Adaptive Control of Multivariable Systems with Unknown Sign of the High Frequency Gain Matrix[1)]

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Abstract A new adaptive control scheme is proposed for multivariable MRAC systems based on the nonlinear backstepping approach in vector form. The assumption on a prior knowledge of the high frequency gain matrix in the existing results is weakened and can be easily checked for certain plants so that the proposed method is widely applicable. This control scheme guarantees the global stability of the closed-loop system and makes the tracking error tend to be zero and quadratically integrable.

Key words Vector backstepping method, global stability, multivariable systems

1 Introduction

The model reference adaptive control (MRAC) guarantees globally asymptotic stability for linear time invariant multi-input and multi-output (MIMO) systems when there are no uncertain dynamics and disturbances[1~3]. All the existing results are based on the linear control design methods which require some auxiliary error signals. These design schemes are usually complex when the generalized relative degree is greater than one. The backstepping approach is very effective in dealing with nonlinear systems in parametric-strict-feedback form[4,5] and it is also applicable to linear systems[6]. For linear MIMO adaptive control systems, the existing results using the above linear design schemes and nonlinear backstepping approach usually require a known interacting polynomial matrix $P(s)$ and a known nonsingular matrix S_p such that the high frequency gain matrix K_p satisfyies $K_p S_p = (K_p S_p)^T > 0$. How to weaken this assumption is crucial in the adaptive control design for multivariable systems. In [7], based on the nonlinear backstepping design scheme, an adaptive controller which guarantees the global stability of the closed-loop system is proposed for MIMO systems, whose high frequency gain matrix is not necessarily positive definite, but can be transformed into a lower or upper triangular matrix of which the signs of diagonal elements are known. In [8,9], a design scheme using the SDU or LDU factorization of the high frequency gain matrix K_p is given but the signs of the leading principal minors of K_p are required to be known. In this paper, an adaptive control design scheme for linear MIMO systems is proposed by using the backstepping approach in vector form, and the high frequency gain matrix K_p(or K_pS_p) is not required to be positive when the relative degree to be one, and the nonsingular matrix S_p is not required to be known when the relative degree is greater than one. In many cases, the assumption of Hurwitz holds while the factorization $K_p = LDU$ is not available. The proposed control scheme guarantees the global stability of the closed-loop systems and makes the tracking error tend to be zero and quadratically integrable.

2 System description

Consider the following observable and controllable MIMO linear time-invariant plant

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$$y = G(s)u \quad (1)$$

where $G(s)$ is the $m\times m$ transfer function matrix, $u(t)\in R^m$ and $y(t)\in R^m$ are the system input and output respectively. The control objective is to design an adaptive controller such that the output signal $y(t)$ tracks asymptotically the output $y_M(t)$ of a reference model $y_M = W_M(s)r$ where $W_M(s)$ and $W_M^{-1}(s)$ are stable, and keeps all the signals in the closed-loop bounded. In order to give a detailed error dynamic equation, the following lemma is used.

Lemma 1[10]. For strictly proper and full rank $m\times m$ transfer matrix $G(s)$, there exists an upper triangular polynomial matrix $\xi_m(s)$ such that $\lim\limits_{s\to\infty} G(s)\xi_m(s) = K_p$, where K_p is a nonsingular matrix, and the elements on the diagonal of $\xi_m(s)$ are monic Hurwitz polynomials of certain degree.

For system (1) the following assumptions are made.

Assumption 1. The transmission zeros of $G(s)$ have negative real parts.

Assumption 2. $G(s)$ is strictly proper and full rank.

Assumption 3. An upper bound $\bar{\nu}_0$ of the observability index ν_0 of $G(s)$ is known.

Assumption 4. There exists a known interacting polynomial matrix $P(s)$ such that $K_p = \lim\limits_{s\to\infty} P(s)G(s)$, where K_p is Hurwitz, otherwise there exists a known nonsingular matrix S_p such that K_pS_p is Hurwitz. Without loss of generality, let K_p be Hurwitz in the latter discussion. When the generalized relative degree of system (1) is greater than one, there exists a positive definite matrix P (unknown) such that PK_p is negative.

According to the Lemma 1 and the Assumption 4, $\xi_m(s)$ is full rank and $\xi_m^{-1}(s)$ is stable. Let n^* be the maximum order of the polynomials in $\xi_m(s)$ and $d(s) = (s+\alpha)^{n^*}$, where $\alpha > 0$ is a known constant. Define $v = d(s)\xi_m^{-1}(s)u$. Then we have

$$u = \xi_m(s)d^{-1}(s)I_{m\times m}v \quad (2)$$

$$y = G(s)\xi_m(s)\xi_m^{-1}(s)u = d^{-1}(s)G(s)\xi_m(s)v \triangleq d^{-1}(s)\bar{G}(s)v \quad (3)$$

Now the observability index of $d^{-1}(s)\bar{G}(s)$ is $\nu_0 + n^*$, and the upper bound is $\nu = \bar{\nu}_0 + n^*$.

Let the transfer matrix of the reference model be

$$W_M(s) = \mathrm{diag}\{1/(s+\alpha)^{n^*}, \cdots, 1/(s+\alpha)^{n^*}\}$$

and define the tracking error as

$$e(t) = y(t) - y_M(t) \quad (4)$$

In general, the zero structure at infinity of $W_M(s)$ is the same as that of $G(s)$[10], i. e., $K = \lim\limits_{s\to\infty} W_M(s)\xi_m(s)$ is finite and nonsingular. In this paper, the relative degree of the reference model is the maximum degree of the polynomials in $\xi_m(s)$ so that it is possible that $K = \lim\limits_{s\to\infty} W_M(s)\xi_m(s)$ is singular. If in the transformation (2), $d(s)\xi_m^{-1}(s)$ is proper, it is guaranteed that $K = \lim\limits_{s\to\infty} W_M(s)\xi_m(s)$ is nonsingular.

If $\bar{G}(s)$ is known, for system (3), by the MRAC method, one can choose a control law v^* such that

$$y = d^{-1}(s)\bar{G}(s)v^* = W_M(s)r = y_M \quad (5)$$

$$v^* = \theta^{*T}\omega, \quad \theta^* = [\bar{\theta}_1{}^T \quad \bar{\theta}_2{}^T \quad \bar{\theta}_3 \quad \bar{\theta}_4]^T, \quad \omega = [\omega_1^T \quad \omega_2^T \quad y^T \quad r^T]^T \quad (6)$$

$$\bar{\theta}_1, \bar{\theta}_2 \in R^{m(\nu-1)\times m}, \quad \bar{\theta}_3 \in R^{m\times m}, \quad \bar{\theta}_4 = K_p^{-1}, \omega_1 = \frac{\gamma(s)}{p(s)}v, \quad \omega_2 = \frac{\gamma(s)}{p(s)}y, \quad \omega_1, \omega_2 \in R^{m(\nu-1)}$$

where $\gamma(s) = [I \quad Is \quad \cdots \quad Is^{\nu-2}]^T$, and $p(s) = \lambda_0 + \lambda_1 s + \cdots + s^{\nu-1}$ is Hurwitz polynomial. When $\bar{G}(s)$ is unknown, we use the adaptive control law v instead of v^*. Then from the plant and by matching Equations (3) and (5), we obtain the following output tracking error dynamics[10]

$$e = W_M(s)K_p[v - \theta^{*T}\omega] \quad (7)$$

3 Adaptive controller

In view of the form of $W_M(s)$ and (7), we obtain

$$\dot{e}(t) = -\alpha e(t) + K_p \frac{v}{(s+\alpha)^{n^*-1}} + K_p\theta^{*T}\boldsymbol{\varphi} \tag{8}$$

where $-\frac{\boldsymbol{\omega}(t)}{(s+\alpha)^{n^*-1}} = \boldsymbol{\varphi}(t)$. We define $\frac{v}{(s+\alpha)^{n^*-1}} \triangleq \boldsymbol{x}_1$, then its state space realization together with (8) is denoted as follows

$$\begin{cases} \dot{e}(t) = -\alpha e(t) + K_p\boldsymbol{x}_1(t) + K_p\theta^{*T}\boldsymbol{\varphi}(t) \\ \dot{\boldsymbol{x}}_i = \boldsymbol{x}_{i+1} & (i = 1,2,\cdots,n^*-2) \\ \dot{\boldsymbol{x}}_{n^*-1} = -\beta_1\boldsymbol{x}_1 - \beta_2\boldsymbol{x}_2 - \cdots - \beta_{n^*-1}\boldsymbol{x}_{n^*-1} + v \end{cases} \tag{9}$$

where $\beta_i > 0 (i=1,2,\cdots,n^*-1)$ are the coefficients of the polynomial $(s+\alpha)^{n^*-1}$, and

$\theta^* = [\boldsymbol{\theta}_1^* \quad \boldsymbol{\theta}_2^* \quad \cdots \quad \boldsymbol{\theta}_{2mv}^*]^T$, $K_p = [\boldsymbol{K}_{p1} \quad \boldsymbol{K}_{p2} \quad \cdots \quad \boldsymbol{K}_{pm}]$, $\boldsymbol{\varphi}(t) = [\varphi_1 \quad \varphi_2 \quad \cdots \quad \varphi_{2mv}]^T$

where $\boldsymbol{\theta}_i^* \in R^m, \boldsymbol{K}_{pi} \in R^m$, $\varphi_i\ (i=1,\cdots,2mv)$ are scalar signals. Based on the backstepping approach in the vector form, a controller will be designed by using the following transformation of variables

$$\begin{cases} z_0(t) = e(t) \\ z_i(t) = \boldsymbol{x}_i(t) - \boldsymbol{\alpha}_{i-1}(t) \qquad (i = 1,2,\cdots,n^*-1) \end{cases} \tag{10}$$

where $\boldsymbol{\alpha}_i(t) (i=0,1,\cdots,n^*-2)$ are referred to as intermediate control functions. The first intermediate control function is taken as

$$\boldsymbol{\alpha}_0(z_0, \hat{\theta}^*, t) = c_0' z_0 - \hat{\theta}^{*T}(t)\boldsymbol{\varphi}(t) \tag{11}$$

where $\hat{\theta}^*$ is the estimation of θ^*, $\tilde{\theta}^* = \hat{\theta}^* - \theta^*$, from (9) and (10), we have

$$\dot{z}_0 = -\alpha z_0 + c_0' K_p z_0 + K_p z_1 - K_p \tilde{\theta}^{*T}(t)\boldsymbol{\varphi}(t) \tag{12}$$

The update law of $\tilde{\boldsymbol{\theta}}_i^*$ is given by

$$\dot{\hat{\boldsymbol{\theta}}}_i^*(t) = -\varphi_i(t) z_0(t), \quad (i = 1,2,\cdots,2mv) \tag{13}$$

With the transformations (10) for dynamics (9), we get

$$\dot{z}_1 = \dot{\boldsymbol{x}}_1 - \dot{\boldsymbol{\alpha}}_0 = z_2 + \boldsymbol{\alpha}_1 - \frac{\partial\boldsymbol{\alpha}_0}{\partial t} - \frac{\partial\boldsymbol{\alpha}_0}{\partial z_0^T}(-\alpha z_0 + K_p\boldsymbol{x}_1 + K_p\theta^{*T}\boldsymbol{\varphi}(t)) - \sum_{i=1}^{2mv}\frac{\partial\boldsymbol{\alpha}_0}{\partial\hat{\boldsymbol{\theta}}_i^{*T}}\dot{\hat{\boldsymbol{\theta}}}_i^* \tag{14}$$

where $K_p\theta^{*T} = \theta^T = [\boldsymbol{\theta}_1 \quad \boldsymbol{\theta}_2 \quad \cdots \quad \boldsymbol{\theta}_{2mv}]$, $\hat{\theta}^T(t)$ and $\hat{K}_p$ are the estimations of $K_p\theta^{*T}$ and K_p respectively, and $\tilde{\theta}^T(t) = \hat{\theta}^T(t) - \theta^T$, $\tilde{K}_p(t) = \hat{K}_p(t) - K_p$. Choose the second intermediate control function as

$$\boldsymbol{\alpha}_1(z_0, \boldsymbol{x}_1, \hat{K}_p, \hat{\theta}, t) = -(c_1 + c_1')z_1 + \frac{\partial\boldsymbol{\alpha}_0}{\partial t} + \sum_{i=1}^{2mv}\frac{\partial\boldsymbol{\alpha}_0}{\partial\hat{\boldsymbol{\theta}}_i^{*T}}\dot{\hat{\boldsymbol{\theta}}}_i^* + \frac{\partial\boldsymbol{\alpha}_0}{\partial z_0^T}(-\alpha z_0 + \hat{K}_p(t)\boldsymbol{x}_1 + \hat{\theta}^T(t)\boldsymbol{\varphi}(t)) \tag{15}$$

In general, the $(k+1)$-th intermediate control function is taken as

$$\boldsymbol{\alpha}_k(z_0, \boldsymbol{x}_1, \cdots, \boldsymbol{x}_k, \hat{K}_p, \hat{\theta}, t) = -c_k z_k - 2z_k + \frac{\partial\boldsymbol{\alpha}_{k-1}}{\partial t} + \frac{\partial\boldsymbol{\alpha}_{k-1}}{\partial z_0^T}(-\alpha z_0 + \hat{K}_p(t)\boldsymbol{x}_1 + \hat{\theta}^T(t)\boldsymbol{\varphi}(t)) + \sum_{i=1}^{k-1}\frac{\partial\boldsymbol{\alpha}_{k-1}}{\partial\boldsymbol{x}_i^T}\boldsymbol{x}_{i+1} + \sum_{i=1}^{m}\frac{\partial\boldsymbol{\alpha}_{k-1}}{\partial\hat{\boldsymbol{K}}_{pi}^T}\boldsymbol{\sigma}_{ki} + \sum_{i=1}^{2mv}\frac{\partial\boldsymbol{\alpha}_{k-1}}{\partial\hat{\boldsymbol{\theta}}_i^T}\boldsymbol{\tau}_{ki} + \boldsymbol{f}_k + \boldsymbol{g}_k, \ c_k > 0 \tag{16}$$

$$\begin{aligned} \boldsymbol{\sigma}_{ki} &= \boldsymbol{\sigma}_{k-1,i} - x_{1i}\frac{\partial\boldsymbol{\alpha}_{k-1}}{\partial z_0^T}z_k (i = 1,2,\cdots,m) \\ \boldsymbol{\tau}_{ki} &= \boldsymbol{\tau}_{k-1,i} - \varphi_i\frac{\partial\boldsymbol{\alpha}_{k-1}}{\partial z_0^T}z_k (i = 1,2,\cdots,2mv) \end{aligned} \tag{17}$$

$$\begin{aligned} f_k^{\mathrm{T}} &= -\sum_{i=1}^{m}\left[\sum_{j=1}^{k-2} z_{j+1}^{\mathrm{T}} \frac{\partial \boldsymbol{\alpha}_j}{\partial \hat{\boldsymbol{K}}_{pi}^{\mathrm{T}}}\right] x_{1i} \frac{\partial \boldsymbol{\alpha}_{k-1}}{\partial \boldsymbol{z}_0^{\mathrm{T}}} \\ g_k^{\mathrm{T}} &= -\sum_{i=1}^{2mv}\left[\sum_{j=1}^{k-2} z_{j+1}^{\mathrm{T}} \frac{\partial \boldsymbol{\alpha}_j}{\partial \hat{\boldsymbol{\theta}}_i^{\mathrm{T}}}\right] \varphi_i \frac{\partial \boldsymbol{\alpha}_{k-1}}{\partial \boldsymbol{z}_0^{\mathrm{T}}} \end{aligned} \quad k \geqslant 3 \tag{18}$$

For the last step n^*, the adaptive control law $\boldsymbol{v}(t)$ now is taken as

$$\begin{aligned} \boldsymbol{v}(t) = {} & \sum_{i=1}^{n^*-1} \beta_i \boldsymbol{x}_i - (c_{n^*-1}+1)\boldsymbol{z}_{n^*-1} + \frac{\partial \boldsymbol{\alpha}_{n^*-2}}{\partial t} + \frac{\partial \boldsymbol{\alpha}_{n^*-2}}{\partial \boldsymbol{z}_0^{\mathrm{T}}}(-\alpha \boldsymbol{z}_0 + \hat{K}_p(t)\boldsymbol{x}_1 + \hat{\theta}^{\mathrm{T}}(t)\boldsymbol{\varphi}(t)) + \\ & \sum_{i=1}^{n^*-2} \frac{\partial \boldsymbol{\alpha}_{n^*-2}}{\partial \boldsymbol{x}_i^{\mathrm{T}}} \boldsymbol{x}_{i+1} + \sum_{i=1}^{m} \frac{\partial \boldsymbol{\alpha}_{n^*-2}}{\partial \hat{\boldsymbol{K}}_{pi}^{\mathrm{T}}} \boldsymbol{\sigma}_{n^*-1,i} + \sum_{i=1}^{2mv} \frac{\partial \boldsymbol{\alpha}_{n^*-2}}{\partial \hat{\boldsymbol{\theta}}_i^{\mathrm{T}}} \boldsymbol{\tau}_{n^*-1,i} + \boldsymbol{f}_{n^*-1} + \boldsymbol{g}_{n^*-1} \end{aligned} \tag{19}$$

$$f_{n^*-1}^{\mathrm{T}} = -\sum_{i=1}^{m}\left[\sum_{j=1}^{n^*-3} \boldsymbol{z}_{j+1}^{\mathrm{T}} \frac{\partial \boldsymbol{\alpha}_j}{\partial \hat{\boldsymbol{K}}_{pi}^{\mathrm{T}}}\right] x_{1i} \frac{\partial \boldsymbol{\alpha}_{n^*-2}}{\partial \boldsymbol{z}_0^{\mathrm{T}}} \tag{20a}$$

$$g_{n^*-1}^{\mathrm{T}} = -\sum_{i=1}^{2mv}\left[\sum_{j=1}^{n^*-3} \boldsymbol{z}_{j+1}^{\mathrm{T}} \frac{\partial \boldsymbol{\alpha}_j}{\partial \hat{\boldsymbol{\theta}}_i^{\mathrm{T}}}\right] \varphi_i \frac{\partial \boldsymbol{\alpha}_{n^*-2}}{\partial \boldsymbol{z}_0^{\mathrm{T}}} \tag{20b}$$

$$\boldsymbol{\sigma}_{n^*-1,i} = \boldsymbol{\sigma}_{n^*-2,i} - x_{1i} \frac{\partial \boldsymbol{\alpha}_{n^*-2}}{\partial \boldsymbol{z}_0^{\mathrm{T}}} \boldsymbol{z}_{n^*-1} \tag{21a}$$

$$\boldsymbol{\tau}_{n^*-1,i} = \boldsymbol{\tau}_{n^*-2,i} - \varphi_i \frac{\partial \boldsymbol{\alpha}_{n^*-2}}{\partial \boldsymbol{z}_0^{\mathrm{T}}} \boldsymbol{z}_{n^*-1} (i = 1,2,\cdots,2mv) \tag{21b}$$

The update laws are chosen as

$$\begin{aligned} \dot{\hat{\boldsymbol{K}}}_{pi} &= \boldsymbol{\sigma}_{n^*-1,i} = -\sum_{j=1}^{n^*-1} x_{1i} \frac{\partial \boldsymbol{\alpha}_{j-1}}{\partial \boldsymbol{z}_0^{\mathrm{T}}} \boldsymbol{z}_j \\ \dot{\hat{\boldsymbol{\theta}}}_i &= \boldsymbol{\tau}_{n^*-1,i} = -\sum_{j=1}^{n^*-1} \varphi_i \frac{\partial \boldsymbol{\alpha}_{j-1}}{\partial \boldsymbol{z}_0^{\mathrm{T}}} \boldsymbol{z}_j \end{aligned} \quad (i = 1,2,\cdots,2mv) \tag{22}$$

The above analysis can be summarized into the following theorem.

Theorem 1. For system (1) and the reference model (2), if Assumptions 1~4 hold, then, given the adaptive control law $\boldsymbol{v}(t) = [v_1(t), v_2(t), \cdots, v_m(t)]^{\mathrm{T}}$ by (19) and the adaptive laws by (13) and (22), all the closed-loop signals are bounded and the tracking error $\boldsymbol{e}(t)$ converges to zero and belongs to L_2.

4 Conclusion

This paper has proposed a new adaptive control scheme for linear multivariable MRAC systems based on the vector form backstepping approach. This control scheme guarantees the global stability of the closed loop systems and makes the tracking error tend to be zero and quadratically integrable. We no longer require that there exist a known nonsingular matrix S_p such that $K_pS_p = (K_pS_p)^{\mathrm{T}} > 0$. For many classes of dynamic systems, our proposed assumptions are easier to check than those in the literature.

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Appendix

The proof of the Theorem 1. Only consider the case that the generalized relative degree is greater than one. Since in Assumption 4, K_p is Hurwitz, and there exists a positive definite matrix P such that

$$K_p^{\mathrm{T}}P + PK_p = -Q$$

if taking a Lyapunov function candidate

$$V_0 = \frac{1}{2}z_0^{\mathrm{T}}Pz_0 - \frac{1}{2}\sum_{i=1}^{2mv}\tilde{\boldsymbol{\theta}}_i^{*\mathrm{T}}K_p^{\mathrm{T}}P\,\tilde{\boldsymbol{\theta}}_i^*$$

we obtain that

$$-\sum_{i=1}^{2mv}\tilde{\boldsymbol{\theta}}_i^{*\mathrm{T}}K_p^{\mathrm{T}}P\,\tilde{\boldsymbol{\theta}}_i^* = -\frac{1}{2}\sum_{i=1}^{2mv}\tilde{\boldsymbol{\theta}}_i^{*\mathrm{T}}(K_p^{\mathrm{T}}P + PK_p)\,\tilde{\boldsymbol{\theta}}_i^* = \frac{1}{2}\sum_{i=1}^{2mv}\tilde{\boldsymbol{\theta}}_i^{*\mathrm{T}}Q\,\tilde{\boldsymbol{\theta}}_i^*$$

is positive definite and so is V_0. Since Q is positive definite, there exist constants l_i $(i=1,2,3)$ such that

$$z_0^{\mathrm{T}}Qz_0 \geqslant l_3 z_0^{\mathrm{T}}z_0,\ z_0^{\mathrm{T}}PK_pz_1 \leqslant l_1 z_0^{\mathrm{T}}z_0 + l_2 z_1^{\mathrm{T}}z_1$$

In (11), let $c_0' l_3 > l_1 + c_0$ where c_0 is positive constant, then

$$\left(\frac{1}{2}z_0^{\mathrm{T}}Pz_0\right)' \leqslant -c_0 z_0^{\mathrm{T}}z_0 + l_2 z_1^{\mathrm{T}}z_1 - \sum_{i=1}^{2mv}\tilde{\boldsymbol{\theta}}_i^{*\mathrm{T}}K_p^{\mathrm{T}}P\varphi_i z_0,\quad \dot{V}_0 \leqslant -c_0 z_0^{\mathrm{T}}z_0 + l_2 z_1^{\mathrm{T}}z_1 \tag{A1}$$

For (14), we consider the following Lyapunov function

$$V_1 = V_0 + \frac{1}{2}z_1^{\mathrm{T}}z_1 + \frac{1}{2}\sum_{i=1}^{m}\tilde{\boldsymbol{K}}_{pi}^{\mathrm{T}}\tilde{\boldsymbol{K}}_{pi} + \frac{1}{2}\sum_{i=1}^{2mv}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_i \tag{A2}$$

From (14) and (A1) we have

$$\dot{V}_1 \leqslant -c_0 z_0^{\mathrm{T}}z_0 - c_1 z_1^{\mathrm{T}}z_1 + z_2^{\mathrm{T}}z_2 + \sum_{i=1}^{m}\tilde{\boldsymbol{K}}_{pi}^{\mathrm{T}}(\dot{\hat{\boldsymbol{K}}}_{pi} - \boldsymbol{\sigma}_{1i}) + \sum_{i=1}^{2mv}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(\dot{\hat{\boldsymbol{K}}}_i - \boldsymbol{\tau}_{1i}) \tag{A3}$$

where $c_1' \geqslant 1 + l_2$, $\boldsymbol{\sigma}_{1i} = -x_{1i}\frac{\partial\boldsymbol{\alpha}_0}{\partial\boldsymbol{z}_0^{\mathrm{T}}}z_1$ $(i=1,2,\cdots,m)$, $\boldsymbol{\tau}_{1i} = -\varphi_i\frac{\partial\boldsymbol{\alpha}_0}{\partial\boldsymbol{z}_0^{\mathrm{T}}}z_1$ $(i=1,2,\cdots,2mv)$. In general, we choose a Lyapunov functuion candidate as $V_k = V_{k-1} + \frac{1}{2}z_k^{\mathrm{T}}z_k$, then we have

$$\begin{aligned}\dot{V}_k \leqslant & -\sum_{i=0}^{k}c_i z_i^{\mathrm{T}}z_i + z_{k+1}^{\mathrm{T}}z_{k+1} + \sum_{i=1}^{m}\tilde{\boldsymbol{K}}_{pi}^{\mathrm{T}}(\dot{\hat{\boldsymbol{K}}}_{pi} - \boldsymbol{\sigma}_{k,i}) + \sum_{i=1}^{2mv}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(\dot{\hat{\boldsymbol{\theta}}}_i - \boldsymbol{\tau}_{k,i}) - \\ & \sum_{i=1}^{m}\left(\sum_{j=1}^{k-2}z_{j+1}^{\mathrm{T}}\frac{\partial\boldsymbol{\alpha}_j}{\partial\hat{\boldsymbol{K}}_{pi}^{\mathrm{T}}}\right)(\dot{\hat{\boldsymbol{K}}}_{pi} - \boldsymbol{\sigma}_{k,i}) - \sum_{i=1}^{2mv}\left(\sum_{j=1}^{k-2}z_{j+1}^{\mathrm{T}}\frac{\partial\boldsymbol{\alpha}_j}{\partial\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}}\right)(\dot{\hat{\boldsymbol{\theta}}}_i - \boldsymbol{\tau}_{k-i})\end{aligned} \tag{A4}$$

The last Lyapunov function is taken as

$$V_{n^*-1} = V_{n^*-2} + \frac{1}{2}z_{n^*-1}^{\mathrm{T}}z_{n^*-1} \tag{A5}$$

Then we obtain

$$\dot{V}_{n^*-1} \leqslant -\sum_{i=0}^{n^*-1}c_i z_i^{\mathrm{T}}z_i \tag{A6}$$

From (A5) and (A6), V_{n^*-1} is bounded and

$$V_{n^*-1}(t) + \int_0^t\sum_{i=0}^{n^*-1}c_i z_i^{\mathrm{T}}z_i \leqslant V_{n^*-1}(0)$$

This implies $z_i(t)\in L_2$ and $\tilde{\boldsymbol{K}}_{pi}(t)\in L_\infty$ $(i=1,2,\cdots,m)$, $z_i(t)\in L_2\cap L_\infty$ $(i=0,1,\cdots,n^*-1)$, and $\tilde{\boldsymbol{\theta}}_i(t)$, $\tilde{\boldsymbol{\theta}}_i^*(t)\in L_\infty$ $(i=1,2,\cdots,2mv)$. It follows from $z_0\in L_\infty$ that $e(t)$ and $y(t)$ are bounded. Since $\boldsymbol{u}=G^{-1}(s)\boldsymbol{y}$, and from (3) we have $\boldsymbol{v}=d(s)\bar{G}^{-1}(s)\boldsymbol{y}$ where the relative degree of $\bar{G}(s)$ is zero ($K_p=\lim_{s\to\infty}\bar{G}(s)=\lim_{s\to\infty}G(s)\xi_m(s)$). According to the assumptions, the transmission zeros of $\bar{G}(s)$ have negative real parts. This guarantees that for $\alpha>0$, $(s+\alpha)^{-n^*}\boldsymbol{v}=\bar{G}^{-1}(s)\boldsymbol{y}$ is bounded. This illustrates that for $\alpha>0$, $(s+\alpha)^{-j}\boldsymbol{v}(j\geqslant n^*)$ are bounded. And thus $-(s+\alpha)^{n^*-1}\boldsymbol{\omega}=\boldsymbol{\varphi}$ is bounded. Since $\boldsymbol{z}_0$ and $\hat{\theta}^*$ are bounded, it follows from (10) and (11) that $\boldsymbol{\alpha}_0(t)$ and $\boldsymbol{x}_1(t)$ are bounded. Using the same arguments, we can prove the boundness of $\boldsymbol{\alpha}_i(t)$ and $\boldsymbol{x}_i(t)$. Then each component of the control law in (19) is bounded and thus the adaptive control law $\boldsymbol{v}(t)$ is bounded. From the transformation (2), $\boldsymbol{u}(t)$ is bounded. This means that all the closed-loop signals are bounded. From (12), every element in $\dot{\boldsymbol{z}}_0$ is bounded, thus $\dot{\boldsymbol{e}}(t)\in L_\infty$. Because it was proved in the previous that $\boldsymbol{e}(t)=\boldsymbol{z}_0(t)\in L_2\cap L_\infty$, we can conclude that the tracking error $\boldsymbol{e}(t)$ converges to zero. The proof of the theorem is completed.

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高频增益矩阵符号未知的多变量系统自适应控制

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摘　要　基于向量形式的非线性 Backstepping 方法,对多变量模型参考自适应控制(MRAC)系统设计出新的自适应控制器.该设计机制减弱了现有研究中对高频增益矩阵的假定条件,使某些系统关于高频增益矩阵的假定条件便于检验,从而该控制器的实用范围更广.该控制器能保证系统的全局稳定性,跟踪误差趋于零并且是平方可积的.

关键词　向量 Backstepping 方法,全局稳定性,多变量系统

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