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Observer-Based H_∞ Controller Designs for T-S Fuzzy Systems[1)]

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Abstract H_∞ control designs for T-S (Takagi-Sugeno) fuzzy systems have been studied, a new quadratic stability condition which is more simple and relaxed than Kim E.'s and a new observer design method for T-S fuzzy systems have been proposed. Then two new sufficient conditions in terms of linear matrix inequalities, which guarantee the existence of observer-based H_∞ controller for the T-S fuzzy systems, have been proposed. The design methods are simple and consider the interactions among the fuzzy subsystems. Finally, we show by examples that observer-based H_∞ controller designs for T-S fuzzy systems are very practical and efficient.

Key words H_∞ control, T-S fuzzy system, quadratic stability, observer, linear matrix inequalities (LMI), state feedback

1 Introduction

T-S (Takagi-Sugeno) fuzzy systems are nonlinear systems described by a set of if-then rules which give a local linear representation of an underlining system. The authors of [2~4] have proved that the T-S fuzzy system can approximate any continuous functions in a compact set of R^n at any preciseness, and that the method based on linear uncertain system theory can convert the stability analysis of a fuzzy control system to the stability analysis of linear time-varying "extreme" subsystems. This allows the designers to take advantage of conventional linear system to analyze and design the fuzzy control systems. Because of these excellent work, fuzzy logic control has been applied to many industry and technology fields, especially the model-based fuzzy logic control has been one of main methods for solving nonlinear control problems.

H_∞ control has been an attractive research topic during the last decade[5~7]. So far some papers[8~10] have discussed the H_∞ feedback control for fuzzy systems. [9]and[10] deal with a state feedback control design that requires all system states to be measured. In many cases, this requirement is too restrictive. The existence conditions of H_∞ feedback control in [9] did not include the interactions among the fuzzy subsystems, so that the conditions are conservative. In [10], the state space has to be divided to some independent subspaces according to the fuzzy rules, then one needs to set up fuzzy subsystem for each subspace and deal with the H_∞ control problem of each subsystem. As the dimension of the state space increases, the complexity of state space division increases exponentially. In [8], since the authors did not consider the interactions among the fuzzy subsystems when they studied the H_∞ control based on observers, the conditions were conservative.

In this paper, first, we propose a new quadratically stable condition which is more simple and relaxed than that in [1]. In each linear matrix inequality of the new condition composed by the coefficient matrices of the subsystems, the interactions among the fuzzy subsystems are adequately considered, hence the new condition is more relaxed. Based on the new quadratic stability condition, we propose a new observer design for the T-S fuzzy

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system. Secondly, in each negative definite matrix composed by the coefficient matrices of the subsystems, we study the interactions among the fuzzy subsystems and propose two new conditions of the existence of H_∞ control based on the observers, which are more simple and holistic and are different from [17~19], the new conditions are in the forms of linear matrix inequalities which can be directly solved by the MATLAB, so that we can design the H_∞ controller based on the observers using the LMI technique and propose a new approach for the study of T-S fuzzy systems. Finally, examples show the efficiency and practicality of the design methods.

2 The stability of T-S fuzzy systems

In this section, firstly we recall the T-S fuzzy systems [11,12] and give a new relaxed quadratic stability condition which is more simple and relaxed than that of [1]. Consider the following "IF-THEN" fuzzy rules:

IF ξ_1 is M_{1i} and…and ξ_p is M_{pi} THEN

$$\begin{cases}\dot{\boldsymbol{x}}(t) = A_i\boldsymbol{x}(t) + B_{1i}\boldsymbol{w}(t) + B_{2i}\boldsymbol{u}(t) \\ \boldsymbol{z}(t) = C_{1i}\boldsymbol{x}(t) + D_{12i}\boldsymbol{u}(t) \qquad\qquad i = 1,2,\cdots,r \\ \boldsymbol{y}(t) = C_{2i}\boldsymbol{x}(t) + D_{21i}\boldsymbol{w}(t)\end{cases}$$

where $\boldsymbol{x}(t)\in R^n$ is the state variable, $\boldsymbol{z}(t)\in R^q$ is the controlled output variable, $\boldsymbol{w}(t)\in R^l$ is the disturbance variable, $\boldsymbol{u}(t)\in R^m$ is the input variable, $\boldsymbol{y}(t)\in R^h$ is the output variable, $A_i\in R^{n\times n}$, $B_{1i}\in R^{n\times l}$, $B_{2i}\in R^{n\times m}$, $C_{1i}\in R^{q\times n}$, $D_{12i}\in R^{q\times m}$, $C_{2i}\in R^{h\times n}$, $D_{21i}\in R^{h\times l}$, $\boldsymbol{\xi}=(\xi_1,\cdots,\xi_p)^{\mathrm{T}}$ are premise variables. It is assumed that the premise variables do not depend on the control variables and disturbance. Then the fuzzy systems are defined as follows:

$$\begin{cases}\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r}\lambda_i(\boldsymbol{\xi}(t))(A_i\boldsymbol{x}(t) + B_{1i}\boldsymbol{w}(t) + B_{2i}\boldsymbol{u}(t)) \\ \boldsymbol{z}(t) = \sum_{i=1}^{r}\lambda_i(\boldsymbol{\xi}(t))(C_{1i}\boldsymbol{x}(t) + D_{12i}\boldsymbol{u}(t)) \\ \boldsymbol{y}(t) = \sum_{i=1}^{r}\lambda_i(\boldsymbol{\xi}(t))(C_{2i}\boldsymbol{x}(t) + D_{21i}\boldsymbol{w}(t))\end{cases} \tag{1}$$

where $\lambda_i(\boldsymbol{\xi}(t))=\dfrac{\beta_i(\boldsymbol{\xi}(t))}{\sum_{j=1}^{r}\beta_j(\boldsymbol{\xi}(t))}$, $\beta_i(\boldsymbol{\xi}(t))=\prod_{j=1}^{p}M_{ji}(\boldsymbol{\xi}(t))$, $M_{ij}(\cdot)$ is the membership function of the fuzzy set M_{ij}. In the following, we always assume $\sum_{i=1}^{r}\lambda_i(\boldsymbol{\xi}(t))=1$, $\lambda_k(\boldsymbol{\xi}(t))\geqslant 0$, $\forall k=1,2,\cdots,r$, $\forall t$.

Definition 1. For (1), when $\boldsymbol{w}(t)\equiv\boldsymbol{0}$, $\boldsymbol{u}(t)\equiv\boldsymbol{0}$, if there exist $\alpha>0$ and a positive definite matrix X such that $\dot{V}(\boldsymbol{x}(t))\leqslant-\alpha\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{x}(t)$, $\forall t>0$, where $V(\boldsymbol{x}(t))=\boldsymbol{x}^{\mathrm{T}}(t)X\boldsymbol{x}(t)$, then (1) is called quadratically stable.

Lemma 1. If there exist matrices F_i, $i=1,\cdots,r$ and a symmetry positive definite matrix X such that

$$(Q_{ij})_{r\times r} < -\alpha I \tag{2}$$

$$Q_{ii} = (A_i + B_{2i}F_i)^{\mathrm{T}}X + X(A_i + B_{2i}F_i) \tag{3}$$

$$Q_{ij} = X(A_i + B_{2i}F_j + A_j + B_{2j}F_i),\ Q_{ji} = Q_{ij}^{\mathrm{T}},\ i<j,\ i,\ j = 1,2,\cdots,r \tag{4}$$

where $\alpha>0$, then the state feedback $\boldsymbol{u}(t)=\sum_{j=1}^{r}\lambda_j(\xi(t))F_j\boldsymbol{x}(t)$ stabilizes the following closed-loop system

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t))((A_i + B_{2i}F_j)\boldsymbol{x}(t)) \tag{5}$$

Proof. We construct Lyapunov function $V(t)=\boldsymbol{x}^{\mathrm{T}}(t)X\boldsymbol{x}(t)$.

$$\dot{V}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i\lambda_j\boldsymbol{x}^{\mathrm{T}}(A_i^{\mathrm{T}}X+F_j^{\mathrm{T}}B_{2i}^{\mathrm{T}}X+XA_i+XB_{2i}F_j)\boldsymbol{x}=\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i\lambda_j\boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}_{ij}\boldsymbol{x}\leqslant$$
$$-\alpha\sum_{i=1}^{r}\lambda_i^2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}\leqslant-\alpha\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}$$
□

Theorem 1. If there exist matrices M_i, Z, Y_{ij}, where Z is a symmetry positive definite matrix, Y_{ii} is a symmetry matrix, $Y_{ji}=Y_{ij}^{\mathrm{T}}$, i, $j=1$, $\cdots$, r, $i\neq j$ satisfy the following linear matrix inequalities

$$ZA_i^{\mathrm{T}}+A_iZ+M_i^{\mathrm{T}}B_{2i}^{\mathrm{T}}+B_{2i}M_i<Y_{ii} \tag{6}$$
$$A_iZ+A_jZ+ZA_i^{\mathrm{T}}+ZA_j^{\mathrm{T}}+B_{2i}M_j+B_{2j}M_i+M_j^{\mathrm{T}}B_{2i}^{\mathrm{T}}+M_i^{\mathrm{T}}B_{2j}^{\mathrm{T}}\leqslant Y_{ij}+Y_{ij}^{\mathrm{T}} \tag{7}$$
$$(Y_{ij})_{r\times r}<0,\quad i<j,\ i,\ j=1,2,\cdots,r \tag{8}$$

then for (1), when $\boldsymbol{w}(t)\equiv\boldsymbol{0}$, $\boldsymbol{u}(t)=\sum_{j=1}^{r}\lambda_j(\xi(t))F_j\boldsymbol{x}(t)$ stabilizes (5), where $F_i=M_iZ^{-1}$, $i=1,2,\cdots,r$.

Proof. Let $X=Z^{-1}$, $F_i=M_iZ^{-1}$. Premultiply and postmultiply (6,7) by X, and premultiply and postmultiply (8) by diag $(X,\cdots,X)$, and we have $A_i^{\mathrm{T}}X+F_i^{\mathrm{T}}B_{2i}^{\mathrm{T}}X+XA_i+XB_{2i}F_i<XY_{ii}X$, $X(A_i+B_{2i}F_j+A_j+B_{2j}F_i)+(A_i^{\mathrm{T}}+F_j^{\mathrm{T}}B_{2i}^{\mathrm{T}}+A_j^{\mathrm{T}}+F_i^{\mathrm{T}}B_{2j}^{\mathrm{T}})X\leqslant XY_{ij}X+XY_{ij}^{\mathrm{T}}X$, $(XY_{ij}X)_{r\times r}<0$, $i<j$, i, $j=1,2,\cdots,r$. By Lemma 1, we can prove closed loop system (5) is quadratically stable. □

Remark 1. Theorem 1 relaxes the conditions of Theorems 7 and 11 in [1]. In example 2 of [1] parameters a and b for Theorems 7 and 11 to ensure existence of the controller stabilizing the system in question are $2\leqslant a\leqslant 6$, $2\leqslant b<9.75$. But Theorem 1 in this paper to ensure existence of the fuzzy controller stabilizing the same system, parameters a and b are $0.25\leqslant a\leqslant 100$, $0.25\leqslant b<100$.

3 The H_∞ Controller Based on the Observer

In the following, based on Section 2, we study the H_∞ controller based on observer for T-S fuzzy systems.

Theorem 2. If there exist matrices L_i, $i=1$, $\cdots$, r and a symmetry positive definite matrix X such that

$$(\boldsymbol{Q}_{ij})_{r\times r}<-\alpha I,\ \alpha>0 \tag{9}$$
$$\boldsymbol{Q}_{ii}=(A_i+L_iC_{2i})^{\mathrm{T}}X+X(A_i+L_iC_{2i}) \tag{10}$$
$$\boldsymbol{Q}_{ij}=X(A_i+L_jC_{2i}+A_j+L_iC_{2j}),\ \boldsymbol{Q}_{ji}=\boldsymbol{Q}_{ij}^{\mathrm{T}},\ i<j,\ i,\ j=1,2,\cdots,r \tag{11}$$

then
$$\dot{\boldsymbol{\eta}}(t)=\sum_{i=1}^{r}\lambda_i\left(A_i\boldsymbol{\eta}+B_{1i}\boldsymbol{w}+B_{2i}\boldsymbol{u}-L_i\left(\boldsymbol{y}-\sum_{j=1}^{r}\lambda_j(C_{2j}\boldsymbol{\eta}+D_{21j}\boldsymbol{w})\right)\right) \tag{12}$$
is the fuzzy observer of fuzzy system (1), i.e., $\boldsymbol{\eta}(t)\to\boldsymbol{x}(t)$ exponentially as $t\to\infty$.

Proof. By (1) and (12), we have $\dot{\boldsymbol{e}}=\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i\lambda_j(A_i+L_iC_{2j})\boldsymbol{e}$, where $\boldsymbol{e}(t)=\boldsymbol{x}(t)-\boldsymbol{\eta}(t)$. Construct Lyapunov $V(t)=\boldsymbol{e}^{\mathrm{T}}(t)\ X\boldsymbol{e}(t)$. By the similar method to Lemma 1, we can prove $\dot{V}(t)<-\alpha\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t)$. Therefore, (12) is the fuzzy observer of the fuzzy system (1), i.e., $\boldsymbol{\eta}(t)\to\boldsymbol{x}(t)$ exponentially as $t\to\infty$. □

Theorem 3. If there exist matrices N_i, X, X_{ij}, where X_{ii} is the symmetry matrix, $X_{ji}=X_{ij}^{\mathrm{T}}$, $i,j=1,\cdots,r$, $i\neq j$, such that the following matrix inequalities

$$A_i^{\mathrm{T}}X+C_{2i}^{\mathrm{T}}N_i^{\mathrm{T}}+XA_i+N_iC_{2i}<X_{ii} \tag{13}$$
$$XA_i+N_iC_{2j}+XA_j+N_jC_{2i}+A_i^{\mathrm{T}}X+C_{2j}^{\mathrm{T}}N_i^{\mathrm{T}}+A_j^{\mathrm{T}}X+C_{2i}^{\mathrm{T}}N_j^{\mathrm{T}}\leqslant X_{ij}+X_{ij}^{\mathrm{T}} \tag{14}$$
$$(X_{ij})_{r\times r}<-\alpha I,\ \alpha>0,\quad i,j=1,\cdots,r,\ i\neq j \tag{15}$$

are satisfied, then (12) is the fuzzy observer of the fuzzy system (1), i.e., $\boldsymbol{\eta}(t)\to\boldsymbol{x}(t)$ exponentially as $t\to\infty$, where $L_i=X^{-1}N_i$.

Proof. Refer to the proof of Lemma 1 and Theorem 1. □

Definition 2[13]. For (1) and given constant $\gamma>0$, a closed loop system is stable with the H_∞ performance bound γ if 1) the closed loop system is asymptotically stable, and 2) with the zero initial condition, the controlled output satisfies $\|z\|_2 \leqslant \gamma \|w\|_2$, where $\|x(t)\|_2 = (\int_0^\infty x^T(t)x(t)dt)^{\frac{1}{2}}$, i.e., the L_2 norm.

Lemma 2. For a given constant $\gamma > 0$, if there exist matrices K_i, L_i, X, Y, X_{ij}, where X, Y are the symmetry positive definite matrices, X_{ii} is symmetry matrix, $X_{ji} = X_{ij}^T$, $i, j = 1, \cdots, r$, $i \neq j$, satisfy the following matrix inequalities

$$\begin{bmatrix} \Lambda_{ii}^T X + X\Lambda_{ii} + \frac{1}{\gamma^2} XB_{1i}B_{1i}^T X & XB_{2i}K_i \\ K_i^T B_{2i}^T X & \Gamma_{ii}^T Y + Y\Gamma_{ii} \end{bmatrix} < X_{ii} \tag{16}$$

$$\begin{bmatrix} X\Lambda_{ij} + \frac{1}{\gamma^2} XB_{1i}B_{1j}^T X & X\Delta_{ij} \\ 0 & Y\Gamma_{ij} \end{bmatrix} + \begin{bmatrix} X\Lambda_{ij} + \frac{1}{\gamma^2} XB_{1i}B_{1j}^T X & X\Delta_{ij} \\ 0 & Y\Gamma_{ij} \end{bmatrix}^T \leqslant X_{ij} + X_{ij}^T, i \neq j \tag{17}$$

$$H_k = \begin{bmatrix} X_{11} & \cdots & X_{1r} & U_{1k}^T \\ \vdots & \ddots & \vdots & \vdots \\ X_{r1} & \cdots & X_{rr} & U_{rk}^T \\ U_{1k} & \cdots & U_{rk} & -I \end{bmatrix} < 0, \ k = 1, \cdots, r \tag{18}$$

where $\Lambda_{ii} = A_i + B_{2i}K_i$, $\Lambda_{ij} = A_i + B_{2i}K_j + A_j + B_{2j}K_i$, $\Gamma_{ii} = A_i + L_iC_{2i}$, $\Gamma_{ij} = A_i + L_iC_{2j} + A_j + L_jC_{2i}$, $\Delta_{ii} = B_{2i}K_i$, $\Delta_{ij} = B_{2i}K_j + B_{2j}K_i$, $U_{ik} = [C_{1i} + D_{12i}K_k, \ D_{12i}K_k]$, $i, k = 1, 2, \cdots, r$, then for (1), the following fuzzy controller based on observer (12)

$$\begin{cases} \dot{\eta} = \sum_{i=1}^r \lambda_i (A_i\eta + B_{1i}w + B_{2i}u - L_i(y - \sum_{j=1}^r \lambda_j (C_{2j}\eta - D_{21j}w))) \\ u = \sum_{i=1}^r \lambda_i K_i \eta \end{cases} \tag{19}$$

makes fuzzy system (1) asymptotically stable with H_∞ performance bound γ.

Proof. Linking the fuzzy controller (19) to fuzzy system (1), we have the following closed loop system

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{\eta} \end{bmatrix} = \sum_{i=1}^r \sum_{j=1}^r \lambda_i\lambda_j \left(\begin{bmatrix} A_i & B_{2i}K_j \\ -L_iC_{2j} & A_i + B_{2i}K_j + L_iC_{2j} \end{bmatrix} \begin{bmatrix} x \\ \eta \end{bmatrix} + \begin{bmatrix} B_{1i} \\ B_{1i} \end{bmatrix} w \right) \\ z = \sum_{i=1}^r \sum_{j=1}^r \lambda_i\lambda_j [C_{1i} \ \ D_{12i}K_j] \begin{bmatrix} x \\ \eta \end{bmatrix} \end{cases} \tag{20}$$

Let

$$\begin{bmatrix} x \\ \eta \end{bmatrix} = \begin{bmatrix} I & 0 \\ I & I \end{bmatrix} \begin{bmatrix} \omega \\ \psi \end{bmatrix} \tag{21}$$

Then substituting (21) into (20) yields

$$\begin{cases} \begin{bmatrix} \dot{\omega} \\ \dot{\psi} \end{bmatrix} = \sum_{i=1}^r \sum_{j=1}^r \lambda_i\lambda_j \left(\begin{bmatrix} A_i + B_{2i}K_j & B_{2i}K_j \\ 0 & A_i + L_iC_{2j} \end{bmatrix} \begin{bmatrix} \omega \\ \psi \end{bmatrix} + \begin{bmatrix} B_{1i} \\ 0 \end{bmatrix} w \right) \\ z = \sum_{i=1}^r \sum_{j=1}^r \lambda_i\lambda_j [C_{1i} + D_{12i}K_j \ \ D_{12i}K_j] \begin{bmatrix} \omega \\ \psi \end{bmatrix} \end{cases} \tag{22}$$

Since any main square sub-block of a negative definite matrix is also negative definite and since (16~18), we have

$$\Gamma_{ii}^T Y + Y\Gamma_{ii} < Y_{ii}, \ \Gamma_{ij}^T Y + Y\Gamma_{ij} \leqslant Y_{ij}^T + Y_{ij}, \ i, j = 1, \cdots, r, \ i < j \tag{23}$$

$$(Y_{ij})_{r \times r} < 0 \tag{24}$$

where Y_{ij}, is the $n \times n$ main sub-block of X_{ij} in the low right-hand corner, $i, j = 1, 2, \cdots, r$. By (21) and (22), we have $\omega = x$, $\psi = \eta - x$ and

$$\dot{\psi} = \dot{\eta} - \dot{x} = \sum_{i=1}^r \sum_{j=1}^r \lambda_i\lambda_j (A_i + L_iC_{2j})\psi$$

Let $N_i = YL_i$. By (23), we have

$$A_i^T Y + C_{2i}^T N_i^T + YA_i + N_iC_{2i} < Y_{ii},$$

$$YA_i + N_iC_{2j} + YA_j + N_jC_{2i} + A_i^TY + C_{2j}^TN_i^T + A_j^TY + C_{2i}^TN_j^T \leqslant Y_{ij} + Y_{ij}^T,\ i<j$$

By (24) and Theorem 3, we know in (22), $\boldsymbol{\psi}(t)=\boldsymbol{\eta}(t)-\boldsymbol{x}(t)\to\mathbf{0}$, as $t\to\infty$, under any disturbance $\boldsymbol{w}(t)$ and any feedback gain K_j. By (18), we have

$$0>\sum_{k=1}^{r}\lambda_k\begin{bmatrix}X_{11} & \cdots & X_{1r} & U_{1k}^T\\ \vdots & \ddots & \vdots & \vdots\\ X_{r1} & \cdots & X_{rr} & U_{rk}^T\\ U_{1k} & \cdots & U_{rk} & -I\end{bmatrix}=\begin{bmatrix}X_{11} & \cdots & X_{1r} & \sum_{k=1}^{r}\lambda_kU_{1k}^T\\ \vdots & \ddots & \vdots & \vdots\\ X_{r1} & \cdots & X_{rr} & \sum_{k=1}^{r}\lambda_kU_{rk}^T\\ \sum_{k=1}^{r}\lambda_kU_{1k} & \cdots & \sum_{k=1}^{r}\lambda_kU_{rk} & -I\end{bmatrix}$$

By Shur complement, we have

$$\begin{bmatrix}X_{11} & \cdots & X_{1r}\\ \vdots & \ddots & \vdots\\ X_{r1} & \cdots & X_{rr}\end{bmatrix}<-\begin{bmatrix}\sum_{k=1}^{r}\lambda_kU_{1k}^T\\ \vdots\\ \sum_{k=1}^{r}\lambda_kU_{rk}^T\end{bmatrix}\begin{bmatrix}\sum_{k=1}^{r}\lambda_kU_{1k}^T\\ \vdots\\ \sum_{k=1}^{r}\lambda_kU_{rk}^T\end{bmatrix}^T\leqslant 0 \qquad (25)$$

Therefore for any $t>0$ and state vector $[\boldsymbol{\omega}^T(t),\boldsymbol{\psi}^T(t)]\neq\mathbf{0}$, we have

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i\lambda_j[\boldsymbol{\omega}^T\quad \boldsymbol{\psi}^T]X_{ij}\begin{bmatrix}\boldsymbol{\omega}\\ \boldsymbol{\psi}\end{bmatrix}< -\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r}\lambda_i\lambda_j\lambda_u\lambda_v[\boldsymbol{\omega}^T\quad \boldsymbol{\psi}^T]U_{iu}^TU_{jv}\begin{bmatrix}\boldsymbol{\omega}\\ \boldsymbol{\psi}\end{bmatrix}=-\boldsymbol{z}^T\boldsymbol{z} \qquad (26)$$

By (25), there exists $\alpha>0$ such that

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i\lambda_j[\boldsymbol{\omega}^T\quad \boldsymbol{\psi}^T]X_{ij}\begin{bmatrix}\boldsymbol{\omega}\\ \boldsymbol{\psi}\end{bmatrix}<-\alpha[\boldsymbol{\omega}^T\quad \boldsymbol{\psi}^T]\begin{bmatrix}\boldsymbol{\omega}\\ \boldsymbol{\psi}\end{bmatrix} \qquad (27)$$

Construct Lyapunov function $V(t)=[\boldsymbol{\omega}^T\quad \boldsymbol{\psi}^T]\begin{bmatrix}X & 0\\ 0 & Y\end{bmatrix}\begin{bmatrix}\boldsymbol{\omega}\\ \boldsymbol{\psi}\end{bmatrix}$. By (22), we have

$$\begin{aligned}\dot V(t) = &\sum_{i=1}^{r}\lambda_i^2[\boldsymbol{\omega}^T\quad \boldsymbol{\psi}^T]\begin{bmatrix}\Lambda_{ii}^TX+X\Lambda_{ii}+\frac{1}{\gamma^2}XB_{1i}B_{1i}^TX & X\Delta_{ii}\\ \Delta_{ii}^TX & \Gamma_{ii}^TY+Y\Gamma_{ii}\end{bmatrix}\begin{bmatrix}\boldsymbol{\omega}\\ \boldsymbol{\psi}\end{bmatrix}+\\ &\sum_{i=1}^{r}\sum_{i<j}^{r}\lambda_i\lambda_j[\boldsymbol{\omega}^T\quad \boldsymbol{\psi}^T]\left(\begin{bmatrix}X\Lambda_{ij}+\frac{1}{\gamma^2}XB_{1i}B_{1j}^TX & X\Delta_{ij}\\ 0 & Y\Gamma_{ij}\end{bmatrix}+\begin{bmatrix}X\Lambda_{ij}+\frac{1}{\gamma^2}XB_{1i}B_{1j}^TX\\ 0\end{bmatrix}^T\right)\begin{bmatrix}\boldsymbol{\omega}\\ \boldsymbol{\psi}\end{bmatrix}-\\ &\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i\lambda_j\frac{1}{\gamma^2}\boldsymbol{\omega}^TXB_{1i}B_{1j}^TX\boldsymbol{\omega}+\sum_{i=1}^{r}\lambda_i(\boldsymbol{w}^TB_{1i}^TX\boldsymbol{\omega}+\boldsymbol{\omega}^TXB_{1i}\boldsymbol{w}).\end{aligned}$$

By (16,17), we have

$$\dot V(t)<\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i\lambda_j[\boldsymbol{\omega}^T\quad \boldsymbol{\psi}^T]X_{ij}\begin{bmatrix}\boldsymbol{\omega}\\ \boldsymbol{\psi}\end{bmatrix}+\gamma^2\boldsymbol{w}^T\boldsymbol{w}-\left(\gamma\boldsymbol{w}-\frac{1}{\gamma}\sum_{i=1}^{r}\lambda_iB_{1i}^TX\boldsymbol{\omega}\right)^T\left(\gamma\boldsymbol{w}-\frac{1}{\gamma}\sum_{i=1}^{r}\lambda_iB_{1i}^TX\boldsymbol{\omega}\right) \qquad (28)$$

When $\boldsymbol{w}(t)\equiv\mathbf{0}$, by (27,28), we have

$$\dot V(t)\leqslant\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i\lambda_j[\boldsymbol{\omega}^T\quad \boldsymbol{\psi}^T]X_{ij}\begin{bmatrix}\boldsymbol{\omega}\\ \boldsymbol{\psi}\end{bmatrix}<-\alpha[\boldsymbol{\omega}^T\quad \boldsymbol{\psi}^T]\begin{bmatrix}\boldsymbol{\omega}\\ \boldsymbol{\psi}\end{bmatrix}.$$

By Definition 1, when $\boldsymbol{w}(t)\equiv\mathbf{0}$, we know (20) and (22) are quadratically stable. By (26~28), for any $t>0$, we have

$$\dot{V}(t) < -\alpha x^{\mathrm{T}} x - \alpha \psi^{\mathrm{T}} \psi + \gamma^2 w^{\mathrm{T}} w - \left(\gamma w - \frac{1}{\gamma}\sum_{i=1}^{r}\lambda_i B_{1i}^{\mathrm{T}} X\omega\right)^{\mathrm{T}}\left(\gamma w - \frac{1}{\gamma}\sum_{i=1}^{r}\lambda_i B_{1i}^{\mathrm{T}} X\omega\right)$$

$$\dot{V}(t) < -z^{\mathrm{T}} z + \gamma^2 w^{\mathrm{T}} w - \left(\gamma w - \frac{1}{\gamma}\sum_{i=1}^{r}\lambda_i B_{1i}^{\mathrm{T}} X\omega\right)^{\mathrm{T}}\left(\gamma w - \frac{1}{\gamma}\sum_{i=1}^{r}\lambda_i B_{1i}^{\mathrm{T}} X\omega\right)$$

Since zero initial condition ($x(0)=\mathbf{0}$), $V(x(0))=0$. Integrating both sides of the above inequalities from 0 to ∞, we have

$$0 \leqslant -\alpha \| x \|_2^2 - \alpha \| \psi \|_2^2 + \gamma^2 \| w \|_2^2 - \gamma^2 \left\| w - \frac{1}{\gamma^2}\sum_{i=1}^{r}\lambda_i B_{1i}^{\mathrm{T}} X\omega \right\|_2^2 \Rightarrow$$

$$\alpha \| x \|_2^2 + \alpha \| x - \eta \|_2^2 \leqslant \gamma^2 \| w \|_2^2$$

$$0 \leqslant - \| z \|_2^2 + \gamma^2 \| w \|_2^2 - \gamma^2 \left\| w - \frac{1}{\gamma^2}\sum_{i=1}^{r}\lambda_i B_{1i}^{\mathrm{T}} X\omega \right\|_2^2 \Rightarrow \| z \|_2^2 \leqslant \gamma^2 \| w \|_2^2 \qquad \square$$

The following theorem gives the method of designing the observer and the H_∞ controller in two steps. In step 1, by solving the LMIs (29~31), we find K_i($i=1,2,\cdots,r$) for the H_∞ controller. In step 2, using Z, N_{ij} obtained in step 1 and by solving LMI (32, 33), we find L_i($i=1,2,\cdots,r$) for the observer.

Theorem 4. Let's consider fuzzy system (1). For a given $\gamma>0$, If in step 1 there exist matrices Z, M_i, N_{ij}, Z_{ij}, where Z is a symmetry positive definite matrix, N_{ii} and Z_{ii} are symmetry matrices, $N_{ji}=N_{ij}^{\mathrm{T}}$, $Z_{ji}=Z_{ij}^{\mathrm{T}}$, i, $j=1,\cdots,r$, $i\neq j$, such that the following linear matrix inequalities

$$\begin{bmatrix} ZA_i^{\mathrm{T}} + M_i^{\mathrm{T}} B_{2i}^{\mathrm{T}} + A_i Z + B_{2i} M_i + \frac{1}{\gamma^2} B_{1i} B_{2i}^{\mathrm{T}} & B_{2i} M_i \\ M_i^{\mathrm{T}} B_{2i}^{\mathrm{T}} & N_{ii} \end{bmatrix} < Z_{ii} \tag{29}$$

$$S_{ij} + S_{ij}^{\mathrm{T}} \leqslant Z_{ij} + Z_{ij}^{\mathrm{T}} \tag{30}$$

are satsified, where

$$S_{ij} = \begin{bmatrix} A_i Z + B_{2i} M_j + A_j Z + B_{2j} M_i + \frac{1}{\gamma^2} B_{1i} B_{1j}^{\mathrm{T}} & B_{2i} M_j + B_{2j} M_i \\ 0 & N_{ij} \end{bmatrix}$$

$$\begin{bmatrix} Z_{11} & \cdots & Z_{1r} & V_{1k}^{\mathrm{T}} \\ \vdots & \ddots & \vdots & \vdots \\ Z_{r1} & \cdots & Z_{rr} & V_{rk}^{\mathrm{T}} \\ V_{1k} & \cdots & V_{rk} & -I \end{bmatrix} < 0, k = 1,\cdots,r \tag{31}$$

where $V_{ik}=[C_{1i}Z+D_{12i}M_k,\ D_{12i}M_k]$, i, $k=1, 2, \cdots, r$, and if (after solving the LMIs in step 1) in step 2 there exist matrices J_i, $i=1, 2,\cdots, r$ and a symmetry positive definite matrix Y such that the following linear matrix inequalities

$$A_i^{\mathrm{T}} Y + C_{2i}^{\mathrm{T}} J_i^{\mathrm{T}} + YA_i + J_i C_{2i} < Z^{-1} N_{ii} Z^{-1} \tag{32}$$

$$(A_i^{\mathrm{T}}+A_j^{\mathrm{T}})Y+Y(A_i+A_j)+C_{2j}^{\mathrm{T}}J_i^{\mathrm{T}}+C_{2i}^{\mathrm{T}}J_j^{\mathrm{T}}+J_iC_{2j}+J_jC_{2i}\leqslant Z^{-1}N_{ij}Z^{-1}+Z^{-1}N_{ij}^{\mathrm{T}}Z^{-1},\ i<j \tag{33}$$

are satisfied, then for (1), the controller (19) based on the observer (12) makes fuzzy system (1) asymptotically stable with H_∞ performance bound γ, where $K_i=M_iZ^{-1}$, $L_i=Y^{-1}J_i$, $i=1,2,\cdots,r$.

Proof. Let $X^{-1}=Z$. Premultiply and postmultiply (32,33) by X^{-1}, and we have

$$ZA_i^{\mathrm{T}}YZ + ZC_{2i}^{\mathrm{T}}J_i^{\mathrm{T}}Z + ZYA_iZ + ZJ_iC_{2i}Z < N_{ii}$$

$$Z(A_i^{\mathrm{T}} + A_j^{\mathrm{T}})YZ + ZY(A_i + A_j)Z + ZC_{2j}^{\mathrm{T}}J_i^{\mathrm{T}}Z + ZC_{2i}^{\mathrm{T}}J_j^{\mathrm{T}}Z + ZJ_iC_{2j}Z + ZJ_jC_{2i}Z \leqslant N_{ij} + N_{ij}^{\mathrm{T}}, i<j$$

Let $E_{ii}=ZA_i^{\mathrm{T}}YZ+ZC_{2i}^{\mathrm{T}}J_i^{\mathrm{T}}Z+ZYA_iZ+ZJ_iC_{2i}Z$, $E_{ij}=ZY(A_i+A_j)Z+ZJ_iC_{2j}Z+ZJ_jC_{2i}Z$. Then we have $E_{ii}<N_{ii}$, $E_{ij}+E_{ij}^{\mathrm{T}}\leqslant N_{ij}+N_{ij}^{\mathrm{T}}$, i, $j=1,\cdots,r$ and

$$\begin{bmatrix} ZA_i^{\mathrm{T}} + M_i^{\mathrm{T}} B_{2i}^{\mathrm{T}} + A_i Z + B_{2i} M_i + \frac{1}{\gamma^2} B_{1i} B_{1i}^{\mathrm{T}} & B_{2i} M_i \\ M_i^{\mathrm{T}} B_{2i}^{\mathrm{T}} & E_{ii} \end{bmatrix} < Z_{ii} \tag{34}$$

Similarly, we have
$$P_{ij}+P_{ij}^{\mathrm{T}}\leqslant Z_{ij}+Z_{ij}^{\mathrm{T}} \tag{35}$$
where $P_{ij}=\begin{bmatrix} A_iZ+B_{2i}M_j+A_jZ+B_{2j}M_i+\frac{1}{\gamma^2}B_{1i}B_{1j}^{\mathrm{T}} & B_{2i}M_j+B_{2j}M_i \\ 0 & E_{ij}\end{bmatrix}$. Premultiply and postmultiply (34, 35) by diag(Z^{-1}, Z^{-1}), premultiply and postmultiply (31) by diag $(Z^{-1},\cdots,Z^{-1},I)$ and let $X=Z^{-1}$, $X_{ij}=Z^{-1}Z_{ij}Z^{-1}$, $K_i=M_iZ^{-1}$, $L_i=Y^{-1}J_i$, $i, j=1,2,\cdots,r$. By (31,34,35), we can verify X, X_{ij}, K_i, L_i, $i, j=1,2,\cdots, r$ satisfy Lemma 2. By Lemma 2, we know that for fuzzy system (1), the fuzzy controller (19) based on observer (12) is asymptotically stable with H_∞ performance bound γ. □

By the similar methods, we also can prove the following lemma and theorem

Lemma 3. Substituting $U_{ik}=[C_{1k}+D_{12k}K_i,\ D_{12k}K_i]$, $i, k=1,2,\cdots, r$, for $U_{ik}=[C_{1i}+D_{12i}K_k,\ D_{12i}K_k]$ in Lemma 2 the lemma also holds.

Theorem 5. Substituting $V_{ik}=[C_{1k}Z+D_{12k}M_i,\ D_{12k}M_i]$, $i, k=1,2,\cdots,r$, for $V_{ik}=[C_{1i}Z+D_{12i}M_k,\ D_{12i}M_k]$ in Theorem 4 the theorem also holds.

4 Numerical Example

We consider the following problem of balancing an inverted pendulum on a cart. The motion equations[13] for the pendulum are
$$\dot{x}_1=x_2,\ \dot{x}_2=\frac{g\sin(x_1)-amlx_2^2\sin(2x_1)/2-a\cos(x_1)u}{4l/3-aml\cos^2(x_1)}+w \tag{36}$$
where x_1 denotes the angle of the pendulum from the vertical, x_2 is the angular velocity, $g=9.8\mathrm{m/s^2}$ is the gravity constant, w is the external disturbance variable. m is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pendulum, and u is the force applied to the cart, $a=1/(m+M)$. We choose $m=2.0\mathrm{kg}$, $M=8.0\mathrm{kg}$, $2l=1.0m$ in the simulation. We apply T-S fuzzy model (1) to the design of H_∞ fuzzy controller (19) based on the observer (12), where $r=2$,
$$A_1=\begin{bmatrix}0 & 1\\ 17.2941 & 0\end{bmatrix},\quad B_{21}=\begin{bmatrix}0\\ -0.1765\end{bmatrix},\quad B_{11}=\begin{bmatrix}0\\1\end{bmatrix}$$
$$C_{21}=C_{11}=\begin{bmatrix}1\\0\end{bmatrix}^{\mathrm{T}},\quad D_{121}=0.008,\quad D_{211}=0.07$$
$$A_2=\begin{bmatrix}0 & 1\\ 12.6305 & 0\end{bmatrix},\quad B_{22}=\begin{bmatrix}0\\ -0.0779\end{bmatrix},\quad B_{12}=\begin{bmatrix}0\\1\end{bmatrix}$$
$$C_{21}=C_{11}=\begin{bmatrix}1\\0\end{bmatrix}^{\mathrm{T}},\quad D_{122}=0.006,\quad D_{212}=0.08$$
$$\lambda_1(x_1(t))=(1-1/(1+\exp(-7(x_1(t)-\pi/4))))\cdot(1/(1+\exp(-7(x_1(t)+\pi/4))))$$
$$\lambda_2(x_1(t))=1-\lambda_1(x_1(t))$$
Apply Theorem 4 ($\gamma=1$). In the first step, we get
$$Z=\begin{bmatrix}0.3061 & -0.8721\\ -0.8721 & 3.5927\end{bmatrix},\quad N_{11}=\begin{bmatrix}-25.3357 & 0.3497\\ 0.3497 & -21.466\end{bmatrix}$$
$$N_{22}=\begin{bmatrix}-26.9294 & 0.7850\\ 0.7850 & -21.6333\end{bmatrix},\quad M_1=[38.8696\ \ -0.8382]$$
$$M_2=[50.7692\ \ -9.7561],\quad N_{12}=0$$
$$Z_{11}=\begin{bmatrix}-1.1803 & 1.1964 & -0.2020 & 0.0842\\ 1.1964 & -16.2144 & -0.3447 & -0.2705\\ -0.2020 & -0.3447 & -12.0890 & 0.1061\\ 0.0842 & -0.2705 & 0.1061 & -11.3996\end{bmatrix}$$

$$Z_{22} = \begin{bmatrix} -1.1030 & 2.0057 & 0.1192 & 0.0132 \\ 2.0057 & -10.9215 & -3.0831 & 0.4477 \\ 0.1192 & -3.0831 & -16.6905 & 0.7319 \\ 0.0132 & 0.4477 & 0.7319 & -11.5527 \end{bmatrix}$$

$$Z_{12} = \begin{bmatrix} -0.0642 & 0.4754 & 0.2313 & -0.3883 \\ 0.9150 & -4.9682 & -9.1021 & 0.9487 \\ 0.0454 & 0.2177 & 4.4617 & -0.1195 \\ 0.0048 & -0.1696 & -0.2369 & 3.9361 \end{bmatrix}$$

Apply Z and N_{ij} obtained in the first step, and we get

$$J_1 = \begin{bmatrix} -0.1329 \\ -7.7936 \end{bmatrix} \times 10^5, \quad J_2 = \begin{bmatrix} -0.1336 \\ -5.8092 \end{bmatrix} \times 10^5, \quad L_1 = \begin{bmatrix} -0.7283 \\ -18.3060 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -0.6242 \\ -13.6468 \end{bmatrix}, \quad K_1 = [409.7513 \quad 99.2353], \quad K_2 = [512.9116 \quad 121.7956]$$

By Theorem 4, we know fuzzy controller (19) based on observer (12) makes the fuzzy system (1) asymptotically stable with H_∞ performance bound γ. In Figure 1, $0 \leqslant t \leqslant 15$, $\boldsymbol{x}(0)=[1.2,2]$, $\boldsymbol{\eta}(0)=[1.5,2.2]$, $w(t)=3\sin(2\pi t)$. In Figure 1(a) the solid line is the graph of $e_1(t)=\eta_1(t)-x_1(t)$, the broken line is the graph of $e_2(t)=\eta_2(t)-x_2(t)$; in figure 1(b) the solid line is the graph of $z(t)$, the broken line is the graph of $w(t)$.

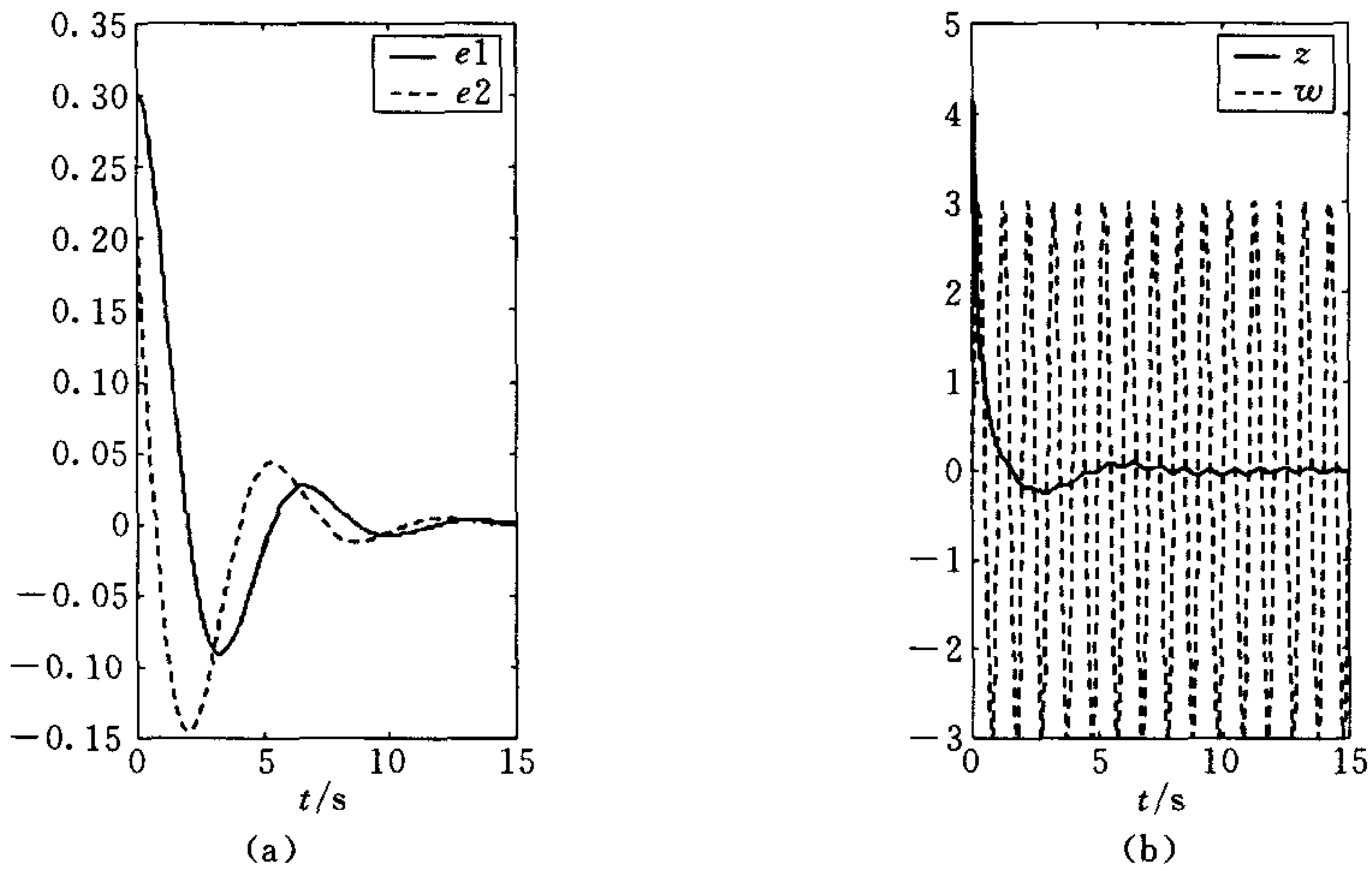

Fig. 1 Response of system (36) for an initial condition $\boldsymbol{x}(0)=[1.2,2]$, $\eta(0)=[1.5,2.2]$

5 Conclusion

Two new sufficient conditions , which guarantee the existence of H_∞ controller based on observer for the T-S fuzzy systems, fully of consider the interactions among the subsystems in terms of linear matrix inequalities and we can apply them to the study of similar problems. By numerical example, we show that the designing methods are very practical and efficient.

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T-S 模糊系统的基于观测器的 H_∞ 控制设计

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摘　要　研究了 T-S (Takagi-Sugeno)模糊系统 H_∞ 控制设计问题. 放宽了 Kim E. 等的 T-S 模糊系统可二次稳定的条件. 给出了 T-S 模糊系统新的观测器设计方法. 然后给出了 T-S 模糊系统基于观测器的 H_∞ 控制存在的两个新的充分条件. 新方法不但简单,而且充分考虑了模糊子系统间的相互作用. 最后通过例子, 应用 LMI 技术, 说明了本文给出的 T-S 模糊系统的基于观测器的 H_∞ 控制器的设计方法简便易行.

关键词　H_∞ 控制, T-S 模糊系统, 二次稳定, 观测器, 线性矩阵不等式(LMI), 状态反馈

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