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The Application of the Control Lyapunov Function to Stabilization Designs[1)]

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Abstract The notion of control Lyapunov functions(CLF), originated from relaxed controls, is newly understood as the characterization of zero-state detectability property, thus the difference between the Sontag-type control[1] and the passivity-based controller is clarified accordingly. Then, based on a CLF under appropriate conditions, a Sontag-type stabilizer is designed for a class of typical cascaded systems which have ever been stabilized by passivity-based controllers. Furthermore, based on an input-to-state stable CLF under appropriate conditions, a Sontag-type input-to-state stabilizer is designed for this kind of cascaded systems with perturbation.

Key words Control Lyapunov functions, zero-state detectability, passivity, input-to-state stabilization

1 Introduction

The control Lyapunov function(CLF) of affine systems, often combined with Sontag's formula[1], has been playing an important role in stabilization designs[2~4], and the notion of zero-state-detectability[5,6] is also a central concept in current stabilization research. In fact, the two notions are closely related, the CLF is just a characterization of zero-state-detectability property of a specified state-output system.

To investigate the application of the CLF, we will address the robust stabilization problem of a class of typical cascaded systems. Although these systems have been discussed in many references such as [7,8] and have been given passivity-based controllers, we intend to design CLFs-based controllers. Noting that the energy function in [7,8] is also the CLF of this kind of cascaded systems, we will design a Sontag-type stabilizer. Further, we consider this kind of cascaded system with perturbation, prescribe corresponding constrains so that the preceding CLF becomes the input-to-state stable CLF (ISS-CLF), and accordingly design a Sontag-type input-to-state stabilizer. It deserves to be remarked that the input-to-state stabilization design is more significant than the stabilization design, since the former implies that the closed-loop is globally asymptotically stable, and is also capable of attenuating disturbance.

2 Essentials to the CLF

In this section, we reveal essentials of the CLF from several aspects. In particular, we point out that the CLF is just a characterization of zero-state-detectability property, and consequently compare the Sontag-type controller with the passivity-based controller.

Consider the affine system

$$\dot{x} = f(x) + g(x)u, \quad x \in R^n, u \in R^m, f(\mathbf{0}) = 0 \tag{1}$$

We will pay attention to the following state-output system later, which is related to some C^1 positive definite and radially-unbounded energy function V:

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$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}), \quad \boldsymbol{y} = [L_g V(\boldsymbol{x})]^{\mathrm{T}} \tag{2}$$

Here $L_g V(\boldsymbol{x}) := (\partial V/\partial \boldsymbol{x}) g(\boldsymbol{x})$. V being positive definite implies $V(\boldsymbol{x}) \geqslant 0$ for all $\boldsymbol{x}$, $V(\boldsymbol{x}) = 0 \Leftrightarrow \boldsymbol{x} = \boldsymbol{0}$; V being radially-unbounded implies $V(\boldsymbol{x}) \to \infty$ as $|\boldsymbol{x}| \to \infty$. From now on we always assume an energy function to be C^1 positive definite and radially-unbounded.

Definition 1[1]. An energy function V is said to be a control Lyapunov function (CLF) of (1) if

$$\forall\, \boldsymbol{x} \neq \boldsymbol{0},\ [L_g V(\boldsymbol{x})]^{\mathrm{T}} = \boldsymbol{0} \Rightarrow L_f V(\boldsymbol{x}) < 0$$

Now, we point out the difference between the CLF characterized by Definition 1 and the CLF characterized by some Riccati-like inequalities. It is easy to test that V is a CLF of (1) if

$$\forall\, \boldsymbol{x} \neq \boldsymbol{0},\ L_f V(\boldsymbol{x}) - |L_g V(\boldsymbol{x})|^{1+p} < 0,\ p \geqslant 0 \tag{3}$$

Actually, the CLF satisfying (3) does exist. For instance, consider a system

$$\dot{x}_1 = -x_1 + x_2, \quad \dot{x}_2 = (1 + 3x_1^2)(-x_1 + x_2) + u$$

With $W = 2V + 3V^2 + 3V^3$, $V = \frac{1}{2}(x_1^2 + x_2^2)$ as an energy function, we get

$$L_f W(\boldsymbol{x}) = (2 + 6V + 9V^2)[-x_1^2 - 3x_1^3 x_2 + (1 + 3x_1^2)x_2^2], \quad L_g W(\boldsymbol{x}) = (2 + 6V + 9V^2)x_2$$

It is easy to test that $L_f W(\boldsymbol{x}) < |L_g W(\boldsymbol{x})|^2$ for all $\boldsymbol{x} \neq 0$. In fact, when $x_2 \neq 0$ we get

$$-x_1^2 - 3x_1^3 x_2 + (1 + 3x_1^2)x_2^2 \leqslant \frac{9}{4}x_1^4 x_2^2 + 3x_1^2 x_2^2 + x_2^2 \leqslant$$

$$(1 + 6V + 9V^2)x_2^2 < (2 + 6V + 9V^2)x_2^2$$

When $x_2 = 0$ but $x_1 \neq 0$, we get $-x_1^2 - 3x_1^3 x_2 + (1 + 3x_1^2)x_2^2 < (2 + 6V + 9V^2)x_2^2$.

However, that system (2) has a CLF V does not imply that there exists a Riccati-like inequality as (3). For instance, consider the following system

$$\dot{x}_1 = -x_2, \quad \dot{x}_2 = x_1 + u x_2 \tag{4}$$

$V(\boldsymbol{x}) = x_1^2 - x_1 x_2 + x_2^2$ can be proved to be a CLF of (4). Here

$$L_g V(\boldsymbol{x}) = (-x_1 + 2x_2)x_2, \quad L_f V(\boldsymbol{x}) = x_2^2 - x_1^2$$

"$\forall\, \boldsymbol{x} \neq \boldsymbol{0}, L_g V(\boldsymbol{x}) = 0$" includes "$x_2 = 0$, $x_1 \neq 0$" and "$x = 2x_2 \neq 0$". Now we have

$$x_2 = 0,\ x_1 \neq 0 \Rightarrow L_f V(\boldsymbol{x}) = -x_1^2 < 0;\ x_1 = 2x_2 \neq 0 \Rightarrow L_f V(\boldsymbol{x}) = -3x_2^2 < 0$$

Hence $V(\boldsymbol{x}) = x_1^2 - x_1 x_2 + x_2^2$ is a CLF of (4). However, there is some $\boldsymbol{x} \neq \boldsymbol{0}$, for instance, $(x_1, x_2) = \left(\frac{1}{8}, \frac{1}{4}\right)$ such that $L_f V(\boldsymbol{x}) - |L_g V(\boldsymbol{x})|^2 > 0$.

The discussion above actually indicates that having a CLF V characterized by Riccati-like inequalities (3) implies that system (1) possesses stronger stabilizability properties, and that a simple control law $\boldsymbol{u} = -[L_g V(\boldsymbol{x})]^{\mathrm{T}}$ is a stabilizer. However, based on CLF V characterized as Definition 1, the complicated law called Sontag-type control is needed for stabilization:

$$\boldsymbol{u}(\boldsymbol{x}) = -P(\boldsymbol{x})\boldsymbol{b},\ P(\boldsymbol{x}) = \begin{cases} \dfrac{a(\boldsymbol{x}) + \sqrt{a^2(\boldsymbol{x}) + |\boldsymbol{b}(\boldsymbol{x})|^4}}{|\boldsymbol{b}(\boldsymbol{x})|^2}, & \boldsymbol{b}(\boldsymbol{x}) \neq 0 \\ c, & \boldsymbol{b}(\boldsymbol{x}) = 0 \end{cases} \tag{5}$$

Here $c > 0$, $a(\boldsymbol{x}) = L_f V(\boldsymbol{x})$, $\boldsymbol{b}(\boldsymbol{x}) = [L_g V(\boldsymbol{x})]^{\mathrm{T}}$.

Lemma 1[2]. Sontag-type control (5) is an optimal stabilizer of (1) with respect to the following cost functions:

$$J = \int_0^\infty [L(\boldsymbol{x}) + \frac{1}{2P(\boldsymbol{x})}\boldsymbol{u}^{\mathrm{T}}\boldsymbol{u}]\mathrm{d}t, \quad L(\boldsymbol{x}) = \frac{1}{2}P(\boldsymbol{x})b(\boldsymbol{x})^{\mathrm{T}}b(\boldsymbol{x}) - a(\boldsymbol{x})$$

and this control law is continuous in $R^n \backslash \{0\}$.

Now we point out that the CLF is just a characterization of the zero-state-detectability property.

Definition 2[5,6]. A state-output system

$$\dot{x} = f(x), \quad y = h(x), \quad x \in R^n, \quad y \in R^m, \quad f(0) = 0, \quad h(0) = 0$$

is said to be zero-state detectable (ZSD) if for all $x_0 \in R^n$,

$$\forall t > 0, \ h(x(t;x_0)) = 0 \Rightarrow \lim_{t\to\infty} x(t;x_0) = 0$$

and is said to be zero-state observable (ZSO) if for all $x_0 \in R^n$,

$$\forall t \geqslant 0, \ h(x(t;x_0)) = 0 \Rightarrow x(t;x_0) = 0$$

Here $x(t;x_0)$ is the solution of the state-output system, x_0 is the initial condition. Obviously, a ZSO system must be a ZSD system.

According to Definitions 1 and 2, system (1) having a CLFV implies actually that system (2) is ZSD.

Due to this observation, and if putting aside the continuity problem of control laws, we can define a class of special CLF, based on which Sontag-type control is a stabilizer of system (1).

Definition 1*. An energy function V is said to be a CLF of system (1) if system (2) is ZSO.

Based on such a CLF, the Sontag-type control is indeed a stabilizer, since we have

$$\dot{V}_{(1)} = -\sqrt{[L_f V(x)]^2 + |L_g V(x)|^4}, \quad E = \{x \mid \dot{V}_{(1)} = 0\} = \{0\}$$

Observing that the following limit does not exist necessarily, we can not ensure the Sontag-type control is continuous with regard to $L_g V(x)$, let alone $x \in R^n \backslash \{0\}$.

$$\lim_{|L_g V(x)| \to 0} u(x) = \lim_{|L_g V(x)| \to 0} \frac{|L_g V(x)|^2 [L_g V(x)]^{\mathrm{T}}}{\sqrt{(L_f V(x))^2 + |L_g V(x)|^4} - L_f V(x)}$$

However, the Sontag-type control based on the CLF characterized by Definite 1* is possible to be continuous. For instance, consider the system

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -e^{x_2}\left(x_1 + \frac{1}{2}x_2\right) + u \tag{6}$$

With $V(x) = x_1^2 + x_2^2$ as an energy function, we have a state-output system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -e^{x_2}\left(x_1 + \frac{1}{2}x_2\right) \end{cases}, \quad y = L_g V(x) = 2x_2$$

which is ZSO. So $V(x) = x_1^2 + x_2^2$ is a CLF of (6) in terms of Definite 1*, though we only get $\forall x \neq 0$, $L_g V(x) = 0 \Rightarrow L_f V(x) = 0$, where $L_f V(x) = x_2^2(2x_1 p - e^{x_2})$, $p = (1 - e^{x_2})/x_2$. Moreover, the corresponding Sontag-type control

$$u = -\frac{1}{2}x_2\left(2x_1 p - e^{x_2} + \sqrt{4x_1^2 p^2 - 4x_1 p e^{x_2} + e^{2x_2} + 16}\right)$$

is obviously continuous.

Finally, we clarify simply the difference between the CLF-based design and the passivity-based design. Based on the ZSD precondition (CLF), the Sontag-type control becomes a universal stabilizer of system (1). If the additional condition that the unforced dynamics of (1) is critical-stable is provided, a simple passivity-based controller is obtained.

Assumption 1. System (2) is ZSD; $L_f V(x) \leqslant 0$, $\forall x \in R^n$.

Lemma 2[5]. If Assumption 1 holds, then the system $\dot{x} = f(x) - g(x)[L_g V(x)]^{\mathrm{T}}$ is globally asymptotically stable, *i.e.*, $u = -[L_g V(x)]^{\mathrm{T}}$ stabilizes system (1).

3 CLFs in robust stabilization of cascaded systems

A class of typical cascaded systems have been discussed in many references such as [7,8], here we continue to study the robust stabilization problem of this kind of cascaded systems. Based on a CLF under appropriate conditions, we design firstly a Sontag-type stabilizer, which is different from the one in [7]. Then for this kind of cascaded systems with perturbation, we design a Sontag-type input-to-state stabilizer, based on an input-to-state stable CLF (ISS-CLF) under appropriate conditions.

Consider the cascaded system

$$\begin{aligned}\dot{x}_1 &= f(x_1, x_2) \\ \dot{x}_2 &= u\end{aligned} \quad i.e., \ \dot{x} = \begin{pmatrix} f(x_1, x_2) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u := F(x) + G(x)u \tag{7}$$

where $(x_1, x_2) \in R^n \times R$.

Assumption 2A. There exist a C^1 function $v, v(0)=0$ and an energy function V such that $L_{f(x_1, v(x_1))} V(x_1) < 0$, for all $x_1 \neq 0$.

Assumption 2B. $f \in C^1$, as a result, there exists a function M such that

$$f(x_1, x_2) - f(x_1, v(x_1)) = (x_2 - v(x_1)) M(x_1, x_2, v(x_1)), \ \forall (x_1, x_2) \in R^{n+1}.$$

Lemma 3[7]. If Assumptions 2A and 2B hold, then the following continuous control law stabilizes system (7):

$$u = \frac{\partial v}{\partial x_1} f(x_1, x_2) - L_{M(x_1, x_2, v(x_1))} V(x_1) - (x_2 - v(x_1)) \tag{8}$$

Theorem 1. If Assumption 2A holds, Sontag-type control (5) is an optimal stabilizer of (7). Here $a(x) = L_{f(x_1, x_2)} V(x_1) - (x_2 - v(x_1)) \frac{\partial v}{\partial x_1} f(x_1, x_2), b(x) = x_2 - v(x_1)$.

Proof. Setting an energy function $W(x_1, x_2) = V(x_1) + \frac{1}{2}(x_2 - v(x_1))^2$, we get

$$L_F W(x) = L_{f(x_1, x_2)} V(x_1) - (x_2 - v(x_1)) \frac{\partial v}{\partial x_1} f(x_1, x_2), \quad L_G W(x) = x_2 - v(x_1)$$

"$v(0)=0, x \neq 0$ and $L_G W(x) = 0$" imply "$x_2 = v(x_1), x_1 \neq 0$", and thus

$$\forall x \neq 0, L_G W(x) = 0 \Rightarrow L_F W(x) = L_{f(x_1, v(x_1))} V(x_1) < 0.$$

Hence W is a CLF of system (7). In terms of Lemma 1, Sontag-type control (5) based on W is an optimal stabilizer of (7). □

Remark 1. Lemma 3 uses the C^1 regularity of f and v, and [8] even points out that Lemma 3 does not hold if f is only continuous. However, Theorem 1 shows that the Sontag-type control is a stabilizer without the condition $f \in C^1$.

The controller in Lemma 3 is obviously simpler and smoother than the one in Theorem 1. Why? The former is obtained by using passivity theory (Lemma 2), the following is a brief explanation.

Setting initially $u = \frac{\partial v}{\partial x_1} f(x_1, x_2) - L_{M(x_1, x_2, v(x_1))} V(x_1) + \tilde{u}$, we have

$$\dot{x} = \begin{pmatrix} f(x_1, x_2) \\ \frac{\partial v}{\partial x_1} f(x_1, x_2) - L_{M(x_1, x_2, v(x_1))} V(x_1) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tilde{u} := \widetilde{F}(x) + \widetilde{G}(x)\tilde{u}$$

With $W(x) = V(x_1) + \frac{1}{2}(x_2 - v(x_1))^2$ as an energy function, we get

$$L_{\widetilde{F}} W(x) = L_{f(x_1, v(x_1))} V(x_1) \leqslant 0, \quad L_{\widetilde{G}} W(x) = x_2 - v(x_1)$$

and the state-output system $\dot{x} = \widetilde{F}(x), y = L_{\widetilde{G}} W(x) = x_2 - v(x_1)$ is ZSD. In fact,

$$\forall t \geqslant 0, \forall x \neq 0, \ y = 0 \Rightarrow L_{\widetilde{F}} W(x) = L_{f(x_1, v(x_1))} V(x_1) < 0$$

Thus, by Lemma 2 it is not hard to obtain stabilizer (8). □

Now we come to input-to-state to stabilize the preceding cascaded systems with perturbation:

$$\begin{aligned}\dot{x}_1 &= f(x_1, x_2, d) \\ \dot{x}_2 &= u\end{aligned} \quad i.e., \ \dot{x} = \begin{pmatrix} f(x_1, x_2, d) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u := F(x, d) + G(x)u \tag{9}$$

where $(x_1, x_2) \in R^n \times R, d \in R^r$.

Lemma 4[9]. The system $\dot{x} = f(x, u), x \in R^n, u \in R^m$ is input-to-state stable (ISS) if and only if there is an energy function V such that $|x| \geqslant \rho(|u|), \rho \in K_\infty \Rightarrow L_{f(x,u)} V(x) < 0$.

Thereafter, the control law $u = \alpha(x)$ is said to ISS stabilize the system $\dot{x} = f(x, u)$, if

$\dot{x}=f(x,\alpha(x)+v)$ is ISS with respect to v.

Definition 3. Consider the affine system with perturbation

$$\dot{x} = f(x,d) + g(x)u,\ x \in R^n,\ u \in R^m,\ d \in R^p,\ f(0,0) = 0 \tag{10}$$

An energy function V is said to be an ISS-CLF of (10) if

$$\{(x,d) \mid |x| \geqslant \rho(|d|), L_gV(x) = 0, \rho \in K_\infty\} \Rightarrow \sup_{d \in \{d \mid |d| \leqslant \rho^{-1}(|x|)\}} L_{f(x,d)}V(x) := \omega(x) < 0.$$

Lemma 5. If V is an ISS-CLF of system (10), then the following Sontag-type control input-to-state stabilizes system (10).

$$k(x) = \begin{cases} -\dfrac{\omega + \sqrt{|\omega|^2 + |b(x)|^4}}{|b(x)|^2} b(x), & b(x) \neq 0 \\ 0, & b(x) = 0 \end{cases} \tag{11}$$

where $\omega(x)$ refers to the above formula, $b(x)=[L_gV(x)]^{\mathrm{T}}$.

Proof. Suppose $|x| \geqslant \rho(|d|)$.

Case (i). $[L_gV(x)]^{\mathrm{T}} \neq 0$. Calculating the time derivative of V along the solutions of the closed-loop, we have

$$\begin{aligned} \dot{V} &= L_{f(x,d)}V(x) - \omega - \sqrt{|\omega|^2 + |L_gV(x)|^4} \\ &\leqslant \sup_{d \in \{d \mid |d| \leqslant \rho^{-1}(|x|)\}} L_{f(x,d)}V(x) - \omega - \sqrt{|\omega|^2 + |L_gV(x)|^4} \\ &= -\sqrt{|\omega|^2 + |L_gV(x)|^4} < 0 \end{aligned}$$

Case (ii). $[L_gV(x)]^{\mathrm{T}}=0$. According to the definition of ISS-CLF, we get

$$\dot{V} = L_{f(x,d)}V(x) \leqslant \sup_{d \in \{d \mid |d| \leqslant \rho^{-1}(|x|)\}} L_{f(x,d)}V(x) = \omega < 0.$$

Therefore, $|x| > \rho(|d|) \Rightarrow \dot{V} < 0$. By Lemma 4, the closed-loop is ISS with respect to d. □

Now we design the input-to-state stabilizer of (9) by using Lemma 5.

Assumption 3. A C^1 control law $v(x_1), v(0)=0$ input-to-state stabilizes system $\dot{x}_1 = f(x_1, x_2, d)$, that is, there exists an energy function $V(x_1)$ such that $\sup_{d \in \{d \mid |d| \leqslant \rho^{-1}(|x_1|)\}} L_{f(x_1, v(x_1), d)}V(x_1) := \bar{\omega}(x_1) < 0$.

Theorem 2. If Assumption 3 holds, then the Sontag-type control (11) input-to-state stabilizes system (9). Here $\omega(x)=\bar{\omega}(x_1)$, $b(x)=x_2-v(x_1)$.

Proof. It is known that

$$L_FW(x) = L_{f(x_1,x_2,d)}V(x_1) - (x_2 - v(x_1)) \frac{\partial v}{\partial x_1} f(x_1, x_2, d),\ L_GW(x) = x_2 - v(x_1).$$

Noting that $v \in C^1, v(0)=0 \Rightarrow \exists \kappa \in K_\infty, |x| = \sqrt{|x_1|^2 + |v(x_1)|^2} \leqslant \kappa(|x_1|)$, it follows that

$$|x| \geqslant \kappa \circ \rho(|d|) := \gamma(|d|) \Rightarrow |x_1| \geqslant \rho(|d|)$$

Hence, $|x| \geqslant \gamma(|d|), L_GW(x)=0$ implies $|x_1| \geqslant \rho(|d|), x_2 = v(x_1)$, and then

$$\begin{aligned} |x| \geqslant \gamma(|d|),\ L_GW(x) = 0 \Rightarrow L_FW(x) &= L_{f(x_1, v(x_1), d)}V(x_1) \\ &\leqslant \sup_{d \in \{d \mid |d| \leqslant \rho^{-1}(|x_1|)\}} L_{f(x_1, v(x_1), d)}V(x_1) := \bar{\omega}(x_1) < 0 \end{aligned}$$

So, $W(x)=V(x_1)+\frac{1}{2}(x_2-v(x_1))^2$ is an ISS-CLF of system (9). By Lemma 5 the result is proved. □

4 Conclusions

Due to Sontag's formula, the existence of CLFs implies affine systems are stabilizable. For a class of typical cascaded systems, based on a CLF under appropriate conditions we have designed a stabilizer which is different from the passivity-based controller. Similarly, due to Sontag's formula, the existence of ISS-CLFs implies affine systems with perturbation are input-to-state stabilizable. For the preceding cascaded system with perturbation, based on an ISS-CLF under appropriate conditions we have designed an input-to-state stabilizer.

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控制李雅普诺夫函数的镇定应用

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摘 要 指出控制李雅普诺夫函数（源于松弛控制）正是零状态可检测性的一种刻画，并由此区别两类镇定控制：无源性控制和 Sontag 型控制. 然后考察一类典型的级联系统，基于一定条件下的控制李雅普诺夫函数，给出了不同于无源性控制的 Sontag 型镇定设计；考察这类系统的扰动情形，基于一定条件下的输入到状态稳定控制李雅普诺夫函数，给出了 Sontag 型输入到状态镇定设计.

关键词 控制李雅普诺夫函数，零状态可检测，无源性，输入到状态镇定

中图分类号 TP202