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New Iterative Learning Control Algorithms Based on Vector Plots Analysis[1)]

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Abstract Based on vector plots analysis, this paper researches the geometric frame of iterative learning control method. New structure of iterative learning algorithms is obtained by analyzing the vector plots of some general algorithms. The structure of the new algorithm is different from those of the present algorithms. It is of faster convergence speed and higher accuracy. Simulations presented here illustrate the effectiveness and advantage of the new algorithm.

Key words Iterative learning control, new algorithm, convergence analysis, numerical simulation

1 Introduction

Iterative learning control was proposed by Arimoto (1984)[1]. Up to now, it has been developed and become an active branch in the intelligent control field with applications covering more and more aspects, such as discrete systems[2,3], general systems[4], systems with distributed parameters[5], and 2-D systems[6]. The structures of the algorithms are focused on P-type[7,8], D-type[9,10], PD-type and PID-type[11]. Also, there are the improved forms of the algorithms. Though each of the algorithms has the feature of its own, the algorithms are evolved from the algorithm

$$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k(t) + L\boldsymbol{e}_k(t), \quad t \in [0, T] \tag{1}$$

proposed by Arimoto (1984). So we can consider them as one type of algorithms. Up to now, iterative learning control has been developed for over one decade. The remaining problems are whether there exist better algorithms of other types, and whether a completed theoretical frame can be established to instruct the development of iterative learning control, and to direct one to seek more effective fast algorithms.

Herein, by the above motivation, we make effort to establish a geometric theory of iterative learning control in this paper. Upon this theoretical frame, we clarify how to design a more effective iterative learning algorithm to converge fast. Thus, a new approach is proposed to the advanced development of iterative learning control.

2 Vector plots analysis and the structure of new algorithms

Consider a nonlinear control system as follows

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) \\ \boldsymbol{y}(t) = C(\boldsymbol{x}, t) + B(t)\boldsymbol{u}(t) \end{cases}, \quad t \in [0, T] \tag{2}$$

and initial condition

$$\boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{3}$$

where $\boldsymbol{x} \in R^n, \boldsymbol{u} \in R^m, \boldsymbol{y} \in R^l$, f and C are vector functions with appropriate dimensional and Lipschitzian continuous, i.e.,

$$\|\boldsymbol{f}(\boldsymbol{x}_1, \boldsymbol{u}_1, t) - \boldsymbol{f}(\boldsymbol{x}_2, \boldsymbol{u}_2, t)\| \leqslant L_f(\|\boldsymbol{x}_1 - \boldsymbol{x}_2\| + \|\boldsymbol{u}_1 - \boldsymbol{u}_2\|) \tag{4}$$

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$$\|C(\boldsymbol{x}_1,t)-C(\boldsymbol{x}_2,t)\|\leqslant L_C\|\boldsymbol{x}_1-\boldsymbol{x}_2\| \tag{5}$$

where L_f, L_C are not specificly known. The ideal output of the system is $\boldsymbol{y}_d(t)$.

It is difficult to seek an ideal input $\boldsymbol{u}_d(t)$ such that the output of the system is just $\boldsymbol{y}_d(t)$, because of the uncertainty of the systems. Therefore, people intend to seek $u_d(t)$ using the following iteration process

$$\boldsymbol{u}_{k+1}(t)=\boldsymbol{u}_k(t)+L(t)\boldsymbol{e}_k(t),\quad t\in[0,T] \tag{6}$$

where $\boldsymbol{e}_k(t)=\boldsymbol{y}_d(t)-\boldsymbol{y}_k(t)$, $\boldsymbol{y}_k(t)$ is the output of system (2) according to the input $\boldsymbol{u}_k(t)$, $L(t)$ is the gain matrix which is remained to be decided. If the sequence $\{\boldsymbol{u}_k(t)\}$ determined by (6) is convergent, the limit $\boldsymbol{u}^*(t)$ must be the ideal input $\boldsymbol{u}_d(t)$.

From (6), the process of learning is to decide the next (the $(k+1)$th step) input through the present (the kth step) input associated with the output error so that the sequence $\{\boldsymbol{u}_k(t)\}$ converges as soon as possible. Denoting $\hat{\boldsymbol{u}}_k(t)=\boldsymbol{u}_k(t)-\boldsymbol{u}_d(t)$, algorithm (6) can be rewritten as

$$\hat{\boldsymbol{u}}_{k+1}(t)=\hat{\boldsymbol{u}}_k(t)+L\boldsymbol{e}_k(t),\quad t\in[0,T] \tag{7}$$

To get $\hat{\boldsymbol{u}}_k(t)\to\boldsymbol{0}$, it is equivalent to have $\|\hat{\boldsymbol{u}}_k(t)\|\to 0$. In another word, the algorithm defined by (7) requires that should decay to zero. From (7), it is easy to get the geometric relation shown in Fig. 1.

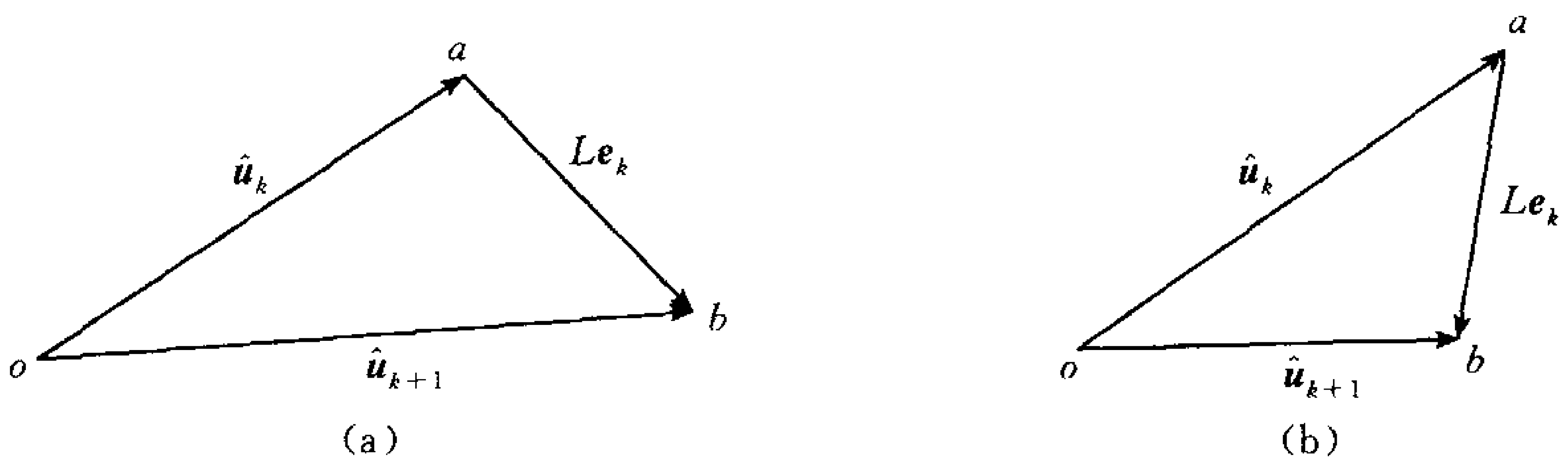


Fig. 1 Graph of vector analysis

If we add the vector $\hat{\boldsymbol{u}}_{k-1}$, then vector plots are as follows.

Thus, Fig. 1(a) corresponds with Fig. 2(a) and Fig. 2(b), Fig. 1(b) corresponds with Fig. 2(c) to Fig. 2(f). To get an effective fast algorithm, we seek a scheme to adjust $\hat{\boldsymbol{u}}_{k+1}$ via the analysis of the above figures.

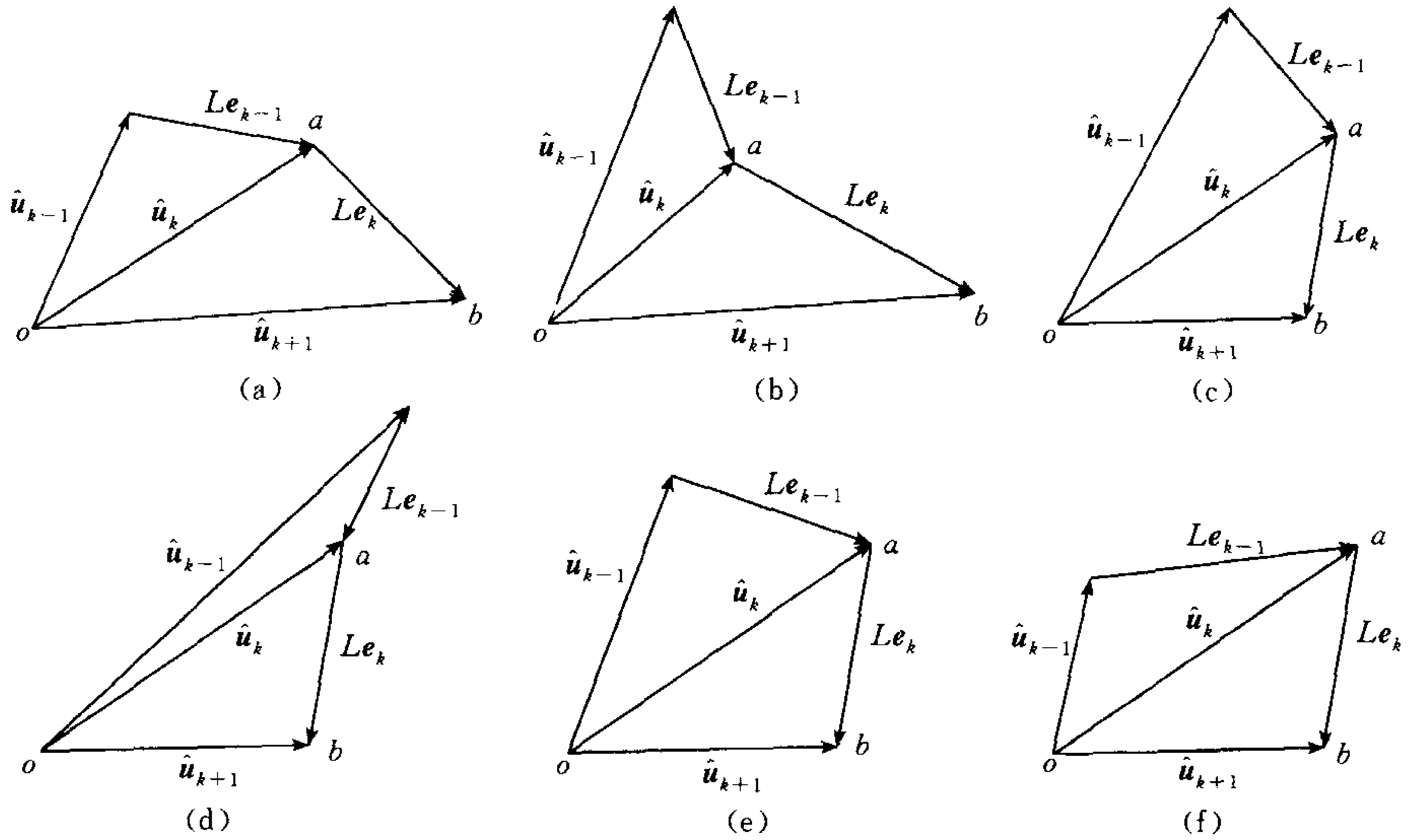


Fig. 2 Graph of vector analysis

At first, we consider the case of Fig. 2(a) and Fig. 2(b). In Fig. 2(a), we draw a vertical line of vector $\hat{\boldsymbol{u}}_k$ through point a. The crossing point of the vertical line with $\hat{\boldsymbol{u}}_{k+1}$ is c, shown in Fig. 3. Thus, we find that $\|\boldsymbol{oc}\| \leqslant \|\hat{\boldsymbol{u}}_{k+1}(t)\|$ and $\|\boldsymbol{oc}\| > \|\hat{\boldsymbol{u}}_k(t)\|$. Through point a, we draw a segment $\boldsymbol{ad}$ crossing $\boldsymbol{ob}$ at point d. When $\angle\beta < 90°$, the length $\|\boldsymbol{od}\|$ is possible to be less than $\|\hat{\boldsymbol{u}}_k(t)\|$. Letting $\boldsymbol{ad} = \hat{\boldsymbol{e}}_k$ and taking $\hat{\boldsymbol{u}}^*_{k+1} = \boldsymbol{od}$ to be the adjustment of $\hat{u}_{k+1}$, we get an algorithm

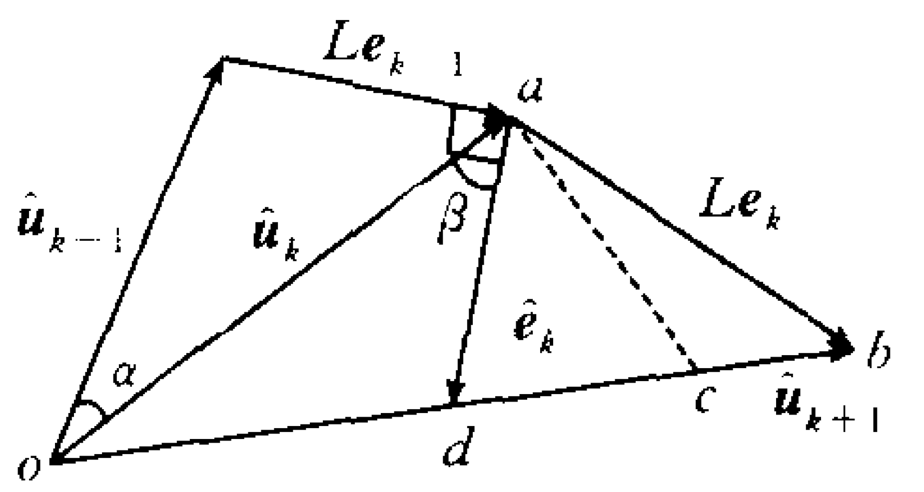


Fig. 3 Graph of vector analysis

$$\hat{\boldsymbol{u}}^*_{k+1} = \hat{\boldsymbol{u}}_k + \hat{\boldsymbol{e}}_k, \quad t \in [0, T] \tag{8}$$

It is clear that the condition $\|\hat{\boldsymbol{u}}^*_{k+1}\| < \|\hat{\boldsymbol{u}}_k\|$ is satisfied.

The problem is how to determine the vector $\hat{\boldsymbol{e}}_k$ so that $\angle\beta < 90°$. We will seek their relations by using the information of $\hat{\boldsymbol{u}}_{k-1}$. Selecting $\boldsymbol{ad}$ to be vertical to $L\boldsymbol{e}_{k-1}$, if $\angle\alpha > 0$, we get $\angle\beta < 90°$. Hence, we select $\hat{\boldsymbol{e}}_k$ to be the vertical of vector $L\boldsymbol{e}_{k-1}$. Obviously, the vector

$$\hat{\boldsymbol{e}}_k = L\boldsymbol{e}_k - \frac{(L\boldsymbol{e}_{k-1})^{\mathrm{T}} L\boldsymbol{e}_k}{\|L\boldsymbol{e}_{k-1}\|^2} L\boldsymbol{e}_{k-1} \tag{9}$$

is vertical to the vector $L\boldsymbol{e}_{k-1}$.

Therefore, when $\|\hat{\boldsymbol{u}}_{k-1}\| < \|\hat{\boldsymbol{u}}_k\|$, we obtain a new algorithm structure

$$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k + \hat{\boldsymbol{e}}_k = \boldsymbol{u}_k + L\left(\boldsymbol{e}_k - \frac{(L\boldsymbol{e}_{k-1})^{\mathrm{T}} L\boldsymbol{e}_k}{\|L\boldsymbol{e}_{k-1}\|^2} \boldsymbol{e}_{k-1}\right), \quad t \in [0, T] \tag{10}$$

For the case of Fig. 2(b), analogous to the above, when $\|\hat{\boldsymbol{u}}_{k-1}\| \geqslant \|\hat{\boldsymbol{u}}_k\|$, we obtain a new algorithm structure

$$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k + \hat{\boldsymbol{e}}_k = \boldsymbol{u}_k + L\left(\boldsymbol{e}_k - \frac{\|L\boldsymbol{e}_k\|^2}{(L\boldsymbol{e}_k)^{\mathrm{T}} L\boldsymbol{e}_{k-1}} \boldsymbol{e}_{k-1}\right), \quad t \in [0, T] \tag{11}$$

In a word, in the case of Fig. 1(a), we can obtain a preliminary algorithm structure, namely, when $\|\boldsymbol{u}_k + L\boldsymbol{e}_k\| > \|\boldsymbol{u}_k\|$,

$$\boldsymbol{u}_{k+1} = \begin{cases} \boldsymbol{u}_k + L\left(\boldsymbol{e}_k - \dfrac{(L\boldsymbol{e}_{k-1})^{\mathrm{T}} L\boldsymbol{e}_k}{\|L\boldsymbol{e}_{k-1}\|^2} \boldsymbol{e}_{k-1}\right), & \|\hat{\boldsymbol{u}}_{k-1}\| < \|\hat{\boldsymbol{u}}_k\| \\ \boldsymbol{u}_k + L\left(\boldsymbol{e}_k - \dfrac{\|L\boldsymbol{e}_k\|^2}{(L\boldsymbol{e}_k)^{\mathrm{T}} L\boldsymbol{e}_{k-1}} \boldsymbol{e}_{k-1}\right), & \|\hat{\boldsymbol{u}}_{k-1}\| \geqslant \|\hat{\boldsymbol{u}}_k\| \end{cases}, \quad t \in [0, T] \tag{12}$$

Next, we consider the case corresponding with Fig. 1(b). First we consider the case of $\|\hat{\boldsymbol{u}}_{k-1}\| > \|\hat{\boldsymbol{u}}_k\|$, that is, Fig. 2(c) and Fig. 2(d). For Fig. 2(c), we can obtain corresponding algorithm structure as same as (10) by analyzing. For Fig. 2(d), the original algorithm (6) remains.

At the end, we consider the case of Fig. 2(e) and Fig. 2(f). For Fig. 2(e), analogous to the above, we can obtain corresponding algorithm structure as same as (10). For Fig. 2 (f), the original algorithm (6) remains.

For the case of Fig. 1(b), we can obtain a preliminary algorithm structure, namely, when $\|\boldsymbol{u}_k + L\boldsymbol{e}_k\| \leqslant \|\boldsymbol{u}_k\|$,

$$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k + L\left(\boldsymbol{e}_k - \sigma \frac{(L\boldsymbol{e}_{k-1})^{\mathrm{T}} L\boldsymbol{e}_k}{\|L\boldsymbol{e}_{k-1}\|^2} \boldsymbol{e}_{k-1}\right), \quad t \in [0, T] \tag{13}$$

where $\sigma = 0, 1$ so that the transferring of Fig. 2(e) and Fig. 2(f) can be better described. Extending the scope of σ, we get $0 \leqslant \sigma \leqslant 1$.

For synthesizing the algorithms of all cases, we also introduce a factor σ in the structure (12). That is, when $\|\boldsymbol{u}_k + L\boldsymbol{e}_k\| > \|\boldsymbol{u}_k\|$,

$$\boldsymbol{u}_{k+1} = \begin{cases} \boldsymbol{u}_k + L\left(\boldsymbol{e}_k - \sigma \dfrac{(L\boldsymbol{e}_{k-1})^{\mathrm{T}} L\boldsymbol{e}_k}{\|L\boldsymbol{e}_{k-1}\|^2} \boldsymbol{e}_{k-1}\right), & \|\hat{\boldsymbol{u}}_{k-1}\| < \|\hat{\boldsymbol{u}}_k\| \\ \boldsymbol{u}_k + L\left(\boldsymbol{e}_k - \sigma \dfrac{\|L\boldsymbol{e}_k\|^2}{(L\boldsymbol{e}_k)^{\mathrm{T}} L\boldsymbol{e}_{k-1}} \boldsymbol{e}_{k-1}\right), & \|\hat{\boldsymbol{u}}_{k-1}\| \geqslant \|\hat{\boldsymbol{u}}_k\| \end{cases}, \quad t \in [0, T] \tag{14}$$

where $\sigma \in [0,1]$.

In the second equation on the right hand in (14), let

$$\sigma = h\,\frac{(L\boldsymbol{e}_{k-1})^{\mathrm T} L\boldsymbol{e}_k (L\boldsymbol{e}_{k-1})^{\mathrm T} L\boldsymbol{e}_k}{\|L\boldsymbol{e}_k\|^2 \|L\boldsymbol{e}_{k-1}\|^2} \tag{15}$$

Because $(L\boldsymbol{e}_{k-1})^{\mathrm T} L\boldsymbol{e}_k (L\boldsymbol{e}_{k-1})^{\mathrm T} L\boldsymbol{e}_k \leqslant \|L\boldsymbol{e}_k\|^2 \|L\boldsymbol{e}_{k-1}\|^2$, we have $\sigma \leqslant h$. So, when $h \in [0,1]$, we get $\sigma \in [0,1]$. Substituting (15) to the second equation on the right hand in (14), we get

$$\boldsymbol{u}_{k+1} = \begin{cases} \boldsymbol{u}_k + L\left(\boldsymbol{e}_k - \sigma \dfrac{(L\boldsymbol{e}_{k-1})^{\mathrm T} L\boldsymbol{e}_k}{\|L\boldsymbol{e}_{k-1}\|^2}\boldsymbol{e}_{k-1}\right), & \|\hat{\boldsymbol{u}}_{k-1}\| < \|\hat{\boldsymbol{u}}_k\| \\ \boldsymbol{u}_k + L\left(\boldsymbol{e}_k - h \dfrac{(L\boldsymbol{e}_{k-1})^{\mathrm T} L\boldsymbol{e}_k}{\|L\boldsymbol{e}_{k-1}\|^2}\boldsymbol{e}_{k-1}\right), & \|\hat{\boldsymbol{u}}_{k-1}\| \geqslant \|\hat{\boldsymbol{u}}_k\| \end{cases}, \quad t \in [0,T] \tag{16}$$

Because σ and h are undetermined parameters in $[0,1]$, (16) can be rewritten

$$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k + L\left(\boldsymbol{e}_k - \sigma \frac{(L\boldsymbol{e}_{k-1})^{\mathrm T} L\boldsymbol{e}_k}{\|L\boldsymbol{e}_{k-1}\|^2}\boldsymbol{e}_{k-1}\right), \quad \|\boldsymbol{u}_k + L\boldsymbol{e}_k\| > \|\boldsymbol{u}_k\| \tag{17}$$

At last, from (13) and (17), we obtain a general new algorithm structure

$$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k + L\left(\boldsymbol{e}_k - \sigma \frac{(L\boldsymbol{e}_{k-1})^{\mathrm T} L\boldsymbol{e}_k}{\|L\boldsymbol{e}_{k-1}\|^2}\boldsymbol{e}_{k-1}\right), \quad t \in [0,T] \tag{18}$$

where $\sigma \in [0,1]$ is an undetermined parameter. According to early discussion, parameter σ is variable in the iteration. We will discuss the variations of the parameter σ later.

3 Convergence analysis of the new algorithm

In the above section, based on the geometrical analysis of the vector plots yielded from the ordinary algorithm, we derive new algorithm structure (18). The new algorithm structure is entirely different from the ordinary algorithm and it is nonlinear. Because the new structure is derived by directly observing and synthesizing the vector plots, the convergence and efficiency of the algorithm need to be analyzed and simulated. In this section, we only analyze the convergence of the new algorithm, numerical simulation will be given in the next section.

Before the convergence of the new algorithm is discussed, two lemmas are introduced (their proofs are omitted).

We assume $\boldsymbol{x}_k(0) = \boldsymbol{x}_0$, $k = 1,2,3,\cdots$, $\Delta\boldsymbol{u}_k(t) = \boldsymbol{u}_{k+1}(t) - \boldsymbol{u}_k(t)$. In this paper, the definitions of the norms are the same as in [4].

Lemma 1. If $\rho, \sigma \in (0,1)$, for positive constants d, l, inequality $\frac{dl\rho\sigma}{1-\rho} < 1$ is satisfied, then there exist $\xi > 1$ and appropriate large positive number λ, so that for positive constants c, a, we have $F(\lambda, \xi) \in (0,1)$, where,

$$F(\lambda,\xi) = \frac{\xi\rho}{1-\xi\rho}\left[\frac{\xi c a l(1+\sigma)}{\lambda(\lambda - a)} + \sigma\xi d l\right]$$

Lemma 2. For an appropriate large λ and $\xi > 1$, we have the following estimation

$$\|\boldsymbol{x}_{k+1} - \boldsymbol{x}_k\|_{(\lambda,\xi)} \leqslant \frac{L_f}{\lambda(\lambda - L_f)} \|\Delta\boldsymbol{u}_k\|_{(\lambda,\xi)}$$

Now we discuss the main problem of this section——the convergence analysis of the new algorithm.

Theorem 3. If the parameter σ and the gain matrix $L(t)$ in (18) satisfy

1) $\|I - B(t)L(t)\| \leqslant \rho \in [0,1)$, 2) $\frac{\rho\sigma L_B l}{1-\rho} < 1$,

then input sequence $\{\boldsymbol{u}_k(t)\}$ of system (2) determined by (18) is uniformly convergent on $[0,T]$ and $\lim_{k\to\infty} \boldsymbol{e}_k(t) = 0$, $\forall t \in [0,T]$, where $l = \sup_{0\leqslant t\leqslant T}\|L(t)\|$, $L_B = \sup_{0\leqslant t\leqslant T}\|B(t)\|$.

Proof. From the definition of $e_k(t)$, (2) and (18), we have

$$\begin{aligned} e_{k+1}(t) &= e_k(t) - (C(x_{k+1}(t),t) - C(x_k(t),t)) - B(t)(u_{k+1}(t) - u_k(t)) \\ &= (I - B(t)L)e_k(t) - (C(x_{k+1}(t),t) - C(x_k(t),t)) + \\ &\quad \sigma B(t)\frac{(Le_{k-1}(t))^{\mathrm{T}} Le_k(t)}{\|Le_{k-1}(t)\|^2} Le_{k-1}(t) \end{aligned} \tag{19}$$

Taking the norm for the above equation and using conditions 1) and (5), we get

$$\|e_{k+1}(t)\| \leqslant \rho\|e_k(t)\| + \|L_C\|\|x_{k+1}(t) - x_k(t)\| + \sigma\|B(t)\|\|Le_k(t)\| \tag{20}$$

Let $\|L_C\| \leqslant c$; then

$$\|e_k(t)\| \leqslant \rho^k\|e_0(t)\| + \sum_{i=1}^{k-1}\rho^{k-i-1}(c\|x_{i+1}(t) - x_i(t)\| + \sigma L_B\|Le_i(t)\|) \tag{21}$$

At first, we choose an appropriate large λ so that $\lambda - L_f > 0$. Since $\rho,\sigma \in (0,1)$ and condition 2), from Lemma 1, there exists $\xi > 1$ so that $F(\lambda,\xi) \in (0,1)$, where

$$F(\lambda,\xi) = \frac{\xi\rho}{1 - \xi\rho}\left[\frac{\xi c L_f l(1+\sigma)}{\lambda(\lambda - L_f)} + \sigma\xi L_B l\right] \tag{22}$$

For the above chosen λ and ξ, from Lemma 2, it follows that

$$(\|e_k(t)\|\xi^k)\mathrm{e}^{-\lambda t} \leqslant \|e_0\|_\lambda + \sum_{i=1}^{k-1}(\rho\xi)^{k-i-1}\xi\left(\frac{cL_f}{\lambda(\lambda - L_f)}\|\Delta u_i\|_{(\lambda,\xi)} + \sigma L_B l\|e_i\|_{(\lambda,\xi)}\right) \tag{23}$$

From (18), we get

$$\|\Delta u_k(t)\| \leqslant l\|e_k(t)\| + \sigma l\|e_k(t)\| = (1+\sigma)l\|e_k(t)\| \tag{24}$$

thus

$$\|\Delta u_k\|_{(\lambda,\xi)} \leqslant (1+\sigma)l\|e_k\|_{(\lambda,\xi)} \tag{25}$$

Substituting (25) into (23), we obtain

$$(\|e_k(t)\|\xi^k)\mathrm{e}^{-\lambda t} \leqslant \|e_0\|_\lambda + F(\lambda,\xi)\sup_{1\leqslant i\leqslant k}\{\|e_i\|_{(\lambda,\xi)}\} \tag{26}$$

Since the right hand of inequality (26) is independent of t, taking the supremum on both hands for t, we get

$$\|e_k\|_{(\lambda,\xi)} \leqslant \|e_0\|_\lambda + F(\lambda,\xi)\sup_{1\leqslant i\leqslant k}\{\|e_i\|_{(\lambda,\xi)}\} \tag{27}$$

From Lemma 1, we have $F(\lambda,\xi) \in (0,1)$, and $\sup_{1\leqslant i\leqslant k}\{\|e_i\|_{(\lambda,\xi)}\} \leqslant \frac{\|e_0\|_\lambda}{1 - F(\lambda,\xi)}$.

So, for $t \in [0,T]$, we obtain

$$\|e_k(t)\| \leqslant \xi^{-k}\mathrm{e}^{\lambda t}\sup_{1\leqslant i\leqslant k}\{\|e_i\|_{(\lambda,\xi)}\} \leqslant \xi^{-k}\mathrm{e}^{\lambda T}\frac{\|e_0\|_\lambda}{1 - F(\lambda,\xi)} \tag{28}$$

Since $\xi > 1$, $\lim_{k\to\infty}\|e_k(t)\| = 0$, $\forall t \in [0,T]$. The proof of uniform convergence of $\{u_k(t)\}$ is omitted. □

The structure of the algorithm (18) is obtained from all the cases in the iteration process. The value of the parameter σ corresponds with all the cases in the iteration process. Since each case is possibly alternate in the process of iteration, σ should be a variable number and an adaptive factor along with the convergence of the learning process. In this paper, we select $\sigma = \alpha(1 - \mathrm{e}^{-\beta\|e_k\|})$, where $(\alpha,\beta) \in (0,1) \times [0,+\infty)$ is a pair of adjustable constants. The pair determines the variation amplitude of adaptive factor σ along with the error.

Since $(\alpha,\beta) \in (0,1) \times [0,+\infty)$, $\sigma \in (0,1)$. The new parameter factor does not influence the convergence analysis.

Consequently, we obtain a new learning algorithm with an adaptive adjusting factor

$$u_{k+1} = u_k + L\left(e_k - \alpha(1 - \exp(-\beta\|e_k\|))\frac{(Le_{k-1})^{\mathrm{T}} Le_k}{\|Le_{k-1}\|^2}e_{k-1}\right), \quad t \in [0,T] \tag{29}$$

4 Comparison with other algorithm by numerical simulations

To explain the superiority of the new algorithm, we make simulations on the following example via the new algorithm and the ordinary algorithm.

Example 1. Consider multi-input and multi-output nonlinear system

$$\begin{cases} \dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + f(\boldsymbol{x}(t)) + B\boldsymbol{u}(t) \\ \boldsymbol{y}(t) = C\boldsymbol{x}(t) + D\boldsymbol{u}(t) \end{cases} \tag{30}$$

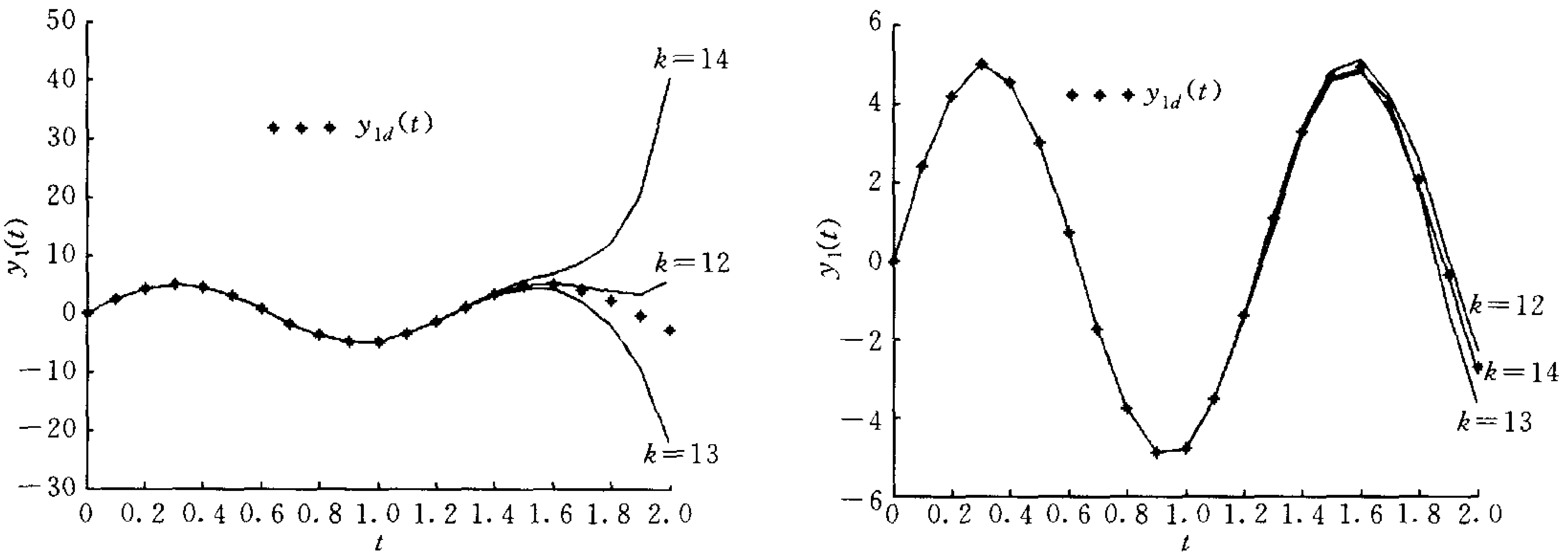
where $\boldsymbol{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$, $\boldsymbol{u}(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}$, $\boldsymbol{y}(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$, $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $f(\boldsymbol{x}(t)) = \begin{pmatrix} \sin x_1(t) \\ \cos x_2(t) \end{pmatrix}$

The aim is to track the expected trajectory $\boldsymbol{y}_d = \begin{pmatrix} y_{1d}(t) \\ y_{2d}(t) \end{pmatrix} = \begin{pmatrix} 5\sin 5t \\ 4t^3 \end{pmatrix}$ within $t \in [0,2]$. Taking $\alpha = 0.8$, $\beta = 0.9$, $L = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, we have the corresponding algorithm

$$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k + L\left(\boldsymbol{e}_k - 0.8(1 - \exp(-0.9\|\boldsymbol{e}_k\|))\frac{(L\boldsymbol{e}_{k-1})^{\mathrm{T}} L\boldsymbol{e}_k}{\|L\boldsymbol{e}_{k-1}\|^2}\boldsymbol{e}_{k-1}\right) \tag{31}$$

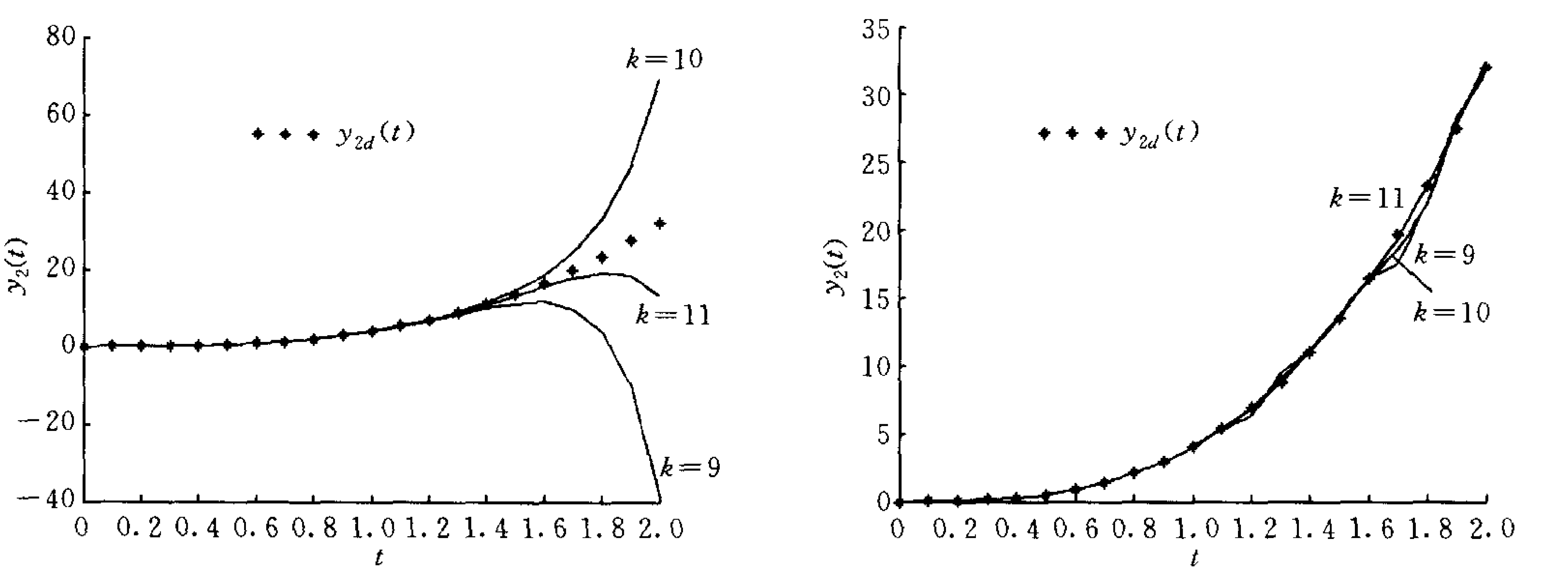
where $\boldsymbol{u}_k = (u_{1k}, u_{2k})^{\mathrm{T}}$, $\boldsymbol{e}_k = \boldsymbol{y}_d - \boldsymbol{y}_k$, $\boldsymbol{y}_k = (y_{1k}, y_{2k})^{\mathrm{T}}$.

Let the initial state and initial input of system (30) are $\boldsymbol{x}_0 = (0,0)^{\mathrm{T}}$, $\boldsymbol{u}_0 = (0,0)^{\mathrm{T}}$, $t \in [0,2]$. Fig. 4 shows the tracking process (at 12th, 13th and 14th iterations, the tracking aim is $y_{1d}(t)$) for the ordinary algorithm (6) and the new algorithm (31). Fig. 5 shows the tracking process (at 9th, 10th and 11th iterations, the tracking aim is $y_{1d}(t)$). The same results can also be found obviously from the tracking error diagrams (See Fig. 6).



(a) ordinary algorithm (b) new algorithm

Fig. 4 The first component of output of the learning control system



(a) ordinary algorithm (b) new algorithm

Fig. 5 The second component of output of the learning control system

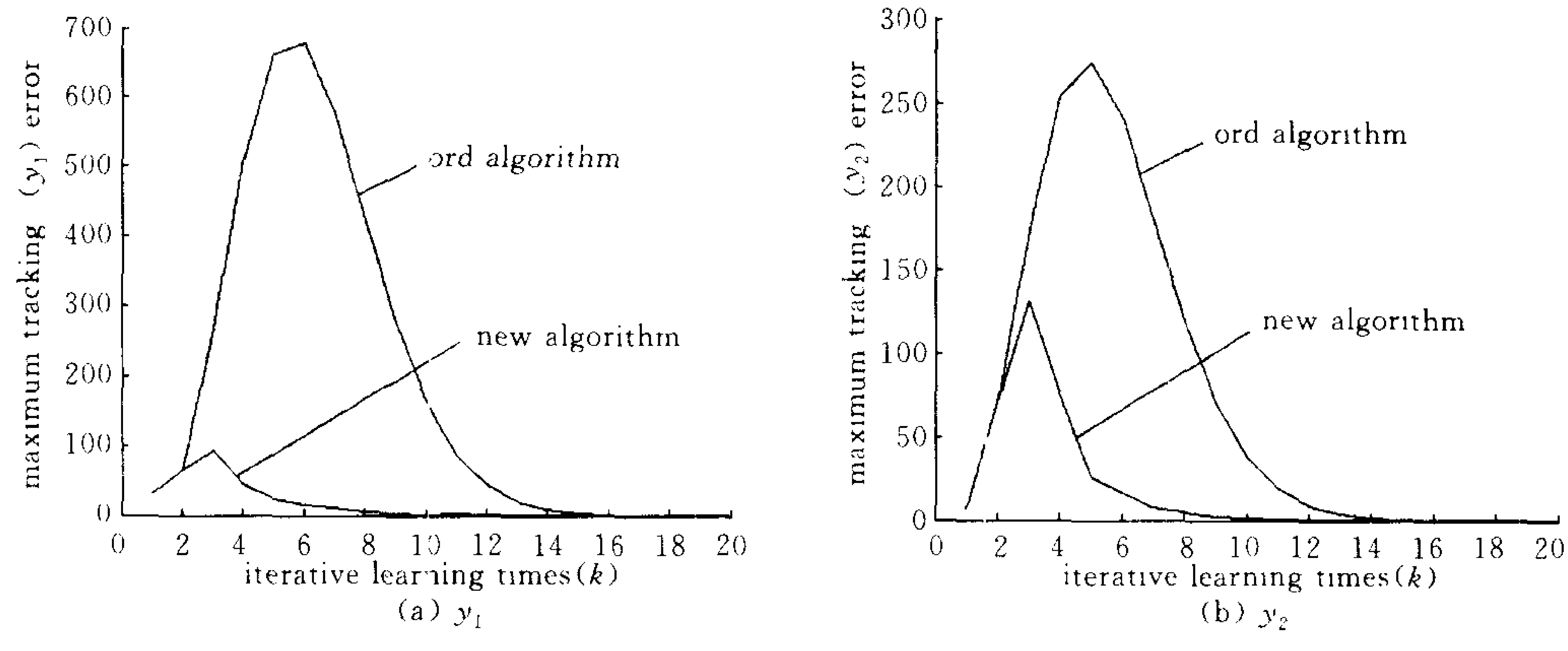


Fig. 6 The maximum tracking error curve in every iteration

5 Conclusion

In this paper, we intend to establish a new theoretical frame for iterative learning control by using the mathematical vector plots analysis. A class of new iterative learning control algorithm is obtained by analyzing the vector plots of the ordinary algorithm. The numerical simulations show the advantage of the new algorithm. In realizing the new algorithm (29), we only give an approximate expression. More accurate realization of the algorithm needs joint efforts made by more experts. Our work is just intended for establishing the geometric theory frame of iterative learning control.

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基于向量图分析的迭代学习控制新算法

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摘　要　基于向量图分析,对迭代学习控制方法的几何框架进行探索.首先通过对通常算法所构成的向量图进行几何分析,导出了新的迭代学习控制算法的结构,然后从理论上对所导出的新算法进行了完整的收敛性分析.所得算法结构的形式与已有算法完全不同,但其收敛速度和精度有明显提高,仿真结果表明了新算法的有效性与优越性.

关键词　迭代学习控制,新算法,收敛性分析,数值仿真

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