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Separability Theory for Blind Signal Separation[1)]

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Abstract The basic theory of blind signal separation is discussed. Not only the sufficiency-necessity condition is given for the extraction of single source but also the problem how many signals can be extracted is answered. Moreover, the blind separation algorithm based on this theory is proposed. The method can also be applied to blind extraction in the case of morbid mixture. At last, simulation is done in different mixed conditions, and the results show that the separation theory is correct and the algorithm is valid.

Key words Blind signal separation, basic theory, morbid mixed, fourth-order cumulants, penalty function

1 Introduction

Blind signal separation has many potential applications since it retrieves original signals from the observed mixture of signals without any transcendental knowledge of the original signals or the channels. It becomes one of the hot topics in the signal processing field. Blind signal separation problem was proposed one decade ago. By the great effort of the research, various effective approaches and corresponding adaptive algorithms are developed[1~11].

Most of these algorithms are based on the precondition "the number of the observers is greater or equal to the number of the source signals". If the mixture matrix is of full rank, the synchronous separation approach is used. If the mixture matrix is not of full rank, the source signals may be extracted one by one. However, we find that the research of the basic theory of blind signal separation is not plentiful, especially for the morbid mixture (*i.e.*, the number of observers is less than the number of the source signals, or the source signals are not independent, *etc*). We should consider and try to solve some problems such as when we could or could not separate all the source signals from the mixed signals, and how many source signals we could separate.

This paper studies the above basic theory and give some basic solution. The paper is composed of five sections. After illustrating our motivation in Section 1, we classify the models of blind source separation in Section 2 and present some theorem on the necessary and sufficient conditions of the separability. Also, we discuss how many source signals could be extracted in this section. In Section 3, some algorithms are established according to the theory proposed in Section2. Some simulations are made in Section 4 for three different cases. The simulation examples illustrate the validity of our theory. In conclusion, Section 5 gives a brief summary of this paper and proposes some problems of the basic theory, which should be further studied.

2 Analysis of extractability

The blind signal separation can be described as the following mathematical model

$$x(t) = As(t) \tag{1}$$

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$$u(t) = Wx(t) \tag{2}$$

where $x(t)=(x_1(t),\cdots,x_m(t))^T$ is the observed signal vector, $s(t)=(s_1(t),\cdots,s_n(t))^T$ is the source signal vector that will be separated. $A=(a_{ij})_{m\times n}$ is the unknown mixture matrix. The aim of blind signal separation is to adjust separation matrix W, according to the observed signals, such that the separated signal vector $u(t)=(u_1(t),u_2(t),\cdots,u_n(t))^T$ is identical to the source signal vector in the waveform. That is,

$$u(t) = WAs(t) = PDs(t) \tag{3}$$

where P is a permutation matrix while D is a diagonal matrix.

There are three cases of the above model: 1) $m\geqslant n$, rank$(A)\doteq n$; 2) $m\geqslant n$, rank$(A)<n$; 3) $m<n$.

The first case is that the number of observers is not less than that of the source signals and the number of the observed independent signals is equal to the number of the source signals. The second case is that although the number of the observers is not less than that of the source signals, the number of the observed independent signals is less than the number of the source signals. The third case is that the number of the observers is less than that of the source signals. In general, case 1) is said to be the normal case while cases 2) and 3) belong to the morbidity. At present, there are few researches who are concerned with the morbid cases.

Next, we discuss the conditions, under which the source signals can be separated from the mixed signals observed.

2.1 The necessary and sufficient conditions of extractability

From the extractive target (3), we can see the essential of the problem. That is, whether there exists a separated matrix W, according to the unknown mixture matrix A, such that WA is a diagonal matrix. Therefore, the problem becomes the existence of the solutions to a group of corresponding equations.

Theorem 1. The necessary and sufficient condition of extracting one source signal from the mixed signals is that, for matrix A, there exists an extended matrix $\overline{A}_i$ of A^T such that

$$\text{rank}(\overline{A}_i) = \text{rank}(A) \tag{4}$$

where

$$\overline{A}_i = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} & 0 \\ & & \cdots\cdots & & \\ a_{1i} & a_{2i} & \cdots & a_{mi} & \lambda_i \\ & & \cdots\cdots & & \\ a_{1n} & a_{2n} & \cdots & a_{mn} & 0 \end{bmatrix}$$

i.e., the last entry of the ith row is λ_i, $\lambda_i\neq 0$, while the last entries of the other rows are all zeros.

Proof. Consider a group of equations

$$\begin{cases} a_{11}w_{i1}+a_{21}w_{i2}+\cdots+a_{m1}w_{im}=0 \\ \cdots\cdots \\ a_{1i}w_{i1}+a_{2i}w_{i2}+\cdots+a_{mi}w_{im}=\lambda_i \\ \cdots\cdots \\ a_{1n}w_{i1}+a_{2n}w_{i2}+\cdots+a_{mn}w_{im}=0 \end{cases} \tag{5}$$

The coefficient matrix of the above group of equations is A^T, its extended matrix is $\overline{A}_i$. The necessary and sufficient condition for existence of solutions to Equations (5) is

$$\text{rank}(\overline{A}_i) = \text{rank}(A^T) = \text{rank}(A) \tag{6}$$

The equivalent condition is that there exists a vector $W_i=(w_{i1},w_{i2},\cdots,w_{im})^T$ such that

$$A^TW_i = (0,\cdots,0,\lambda_i,0,\cdots,0)^T \tag{7}$$

Hence,

$$u_1(t) = W_i^Tx(t) = W_i^TAs(t) = (0,\cdots,0,\lambda_i,0,\cdots,0)s(t) = \lambda_is_i(t) \tag{8}$$

It means a source signal $s_i(t)$ can be extracted. The theorem is complete. □

Since the arbitrariness of the entry λ_i, the above condition is not convenient to be verified. So, we present another equivalent criterion.

Theorem 2. The necessary and sufficient condition of extracting one source signal from the mixed signals is that there exists an $m\times(n-1)$ sub-matrix $\widetilde{A}_i$ of A such that $1+\sigma(\widetilde{A}_i)=\sigma(A)$, where $\sigma(\cdot)$ is the number of the nonsingular values.

Proof. Consider matrices

$$\widetilde{A}_i^{\mathrm{T}}=\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ & \cdots\cdots & & \\ a_{1,i-1} & \cdots & & a_{m,i-1} \\ a_{1,i+1} & \cdots & & a_{m,i+1} \\ & \cdots\cdots & & \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix},\quad \widetilde{\widetilde{A}}_i=\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} & 0 \\ & & \cdots\cdots & & \\ a_{1,i-1} & & \cdots & a_{m,i-1} & 0 \\ a_{1,i+1} & & \cdots & a_{m,i+1} & 0 \\ & & \cdots\cdots & & \\ a_{1n} & a_{2n} & \cdots & a_{mn} & 0 \end{bmatrix}$$

Obviously, one of the following equalities must hold: $\mathrm{rank}(\widetilde{\widetilde{A}}_i)=\mathrm{rank}(\widetilde{A}_i^{\mathrm{T}})$, $\mathrm{rank}(\widetilde{\widetilde{A}}_i)=\mathrm{rank}(\overline{A}_i)$ and $1+\mathrm{rank}(\widetilde{\widetilde{A}}_i)=\mathrm{rank}(\overline{A}_i)$.

Let a_i be the ith row vector of matrix $\overline{A}_i$. If $\mathrm{rank}(\widetilde{\widetilde{A}}_i)=\mathrm{rank}(\overline{A}_i)$, then vector $\boldsymbol{a}_i$ is the linear combination of the other row vectors, *i. e.*, $\boldsymbol{a}_i=\sum_{j=1,j\neq i}^{n}k_j\boldsymbol{a}_j$. This means that $\lambda_i=0$ contradicts to $\lambda_i\neq 0$. There must be

$$1+\mathrm{rank}(\widetilde{\widetilde{A}}_i)=\mathrm{rank}(\overline{A}_i) \tag{9}$$

Recalling that for any matrix B, one has $\mathrm{rank}(B)=\mathrm{rank}(B^HB)$ and B^HB is the Hermit matrix, so there is a matrix U, $U^HU=I$, such that $U^HB^HBU=\mathrm{diag}(l_1^2,\cdots,l_r^2,0,\cdots,0)$, where l_i are the singular value of matrix B. Thus, $\mathrm{rank}(B)=\sigma(B)$ and

$$\mathrm{rank}(A)=\sigma(A),\quad \mathrm{rank}(\widetilde{A}_i^{\mathrm{T}})=\mathrm{rank}(\widetilde{A}_i)=\sigma(\widetilde{A}_i) \tag{10}$$

From conditions (9) and (10), we conclude that the necessary and sufficient condition of extracting a single source signal from the mixed signals is $1+\sigma(\widetilde{A}_i)=\sigma(A)$. The theorem is complete. □

2.2 The number of the extractable signals

After presented the necessary and sufficient condition of extracting a single source signal, we wish to discuss the number of the extractable signals.

Theorem 3. If there are L extended matrices $\overline{A}_i$ such that $\mathrm{rank}(\overline{A}_i)=\mathrm{rank}(A)$, then there are at lest L sequentially extractable source signals.

Proof. Without loss of generality, assume that

$$\overline{A}_1=\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} & \lambda_1 \\ & & \cdots\cdots & & \\ a_{1i} & a_{2i} & \cdots & a_{mi} & 0 \\ & & \cdots\cdots & & \\ a_{1n} & a_{2n} & \cdots & a_{mn} & 0 \end{bmatrix},\quad \cdots,\quad \overline{A}_L=\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} & 0 \\ & & \cdots\cdots & & \\ a_{1L} & a_{2L} & \cdots & a_{mL} & \lambda_L \\ & & \cdots\cdots & & \\ a_{1n} & a_{2n} & \cdots & a_{mn} & 0 \end{bmatrix}$$

Since $\mathrm{rank}(\overline{A}_1)=\mathrm{rank}(A)$, from Theorem 1 we can extract one source signal $s_1(t)$. By eliminant method, the mixture matrix can be transformed into

$$A=\begin{bmatrix} a_{12} & a_{13} & \cdots & a_{1n} \\ & \cdots\cdots & & \\ a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Consider

$$\begin{cases} a_{12}w_1+a_{22}w_2+\cdots+a_{m2}w_m=\lambda_2 \\ a_{13}w_1+a_{23}w_2+\cdots+a_{m3}w_m=0 \\ \cdots\cdots \\ a_{1n}w_1+a_{2n}w_2+\cdots+a_{mn}w_m=0 \end{cases} \tag{11}$$

Since $\mathrm{rank}(\overline{A}_2)=\mathrm{rank}(A)$, again from Theorem 1 there exists the solution to the following equations

$$\begin{cases} a_{11}w_1 + a_{21}w_2 + \cdots + a_{m1}w_m = 0 \\ a_{12}w_1 + a_{22}w_2 + \cdots + a_{m2}w_m = \lambda_2 \\ \cdots\cdots \\ a_{1n}w_1 + a_{2n}w_2 + \cdots + a_{mn}w_m = 0 \end{cases} \tag{12}$$

Obviously, from comparison of (11) and (12), we find that the solution to (12) should be the solution to (11). So, there must exist the solution to (11). This means that one can extract the second source signal. Sequentially, one can extract L source signals from the mixed signals. However, the existence of the solution to (11) does not mean the existence of the solution to (12). Therefore, at lest L source signals can be extracted. □

Also, we present an equivalent theorem

Theorem 4. If there exist L sub-matrices $\widetilde{A}_i$ of the size $m\times(n-1)$ such that $1+\sigma(\widetilde{A}_i)=\sigma(A)$, then, at least L source signals can be extracted.

Proof. Without loss of generality, assume that

$$\widetilde{A}_1 = \begin{bmatrix} a_{12} & a_{13} & \cdots & a_{1n} \\ & \cdots\cdots & & \\ a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}, \quad \cdots, \quad \widetilde{A}_L = \begin{bmatrix} a_{11} & \cdots & a_{1,L-1} & a_{1,L+1} & \cdots & a_{1n} \\ & & \cdots\cdots & & & \\ a_{m1} & \cdots & a_{m,L-1} & a_{m,L+1} & \cdots & a_{mn} \end{bmatrix}$$

Since $1+\sigma(\widetilde{A}_1)=\sigma(A)$, one source signal $s_1(t)$ can be extracted. By eliminant method, the mixture matrix can be transformed into

$$\bar{A} = \begin{bmatrix} a_{12} & a_{13} & \cdots & a_{1n} \\ & \cdots\cdots & & \\ a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

By verifying

$$\bar{\widetilde{A}}_1 = \begin{bmatrix} a_{13} & a_{14} & \cdots & a_{1n} \\ & \cdots\cdots & & \\ a_{m3} & a_{m4} & \cdots & a_{mn} \end{bmatrix}, \quad \widetilde{A}_2 = \begin{bmatrix} a_{11} & a_{13} & \cdots & a_{1n} \\ & \cdots\cdots & & \\ a_{m1} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \tag{13}$$

if the vector $\boldsymbol{a}_2=(a_{12},\cdots,a_{m2})^{\mathrm{T}}$ is a linear combination of the other column vectors of matrix $\bar{\widetilde{A}}_1$, then it can be a linear combination of all the column vectors of $\widetilde{A}_2$. This means $\text{rank}(\widetilde{A}_2)=\text{rank}(A)$. It contradicts to the condition $1+\sigma(\widetilde{A}_2)=\sigma(A)$. Thus, the column vector a_2 is not linearly relative to the each column vector of $\bar{\widetilde{A}}_1$. It causes $1+\text{rank}(\bar{\widetilde{A}}_1)=\text{rank}(\bar{A})$, *i.e.*, $1+\sigma(\bar{\widetilde{A}}_1)=\sigma(\bar{A})$. By Theorem 2, the second source signal can be extracted. By keeping on the procedure, L source signals can be extracted. Clearly, if $1+\sigma(\widetilde{A}_L)=\sigma(A)$, there must be $1+\sigma(\bar{\widetilde{A}}_L)=\sigma(\bar{A}_L)$. However, the inverse could not be held definitely. Therefore, at lest L source signals can be extracted.

3 Algorithm of blind source separation

To illustrate the validity of the extractability theory, we will give some simulations according to some algorithms. Now we present some algorithms.

We verify the fourth-order cumulant of the signal u_1. From the model of extracting signals and the properties of the fourth-order cumulant[12], we have

$$kut(u_1) = kut(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}) = kurt(\boldsymbol{w}^{\mathrm{T}}A\boldsymbol{s}) = kurt(\boldsymbol{z}^{\mathrm{T}}\boldsymbol{s}) = \sum_{i=1}^{n} z_i^4 kurt(s_i) \tag{14}$$

where $\boldsymbol{w}^{\mathrm{T}}A=\boldsymbol{z}$. As in general study, we assume that the source signals are of zero-mean and unit variance, *i.e.*, $E(\boldsymbol{s}\boldsymbol{s}^{\mathrm{T}})=1$. Therefore,

$$\|\boldsymbol{z}\|^2 = \boldsymbol{w}^{\mathrm{T}}AA^{\mathrm{T}}\boldsymbol{w} = \boldsymbol{w}^{\mathrm{T}}AE(\boldsymbol{s}\boldsymbol{s}^{\mathrm{T}})A^{\mathrm{T}}\boldsymbol{w} = \boldsymbol{w}^{\mathrm{T}}E(\boldsymbol{x}\boldsymbol{x}^{\mathrm{T}})\boldsymbol{w} = E(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x})^2 \tag{15}$$

The target of extraction needs to guarantee that the variance of extracted signal u_1 is 1. This is

$$E(u_1)^2 = E(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x})^2 = 1 \tag{16}$$

By (15) and (16), we get the constraint

$$\| z \|^2 = 1 \tag{17}$$

From (14) and (17), when the sources are all super-Gaussian signals and under the constraint, we can extract the signal with the maximal fourth-order cumulant by maximizing $kurt(w^T x)$. Contrarily, when the sources are all sub-Gaussian signals and under the constraint, the signal with minimal fourth-order cumulant can be extracted by minimizing $kurt(w^T x)$. For simplicity, this paper considers only the super-Gaussian signals (the algorithm for the sub-Gaussian signals can be obtained by the same method). So, the optimal problem with the constraint

$$\max_{E(w^T x)^2=1} \quad kurt(\boldsymbol{w}^T \boldsymbol{x}) \tag{18}$$

can extract one signal. By the definition of the fourth-order cumulant, we have

$$kut(\boldsymbol{w}^T \boldsymbol{x}) = E(\boldsymbol{w}^T \boldsymbol{x})^4 - 3E^2(\boldsymbol{w}^T \boldsymbol{x})^2 = E(\boldsymbol{w}^T \boldsymbol{x})^4 - 3 \tag{19}$$

Moreover, setting $V = Exx^T$, the constraint (19) becomes

$$E(\boldsymbol{w}^T \boldsymbol{x})^2 = \boldsymbol{w}^T V \boldsymbol{w} = 1 \tag{20}$$

Thus,(18) is equivalent to

$$\begin{cases} \max J(\boldsymbol{w}) \\ \text{s. t.} \quad \boldsymbol{w}^T V \boldsymbol{w} - 1 = 0 \end{cases} \tag{21}$$

where $J(\boldsymbol{w}) = E(\boldsymbol{w}^T \boldsymbol{x})^4$. The constant term can be ignored since it is irrelevant to w. To solve the optimizing problem (21), we introduce the penalty function[13], that is, (21) is transformed into

$$\max P(\boldsymbol{w}, \beta) \tag{22}$$

where $P(\boldsymbol{w},\beta) = E(\boldsymbol{w}^T \boldsymbol{x})^4 - \beta(\boldsymbol{w}^T V \boldsymbol{w} - 1)^2$, and β is a large positive constant. If the constraint cannot be satisfied, the function $P(\boldsymbol{w},\beta)$ cannot reach the maximum, since the value of β is large. Inversely, if the constraint is satisfied, we have $\max\{P(\boldsymbol{w},\beta)\} = \max\{J(\boldsymbol{w})\}$, *i. e.*, (22) is equivalent to(21). The optimizing problem with the constraint (21) is changed to the problem (22) without the constraint. By using the steepest descent method, we get the following learning algorithm. Because of

$$\nabla \boldsymbol{w} = \frac{\partial P(\boldsymbol{w},\beta)}{\partial \boldsymbol{w}} = 4E(\boldsymbol{w}^T \boldsymbol{x})^3 \boldsymbol{x}^T(t) - 4\beta(\boldsymbol{w}^T V \boldsymbol{w} - 1)E(\boldsymbol{w}^T \boldsymbol{x})\boldsymbol{x}^T(t) \tag{23}$$

the algorithm is

$$\boldsymbol{w}(t+1) = \boldsymbol{w}(t) + \alpha E(\boldsymbol{w}^T(t)\boldsymbol{x}(t))^3 \boldsymbol{x}^T(t) - \alpha\beta(\boldsymbol{w}^T(t) V \boldsymbol{w}(t) - 1)E(\boldsymbol{w}^T(t)\boldsymbol{x}(t))\boldsymbol{x}^T(t) \tag{24}$$

For convenience, we use the ordinary method to transform the above algorithm in to a random gradient algorithm:

$$\boldsymbol{w}(t+1) = \boldsymbol{w}(t) + \alpha(\boldsymbol{w}^T(t)\boldsymbol{x}(t))^3 \boldsymbol{x}^T(t) - \alpha\beta(\boldsymbol{w}^T(t) V \boldsymbol{w}(t) - 1)(\boldsymbol{w}^T(t)\boldsymbol{x}(t))\boldsymbol{x}^T(t) \tag{25}$$

4 Simulating experiments

In the last two sections, we present the basic theory of separability and the algorithms according to the theory. To verify the validity of the algorithms and the extractability theory, we will give the simulations for the normal case and the morbid case. Form voice signals are taken from the website http://www. cis. hut. fi/projects/ica/cocktail/cocktail_en. cgi as the source signals.

To judge the effect of the extraction, we use the similitude coefficient ζ_{ij} as one of the test index as in general cases. That is,

$$\zeta_{ij} = \zeta(u_i, s_j) = \left| \sum_{t=1}^{n} u_i(t) s_j(t) \right| \Big/ \sqrt{\sum_{t=1}^{n} u_i^2(t) \sum_{t=1}^{n} s_j^2(t)} \tag{26}$$

If $u_i = cs_j$, then $\zeta_{ij} = 1$. This means the result of blind source separation can be different from the source signals only in the amplitude. When u_i is independent of s_j, the index satisfies $\zeta_{ij} = 0$.

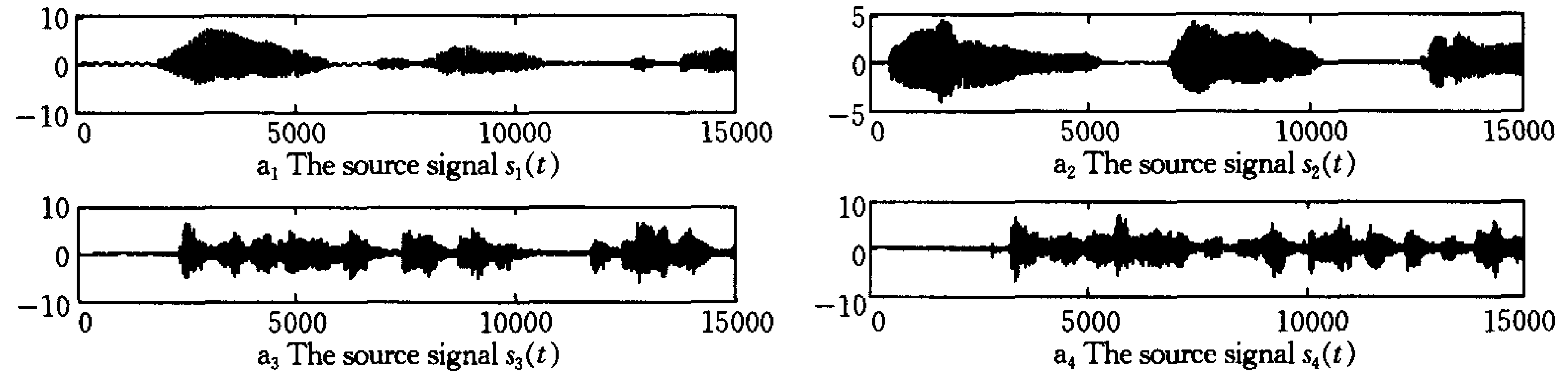

Fig. 1 The source signals in each simulation

Simulation 1. At first, we consider the normal case 1). In this case, the number of the observers is equal to the number of the source signals, and rank$(A)=n$. Take the mixture matrix $A=(\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3, \boldsymbol{A}_4)$, where $\boldsymbol{A}_1=(1, 0.4, 0.2, 0.1)^T$, $\boldsymbol{A}_2=(0.65, 1, 0.5, 0.2)^T$, $\boldsymbol{A}_3=(0.33, 0.43, 1, 0.35)^T$ and $\boldsymbol{A}_4=(0.2, 0.5, 0.6, 1)^T$. By computation, we have rank$(A)=4$, the conditions for complete extraction are satisfied. The result can be seen in the following figures.

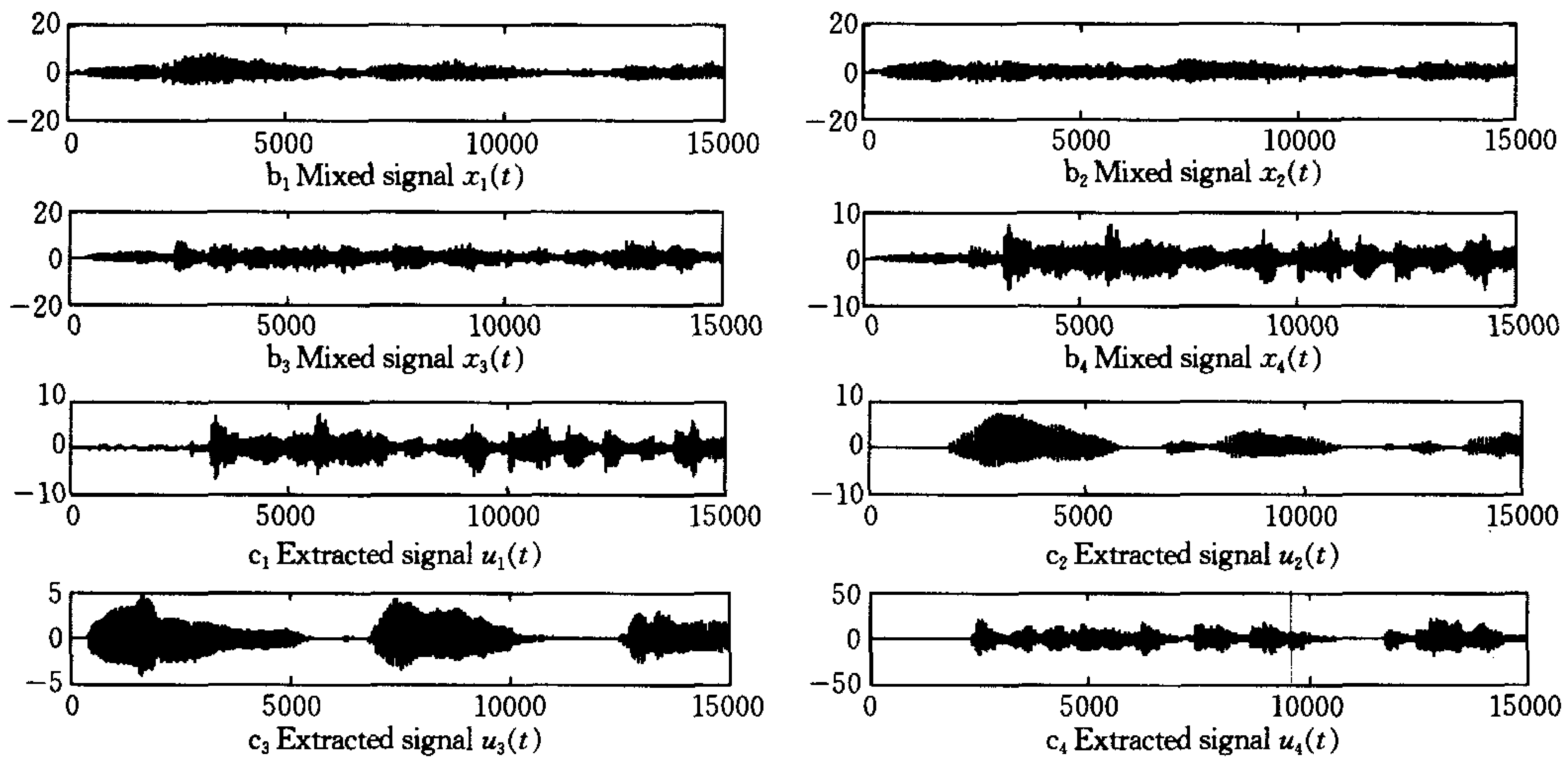

Fig. 2 Mixed signals and separated signals

The similitude coefficient matrix can be computed as $\zeta=(\zeta_1, \zeta_2, \zeta_3, \zeta_4)$, where $\zeta_1=(0.0047, 0.9858, 0.0296, 0.0273)^T$, $\zeta_2=(0.0088, 0.0115, 0.9881, 0.0045)^T$, $\zeta_3=(0.0103, 0.0201, 0.1154, 0.9239)^T$ and $\zeta_4=(0.9911, 0.0105, 0.0384, 0.0037)^T$. One can see the good effect and that the all source signals are complete extracted.

Simulation 2. When the number of the observers is less than the number of the source signals, that is, in the morbid case, $m<n$, we consider 2 observers versus 4 source signals. Taking the mixture matrix $A=\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$, the result of the extraction is

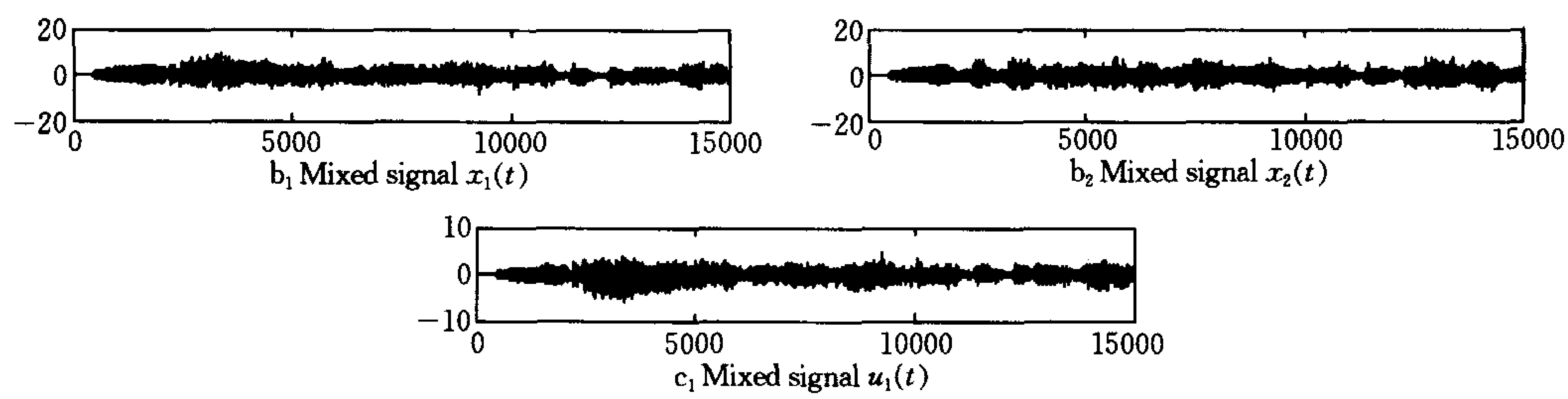

Fig. 3 No signal can be separated

By theoretical analysis, Theorem 1 tells us the reason. Since for any 2×3 sub-matrix A_i of the mixture matrix, we have $\sigma(A_i)=2$ and $\sigma(A)=2$. There is no sub-matrix A_i satisfying $1+\sigma(A_i)=\sigma(A)$. This causes that no signal can be extracted. Computing the similitude coefficient vector, we have $\zeta_1=(0.3248, 0.3560, 0.4219, 0.4954)$. This clearly indicates the failure of extracting signal $u_1(t)$. It illustrates the consistency of the theoretical and simulation analysis.

Simulation 3. At last, we consider the morbid case 3). In this case, it has $m>n$, rank $(A)<n$. Let the mixture matrix be $A=(\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3, \boldsymbol{A}_4)$, where $\boldsymbol{A}_1=(1, 0, 2, 0.5, 1)^{\mathrm{T}}$, $\boldsymbol{A}_2=(1, 1, 2, 0.5, 1)^{\mathrm{T}}$, $\boldsymbol{A}_3=(0, 1, 2, 0, 1)^{\mathrm{T}}$ and $\boldsymbol{A}_4=(1, 1, 2, 0.5, 1)^{\mathrm{T}}$. By computation, we have $\mathrm{rank}(A)=3<4$. The result of extraction is

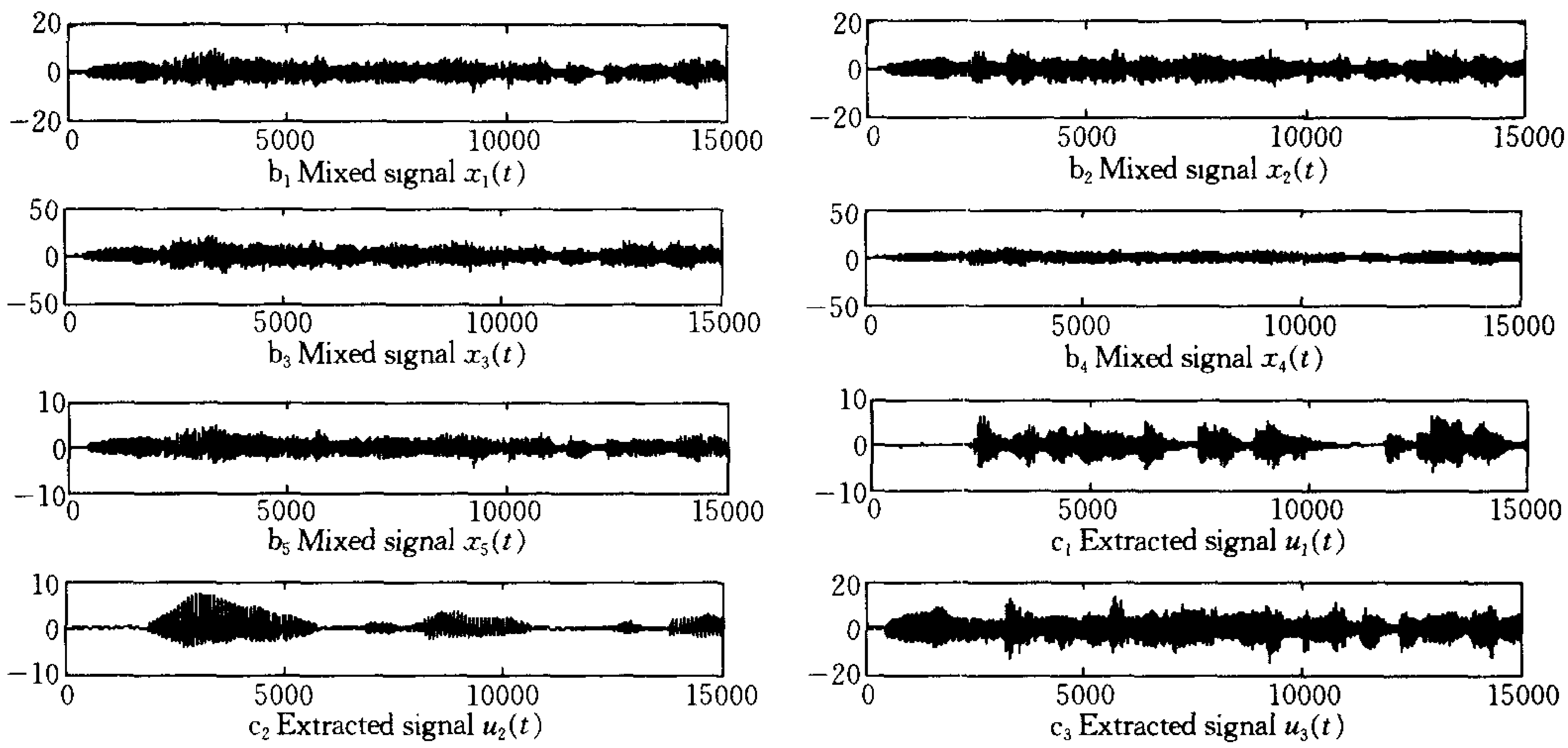

Fig. 4 Simulation 3: mixed signals and separated signals

Similarly, there exist two 2×3 sub-matrices $\widetilde{A}_i$, *i. e.*, $\widetilde{A}_1=(A_2, A_3, A_4)$ and $\widetilde{A}_2=(A_1, A_2, A_4)$. Hence, we obtain $\sigma(\widetilde{A}_i)=2$ and $\sigma(A)=3$. By Theorem 3, there are 2 source signals that can be extracted. From the simulation, we can see that the signals $u_1(t)$ and $u_2(t)$ can be extracted successfully, while the extraction of signal $u_3(t)$ is a failure.

5 Conclusion

This paper discusses the basic theory of blind source signal separation. A necessary and sufficient condition of the separation is presented. Also, we give a result to determine how many source signals can be extracted from the observed signals. Some algorithms based on the basic theory are proposed. Under different cases, the simulating experiments are made to verify the validity of our theory and algorithms. It is worthwhile to say that the basic theory we presented is based on some information of the mixture matrix A. However, in practice, the information of A could not be known. So, this is only a theoretical analysis. Whether we could discuss the basic theory only from the observed signals or not? This is a problem that will be studied in the future.

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盲分离问题的可分性理论

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摘　要　对盲信号分离的基本理论问题进行了探讨,给出了从混叠信号中分离出源信号的充要条件,并回答了源信号能提取的数量问题.同时,还给出了基于该理论的盲分离算法,该方法适应于病态混叠情形下的提取.且就几种不同的情况进行了计算机仿真.仿真结果表明了所提理论的正确性及相应算法的有效性.

关键词　盲信号分离,基本理论,病态混叠,四阶累计量,罚函数

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