Adaptive and Practical Output Tracking Control of Nonlinear Systems[1)]

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Abstract This paper studies a globally adaptive and practical output-tracking control of linearly parameterized nonlinear systems with uncontrollable unstable linearization. These class systems are neither linearizable in the feedback nor affine in the control input. The asymptotic output tracking (even local) is usually not possible because the linearized system has uncontrollable modes whose eigenvalues are on the right half plane. Using the modified adaptively adding a power integrator technique as a basic tool, we construct a smooth adaptive state feedback controller that can ensure all signals of closed-loop systems are globally, uniformly and ultimately bounded and steer the output tracking error to a small neighborhood of the origin. Simulation results show that the controller is feasible and effective.

Key words Adaptive and practical output tracking, adaptive smooth state feedback, uncontrollable unstable linearization

1 Introduction

One of the long-standing problems in adaptive design is the so-call matching condition. This condition implies that uncertainties of the plant model are in the span of control, *i.e.*, they can be directly cancelled by control when they are known. During several decades this condition presented a firm limitation on applicability of the adaptive controllers until it was relaxed to the extended matching condition[1,2]. In turn, the barrier of the extended matching condition was completely overcome with the use of a new recursive design called adaptive backstepping[3]. At the same time, output tracking control based on backstepping has been researched by many experts. The recent survey[4] and two monographs[5,6] provide a fairly complete review and detailed report on the major developments for linear and nonlinear systems.

At present, most of the existing solutions to the problem of output regulation, particularly, the nonlinear regulator theory[6], are derived based on the assumption that the Jacobian linearization of nonlinear systems is stabilizable and detectable. In fact, stabilizability and detectability, as illustrated in [6～9], are two crucial conditions for the nonlinear regulator problem to be solvable by either state or error feedback. To the contrary, when the systems under consideration are inherently nonlinear, little attention has been paid to the adaptive output regulation problem in the literatures, except for the recent papers[10,11].

In this paper, we concentrate on the adaptive output tracking of a class of systems which are inherently nonlinear with uncontrollable unstable linearization. We suppose that the reference signal is a known, bounded and time-varying signal whose first derivative is also bounded. The control objective is to seek a smooth adaptive state feedback control law such that the output of the system globally follows the reference signal. In Section 2, we will first formulate the problem we want to solve. Then two lemmas which will be used constantly are given. Section 3 contains our main results and Section 4 contains a simula-

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tion. Conclusion is drawn in Section 5.

2 Problem formulation and preliminaries

Consider the nonlinear system as follows.

$$\begin{aligned} \dot{x}_1 &= d_1(t,\boldsymbol{x},u)x_2^{p_1} + \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\phi}_1(x_1) \\ &\vdots \\ \dot{x}_n &= d_n(t,\boldsymbol{x},u)u^{p_n} + \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\phi}_n(x_1,\cdots,x_n) \\ y &= x_1 \end{aligned} \tag{1}$$

where $\boldsymbol{x}=(x_1,\cdots,x_n)^{\mathrm{T}}\in R^n$, $u\in R$ and $y\in R$ are the system state, input and output, respectively. For $i=1,\cdots,n$, the function $\boldsymbol{\phi}_i:R^i\to R^q$, $\boldsymbol{\phi}_i(0)=0$ is C^1 and $d_i(t,\boldsymbol{x},u)\neq 0$ is a C^1 real-valued function of its variables (t,x,u). $\boldsymbol{\theta}\in R^q$ is an unknown constant parameter vector and p_i is an odd positive integer. In the case of feedback linearizable systems, *i. e.*, $p_i=1$, and $d_i(t,x,u)=1$ for all $i=1,\cdots,n$, system (1) is feedback equivalent to a chain of linear integrators. Its global stabilization and global asymptotic output tracking have been obtained[5]. In the case $p_i>1$, system (1) becomes a highly nonlinear system whose Jacobian linearization may have uncontrollable modes associated with eigenvalues on the right-half plane. Moreover, the system is not affine in the control input. All of this makes the problem of asymptotic output tracking for nonlinear system (1) far more difficult than for feedback linearizable systems. In particular, unlike in the feedback linearizable case, stabilizability of system (1) does not necessarily imply the existence of a solution to the tracking problem. When $p_i>1$ and $n>1$, Qian C J and Lin W pointed out that it is not possible to solve the global asymptotic output tracking of a time-varying reference signal by any smooth even continuous feedback[12].

Being aware of the above statement, we concentrate on the problem of global adaptive and practical output tracking of system (1). Our objective is to design a smooth adaptive feedback control law that makes the output of system (1) follow a prescribed reference signal, with an arbitrarily small steady output tracking error. More precisely, we consider the following control problem.

Global adaptive practical output tracking: Let $y_r(t)\in C^1$ be a bounded reference signal whose derivative $\dot{y}_r(t)$ is also bounded. For any $\varepsilon>0$, we will find, if possible, an adaptive smooth controller of the form

$$u = \alpha(\boldsymbol{x},\hat{\theta},y_r) \tag{2}$$

$$\dot{\hat{\boldsymbol{\theta}}} = \tau(\boldsymbol{x},\hat{\theta},y_r) \tag{3}$$

where $\hat{\boldsymbol{\theta}}$ is the estimator of $\boldsymbol{\theta}$ such that

a) all signals of the closed-loop system (1), (2) and (3) are well-defined on $[0,+\infty)$ and globally bounded;

b) for every $\boldsymbol{x}(\mathbf{0})\in R^n$, there is a finite-time $T(\varepsilon,\boldsymbol{x}(\mathbf{0}))>0$ such that the output of the closed-loop system (1), (2) and (3) satisfies

$$|y(t)-y_r(t)|<\varepsilon, \quad \forall t\geqslant T>0 \tag{4}$$

In Section 2, we will present a constructive solution to the adaptive practical output-tracking problem of system (1).

We conclude this section with two important lemmas that will be used constantly throughout the rest of the paper.

Lemma 1[12]. For real numbers $a\geqslant 0$, $b\geqslant 0$ and $m\geqslant 1$ the following inequality holds

$$a\leqslant b+\left[\frac{a}{m}\right]^m\left[\frac{m-1}{b}\right]^{m-1} \tag{5}$$

Lemma 2. Let $p_i, i=1,\cdots,n$ be odd positive integers and $p=\max\{p_i, i=1,\cdots,n\}$. Suppose the Lyapunov function is

$$V(z_1,\cdots,z_n\hat{\theta}) = \sum_{i=1}^{n} \frac{z_i^{p-p_i+2}}{p-p_i+2} + \frac{1}{2r}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\tilde{\boldsymbol{\theta}} \tag{6}$$

which is positive-definite and proper and satisfies

$$\frac{\mathrm{d}V}{\mathrm{d}t} \leqslant -\sum_{i=1}^{n} z_i^{p+1} - \frac{\sigma}{2}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\tilde{\boldsymbol{\theta}} + n\delta + \frac{\sigma}{2}\|\boldsymbol{\theta}-\boldsymbol{\theta}^0\|^2 \tag{7}$$

where $0<\delta<1,\sigma>0$ and $r>0$ are real constants, z_i is a C^1 function of t , $\tilde{\boldsymbol{\theta}}=\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}$, $\hat{\boldsymbol{\theta}}$ is the estimate of $\boldsymbol{\theta}\in R^q$, and $\boldsymbol{\theta}^0\in R^q$ is a design constant vector. Then, there exist two constants $c_1>0,c_2>0$ and a finite $T>0$ such that

$$\frac{z_i^{p-p_i+2}}{p-p_i+2} \leqslant V \leqslant \frac{2c_2}{c_1}, \quad \forall t\geqslant T, i=1,\cdots,n \tag{8}$$

Proof. Illuminate by [12], we can easily get the proof of Lemma 2 and thus omit it here. □

3 Globally adaptive and practical output tracking

In this section, we present the main result of this paper, which provides a solution to the globally adaptive output tracking for the nonlinear system (1) which is neither linearizable in feedback nor affine in the control input.

Assumption A. For $i=1,\cdots,n$ there are constants μ_i and $\bar{\mu}_i$, such that

$$0<\mu_i \leqslant d_i(t,xl,u) \leqslant \bar{\mu}_i \tag{9}$$

Theorem 1. Under Assumption A, the problem of globally adaptive and practical output tracking of system (1) is solvable by smooth state feedback adaptive controller of the form (2) and (3).

Proof. We begin the algorithm by introducing the odd positive integer

$$p = \max_{i=1,\cdots,n}\{p_i\} \tag{10}$$

Initial step. Let $y_r(t)\in C^1$ be a bounded reference signal, whose derivative $\dot{y}_r(t)$ is bounded also. Let $z_1=x_1-y_r$ be the output error signal. Construct the Lyapunov function as

$$V_1(z_1,\tilde{\theta}) = \frac{z_1^{p-p_1+2}}{p-p_1+2} + \frac{1}{2r}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\tilde{\boldsymbol{\theta}} \tag{11}$$

which is positive-definite and proper and where $r>0$ is a constant to be selected, $\tilde{\theta}=\theta-\hat{\theta}$, $\hat{\theta}$ is the estimate of θ. A direct calculation gives

$$\begin{aligned}\dot{V}_1 &= z_1^{p-p_1+1}[d_1(t,\boldsymbol{x},u)x_2^{p_1} + \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\phi}_1(x_1) - \dot{y}_r] - \frac{1}{r}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}} \\ &= z_1^{p-p_1+1}[d_1(t,\boldsymbol{x},u)x_2^{p_1} + \hat{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\phi}_1(x_1) - \dot{y}_r] - \frac{1}{r}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(\dot{\hat{\boldsymbol{\theta}}} - rz_1^{p-p_1+1}\boldsymbol{\phi}_1(x_1))\end{aligned} \tag{12}$$

Since $\dot{y}_r(t)$ is bounded, it can be shown that for any real number $\delta>0$, by Lemma 1 [$b=\delta$, $a=|z_1^{p-p_1+1}[\hat{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\phi}_1(x_1)-\dot{y}_r(t)]|$ and $m=(p+1)/(p-p_1+1)$], there always exists a smooth function $\rho_{11}(z_1,\hat{\theta})\geqslant 0$ such that

$$|z_1^{p-p_1+1}[\hat{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\phi}_1(x_1) - \dot{y}_r(t)]| \leqslant \delta + z_1^{p+1}\rho_{11}(z_1,\hat{\theta}) \tag{13}$$

Define $\boldsymbol{\tau}_1=r[z_1^{p-p_1+1}\boldsymbol{\phi}_1(x_1)-\sigma(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}^0)]$. Then

$$\dot{V}_1 \leqslant d_1 z_1^{p-p_1+1}x_2^{p_1} + z_1^{p+1}\rho_{11}(z_1,\hat{\theta}) - \frac{1}{r}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(\dot{\hat{\boldsymbol{\theta}}} - \boldsymbol{\tau}_1) + \sigma\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}^0) + \delta \tag{14}$$

Then the virtual smooth controller

$$\alpha_1 = -z_1\left[\frac{2+\rho_{11}(z_1,\hat{\theta})}{\mu_1}\right]^{1/p_1} = -z_1\beta_1(z_1,\hat{\theta}), \quad \beta_1(z_1,\hat{\theta})>0 \tag{15}$$

renders

$$\dot{V}_1 \leqslant -2z_1^{p+1} + d_1(t,x,u)z_1^{p-p_1+1}x_2^{p_1} - \mu_1 z_1^{p-p_1+1}\alpha_1^{p_1} - \frac{1}{r}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(\dot{\hat{\boldsymbol{\theta}}} - \boldsymbol{\tau}_1) + \sigma\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}^0) + \delta \tag{16}$$

Since $-z_1^{p-p_1+1}\alpha_1^{p_1} \geqslant 0$, we have

$$\dot{V}_1 \leqslant -2z_1^{p+1} + d_1(t,x,u)z_1^{p-p_1+1}[x_2^{p_1} - \alpha_1^{p_1}] - \frac{1}{r}\tilde{\theta}^{\mathrm{T}}(\dot{\hat{\theta}} - \tau_1) + \sigma\tilde{\theta}^{\mathrm{T}}(\hat{\theta} - \theta^0) + \delta \leqslant$$
$$-2z_1^{p+1} + \bar{\mu}_1 |z_1^{p-p_1+1}| \, |x_2^{p_1} - \alpha_1^{p_1}| - \left(\frac{1}{r}\tilde{\theta} + \eta_1\right)^{\mathrm{T}}(\dot{\hat{\theta}} - \tau_1) + \sigma\tilde{\theta}^{\mathrm{T}}(\hat{\theta} - \theta^0) + \delta \tag{17}$$

where $\eta_1 = 0$.

Inductive step: Suppose at step $k-1$ there are a set of smooth virtual controllers α_0, $\cdots,\alpha_k$, defined by

$$\begin{aligned} \alpha_0 &= y_r, & z_1 &= x_1 - \alpha_0 \\ \alpha_1 &= -z_1\beta_1(z_1,\hat{\boldsymbol{\theta}}), & z_2 &= x_2 - \alpha_1 \\ &\vdots & &\vdots \\ \alpha_k &= -z_k\beta_{k-1}(z_1,\cdots,z_k,\hat{\boldsymbol{\theta}}), & z_{k+1} &= x_{k+1} - \alpha_k \end{aligned} \tag{18}$$

with $\beta_1(z_1,\hat{\boldsymbol{\theta}})>0,\cdots,\beta_k(z_1,\cdots z_k,\hat{\boldsymbol{\theta}})>0$ being smooth, such that

$$\dot{V}_{k-1} \leqslant -(z_1^{p+1} + \cdots + z_{k-2}^{p+1}) - 2z_{k-1}^{p+1} + \bar{\mu}_{k-1} |z_{k-1}^{p-p_{k-1}+1}[x_k^{p_{k-1}} - \alpha_{k-1}^{p_{k-1}}]| -$$
$$\left(\frac{1}{r}\tilde{\boldsymbol{\theta}} + \boldsymbol{\eta}_{k-1}\right)^{\mathrm{T}}(\dot{\hat{\boldsymbol{\theta}}} - \boldsymbol{\tau}_{k-1}) + (k-1)\delta + \boldsymbol{\sigma}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}^0) \tag{19}$$

where

$$V_{k-1} = \sum_{i=1}^{k-1} \frac{z_i^{p-p_i+2}}{p - p_i + 2} + \frac{1}{2r}\tilde{\theta}^{\mathrm{T}}\tilde{\theta} \tag{20}$$

is a positive definite and proper Lyapunov function.

Then, in step k, we claim that (19) holds as well. Consider the Lyapunov function

$$V_k = V_{k-1} + \frac{z_k^{p-p_k+2}}{p - p_k + 2} \tag{21}$$

Using (18), we can see

$$\dot{V}_k = \dot{V}_{k-1} + z_k^{p-p_k+1}(\dot{z}_k - \dot{\alpha}_{k-1}) = \dot{V}_{k-1} + z_k^{p-p_k+1}\Big[d_k x_{k+1}^{p_k} + \theta^{\mathrm{T}}\phi_k - \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_i}(d_i x_{i+1}^{p_i} + \theta^{\mathrm{T}}\phi_i) -$$
$$\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} - \frac{\partial\alpha_{k-1}}{\partial y_r}\dot{y}_r\Big] = \dot{V}_{k-1} + z_k^{p-p_k+1}\Big[d_k x_{k+1}^{p_k} + \theta^{\mathrm{T}}w_k - \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_i}d_i x_{i+1}^{p_i} -$$
$$\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} - \frac{\partial\alpha_{k-1}}{\partial y_r}\dot{y}_r\Big] \leqslant -(z_1^{p+1} + \cdots + z_{k-2}^{p+1}) - 2z_{k-1}^{p+1} + \bar{\mu}_{k-1}|z_{k-1}^{p-p_{k-1}+1}[x_k^{p_{k-1}} - \alpha_{k-1}^{p_{k-1}}]| +$$
$$z_k^{p-p_k+1}\Big[d_k x_{k+1}^{p_k} + \hat{\theta}^{\mathrm{T}}w_k - \frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\tau_k - \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_i}d_i x_{i+1}^{p_i} - \frac{\partial\alpha_{k-1}}{\partial y_r}\dot{y}_r - \sum_{i=1}^{k-1} r w_k z_i^{p-p_i+1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\Big] -$$
$$\left(\frac{1}{r}\tilde{\theta} + \eta_k\right)^{\mathrm{T}}(\dot{\hat{\theta}} - \tau_k) + (k-1)\delta \tag{22}$$

where

$$\boldsymbol{\tau}_k = \boldsymbol{\tau}_{k-1} + r\boldsymbol{w}_k z_k^{p-p_k+1}, \ \boldsymbol{w}_k = \boldsymbol{\phi}_k - \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_i}\boldsymbol{\phi}_i, \quad \boldsymbol{\eta}_k^{\mathrm{T}} = \boldsymbol{\eta}_{k-1}^{\mathrm{T}} + z_k^{p-p_k+1}\frac{\partial\alpha_{k-1}}{\partial\hat{\boldsymbol{\theta}}} \tag{23}$$

Note that

$$\bar{\mu}_{k-1}|z_{k-1}^{p-p_{k-1}+1}[x_k^{p_{k-1}} - \alpha_{k-1}^{p_{k-1}}]| \leqslant \frac{\bar{\mu}_{k-1}p_{k-1}}{2}|z_{k-1}^{p-p_{k-1}+1}| \, |x_k - \alpha_{k-1}| (x_k^{p_{k-1}-1} + \alpha_{k-1}^{p_{k-1}-1}) =$$
$$\frac{\bar{\mu}_{k-1}p_{k-1}}{2}|z_{k-1}^{p-p_{k-1}+1}| \, |z_k| ([z_k - z_{k-1}\beta_{k-1}(z_{k-1})]^{p_{k-1}-1} + [z_{k-1}\beta_{k-1}(z_{k-1})]^{p_{k-1}-1}) \leqslant$$

$$\frac{\bar{\mu}_{k-1}p_{k-1}}{2}|z_{k-1}^{p-p_{k-1}+1}||z_k|[2^{p_{k-1}-2}z_k^{p_{k-1}-1}+(1+2^{p_{k-1}-2})(z_{k-1}\beta_{k-1})^{p_{k-1}-1}]\leqslant$$
$$z_{k-1}^{p+1}+z_k^{p+1}\rho_{k1}(z_1,\cdots,z_k,\hat{\theta}) \tag{24}$$

for a smooth function $\rho_{k1}(z_1,\cdots,z_k,\hat{\boldsymbol{\theta}})\geqslant 0$. The last relation of (28) follows from Young's inequality.

Due to the boundedness of $y_r, \dot{y}_r$ and $d_i(t,x,u)$, similar to the proof of (13), there is a smooth function $\rho_{k2}(z_1,\cdots,z_k,\hat{\boldsymbol{\theta}})\geqslant 0$ such that

$$\left|z_k^{p-p_k+1}\left[\hat{\theta}^{\mathrm{T}}w_k-\sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_i}d_ix_{i+1}^{p_i}-\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\tau_k-\frac{\partial\alpha_{k-1}}{\partial y_r}\dot{y}_r-\sum_{i=1}^{k-1}rw_kz_i^{p-p_i+1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\right]\right|\leqslant$$
$$\delta+z_k^{p+1}\rho_{k2}(z_1,\cdots,z_k,\hat{\theta}) \tag{25}$$

Putting (23),(24), and (22) together, we have

$$\dot{V}_k\leqslant-(z_1^{p+1}+\cdots+z_{k-1}^{p+1})+z_k^{p-p_k+1}[d_kx_{k+1}^{p_k}+z_k^{p_k}\rho_{k1}+z_k^{p_k}\rho_{k2}]-\boldsymbol{\lambda}_k^{\mathrm{T}}(\dot{\hat{\boldsymbol{\theta}}}-\boldsymbol{\tau}_k)+k\delta \tag{26}$$

Clearly, the virtual smooth controller

$$\alpha_k=-z_k\left[\frac{2+\rho_{k,1}(z_1,\cdots,z_{k-1})+\rho_{k,2}(z_1,\cdots,z_k,\hat{\theta})}{\mu_k}\right]^{1/p_k}=-z_k\beta_k(z_1,\cdots,z_k,\hat{\theta}),\beta_k(\cdot)>0 \tag{27}$$

yields

$$\dot{V}_k\leqslant-(z_1^{p+1}+\cdots+z_{k-1}^{p+1})-2z_k^{p+1}+\bar{\mu}_k|z_k^{p-p_k+1}[x_{k+1}^{p_k}-\alpha_k^{p_k}]|-$$
$$\lambda_k^{\mathrm{T}}(\dot{\hat{\theta}}-\tau_k)+k\delta+\sigma\tilde{\theta}^{\mathrm{T}}(\hat{\theta}-\theta^0) \tag{28}$$

The induction argument shows that (19) holds for $k=n+1$. Hence, there exist a set of transformations $(z_1,\cdots,z_n)$ of the form (18), a smooth Lyapunov function

$$V_n=\sum_{i=1}^{n}\frac{z_i^{p-p_i+2}}{p-p_i+2}+\frac{1}{2r}\tilde{\theta}^{\mathrm{T}}\tilde{\theta} \tag{29}$$

and a smooth adaptive controller

$$u=-z_n\beta_n(z,\hat{\boldsymbol{\theta}}),\quad \beta_n(z,\hat{\theta})>0$$
$$\dot{\hat{\boldsymbol{\theta}}}=\boldsymbol{\tau}_n \tag{30}$$

such that

$$\dot{V}_n\leqslant-(z_1^{p+1}+\cdots+z_{n-1}^{p+1}+z_n^{p+1})+n\delta+\sigma\tilde{\theta}^{\mathrm{T}}(\hat{\theta}-\theta^0) \tag{31}$$

Because of

$$\sigma\tilde{\theta}^{\mathrm{T}}(\hat{\theta}-\theta^0)=-\frac{\sigma}{2}\tilde{\theta}^{\mathrm{T}}\tilde{\theta}-\frac{\sigma}{2}\|\hat{\theta}-\theta^0\|^2+\frac{\sigma}{2}\|\theta-\theta^0\|^2 \tag{32}$$

(31) becomes

$$\dot{V}_n\leqslant-(z_1^{p+1}+\cdots+z_n^{p+1})-\frac{\sigma}{2}\tilde{\theta}^{\mathrm{T}}\tilde{\theta}+n\delta+\frac{\sigma}{2}\|\theta-\theta^0\|^2 \tag{33}$$

The last inequality, together with Lemma 2, implies immediately that all the solutions of the closed-loop system are globally bounded and well defined over. This leads to the conclusion that the states are globally bounded, because of the relation (18) and boundedness of $y_r(t)$. Moreover, from Lemma 2, for any $\delta>0$, there exist a finite time $T>0$, such that

$$|y-y_r|=|z_1|\leqslant\left[\frac{2c_2}{c_1}(p-p_1+2)\right]^{1/p-p_1+2},\quad \forall t\geqslant T>0 \tag{34}$$

Therefore, for any $\varepsilon>0$, there exist $\delta>0, r>0, \sigma>0$, such that

$$|y-y_r|<\varepsilon,\quad \forall t\geqslant T>0 \tag{35}$$

This completes the proof of Theorem 1. □

Remark 1. Although c_2 contains unknown parameter vector $\boldsymbol{\theta}$, we may select σ to make c_2 very small because $\boldsymbol{\theta}$ is a constant vector.

Remark 2. In many practical control systems the parameter vector $\boldsymbol{\theta}$ is not completely unknown. We may roughly estimate it previously and let $\boldsymbol{\theta}^0$ equal the initial estimate. This may make $\|\boldsymbol{\theta}-\boldsymbol{\theta}^0\|^2$ very small and result in better tracking.

4 Simulation

Consider the following example

$$\begin{aligned}\dot{x}_1 &= (1+\sin^2 u)x_2^3 + \theta_1 x_1 e^{x_1} \\ \dot{x}_2 &= u^5 + \theta_2(x_1+x_2) \\ y &= x_1\end{aligned} \tag{36}$$

which is of the form (1) and where θ_1 and θ_2 are two unknown constant parameters.

Let $y_r=\sin(t)$ be the reference signal. The object is to design a smooth controller such that the output of system (36) practically follows y_r. By Theorem 1, we can design an adaptive controller as follows.

$$u = -z_2(1+\rho_1(z_1,\hat{\theta})+\rho_2(z_1,z_2,\hat{\theta}))^{1/5} \tag{37}$$

$$\dot{\hat{\theta}} = \tau_2 = r\left[z_2\left(\phi_2 - \frac{\partial\alpha_1}{\partial x_1}\phi_1\right) + z_1^3\phi_1 - \sigma(\hat{\theta}-\theta^0)\right] \tag{38}$$

where

$z_1 = x_1 - y_r, z_2 = x_2 - \alpha_1$, $\phi_1 = (x_1 e^{x_1}, 0)^{\mathrm{T}}$, $\phi_2 = (0, x_1+x_2)^{\mathrm{T}}$, $\theta = (\theta_1, \theta_2)^{\mathrm{T}}$, $\hat{\theta}$ is the estimate of θ

$$\alpha_1 = -z_1\beta_1 = -z_1\left(2+\frac{1}{2\delta}(1+(\hat{\theta}^{\mathrm{T}}\phi_1)^2)\right)^{1/3}, \quad \rho_1 = 54 + \frac{9^6}{6}\beta_1^{12}$$

$$\rho_2 = \frac{5^5}{6^6\delta^5}\left[2\left|\hat{\theta}^{\mathrm{T}}\left(\phi_2 - \frac{\partial\alpha_1}{\partial x_1}\phi_1\right) - \frac{\partial\alpha_1}{\partial\hat{\theta}}\tau_2\right|^2 + 16\left(\frac{\partial\alpha_1}{\partial x_1}x_2^3\right)^2 + 4\left(\frac{\partial\alpha_1}{\partial y_r}\right)^2\right]^3$$

The simulation results are shown in Fig. 1 and Fig. 2 are based on the following parameters: $\theta=(1,3)^{\mathrm{T}}, \delta=0.01, \theta^0=(2,2)^{\mathrm{T}}, r=20, \sigma=0.1$. The initial conditions are chosen as $x_1(0)=1$, $x_2(0)=1$, $\hat{\theta}(0)=[0,0]^{\mathrm{T}}$.

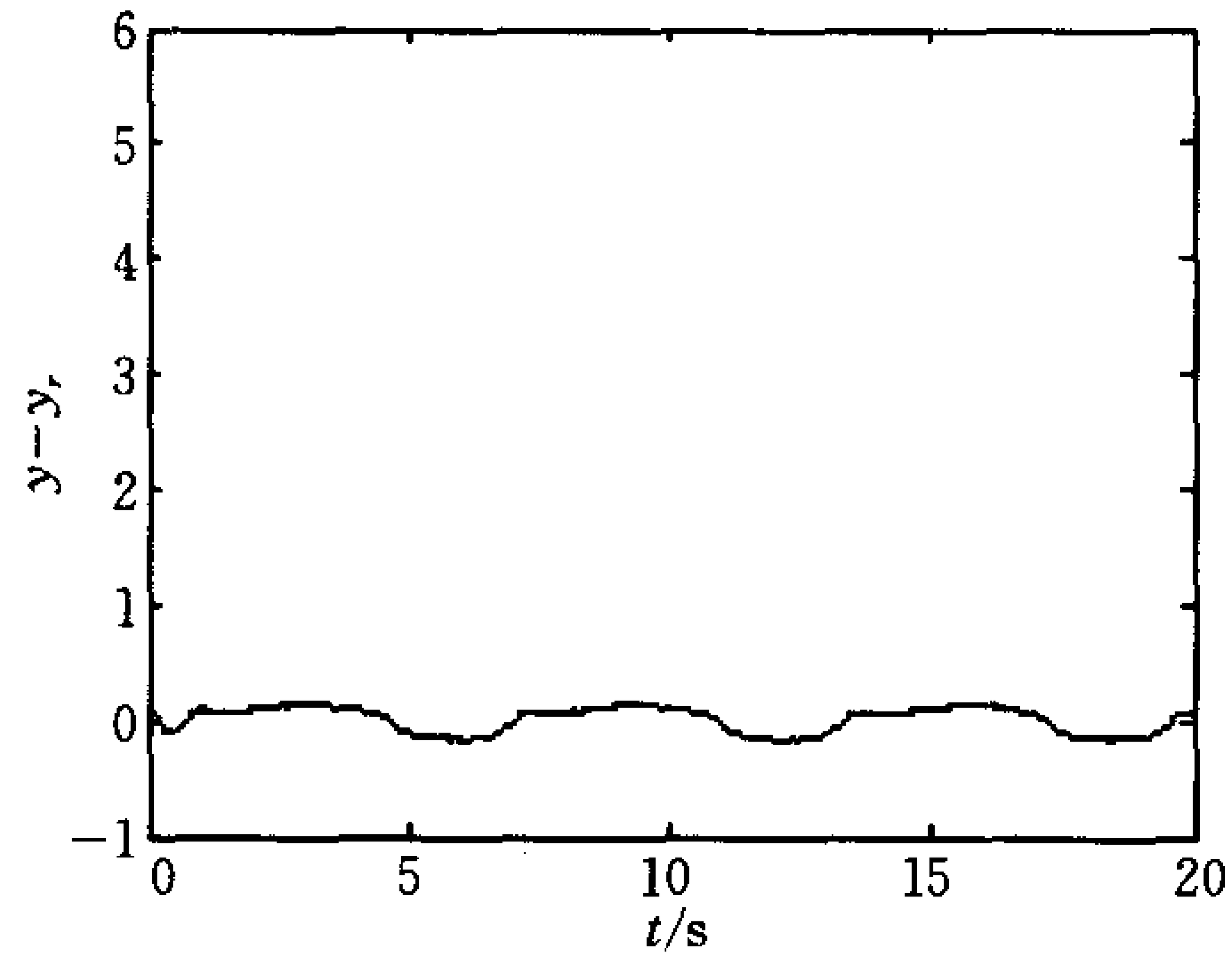

Fig. 1 Tracking error $y-y_r$

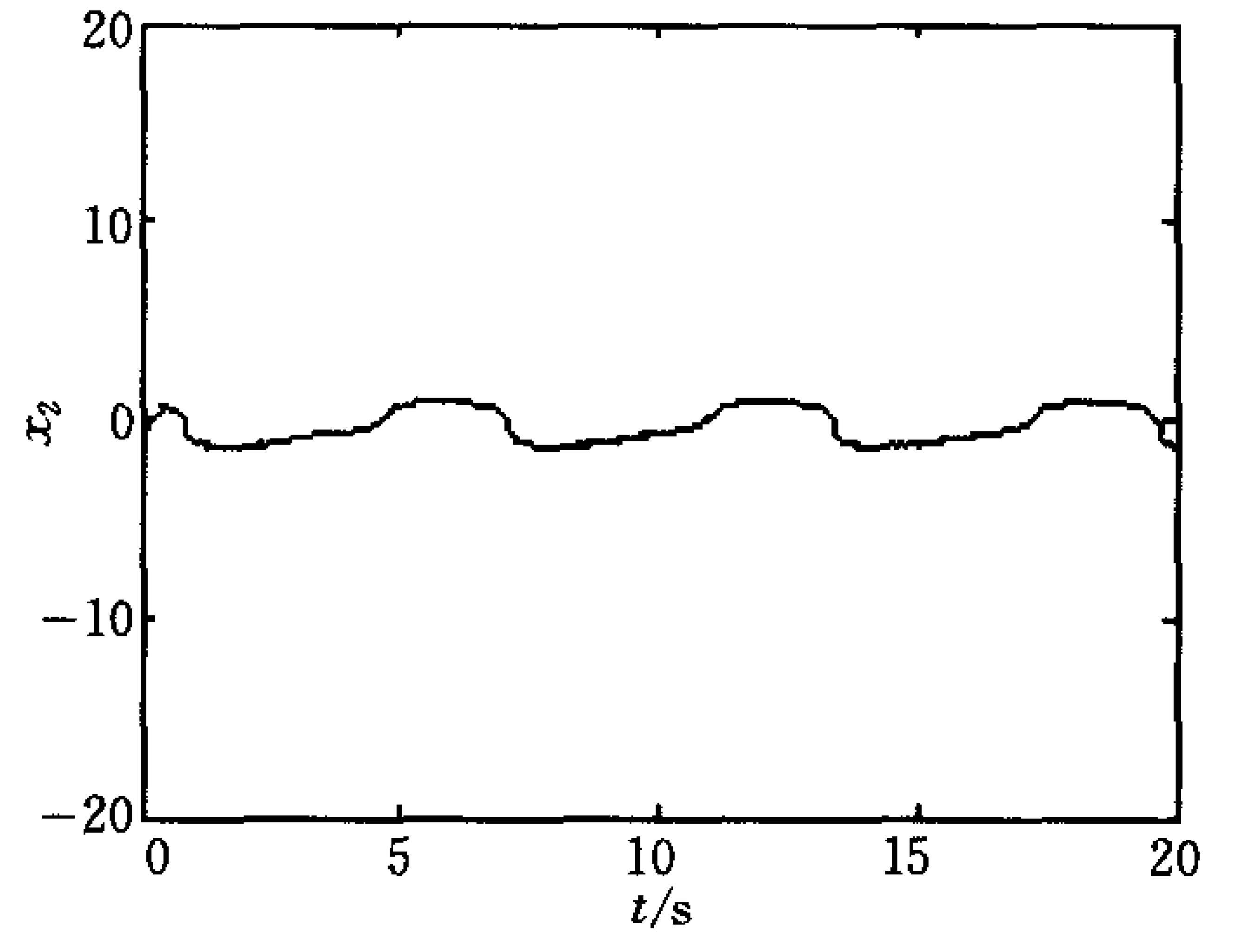

Fig. 2 State x_2

From Fig. 1 and Fig. 2, we see that all the states of the closed-loop system are globally, uniformly and ultimately bounded and the tracking error becomes very small after a finite time.

5 Conclusions

In this paper, the problem of globally adaptive and practical output tracking for a class of inherently nonlinear systems has been studied. The systems may contain an uncontrollable unstable linearization. Although the asymptotic output tracking of the system is impossible, using the modified adaptively adding a power integrator technique as a basic tool we construct a smooth adaptive controller and achieve globals practical output tracking. A simulation is given to illustrate the proposed method.

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非线性系统的自适应实用输出跟踪控制

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摘　要　研究了一类具有不可控不稳定线性化和线性化参数的非线性系统的全局自适应实用输出跟踪控制问题，这类系统既不具有反馈线性化性质，对输入也没有仿射特性. 因为这类系统的线性化系统具有不可控模式且该模式在右半平面内有根，所以全局渐近输出跟踪是不可能的（甚至是局部的）. 应用修正的自适应增加幂积分方法，设计了一种光滑状态反馈控制器，保证闭环系统所有信号全局一致最终有界，且使跟踪误差进入零的小邻域内. 仿真结果表明该控制器是可行的并且是有效的.

关键词　自适应实用输出跟踪，自适应光滑状态反馈，不可控不稳定线性化

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