刻的可行解进行了研究,指出了衰减系数的上界应满足的条件;而后证明了系统输入将最终趋于零,同时该控制器可使处于不变集之外的系统状态趋向于不变集,从而使系统具有鲁棒稳定性. 最后通过仿真实例对文中结论进行了验证.

关键词 预测控制,鲁棒稳定,集结,扰动

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