Satisfactory Filter of Linear Periodic Systems with Parameter Uncertainty[1)]

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Abstract The satisfactory filter problem is studied for linear multi-rate periodic systems with parameter uncertainty. The problem under consideration is to design a satisfactory filter such that the resulting filtering system is asymptotically stable and meets desired H-infinity rejection level for all admissible parameter uncertainty. Based on the satisfactory estimation idea, the problem is firstly converted into a problem of solving Riccati matrix inequalities (RMI). Then the latter is cast into a feasibility problem of a system of linear matrix inequalities (LMI) *via* matrix transform and LMI technique. A design method of satisfactory filter meeting desired multiple objectives is then provided which is effective by using Matlab-LMI computational software. Finally, a simulating example confirms the results obtained.

Key words Linear periodic systems, satisfactory filter, LMI, H_∞

1 Introduction

H_∞ robust filter problem of uncertain systems is an interesting research topic in recent years. Robust sampled-data estimation problems of linear time-invariant (LTI) systems have been studied in time domain *via* Kalman filter[1] and in frequency domain *via* Wiener filter[2]. Usually, high estimation precision for complex signals is hard to achieve *via* LTI filter. Therefore, multi-rate sampled-data approach is now commonly used in data sampling according to characteristics of signal frequencies. Many jointed sampled-data systems are multi-rate periodic systems and are widely studied and used in signal processing and communication. However, most relevant research works in the past were based on optimal control for one synthesis criterion[3], while satisfactory filter[4] with multiple objectives is rarely studied.

In this paper, filter design for a linear periodic multi-rate system with signal delay will be studied. The desired filter must meet required constraints on asymptotical stability and H_∞ rejection level for all admissible parameter uncertainty. The filter designing method provided in this paper is mainly based on satisfactory estimation idea and matrix transform and LMI technique.

2 Problem statement

Consider a digital filter system with periodically sampled channels, as in Fig. 1 where Σ_c is a continuous observed object, $\hat{\Sigma}$ is a discrete-time digital filter, Z^{-d} stands for d-step delay, $T_j (j=1,2,\cdots,l)$ denotes sampling period of each channel.

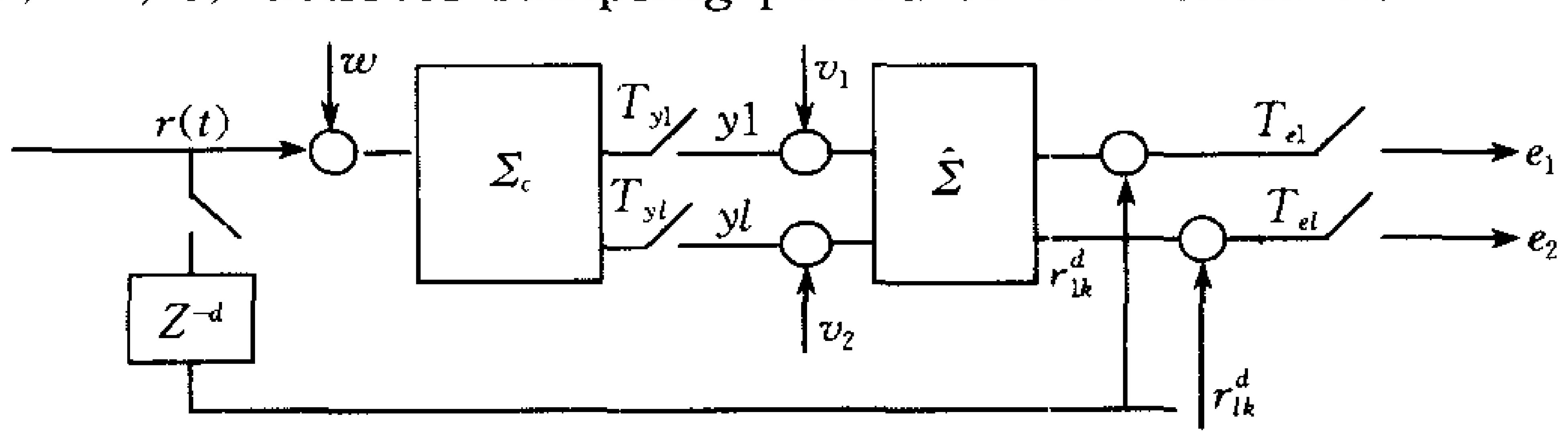

Fig. 1 Filter systems of periodical sampled-data digital channels

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Suppose the observed object can be expressed by the following linear discrete-time periodic system with parameter uncertainty

$$\Sigma_c:\begin{cases}x_{k+1}=(A_k+\Delta A_k)x_k+(B_k+\Delta B_k)r_k+E_kw_k\\ y_k=(C_k+\Delta C_k)x_k+(D_k+\Delta D_k)r_k\end{cases}\tag{1}$$

where $x\in R^n$ is the state vector, $y\in R^1$ is the measured output, $r\in R^1$ is reference input and $w\in R^1$ is external disturbance, $A_k\in R^{n\times n}$, $B_k\in R^{n\times 1}$, $C_k\in R^{1\times n}$, $D_k\in R^{1\times 1}$, $E_k\in R^{n\times 1}$ are N-periodic time-variant matrices and N denotes the number of sampled channels, ΔA_k, ΔB_k, ΔC_k, ΔD_k are uncertain parameters with the following structured forms.

$$\begin{bmatrix}\Delta A_k & \Delta B_k\\ \Delta C_k & \Delta D_k\end{bmatrix}=\begin{bmatrix}M_{1k}\\ M_{2k}\end{bmatrix}F_k[G_{1k}\quad G_{2k}]\tag{2}$$

where M_{1k}, M_{2k}, G_{1k}, G_{2k} are known N-periodic matrices and uncertain matrix $F_k\in F^*$, $F^*:=\{F_k\in R^{(n+1)\times(n+1)}\mid F_k^{\mathrm T}F_k\leqslant\xi^2 I,\forall k\}$, I is the unit matrix, ξ_k is a positive scalar, called uncertainty intensity.

Assumption H_1: Pairs (A_k, C_k) are all observable and (A_k, E_k) disturbable.

Consider periodically sampled behavior of output y and error e, and choose the following digital filter

$$\hat\Sigma:\begin{cases}\hat x_{k+1}=\hat A_k\hat x_k+\hat B_k\hat y_k\\ \hat r_k=\hat C_k\hat x_k+\hat D_k\hat y_k\end{cases}\tag{3}$$

where filter state vector $\hat x\in R^{\hat n}$, filter output $\hat y_k=y_k+v_k$, measurement noise $v_k\in R$, and $\hat A_{k+N}=\hat A_k\in R^{\hat n\times\hat n}$, $\hat B_{k+N}=\hat B_k\in R^{\hat n\times 1}$, $\hat C_{k+N}=\hat C_k\in R^{1\times\hat n}$, $\hat D_{k+N}=\hat D_k\in R$, $k=0,1,\cdots,N-1$.

The delay system of input signal with d-step delay is expressed by

$$\Sigma_d:\begin{cases}x^d_{k+1}=A_dx^d_k+B_dr_k\\ r^d_k=C_dx^d_k\end{cases}\tag{4}$$

where $r^d_k=r_{k-d}=z^{-d}r_k$, $A_d=\{0,e(1),e(2),\cdots,e(d-1)\}\in R^{d\times d}$, $B_d=e(d)\in R^{d\times 1}$, $C_d=e^{\mathrm T}(1)\in R^{1\times d}$, $e(i)$ denotes unit column vector whose ith component is 1 and others are zero.

Denote $\tilde x_k=[x_k, x^d_k, \hat x_k]^{\mathrm T}$, $\tilde u_k=[r_k, w_k, v_k]^{\mathrm T}$ and signal estimation error $e=z^{-d}r-\hat r$. By (1), (3) and (4), we have an augmented system:

$$\tilde\Sigma:\begin{cases}\tilde x_{k+1}=(\tilde A_k+\Delta\tilde A_k)\tilde x_k+(\tilde E_k+\Delta\tilde E_k)\tilde u_k\\ e_k=(\tilde C_k+\Delta\tilde C_k)\tilde x_k+(\tilde D_k+\Delta\tilde D_k)\tilde u_k\end{cases}\tag{5}$$

where $\tilde A_k=\begin{bmatrix}\bar A_k & 0\\ \hat B_k\bar C_k & \hat A_k\end{bmatrix}$, $\bar A_k=\begin{bmatrix}A_k & 0\\ 0 & A_d\end{bmatrix}$, $\bar B_k=\begin{bmatrix}B_k & E_k & 0\\ B_d & 0 & 0\end{bmatrix}$, $\bar D_k=[D_k\quad 0\quad I]$

$\tilde C_k=[\hat D_k\bar C_k+\bar C_d\quad \hat C_k]$, $\bar C_k=[C_k\quad 0]$, $\bar C_d=[0\quad -C_d]$, $\tilde E_k=[\bar B_k^{\mathrm T}\quad \bar D_k^{\mathrm T}\hat B_k^{\mathrm T}]^{\mathrm T}$

$\Delta\tilde A_k=\begin{bmatrix}\Delta\bar A_k & 0\\ \hat B_k\Delta\bar C_k & 0\end{bmatrix}$, $\Delta\bar A_k=\begin{bmatrix}\Delta A_k & 0\\ 0 & 0\end{bmatrix}$, $\Delta\tilde E_k=\begin{bmatrix}\Delta\bar B_k\\ \hat B_k\Delta\bar D_k\end{bmatrix}$, $\Delta\bar B_k=\begin{bmatrix}\Delta B_k & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

$\Delta\tilde C_k=[\hat D_k\Delta\bar C_k\quad 0]$, $\Delta\bar C_k=[\Delta C_k\quad 0]$, $\Delta\tilde D_k=\hat D_k\Delta\bar D_k$, $\Delta\bar D_k=[\Delta D_k\quad 0\quad 0]$,

$\tilde D_k=\hat D_k\cdot\bar D_k$

In order to make sure that the system is of good dynamic behavior and strong robustness, our objective is to design a filter to satisfy the following two requirements.

a) The augmented filter system in (5) is asymptotically stable for all permitted parameter uncertainty $\Delta(\cdot)$;

b) The augmented filter system in (5) meets desired H_∞ disturbance rejection level

$$\|H_{e\tilde u}(\tilde u,\tilde x_{mN},\tilde x_{mN+N-1})\|_\infty=$$

$$\inf_m\left(\sup_{0\neq(\tilde u,x_0)\in(\ell_2[0,N-1],R^{n_x})}\left\{\frac{\|e\|^2_{2,[0,N-1]}+\tilde x^{\mathrm T}_{mN+N-1}Q_U\tilde x_{mN+N-1}}{\|\tilde u\|^2_{2,[0,N-1]}+\|\tilde x_{mN}\|^2_2}\right\}^{1/2}\right)<\gamma\tag{6}$$

where $\gamma\in R$ is a given positive number, $\|\cdot\|_{2,[0,N-1]}$ denotes L_2 norm of $[\cdot]$ at the period of $[mN, mN+N-1]$, $\tilde x_{mN}$, $\tilde x_{mN+N-1}$, Q_U are differently initial, terminal states and its given weighting matrix, respectively.

3 H_∞ robust filter design for periodic systems

Lemma 1[5]. The linear discrete-time N-periodic system

$$\begin{cases} x_{k+1} = A_k x_k + E_k w_k \\ y_k = C_k x_k \end{cases} \tag{7}$$

is stable if and only if there exists symmetrical positive matrix $P_k \in R^{n\times n}$ that meets periodic RMI:

$$\begin{cases} A_k^T P_{k+1} A_k - P_k < 0 \\ P_{k+N} = P_k \end{cases} \quad (k = 0, 1, \cdots, N-1) \tag{8}$$

Lemma 2[6]. Given appropriate dimension matrices Y_k, Z_k and real symmetrical matrix M_k, the following inequalities hold for $\forall F_k \in F^*$

$$M_k + Y_k F_k Z_k + (Y_k F_k Z_k)^T < 0 \tag{9}$$

if and only if there exists positive scalar ε_k such that

$$M_k + \varepsilon_k \xi^2 Y_k Y_k^T + \varepsilon_k^{-1} Z_k^T Z_k < 0 \tag{10}$$

Theorem 1. Supposing Assumption H_1 is met, there exists periodic matrix pairs $(\hat{A}_k, \hat{B}_k, \hat{C}_k, \hat{D}_k)$ such that the filter (3) meets index a) if and only if the following inequalities in $Q_k \in R^{\bar{n}\times\bar{n}}$ and scalar ε_{1k}, *i.e.*,

$$\begin{bmatrix} -Q_{k+1} & \widetilde{A}_k Q_k & 0 & \widetilde{M}_k \\ Q_k \widetilde{A}_k^T & -Q_k & Q_k \widetilde{G}_{1k}^T & 0 \\ 0 & \widetilde{G}_{1k} Q_k & -\varepsilon_{1k} I & 0 \\ \widetilde{M}_k^T & 0 & 0 & -\varepsilon_{1k}^{-1}\xi^{-2} I \end{bmatrix} < 0 \tag{11}$$

have a feasible solution, where $\widetilde{M}_k = [M_{1k}^T \quad 0 \quad M_{2k}^T \hat{B}_k^T]^T$, $\widetilde{G}_{1k} = [G_{1k} \quad 0 \quad 0]$, $Q_{k+N} = Q_k$, $Q_k = (Q_k)^T > 0$, $k = 0, 1, \cdots, N-1$.

Proof. By Lemma 1, for the augmented system (5), if there exist an N-period symmetrical matrix $P_k \in R^{\bar{n}\times\bar{n}}$ and $P_k > 0$ to meet periodic Lyapunov inequalities

$$(\widetilde{A}_k^T + \Delta\widetilde{A}_k^T) P_{k+1} (\widetilde{A}_k + \Delta\widetilde{A}_k) - P_k < 0 \tag{12}$$

or the transforming Lyapunov inequality

$$(\widetilde{A}_k + \Delta\widetilde{A}_k) Q_k (\widetilde{A}_k^T + \Delta\widetilde{A}_k^T) - Q_{k+1} < 0 \tag{13}$$

then system (5) is asymptotically stable and the corresponding digital filter naturally meets requirement *a*). By Schur complements[7], inequalities in (13) are equivalent to the following.

$$\begin{bmatrix} -Q_{k+1} & (\widetilde{A}_k + \Delta\widetilde{A}_k) Q_k \\ Q_k (\widetilde{A}_k^T + \Delta\widetilde{A}_k^T) & -Q_k \end{bmatrix} < 0 \tag{14}$$

By Lemma 2, we can get from (14)

$$\begin{bmatrix} -Q_{k+1} & \widetilde{A}_k Q_k \\ Q_k \widetilde{A}_k^T & -Q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_{1k}\xi^2 \widetilde{M}_k \widetilde{M}_k^T & 0 \\ 0 & \varepsilon_{1k}^{-1} Q_k \widetilde{G}_{1k}^T \widetilde{G}_{1k} Q_k \end{bmatrix} < 0 \tag{15}$$

Using Schur complements again, we know inequality (15) is equivalent to inequality (11).

Inversely following the above deducing process we can have the sufficiency of Theorem 1. □

Lemma 3[5] Given the H_∞ index γ, if there exist positive matrices $P_k \in R^{n\times n}$ meeting the following Riccati matrix inequalities.

$$-P_k + A_k^T P_{k+1} A_k + A_k^T P_{k+1} E_k [\gamma^2 I - E_k^T P_{k+1} E_k]^{-1} E_k^T P_{k+1} A_k + C_k^T C_k < 0 \tag{16}$$

and $P_{N-1} = P_U$, $I - \gamma^{-2} E_k^T P_{k+1} E_k > 0$ $(k = 0, 1, 2, \cdots, N-1)$, then system (7) is asymptotically stable and meets requirement $\| H_{yw}(s) \|_\infty < \gamma$.

By Lemma 3, system (5) meets requirement b) if and only if there exists N-periodic and symmetrical matrices $Q_k \in R^{\bar{n}\times\bar{n}}$ such that $Q_{N-1} = Q_U$ and

$$\begin{bmatrix} -Q_{k+1} + (\widetilde{A}_k + \Delta\widetilde{A}_k) Q_k (\widetilde{A}_k + \Delta\widetilde{A}_k)^T & (\widetilde{A}_k + \Delta\widetilde{A}_k) Q_k (\widetilde{C}_k + \Delta\widetilde{C}_k)^T & \widetilde{E}_k + \Delta\widetilde{E}_k \\ * & -\gamma^2 I + (\widetilde{C}_k + \Delta\widetilde{C}_k) Q_k (\widetilde{C}_k + \Delta\widetilde{C}_k)^T & \widetilde{D}_k + \Delta\widetilde{D}_k \\ * & * & -I \end{bmatrix} < 0 \tag{17}$$

$$-\gamma^2 I+(\widetilde{C}_k+\Delta\widetilde{C}_k)Q_k(\widetilde{C}_k+\Delta\widetilde{C}_k)^{\mathrm{T}}+(\widetilde{D}_k+\Delta\widetilde{D}_k)(\widetilde{D}_k+\Delta\widetilde{D}_k)^{\mathrm{T}}<0 \tag{18}$$

where " * " denotes symmetric entries of matrices.

From the above statement the following theorem gives a satisfactory filter of the periodic system (5) meeting requirements of a) and b).

Theorem 2. Suppose Assumption H_1 is met and that the desired H_∞ index γ of the system (5) is given. If there exist N-period positive matrices $Q_k\in R^{\tilde{n}\times\tilde{n}}$ and matrix pairs $(\hat{A}_k,\hat{B}_k,\hat{C}_k,\hat{D}_k)$ and scalars ε_{2k}, such that

$$\begin{bmatrix} -Q_k & (\widetilde{A}_kQ_k)^{\mathrm{T}} & Q_k\widetilde{C}_k^{\mathrm{T}} & 0 & Q_k\widetilde{G}_{1k}^{\mathrm{T}} & 0\\ * & -Q_{k+1} & 0 & \widetilde{E}_k & 0 & \widetilde{M}_k\\ * & * & -\alpha I & \widetilde{D}_k & 0 & \hat{D}_kM_{2k}\\ * & * & * & -I & \widetilde{G}_{2k}^{\mathrm{T}} & 0\\ * & * & * & * & -\varepsilon_{2k}I & 0\\ * & * & * & * & * & -\varepsilon_{2k}^{-1}\xi^{-2}I \end{bmatrix}<0 \tag{19}$$

then system(5) meets requirements of a) and b), where $Q_{k+N}=Q_k$, $\widetilde{G}_{2k}=[G_{2k}\ \ 0\ \ 0]$, $\alpha=\gamma^2$, $*$ denotes symmetric parts of the matrices, $k=0,1,\cdots,N-1$.

Proof. By Lemma 3 and inequality (17), the proof is similar to that of Theorem 1. □

Notice that inequalities (11) and (19) are nonlinear and it is not easy to find a feasible solution. But if the filter (3) is full of rank, $i.e.$, $\tilde{n}=n+d$, then these inequalities can be formulated into LMI forms by means of matrix transform as in [7]. In fact, if we denote

$$Q_j=\begin{bmatrix} X_j & S_j\\ S_j^{\mathrm{T}} & Y_j\end{bmatrix},\ Q_j^{-1}=\begin{bmatrix} R_j & L_j\\ L_j^{\mathrm{T}} & U_j\end{bmatrix},\ \varphi_{1j}=\begin{bmatrix} X_j & I_n\times\tilde{n}\\ S_j^{\mathrm{T}} & 0\end{bmatrix},\ \varphi_{2j}=\begin{bmatrix} I_n\times\tilde{n} & R_j\\ 0 & L_j^{\mathrm{T}}\end{bmatrix} \tag{20}$$

$$\Theta_j=(R_{j+1}^{\mathrm{T}}\overline{A}_j+L_{j+1}\hat{B}_j\overline{C}_j)X_j+L_{j+1}\hat{A}_jS_j^{\mathrm{T}}\in R^{\tilde{n}\times\tilde{n}} \tag{21}$$

$$\Phi_j=X_j(\hat{D}_j\overline{C}_j+\overline{C}_d)^{\mathrm{T}}+S_j\hat{C}_j^{\mathrm{T}}\in R^{\tilde{n}\times 1} \tag{22}$$

$$\Psi_j=L_{j+1}\hat{B}_j\in R^{\hat{n}\times 1} \tag{23}$$

where X_j, Y_j, R_j, U_j, S_j, L_j are $\tilde{n}\times\tilde{n}$ matrices, then we can get the following equations ($j=0,1,\cdots,N-1$)

$$\varphi_{2(j+1)}^{\mathrm{T}}\widetilde{A}_jQ_j\varphi_{2j}=\Pi_j=\begin{bmatrix}\overline{A}_jX_j & \overline{A}_j\\ \Theta_j & R_{j+1}^{\mathrm{T}}\overline{A}_j+\Psi_j\overline{C}_j\end{bmatrix},\quad \varphi_{2j}^{\mathrm{T}}Q_j\varphi_{2j}=-\Lambda_j=\begin{bmatrix}X_j & I\\ I & R_j\end{bmatrix}$$

$$\varphi_{2j}^{\mathrm{T}}Q_j\widetilde{C}_j^{\mathrm{T}}=\Gamma_j^{\mathrm{T}}=\begin{bmatrix}\Phi_j\\ (\hat{D}_j\overline{C}_j+\overline{C}_d)^{\mathrm{T}}\end{bmatrix},\quad \varphi_{2j}^{\mathrm{T}}Q_j\widetilde{G}_{1j}^{\mathrm{T}}=\Xi_j^{\mathrm{T}}=\begin{bmatrix}X_j\bar{\bar{G}}_{1j}^{\mathrm{T}}\\ \bar{\bar{G}}_{1j}^{\mathrm{T}}\end{bmatrix},$$

$$\bar{\bar{G}}_{1j}=[G_{1j}\ \ 0],\quad Q_j\varphi_{2j}=\varphi_{1j},\quad \varphi_{2(j+1)}^{\mathrm{T}}\widetilde{M}_j=Z_j=\begin{bmatrix}\overline{M}_j\\ R_{j+1}\overline{M}_j+\Psi_jM_{2j}\end{bmatrix},$$

$$\varphi_{2(j+1)}^{\mathrm{T}}\widetilde{E}_j=\Omega_j=\begin{bmatrix}\overline{B}_j\\ R_{j+1}\overline{B}_j+\Psi_j\overline{D}_j\end{bmatrix},\quad \overline{M}_j=[M_{1j}^{\mathrm{T}}\ \ 0]^{\mathrm{T}}$$

Consequently, pre-and post-multiplying (19) by diag$\{(\varphi_{2k})^T, (\varphi_{2(k+1)})^T, I, I, I, I\}$ and diag$\{\varphi_{2k}, \varphi_{2(k+1)}, I, I, I, I\}$, we get its LMI form as follows.

$$\begin{bmatrix}\Lambda_k & \Pi_k^{\mathrm{T}} & \Gamma_k^{\mathrm{T}} & 0 & \Xi_k^{\mathrm{T}} & 0\\ * & \Lambda_{k+1} & 0 & \Omega_k & 0 & Z_k\\ * & * & -\alpha I & \hat{D}_k\overline{D}_k & 0 & \hat{D}_kM_{2k}\\ * & * & * & -I & \widetilde{G}_{2k}^{\mathrm{T}} & 0\\ * & * & * & * & -\varepsilon_{2k}I & 0\\ * & * & * & * & * & -\varepsilon_{2k}^{-1}\xi^{-2}I\end{bmatrix}<0 \tag{24}$$

Proposition 1. By finding a feasible solution to LMI system (24) in (X_k, R_k, Θ_k, Ψ_k, Φ_k, $\hat{D}_k$) *via* Matlab-LMI, and solving equation system consisting of (20)~(23) and $S_kL_k^{\mathrm{T}}=I-X_kR_k$, a satisfactory parameter $(\hat{A}_k,\hat{B}_k,\hat{C}_k,\hat{D}_k)(k=0,1,\cdots,N-1)$ is obtained. They will guarantee the desired requirements of a) and b) to be met for the augmented system (5).

4 Numerical Examples

Consider an synthesized bank with length three and two channels yielding the sub-band signals $r_j(m)$ for $j=1, 2$. Suppose two synthesized digital filter systems are given by

$$H_0(z) = -0.1 + (-0.1967 + 0.02\delta_k)z^{-1} + 0.3933z^{-2} + (-0.1967 + 0.01\delta_k)z^{-3},$$

$$H_1(z) = -0.1920 + 0.04\delta_k - 0.4800z^{-1} + (0.0960 + 0.03\delta_k)z^{-2} - 0.1920z^{-3}$$

where δ_k is the parameter uncertain factor of each subsystem with $\delta_k \in [-1,1]$.

Suppose the system input r_k and model noise and measured noise at each sampled channel are all white noises with zero mean-value and intensity $\sigma^2=1$. Our objective is to design a filter to satisfy: 1) the augment system (5) is asymptotically stable under the permitted parameter uncertainties of $\Delta(\cdot)$, and 2) $\| H_{e\tilde{u}}(\cdot) \|_\infty < 1.9$.

In the form (1) we have $A_k=[0\ \ 1\ \ 0;\ 0\ \ 0\ \ 1;\ 0\ \ 0\ \ 0]$, $\Delta A_k=[0]_{3\times 3}$, $B_k=[0\ \ 0\ \ 1]^T$, $\Delta B_k=[0]_{3\times 1}$, $E_k=[1\ \ 1\ \ 1]^T$; $k=0, 2, 4, \cdots$, $C_k=[-0.1967\ \ 0.3933\ \ -0.1967]$, $\Delta C_k=0.01\times\delta_k\times[1\ \ 0\ \ 2]$, $D_k=-0.1$, $\Delta D_k=0$; $k=1, 3, 5, \cdots$, $C_k=[-0.192\ \ 0.096\ \ -0.48]$, $\Delta C_k=0.01\delta_k\times[0\ \ 3\ \ 0]$, $D_k=-0.192$, $\Delta D_k=0.04$.

According to Theorem 2 and choosing $L_j=I$, we can get a pair of feasible parameters of filter(3) by resolving inequalities (24). Therefore $\| H_{e\tilde{u}}(\cdot) \|_\infty = 1.617$ and the tracking simulation is as follows.

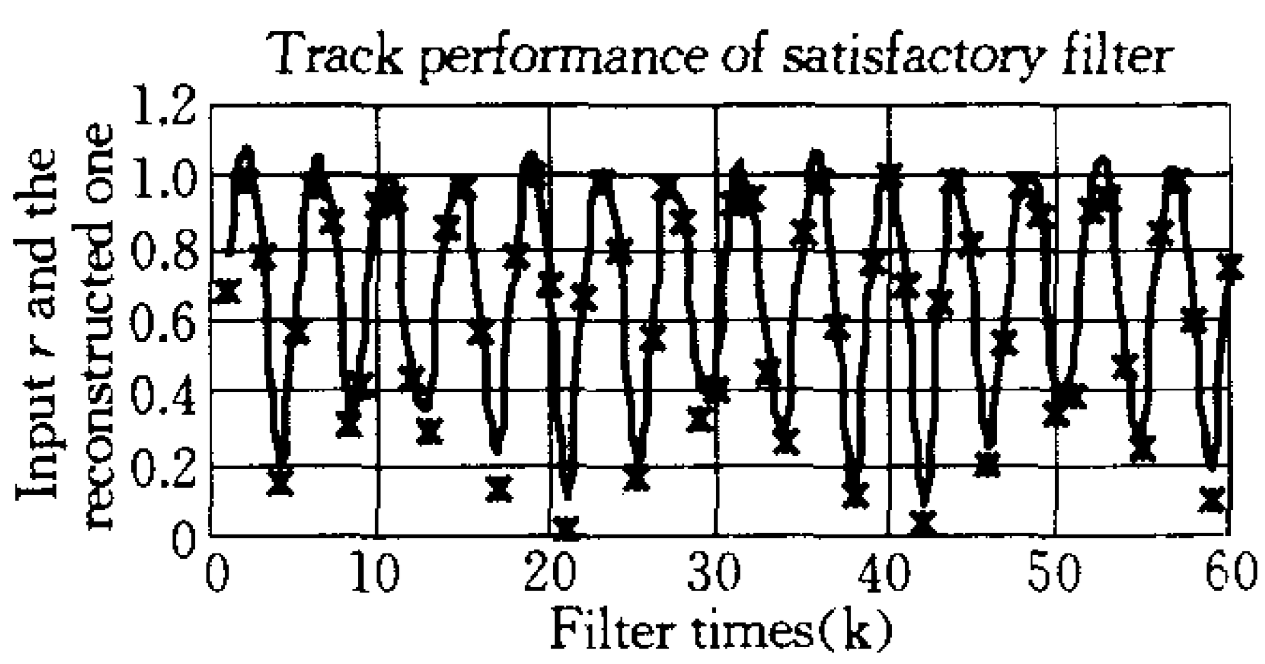

Fig. 2 Satisfactory filter for the signal r_k

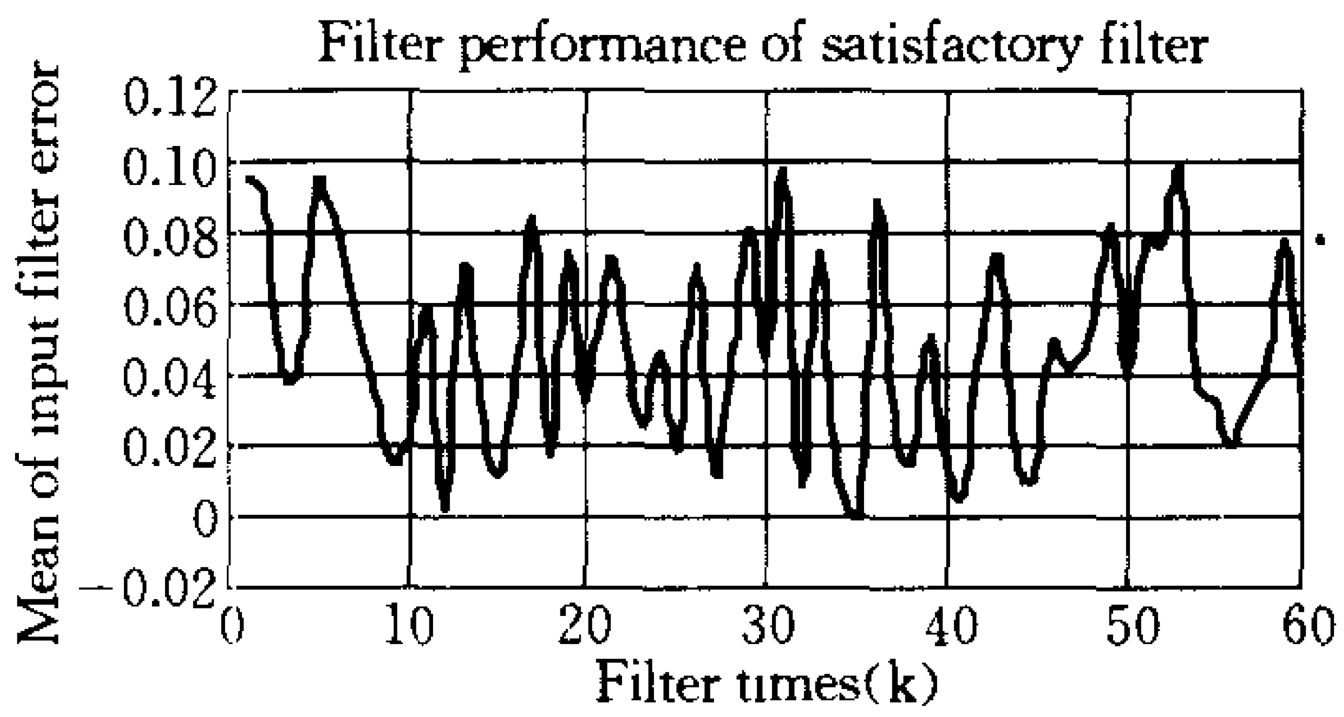

Fig. 3 Satisfactory filter error for the signal r_k

5 Conclusion

In this paper we discuss problems about satisfactory filter of periodical multirate systems, and present a direct method to designate a satisfactory filter under systemic parameter uncertainties and signal time delay and the desired H-infinity index by converting RMIs into LMIs. This approach is more effective and direct than the usual lifting technique. Of course, we only present the above problem at the sample points while have not discussed intersample behaviors or other satisfactory indices such as circular poles and variance.

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不确定线性周期系统的满意滤波

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摘　要　针对线性周期多采样率系统的满意滤波问题,根据满意估计思想,将结构不确定周期系统满足滤波系统稳定与 H_∞ 指标的满意估计问题转化为 Riccati 矩阵不等式求解问题,无须采用通常的提升手段而直接运用数值法与内点法进行周期系统多指标设计. 利用线性矩阵不等式技术(LMI)提出了一种基于 LMI 的满意滤波器设计方法. 仿真例子验证了相关的结论.

关键词　周期系统,满意滤波,LMI,H_∞

中图分类号　TP13