Delay-Dependent Absolute Stability of Uncertain Lur′e Systems with Time-Delays[1)]

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Abstract This paper is concerned with delay-dependent absolute stability for a class of uncertain Lur′e systems with multiple time-delays. By using a descriptor model transformation of the system and by applying a recent result on bounding of cross products of vectors, a new type of Lyapunov-Krasovskii functional is constructed. Based on the new functional, delay-dependent sufficient conditions for absolute stability are derived in terms of linear matrix inequalities. These conditions do not require any parameter tuning, and can be solved numerically using the software LMI Lab. A numerical example is presented which shows that the proposed method can substantially improve the delay bound for absolute stability of Lur′e system with time-delays, compared to the existing ones.

Key words Lur′e system, absolute stability, delay-dependent criteria, time-delay, linear matrix inequality

1 Introduction

Recently, several authors considered the problem of delay-dependent stability of Lur′e system with time-delay[1~3]. In [1,2], the Razumilkin′s approach was applied to the delay-dependent stability analysis of Lur′e system with time-delay, based on the construction of Lyapunov function. However, this approach does not utilize adequately the property of the nonlinearity of Lur′e system, so the results in [1,2] may be conservative. In [3], the Krasovskii′s approach was applied to the same problem and a delay-dependent sufficient condition was obtained in terms of matrix inequality. However, the matrix inequality is not convex, so it is difficult to check the sufficient condition given in [3] in practice. In this paper, inspired by the idea of Fridman in [4], we introduce a new type of Lyapunov-Krasovskii functional to study the problem of delay-dependent stability of Lur′e system with time-delays, which is based on the equivalent descriptor form of the original system. New delay-dependent sufficient conditions are derived in terms of linear matrix inequalities. These conditions can be solved efficiently using the software LMI Lab and are essentially less conservative than the existing ones.

2 Main results

Consider the following uncertain Lur′e system with multiple time-delays

$$\dot{\boldsymbol{x}}(t) = \sum_{i=0}^{m} A_i(t)\boldsymbol{x}(t-\tau_i) + \boldsymbol{b}f(\sigma(t)) \tag{1}$$

$$\sigma(t) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(t) \tag{2}$$

where $\boldsymbol{x}(t)\in R^n$, $\tau_0=0$, $\tau_i>0$, $i=1,2,\cdots,m$, denote time-delays, vectors $\boldsymbol{b}$, $\boldsymbol{c}\in R^n$. For each $i\in\{0, 1, \cdots, m\}$, $A_i(t)=A_i+\Delta A_i(t)$, A_i are known matrices, $\Delta A_i(t)$ are unknown matrices representing time-varying parameter uncertainties in the system model. We assume that the uncertainties are norm-bounded and can be described as $\Delta A_i(t)=E_iF_i(t)H_i$, in

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which E_i and H_i are known constant matrices and $F_i(t)$ is an unknown matrix function satisfying $F_i^{\mathrm{T}}(t)F_i(t)\leqslant I$.

In the proof of our main results, we will need the following two lemmas.

Lemma 1[6]. If D, E, F are properly dimensioned matrices with $F^{\mathrm{T}}F\leqslant I$, for any scalar $\varepsilon>0$, then $DFE+E^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}}\leqslant\varepsilon^{-1}DD^{\mathrm{T}}+\varepsilon E^{\mathrm{T}}E$.

Lemma 2[5]. For any properly dimensioned vectors $\boldsymbol{a}, \boldsymbol{b}$ and matrices N, X, Y, Z, if

$$\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix}\geqslant 0,\ \text{then}\ -2\boldsymbol{a}^{\mathrm{T}}N\boldsymbol{b}\leqslant\begin{bmatrix}\boldsymbol{a}\\ \boldsymbol{b}\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} X & Y-N \\ Y^{\mathrm{T}}-N^{\mathrm{T}} & Z \end{bmatrix}\begin{bmatrix}\boldsymbol{a}\\ \boldsymbol{b}\end{bmatrix}.$$

Theorem 1. Let $f\in F_{[0,\infty]}=\{f(\cdot)\mid f(0)=0,\ 0<\sigma f(\sigma)<\infty,\ \sigma\neq 0\}$. If there exist scalars $\beta\geqslant 0$, $\alpha_i>0$, $i=0,1,\cdots,m$, and matrices $P>0$, P_1, P_2, W_i, M_i, $Q_i\geqslant 0$, $S_i>0$, $i=1,\cdots,m$, such that the following matrix inequalities hold

$$\Pi_i=\begin{bmatrix} W_i & M_i \\ M_i^{\mathrm{T}} & Q_i \end{bmatrix}\geqslant 0,\ i=1,\cdots,m,\quad \Lambda=\begin{bmatrix} \Lambda_0 & L_2 \\ L_2^{\mathrm{T}} & -\Phi \end{bmatrix}<0$$

where

$$\Lambda_0=\begin{bmatrix} \Psi & G^{\mathrm{T}}[0\ \ A_1^{\mathrm{T}}]^{\mathrm{T}}-M_1 & \cdots & G^{\mathrm{T}}[0\ \ A_m^{\mathrm{T}}]^{\mathrm{T}}-M_m & L_1 \\ * & \alpha_1H_1^{\mathrm{T}}H_1-S_1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ * & 0 & \cdots & \alpha_mH_m^{\mathrm{T}}H_m-S_m & 0 \\ * & 0 & \cdots & 0 & L_0 \end{bmatrix}$$

$$G=\begin{bmatrix} P & 0 \\ P_1 & P_2 \end{bmatrix},\quad L_1=\begin{bmatrix} P\boldsymbol{b}+\beta\boldsymbol{c} \\ \boldsymbol{c}/2+\sum_{i=1}^{m}\tau_iQ_i\boldsymbol{b} \end{bmatrix},\quad L_0=\boldsymbol{c}^{\mathrm{T}}\boldsymbol{b}+\sum_{i=1}^{m}\tau_i\boldsymbol{b}^{\mathrm{T}}Q_i\boldsymbol{b}$$

$$L_2=[G^{\mathrm{T}}[0\ \ E_0^{\mathrm{T}}]^{\mathrm{T}}\ \ \cdots\ \ G^{\mathrm{T}}[0\ \ E_m^{\mathrm{T}}]^{\mathrm{T}}],\quad \Phi=\mathrm{diag}\{\alpha_0I,\ \cdots,\ \alpha_mI\}$$

$$\Psi=G^{\mathrm{T}}\begin{bmatrix} 0 & I \\ A_0 & -I \end{bmatrix}+\begin{bmatrix} 0 & A_0^{\mathrm{T}} \\ I & -I \end{bmatrix}G+\alpha_0\begin{bmatrix} H_0^{\mathrm{T}}H_0 & 0 \\ 0 & 0 \end{bmatrix}+\sum_{i=1}^{m}\{\tau_iW_i+[M_i\ \ 0]+$$
$$[M_i\ \ 0]^{\mathrm{T}}+\mathrm{diag}\{S_i,\ \ \tau_iQ_i\}\}$$

then the system described by (1) and (2) is absolutely stable.

Proof. Let $\boldsymbol{y}(t)=\sum_{i=0}^{m}A_i(t)\boldsymbol{x}(t-\tau_i)$. Then system (1) can be represented in the equivalent descriptor form

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{y}(t)+\boldsymbol{b}f(\sigma(t))\tag{3}$$

$$0=-\boldsymbol{y}(t)+\sum_{i=0}^{m}A_i(t)\boldsymbol{x}(t)-\sum_{i=1}^{m}A_i(t)\int_{t-\tau_i}^{t}(\boldsymbol{y}(s)+\boldsymbol{b}f(\sigma(s)))\mathrm{d}s\tag{4}$$

Let the Lyapunov-Krasovskii functional for system (3) and (4) be

$$V(t)=V_1(t)+\sum_{i=1}^{m}\left\{\int_{-\tau_i}^{0}\mathrm{d}\theta\int_{t+\theta}^{t}(\boldsymbol{y}(s)+\boldsymbol{b}f(\sigma(s)))^{\mathrm{T}}Q_i(\boldsymbol{y}(s)+\boldsymbol{b}f(\sigma(s)))\mathrm{d}s+\right.$$
$$\left.\int_{t-\tau_i}^{t}\boldsymbol{x}^{\mathrm{T}}(s)S_i\boldsymbol{x}(s)\mathrm{d}s\right\}+\int_{0}^{\sigma(t)}f(s)\mathrm{d}s+2\beta\int_{0}^{t}\sigma(s)f(\sigma(s))\mathrm{d}s$$

where $V_1(t)=\boldsymbol{x}^{\mathrm{T}}(t)P\boldsymbol{x}(t)$. Using (4), we get

$$\dot{V}_1(t)=2\boldsymbol{\eta}^{\mathrm{T}}(t)G^{\mathrm{T}}[\boldsymbol{y}^{\mathrm{T}}(t)\ \ 0]^{\mathrm{T}}+2\boldsymbol{x}^{\mathrm{T}}(t)P\boldsymbol{b}f(\sigma(t))=$$

$$2\boldsymbol{\eta}^{\mathrm{T}}(t)G^{\mathrm{T}}\left\{\begin{bmatrix} \boldsymbol{y}(t) \\ \sum_{i=0}^{m}A_i(t)\boldsymbol{x}(t)-\boldsymbol{y}(t) \end{bmatrix}-\sum_{i=1}^{m}\begin{bmatrix} 0 \\ A_i(t) \end{bmatrix}\int_{t-\tau_i}^{t}\boldsymbol{y}_1(s)\mathrm{d}s\right\}+u(t)\tag{5}$$

where $\boldsymbol{\eta}^{\mathrm{T}}(t)=[\boldsymbol{x}^{\mathrm{T}}(t)\ \boldsymbol{y}^{\mathrm{T}}(t)]$, $\boldsymbol{y}_1(t)=\boldsymbol{y}(t)+\boldsymbol{b}f(\sigma(t))$, $u(t)=2\boldsymbol{x}^{\mathrm{T}}(t)P\boldsymbol{b}f(\sigma(t))$.

Using Lemma 2, we get

$$-2\int_{t-\tau_i}^{t}\boldsymbol{\eta}^{\mathrm T}(t)G^{\mathrm T}[0\quad A_i^{\mathrm T}]^{\mathrm T}\boldsymbol{y}_1(s)\mathrm{d}s\leqslant\int_{t-\tau_i}^{t}\begin{bmatrix}\boldsymbol{\eta}(t)\\ \boldsymbol{y}_1(s)\end{bmatrix}^{\mathrm T}\begin{bmatrix}W_i & M_i-G^{\mathrm T}[0\quad A_i^{\mathrm T}]^{\mathrm T}\\ * & Q_i\end{bmatrix}\begin{bmatrix}\boldsymbol{\eta}(t)\\ \boldsymbol{y}_1(s)\end{bmatrix}\mathrm{d}s\leqslant$$

$$\tau_i\boldsymbol{\eta}^{\mathrm T}(t)W_i\boldsymbol{\eta}(t)+\int_{t-\tau_i}^{t}\boldsymbol{y}_1^{\mathrm T}(s)Q_i\boldsymbol{y}_1(s)\mathrm{d}s+2\boldsymbol{\eta}^{\mathrm T}(t)(M_i-G^{\mathrm T}[0\quad A_i^{\mathrm T}]^{\mathrm T})(\boldsymbol{x}(t)-\boldsymbol{x}(t-\tau_i))\quad(6)$$

Combining (5) and (6) and using Lemma 1, with some efforts, we get $\dot V(t)\leqslant\boldsymbol{\xi}^{\mathrm T}(t)\Lambda_1\boldsymbol{\xi}(t)$, here $\boldsymbol{\xi}^{\mathrm T}(t)=[\boldsymbol{\eta}^{\mathrm T}(t)\quad \boldsymbol{x}^{\mathrm T}(t-\tau_1)\quad\cdots\quad \boldsymbol{x}^{\mathrm T}(t-\tau_m)\quad f(\sigma(t))]$, Λ_1 is taken from Λ_0 by replacing Ψ with $\Psi+\sum_{i=0}^{m}\alpha_i^{-1}G^{\mathrm T}[0\quad E_i^{\mathrm T}]^{\mathrm T}[0\quad E_i^{\mathrm T}]G$. By Schur complement, $\Lambda<0$ is equivalent to $\Lambda_1<0$, which implies that system (1) and (2) is absolute stable. The proof is complete. □

Similar to the proof of Theorem 1, we can obtain the following theorem.

Theorem 2. Let $f\in F_{[0,k]}=\{f(\cdot)\mid f(0)=0,\ 0<\sigma f(\sigma)\leqslant k\sigma^2,\ \sigma\neq 0\}$. If there exist scalars $\beta\geqslant 0$, $\alpha_i>0$, $i=0,1,\cdots,m$, and matrices $P>0, P_1, P_2, W_i, M_i, Q_i\geqslant 0, S_i>0, i=1,\cdots,m$, such that $\Pi_i<0, i=1,\cdots,m$, and $\Lambda_2<0$, here Λ_2 is taken from Λ by replacing L_0 with $L_0-2\beta/k$, then system (1) and (2) is absolute stable.

3 Numerical example

Consider the following Lur'e system with time-delay

$$\begin{bmatrix}\dot x_1(t)\\ \dot x_2(t)\end{bmatrix}=\begin{bmatrix}-2 & 0\\ -1 & -2\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}-0.2 & -0.5\\ 0.5 & -0.2\end{bmatrix}\begin{bmatrix}x_1(t-\tau)\\ x_2(t-\tau)\end{bmatrix}-\begin{bmatrix}0.2\\ 0.3\end{bmatrix}f(\sigma(t))$$

$$\sigma(t)=0.6x_1(t)+0.8x_2(t),\quad f\in F_{[0,\infty)}$$

By Theorem 1 and using the software package LMI Lab it is obtained that the system is absolute stable for any constant time-delay $\tau\leqslant 898999699999$. We note that the result of [3] guarantees the absolute stability of this system when $\tau\leqslant 2.055$, whereas by the methods of [1] and [2], under the condition of $f\in F_{[0,0.5]}$, the time-delay τ is allowed to be less than 0.3053 and 0.3230, respectively. This example shows that the method of this paper is an improvement of the previous results.

4 Conclusion

Through constructing a new type of Lyapunov-Krasovskii functional, new delay-dependent sufficient conditions for absolute stability of uncertain Lur'e systems with multiple time-delays are derived in terms of linear matrix inequalities. The example has shown that our results are less conservative than the existing stability criteria.

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不确定时滞 Lur′e 型控制系统与时滞相关的绝对稳定性

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摘　要　研究了一类参数不确定的具有多个时滞的 Lur′e 型控制系统与时滞相关绝对稳定性问题. 通过将原系统变换为等价的奇异系统，利用 Moon 不等式放大向量积，构造出一个新的 Lyapunov-Krasovskii 泛函. 并由此基于线性矩阵不等式，得到了系统与时滞相关绝对稳定的充分条件. 这些充分条件无须预调任何参数矩阵，可以直接运用 Matlab 软件中 LMI 工具箱求解. 数值例子表明，与现有结果相比，本文结果较大地改进了保证不确定时滞 Lur′e 型控制系统绝对稳定的时滞界.

关键词　Lur′e 型控制系统，绝对稳定性，时滞系统，时滞相关判据，线性矩阵不等式

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