

A New Method on Single View Metrology¹⁾

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Abstract In the literature, it was shown that distances on a scene plane (referred as the reference plane) can be measured via the homography between this plane and its projected image. We show in this paper that we can push the frontier slightly further under the same condition, in other words, some more pieces of geometrical information on the scene can also be retrieved through such a homography. In particular, we show that in addition to distances on the reference plane, distances on any plane perpendicular to the reference plane, or distance of any pair of points with one point lying on the reference plane and the other on a perpendicular plane can also be measured provided that the used camera is either a calibrated one or an uncalibrated one but with only two unknown parameters. Furthermore, we also have had an investigation on how to estimate the two unknown parameters from the homography. Finally, both simulated data and real images have been used to test our new method and results are satisfactory.

Key words Single view metrology, homography, camera calibration

1 Introduction

Recently visual metrology has attracted a lot of attention in the field of computer vision^[1~6], such as architectural and indoor measurement, reconstruction from paintings and traffic accident investigation. Using only one view to measure world distances is a very important branch^[1~3]. In [1,2], the Euclidean distance of any pair of points on a scene plane (referred as the reference plane) can be measured if the homography between this plane and its projected images is available. In this paper, we propose a new method and show that we can go further under the same condition, in other words, some more geometrical information on the scene can also be retrieved through such a homography. In particular, we show that in addition to distances on the reference plane, distances on any plane perpendicular to the reference plane, or distance of any pair of points with one point lying on the reference plane and the other on a perpendicular plane can also be measured provided that the used camera is either a calibrated one or an uncalibrated one but with only two unknown parameters. In the previous reported methods a common assumption is that the camera's intrinsic and extrinsic parameters are completely unknown. However, with the development of camera manufacturing technology, usually some of camera's parameters are known a priori, such as

- 1) the principal point is always close to the center of image
- 2) the x and y image axes are always almost at 90° close to perpendicular
- 3) the aspect ratio equals 1

By taking into account the availability of such pieces of information, a camera could have only two unknown parameters. In this paper, we also have an investigation on how to estimate the two unknown parameters from a homography.

The paper is organized as follows: In section 2 some preliminaries are given, and the

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proposed method will be described in section 3. In sections 4 and 5, the experiment results are reported both with simulated data and with real images. Some conclusions are listed at the end of the paper.

2 Preliminaries

2.1 Notation

A 2D point is denoted by $\mathbf{m} = [u, v]^T$. A 3D point is denoted by $\mathbf{x} = [X, Y, Z]^T$. Their homogeneous coordinates are in the form $\tilde{\mathbf{m}} = [u, v, 1]^T$ and $\tilde{\mathbf{x}} = [X, Y, Z, 1]^T$. The camera model employed here is the central projection and the relationship between a 3D point \mathbf{x} and its image \mathbf{m} is:

$$\lambda \tilde{\mathbf{m}} = K [R \quad \mathbf{t}] \tilde{\mathbf{x}} \quad \text{with} \quad K = \begin{bmatrix} f & s & u_0 \\ 0 & \alpha f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where λ is an arbitrary scale factor; R is the 3×3 rotation matrix and \mathbf{t} is the 3×1 translation vector from the world coordinate system to the camera coordinate system, called the extrinsic parameters in the field of camera calibration. K is the camera intrinsic matrix, and f is the effective focal length, s the skew, α the aspect ratio, and (u_0, v_0) the principal point.

2.2 Plane to plane homography

A plane, without loss of generality, can be assumed as the plane of $Z=0$. By denoting the i th column of the rotation matrix R by \mathbf{r}_i , then from (1), we have:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K [\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{r}_3 \quad \mathbf{t}] \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix} = K [\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{t}] \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Therefore, a plane point \mathbf{x} and its image \mathbf{m} is related by a plane homography H :

$$\lambda \tilde{\mathbf{m}} = H \tilde{\mathbf{x}} \quad \text{with} \quad H = K [\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{t}] = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \quad (2)$$

The 3×3 matrix H is defined up to a scale factor. There are many ways to estimate the homography between a plane and its image, however, details will be omitted and the reader is referred to [10] for more information. As long as the homography is obtained, an image point can be back-projected to the world plane via H^{-1} , so distances on the world plane can be measured.

3 A new method on single view metrology

In this section, we show how to measure world distances on a plane perpendicular to the reference plane and distance of any pair of points with one lying on the reference plane and the other on a plane perpendicular to the reference plane, if the homography between the reference plane and its projected image is available and the used camera is either a calibrated one or an uncalibrated one but with only two unknown parameters. Firstly, we show how to obtain the homography between a world plane perpendicular to the reference plane and its image if the camera has been calibrated (the extrinsic and intrinsic parameters are available). Once the homography is obtained, any distance on the plane perpendicular to the reference plane can be measured. Secondly, an approach to calibrate a camera with only two unknown parameters is presented¹.

¹ Of the five parameters, the principal point is known and either skew equals 0 or aspect ratio equals 1.

3.1 Homography of a world plane perpendicular to the reference plane

Now we use the orthogonal property of the two planes to get the homography between the plane perpendicular to the reference plane and its projected image. To begin with, we define two coordinate systems as shown in Fig. 1.

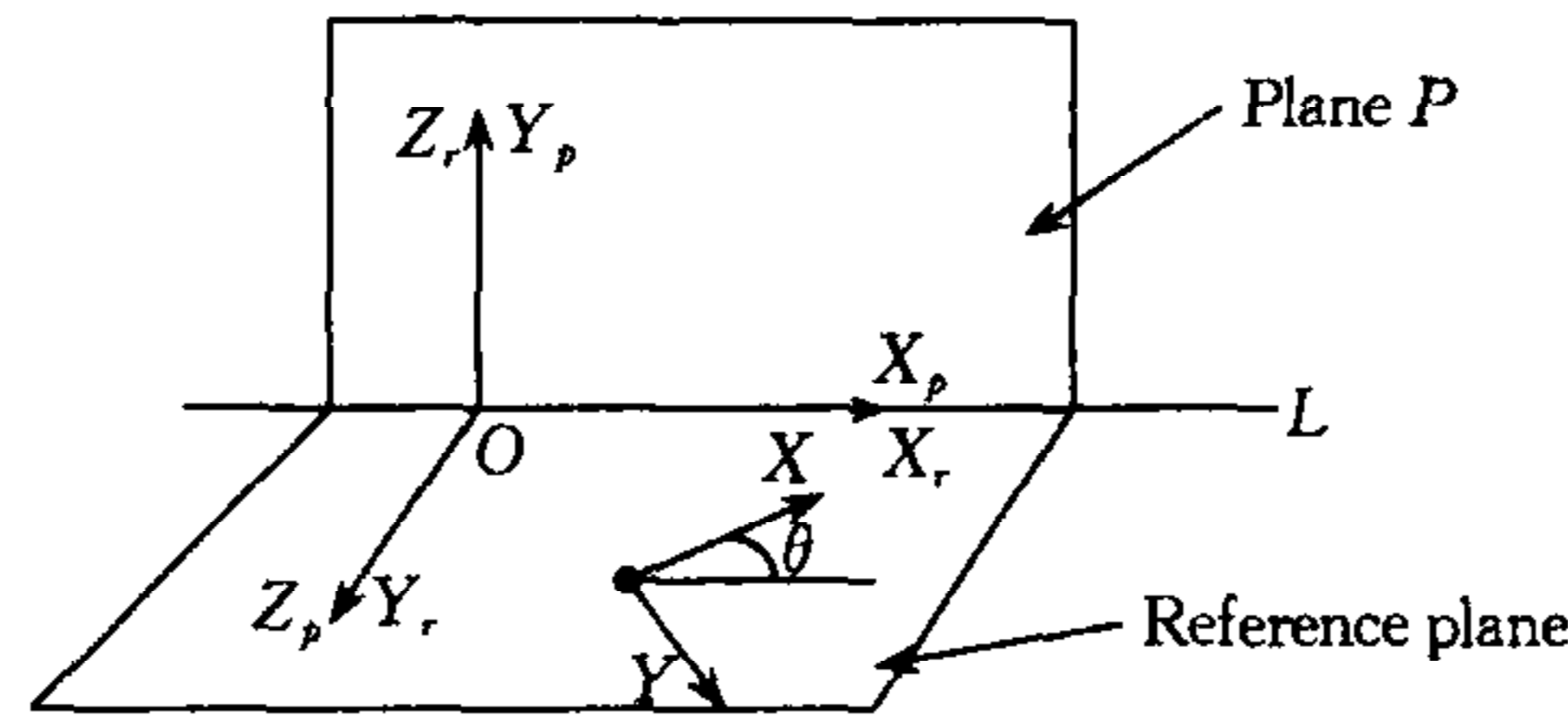


Fig. 1 Coordinate systems

In Fig. 1, X_r, Y_r, Z_r is the coordinate system of the reference plane and X_p, Y_p, Z_p is the coordinate system of the plane P , which is perpendicular to the reference plane. L is the intersection line of the two planes. Let the two coordinate frames have the same origin and let the origin lie on line L , and the X_r and X_p axes superimpose on line L . Note that the coordinate system of the reference plane in practice may not be as what we defined. In fact, its origin and X -axis could not lie on the intersection line L . In appendix, we show how to transform the system of the reference plane to X_r, Y_r, Z_r . But, at present, let us assume the definitions of the two systems in Fig. 1 are in deed realizable. Now we show how to get the homography of plane P shown in Fig. 1.

Clearly, from Fig. 1 and the perpendicularity of the two planes, we know the relationship of the two coordinate systems is:

$$\begin{bmatrix} X_r \\ Y_r \\ Z_r \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \frac{\pi}{2} & \sin \frac{\pi}{2} & 0 \\ 0 & -\sin \frac{\pi}{2} & \cos \frac{\pi}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_p \\ Y_p \\ Z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_p \\ Y_p \\ Z_p \\ 1 \end{bmatrix} \tag{3}$$

Then the relationship of the plane P and its projected image is:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \begin{bmatrix} r_1 & r_2 & r_3 & t \end{bmatrix} \begin{bmatrix} X_r \\ Y_r \\ Z_r \\ 1 \end{bmatrix} = K \begin{bmatrix} r_1 & r_2 & r_3 & t \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_p \\ Y_p \\ Z_p \\ 1 \end{bmatrix} = K \begin{bmatrix} r_1 & -r_3 & t \end{bmatrix} \begin{bmatrix} X_p \\ Y_p \\ 1 \end{bmatrix}$$

i. e., the homography between the plane P and its image (we call it H_p) is:

$$H_p = K \begin{bmatrix} r_1 & -r_3 & t \end{bmatrix}$$

Since the camera is calibrated, and the homography of the reference plane is known, *i. e.*, we know r_1, r_2, t , and K . In addition, $r_3 = r_1 \times r_2$, hence, H_p can be determined. Finally, any distance on the plane P can be measured via H_p as shown in [1,2].

3.2 Camera calibration

Now we show how to calibrate camera with only two unknown parameters from a homography. Certainly, camera with one unknown parameters can be calibrated too, as has been described in [9].

In [7], it was proved that the camera can be calibrated from one homography between

a world plane and its image if skew s equals 0 and the principal point is known. In this case, the camera intrinsic matrix K becomes:

$$K = \begin{bmatrix} f & 0 & 0 \\ 0 & \alpha f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now we have the following proposition.

Proposition. The Camera can be calibrated from a homography between a world plane and its projected image if the camera is of aspect ratio $\alpha=1$ and known principal point².

Proof. If the aspect ratio α equals 1 and the principal point is at the image center, the camera intrinsic matrix K is of the form:

$$K = \begin{bmatrix} f & s & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

and

$$K^{-T}K^{-1} = \begin{bmatrix} \frac{1}{f^2} & -\frac{s}{f^3} & 0 \\ -\frac{s}{f^3} & \frac{s^2}{f^4} + \frac{1}{f^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a & c & 0 \\ c & b & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

Let us denote the homography $H = [h_1 \ h_2 \ h_3]$. From (2) we have^[8]:

$$h_1^T K^{-T} K^{-1} h_2 = 0$$

$$h_1^T K^{-T} K^{-1} h_1 = h_2^T K^{-T} K^{-1} h_2$$

$$i. e., \quad h_{11}h_{12}a + h_{21}h_{12}c + h_{11}h_{22}c + h_{21}h_{22}b + h_{31}h_{32} = 0$$

$$h_{11}^2a + 2h_{11}h_{21}c + h_{21}^2b + h_{31}^2 = h_{12}^2a + 2h_{22}h_{12}c + h_{22}^2b + h_{32}^2$$

These are the two basic constraints on the intrinsic parameters given a homography, but the problem could not be solved only from these two constraints. However, from (5), we see that there is an additional constraint on the parameters a, b and c , that is,

$$a^2 + c^2 - ab = 0$$

Now we have three constraints on three unknown parameters, two of them are linear equation and one is a polynomial of degree 2. So at most two solutions are possible but only one is reasonable, because: I) from (5), a, b must be positive and a must be smaller than b ; II) $|c|$ must be smaller than a . After $K^{-T}K^{-1}$ is determined, K can be derived from (5):

$$K = \begin{bmatrix} \frac{1}{\sqrt{a}} & -c \cdot a^{-3/2} & 0 \\ 0 & \frac{1}{\sqrt{a}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From (2) we have:

$$\lambda K^{-1} H = [r_1 \ r_2 \ t] \quad (6)$$

Because R is a rotation matrix, the scale factor λ can be determined at first, then from (6) we have:

$$r_1 = \lambda K^{-1} h_1, \quad r_2 = \lambda K^{-1} h_2, \quad r_3 = r_1 \times r_2, \quad t = \lambda K^{-1} h_3$$

Therefore, the camera can be calibrated from one homography if the aspect ratio α equals 1 and the principal point is known.

In one word, the camera with only two unknown intrinsic parameters can be calibrated, *i. e.*, the extrinsic parameters (R, t) as well as two unknown intrinsic parameters can

² Without loss of generality, we assume the principal point is at the image center, or $(u_0, v_0) = (0, 0)$.

be derived from one homography. Once the camera is calibrated, the homography between any plane perpendicular to the reference plane and its image can be obtained as shown in section 3.1. Therefore, in addition to distances on the reference plane, distances on any plane perpendicular to the reference plane can also be measured. \square

Remark. Actually, we can also measure the distance of any pair of points with one point lying on the reference plane and the other on a plane perpendicular to the reference plane. The geometry is illustrated in Fig. 2.

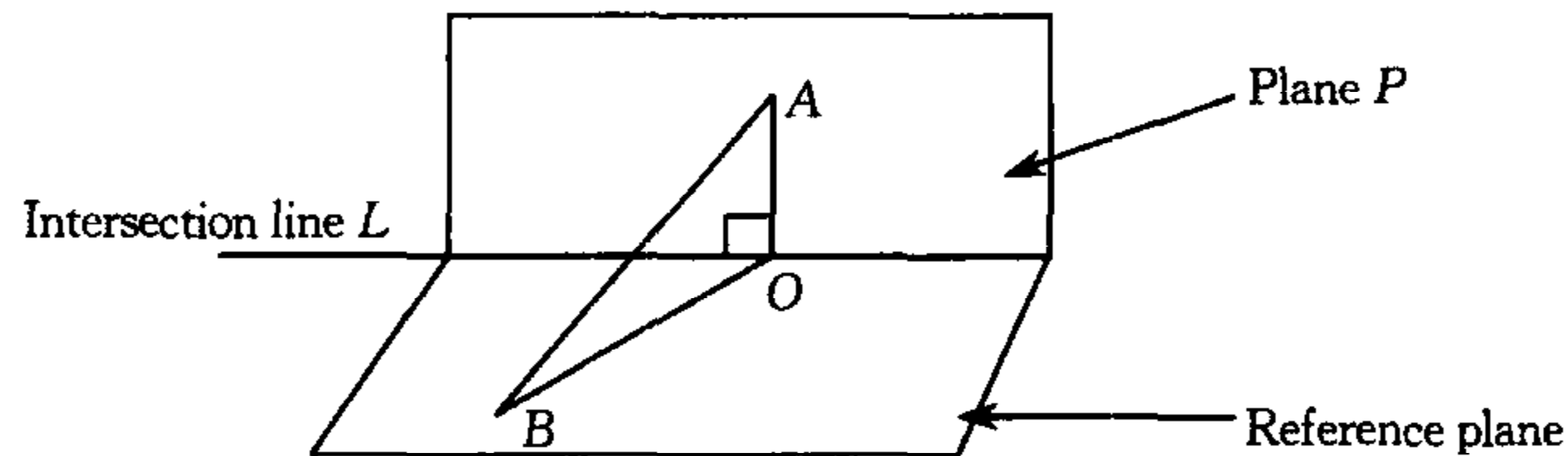


Fig. 2

As shown in Fig. 2, A is a point in the plane P perpendicular to the reference plane; B is a point in the reference plane. Because we have the homography between plane P and its image and the homography between the reference plane and its image, the distance between point A and the intersection line L , in other words, the line segment AO can be calculated, and the line segment BO can be calculated too. Then we can obtain the distance between A and B from $AB^2 = AO^2 + BO^2$ (Of course, the intersecting line L must be physically available in image).

3.3 Summary

The algorithm can be outlined as follows.

- 1) Detect the image of the intersecting line L between the reference plane and the plane P perpendicular to the reference plane.
- 2) Determine the transformation between the coordinate system X_r, Y_r and XY using the method described in Appendix and get the homography between the reference plane and its image under coordinate system X_r, Y_r .
- 3) If the used camera is a calibrated one, go to step 4 directly. Otherwise, calibrate the camera using the method showed in section 3.2.
- 4) Estimate the homography between the plane P and its image using the method presented in section 3.1.

4 Experiments with synthetic data

In this section, the method is assessed with synthetic data. The camera's setup is:

$$K = \begin{bmatrix} f & 0 & 0 \\ 0 & \alpha f & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1200 & 0 & 0 \\ 0 & 900 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

rotation axis $r = [20, 10, 40]^T$, rotation angle equals $\pi/6$, and translation $t = [-5, -10, 800]^T$. The image is 1000×1000 pixels. In all the subsequent experiments, we assume that image is taken by an uncalibrated camera with only two unknown parameters.

4.1 Noise influence

In order to get the homography between the reference plane and its image, we randomly select four points in the reference plane and their corresponding images. Gaussian noise with mean 0 and standard deviation ranging from 0 to 4 pixels is added to the coordinates of the four image points. At each noise level, we randomly select 50 pairs of space points in the world plane perpendicular to the reference plane and use their corresponding

image points to estimate space distance by the proposed method in this paper. The result is the average of 100 independent trials. Relative error and standard deviation of computed distance at different noise level are shown in Fig. 3. From this Figure, we can see that though the noise level increases to as large as 4.0 pixels, the relative error and the standard deviation of the measured distances (the real average distance is 674.2985) are still low. This indicates that the proposed technique is fairly accurate and robust even with the presence of high level noise.

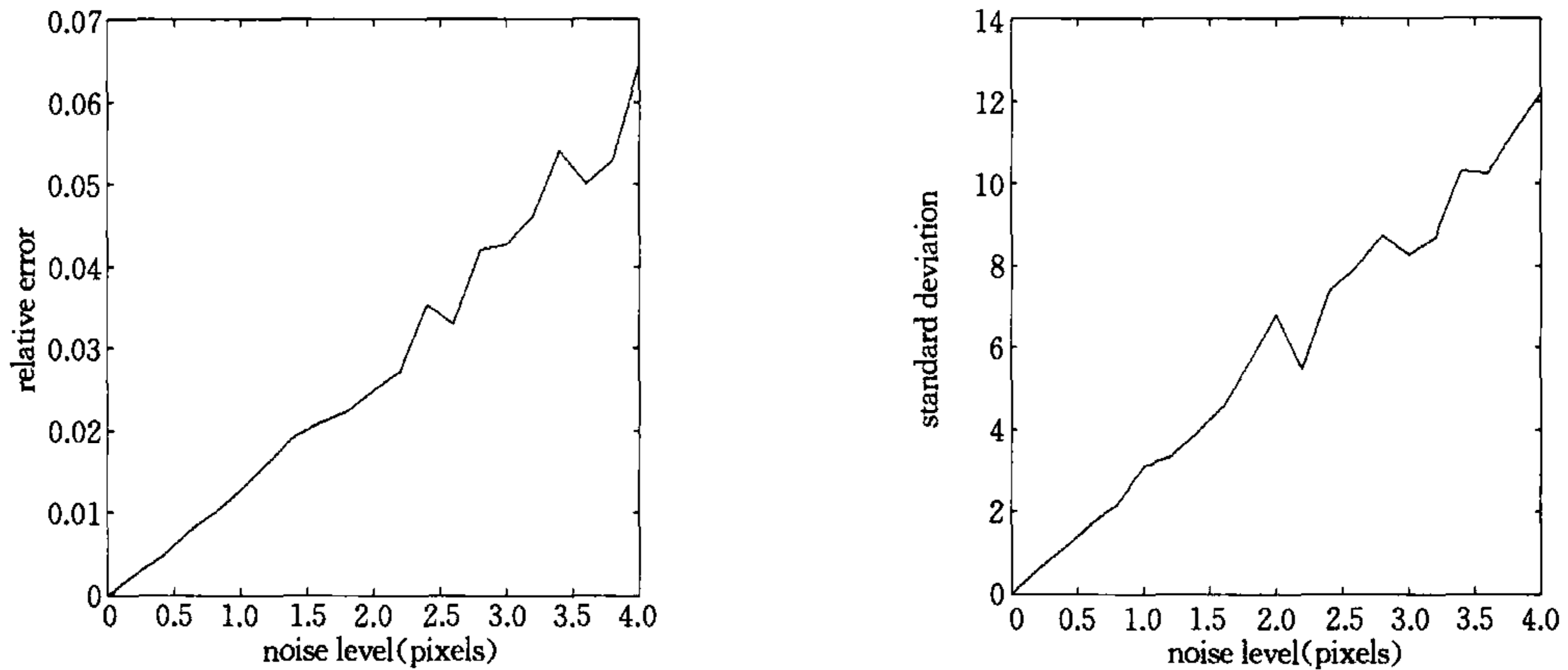


Fig. 3 Relative error and the standard dev vs. the noise level

4.2 Influence of the position error of the principal point

This experiment investigates the performance with respect to the position error of the camera's principal point. In this experiment, we still randomly select four points in the reference plane and their corresponding images to compute the homography of the reference plane, and Gaussian noise with mean 0 and standard deviation 1.0 pixel is added to the coordinates of the four image points. We randomly select 50 pairs of space points in a world plane perpendicular to the reference plane and use their corresponding image points to estimate space distance by the method in this paper. In order to examine the influence of the principal point, we vary the error level of the principal point, *i. e.*, the distance between the principal point and the image center, from 10 to 50 pixels with a step of 2 pixels. At each error level, we randomly select 50 points within the error level as positions of the principal point. For each error level, 100 independent trials are done. The average of relative error and standard deviation of computed distance at different error level are shown in Fig. 4. From this Figure, we can see that generally speaking, the influence of the posi-

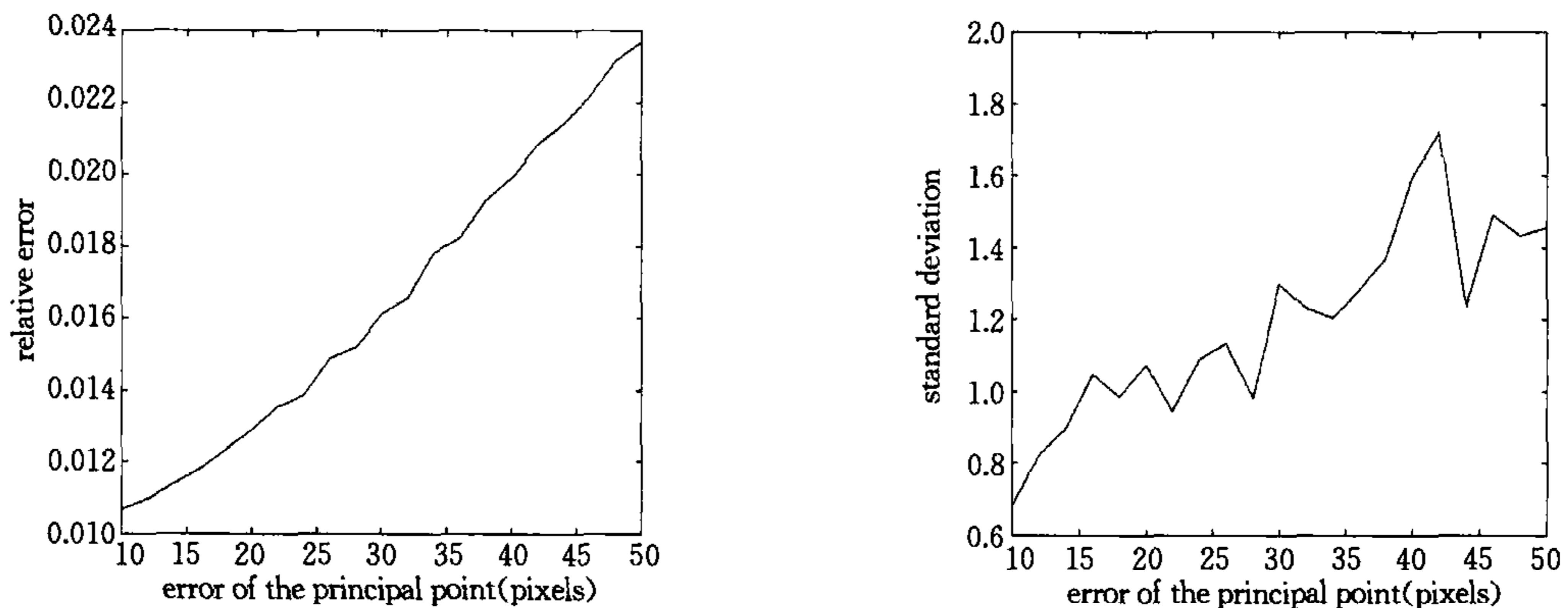


Fig. 4 Relative error and the standard dev vs. the error of the principal point

tion of the principal point is insignificant, as expectedly, the less the principal point error, the better the results.

5 Experiment with real image

In our real image test, the images were obtained by a NIKON-COOLPIX990 CCD digital camera with a resolution of 1024×768 , and we assume the intrinsic matrix of the camera is of form (4). Fig. 5 are the two test images. The ground is the reference plane. We use four points in the ground and their corresponding image points to estimate the homography H between the ground and its image. With the homography H between the ground and image, we calibrate the camera and get the homography between the plane perpendicular to the ground and its image, and then calculate the distance of any two points lying on the perpendicular plane. Table 1 gives some results of the test.

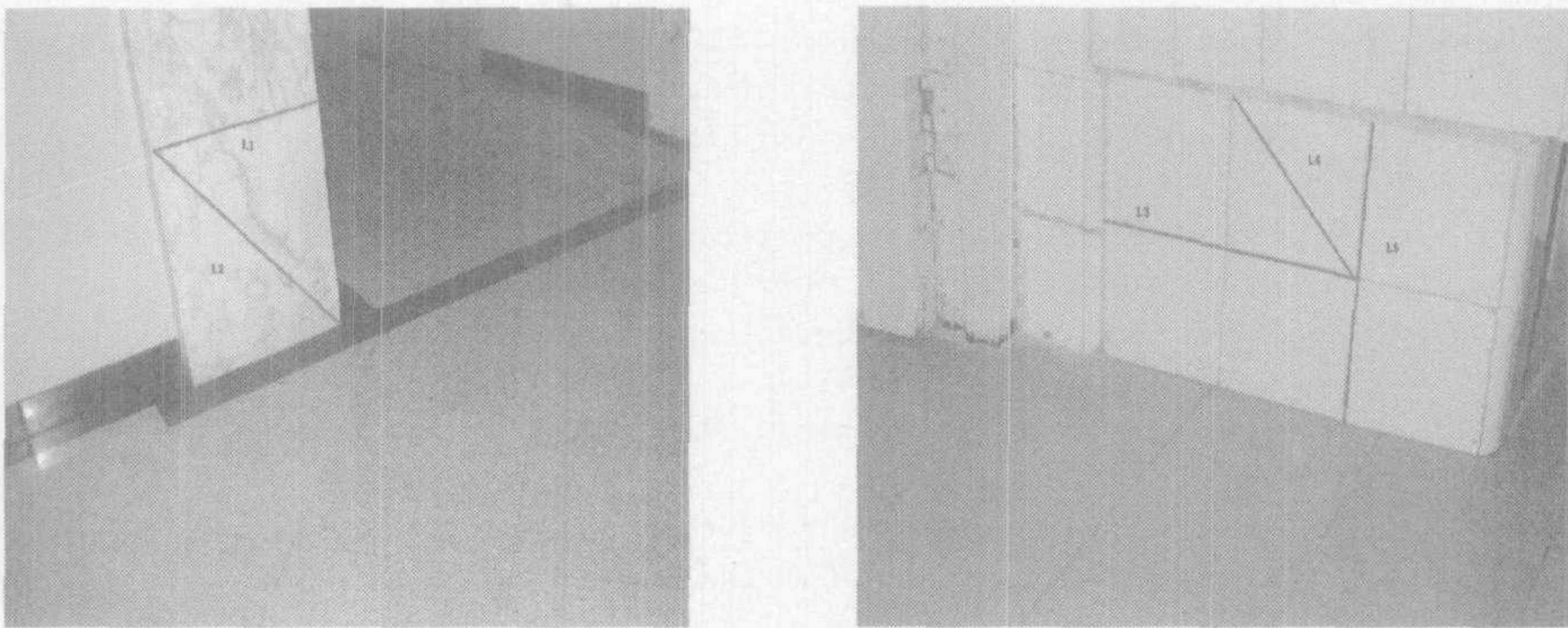


Fig. 5 Two images of test

Table 1 Real image test result (unit: mm)

Line segment	Real distance	Estimated distance	Relative error(%)
L1	600	604.6	0.77
L2	960	930.4	3.08
L3	300	305.7	1.9
L4	214	230.9	7.8
L5	300	280.6	6.5

From table 1, we see that the measuring accuracy is not high, mainly because the camera intrinsic matrix does not faithfully accord with (4). In particular, its aspect ratio is not strictly equal to 1 and the principal point is not exactly at the center of image. Nevertheless, it is not too bad or useless either. On the contrary, we think the proposed method is rather useful. This is because in many real applications, people do not need too high an accuracy but flexible methods, such as measuring huge buildings for modeling, traffic accident investigation, image based animation and so on.

6 Conclusions

In this paper we propose a new method in single view metrology. Using this method, we can retrieve some more pieces of geometrical information from the homography between the reference plane and its projected image. In particular, distances on any plane perpen-

dicular to the reference plane as well as distances on the reference plane, or distance of any pair of points with one point lying on the reference plane and the other on a perpendicular plane, can be measured provided that the used camera is either a calibrated one or an uncalibrated one with only two unknown parameters.

Of course, the measuring accuracy is still a concern with our proposed method. How to further increase measuring accuracy is our future work to undertake.

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Appendix

Coordinate system transformation

Actually, perhaps the coordinate system X_r, Y_r, Z_r is not the one as we defined in section 3.1. More specifically, the origin and the X_r axis could be not exactly on the intersection line L . Let us assume that the coordinate system for H is the coordinate system XY . Therefore, in order to get the homography under the coordinate system X_r, Y_r , we first need to find the relationship between coordinate systems XY and X_r, Y_r . The geometry is illustrated in Fig. 1.

It is evident that the two coordinate systems are related by a 2D rotation R and translation t .

$$\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_r \\ Y_r \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & t_1 \\ -\sin\theta & \cos\theta & t_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_r \\ Y_r \\ 1 \end{bmatrix}$$

From Fig. 1, we know that θ is the angle between the intersection line L and X and translation $t = [t_1, t_2]$ moves the origin of coordinate system XY to L . To get the rotation matrix R and translation t is not a difficult task. The intersection line L in the coordinate system XY can be obtained via homography H because its projection l in the image can be obtained, *i. e.*, $L = H^T l$. As long as L is known in XY coordinate system, the angle between intersection line L and X axis can be computed, so the rotation matrix R and translation t can be easily obtained.

Once the transformation from the coordinate system X_r, Y_r to XY is known, the homography between the reference plane and its image under the X_r, Y_r coordinate system (we call it H_f) is:

$$H_f = H \begin{bmatrix} \cos\theta & \sin\theta & t_1 \\ -\sin\theta & \cos\theta & t_2 \\ 0 & 0 & 1 \end{bmatrix}$$

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单幅图像测量的一种新方法

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摘 要 本文方法表明,由一空间平面(参考平面)与其图像间的单应性矩阵(Homography)不仅此参考平面上的距离可以测量,而且可以测量与此参考平面垂直的平面上的距离.同时,分别位于两平面上的点间的距离也可以测量.这样就可以得到关于场景的更多的几何信息,此结果是在前人的基础上又向前跨了一步.另外,本文提出一种新的基于平面单应性矩阵的摄像机标定方法.模拟和真实图像试验均表明本文方法是可行的,并得到了令人满意的结果.

关键词 单视测量,单应性矩阵,摄像机标定

中图分类号 TP391