

## A New Approach to Automatic Seal Imprint Identification<sup>1)</sup>

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**Abstract** Seal imprints have been extensively used in many oriental countries. The automatic identification of seal imprints is a very difficult problem of pattern recognition. Every phase of the automatic identification of seal imprints is studied carefully and a corresponding series of algorithms is presented. In order to make a distinction between surplus parts of strokes in forgery seal imprint and surplus parts of seal paste in genuine seal imprint, and make a distinction between missing parts of strokes in forgery seal imprint and missing parts of seal paste in genuine seal imprint, several difference images and image processing methods are proposed, *i. e.*, full difference image, inner difference image, surplus difference image, missing difference image, *etc.* On such a basis, an automatic seal imprint identification scheme is presented. Experimental results confirm that the proposed scheme are feasible for practical applications.

**Key words** Pattern recognition, automatic identification of seal imprints, feature extraction

### 1 Introduction

As legal identification of persons, companies, institutions, organizations and government departments, seal imprints have been widely used in China, South Korea, Japan and other Asian countries, and seals are as common and important as signatures in western nations. However, there are several drawbacks in the manual identification of seal imprints, *i. e.*, low speed, inefficiency, and the correctness of identification which may directly be effected by the technical expertise, emotions of operators and operational environment. It is obvious that the manual identification of seal imprints is a bottleneck of automatic banking. Therefore, it has been proposed for many years to automatically verify seal imprints, and several approaches have been proposed in the past 20 years.

In practice, the process of automatic seal imprint identification can be divided into the following steps: 1) input the image of model or standard seal imprint (denoted by *MS*) and establish database of standard seal imprint; 2) input the image of sample seal imprint to be verified (denoted by *SS*); 3) do registration between *SS* and the corresponding *MS*; 4) compute and analyze the differences and classify *SS* as genuine, forgery or ambiguous.

The identification of seal imprints is a very special and difficult issue of pattern recognition. The specialty comes from its application environment. For instance, identification of seal imprint on a bank check must be absolutely reliable, meanwhile, rejection of a genuine imprint cannot happen often.

The difficulty comes from the two following aspects: 1) the intentionally forged seal imprint may be very similar to the standard seal imprint and it is very difficult, even for the experts, to discriminate the differences between *MS* and *SS* in this case; 2) different stamping conditions may seriously affect the quality of the seal imprint image, so that the image of a genuine *SS* looks quite different from the image of *MS*, as shown in Fig. 1.

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Thus, it is almost impossible to design a seal imprint identification method which always yields correct results. However, a good method should detect the forgery seal imprints if there are adequate distinctions between *SS* and *MS* even under different stamping conditions.



(a) an MS      (b) a forged seal print of (a)      (c) two SSes of a same MS

Fig. 1 The examples of similar seal imprints

Several methods have been presented for automatic identification of seal imprints in recent 20 years. Ueda and Naxamura<sup>[1]</sup> proposed an early identification method, which makes statistical decisions based on some local and global features. Their method can only verify very clear seal imprints. Fan and Tsai<sup>[2]</sup> developed an approach by which thinned stroke skeletons are extracted to represent the binary seal image. The middle points of the skeletons and slopes of the square frame are used for registration; a distance-weighted correlation method is then used to measure the similarity between *SS* and *MS*. Lee and Kim<sup>[3]</sup> introduced an advanced approach by which attributed stroke graph is used for registration and a distance measurement criterion is chosen for classification. Hu *et al.*<sup>[4]</sup> proposed a method based on difference images. Their registration is based on vertical points of square seal images. There are other methods for the identification of automatic seal imprints<sup>[5~8]</sup>.

In this paper, we develop a new approach for the identification of seal imprints, which is also based on difference images. Our approach has few constraints on seal imprints. The shape of seals can be circular, square, elliptical or any other closed loop. Furthermore, to make a distinction between surplus parts of strokes in forgery seal imprint and surplus parts of seal paste in genuine seal imprint, and make a distinction between missing parts of strokes in forgery seal imprint and missing parts of seal paste in genuine seal imprint, the difference images are processed very carefully. For this purpose, we propose a series concepts of difference image, such as full difference image, inner difference image, surplus difference image, missing difference image, real difference image, etc. In addition, we also study the techniques of grey level seal imprint images thresholding, present registration image thresholding method and verification image thresholding method.

## 2 Definitions and denotations

Let  $I$  be an  $M \times M$  binary image,  $I(i, j)$  be the value of the pixel at  $i$ th row and  $j$ th column of image  $I$ , where  $0 \leq i \leq M-1$ ,  $0 \leq j \leq M-1$ . The pixel is called "black pixel" (the foreground point) if  $I(i, j) = 1$  and the pixel is called "white pixel" (the background point) if  $I(i, j) = 0$ . Let

$$(I \oplus J)(i, j) = \begin{cases} 1, & \text{if } I(i, j) \neq J(i, j) \\ 0, & \text{if } I(i, j) = J(i, j) \end{cases}$$

The standard seal imprint image is a binary image, denoted by *MS*; the sample seal imprint to be verified is a grey level image, denoted by *GSS* (gray level sample seal). Suppose the size of *MS* and *GSS* is the same. The binary image of *GSS* after registration image thresholding is denoted by *RBS* (registration binary sample seal); and the gray level image after registration is denoted by *RGS* (registration grey level sample seal); and the binary

image after verification image thresholding is denoted by  $VBS$  (verification binary sample seal). In addition,  $\otimes$  is exclusive disjunction operator,  $\vee$  is disjunction operator. The followings are some definitions.

**Definition 1.** Image  $Fdif = MS \oplus VBS$  is called the full difference image, denoted by  $Fdif$ .

**Definition 2.** Let  $MSd$  be the dilation image of image  $MS$  with threshold  $d$ , and the foreground of  $MSd$  be  $MSd_F$ , that is  $MSd_F = \{(i, j) : MSd(i, j) = 1\}$ ; then  $Idif = MSd_F \otimes VBS$  is called the inner difference image with threshold  $d$ , where

$$MSd_F \otimes VBS(i, j) = \begin{cases} MSd(i, j) \oplus VBS(i, j), & \text{if } (i, j) \in MSd_F \\ 0, & \text{if } (i, j) \notin MSd_F \end{cases}$$

**Definition 3.** Image  $Mdif = MS/VBS$  is called the missing difference image, where

$$MS/VBS(i, j) = \begin{cases} 1, & \text{if } MS(i, j) = 1 \text{ and } VBS(i, j) = 0 \\ 0, & \text{otherwise} \end{cases}$$

**Definition 4.** Image  $Sdif = VBS/MS$  is called the surplus difference image, where

$$VBS/MS(i, j) = \begin{cases} 1, & \text{if } VBS(i, j) = 1 \text{ and } MS(i, j) = 0 \\ 0, & \text{otherwise} \end{cases}$$

**Definition 5.** The real difference image between  $MS$  and  $VBS$  is called the real difference image.  $Rdif$  denotes the real difference image.

**Definition 6.** The binary image after registration image thresholding is called the registration binary image.

**Definition 7.** The binary image after verification image thresholding is called the verification binary image.

### 3 Registration

In this section, we will introduce our registration procedure. Registration is an important step in the identification of seal imprints. There will be a marked drop in the recognition rate if the registration is not very accurate. Our registration procedure is based on stroke skeleton features match, which is independent of the shape of seal. In our registration procedure, the input image is  $GSS$ , and the output image is  $RGS$ , which is different to all other methods. The input and output images are binary seal imprint images of the existing methods. The reasons of our procedure are as follows. 1) The  $SS$  will be segmented by thresholding two times, namely registration image thresholding and verification image thresholding, respectively, because the verification result is strongly dependent on the segmented image. The main purpose of the first image thresholding is only for registration and there is not any other additional information to use. The second image thresholding follows the registration step. Then we can fully use the information of  $MS$  and make the "best" image thresholding. 2) The transformation error of binary image is more than that of grey level image, because the grey level image can reduce errors by adopting linear or quadratic interpolation so that the difference image can contain as less false errors as possible. Next we will discuss them in details.

#### 3.1 Registration image thresholding

Otsu method<sup>[9]</sup> is an adaptive nonparametric non-monitor thresholding method based on discrimination measure criterion. The best threshold is achieved when the measure function reaches the maximum. However, Otsu method only uses one-dimensional distribution of grey level histogram. It does not contain the relevant image spacial information, and threshold segmentation results are not ideal. The method is easily extended to two-dimensional grey level histogram. Meanwhile, the grey level distribution of pixels and the average grey level distribution of their neighbor pixels are considered. The threshold  $t^*$  is the maximum under the two-dimensional criterion. Then we further consider it with re-

spect to  $MS$ . In fact, the image is smoothed by  $3 \times 3$  neighborhood, and the smoothed imaged is processed later on. The main purpose is to make the histogram smoother. There are two obvious peaks in the smoothed histogram. The experimental result shows that the  $t^*$  chosen by the extended Otsu method is not ideal and the value is generally smaller than the best one. Therefore, we need the following theorem.

**Theorem 1.** The following convex programming problem

$$(QP) \quad \min\{q(t) = |at^2 + bt + c| \ (|a| + |b| > 0); t^* \leq t \leq T\}$$

exists the optimal solution  $t_{opt}$ , and

i) if the equation

$$at^2 + bt + c = 0 \tag{1}$$

has a real solution  $\bar{t}$  which satisfies  $t^* \leq \bar{t} \leq T$ , then  $t_{opt} = \bar{t}$ ;

ii) if Equation (1) does not have any solution which satisfies  $t^* \leq \bar{t} \leq T$ , then  $t_{opt} = t'$ , where  $t' = \arg \min\{q(t) : t = t^* \text{ or } t = T \text{ or } t = -b/(2a)\}$  if  $a \neq 0$ , otherwise  $t' = \arg \min\{q(t) : t = t^* \text{ or } t = T\}$ .

**Proof.** i) If Equation (1) has a real solution, which satisfies  $t^* \leq \bar{t} \leq T$ , then  $q(\bar{t}) = 0$  and  $q(t) \geq 0$  for any real  $t$ . Thus  $\bar{t}$  is the optimal solution of (QP). ii) If Equation (1) does not have any real solution, then  $a \neq 0$ . Therefore  $at^2 + bt + c$  does not change sign for any real  $t$ . If  $at^2 + bt + c > 0$  for any real  $t$ , then form  $q(t) = at^2 + bt + c$ ,  $q'(t) = 2at + b$ , the solution of  $q'(t) = 0$  is  $t_0 = -b/(2a)$ . If  $t_0 \in [t^*, T]$ , then  $t_0$  is the optimal solution of (QP); if  $t_0 \notin [t^*, T]$ , then  $q(t)$  is monotonous in  $[t^*, T]$ , therefore  $t^*$  or  $T$  is the optimal solution of (QP). If  $at^2 + bt + c < 0$  for any real  $t$ , then  $q(t) = -at^2 - bt - c$ ,  $q'(t) = -2at - b$ . The solution of  $q'(t) = 0$  is  $t_0 = -b/(2a)$ . As the above discussion,  $t_0$  or  $t^*$  or  $T$  is the optimal solution of (QP). iii) If Equation (1) has a real solution, which satisfies  $t^* \leq \bar{t} \leq T$ , then  $at^2 + bt + c$  does not change sign in  $[t^*, T]$  (otherwise, there must exist zero point of  $q(t)$  in  $[t^*, T]$ ). a) If  $q(t)$  is monotonous in  $[t^*, T]$ , then  $t^*$  or  $T$  must be the optimal solution of (QP); b) If  $q(t)$  is not monotonous in  $[t^*, T]$ , then  $a \neq 0$  and  $t_0 = -b/(2a)$  is the maximal point of  $q(t)$ , therefore  $t^*$  or  $T$  is the optimal solution of (QP). The theorem is proved.  $\square$

Let  $sum_0$  be the total number of foreground points of  $MS$ , and  $sum(t)$  be the total number of foreground points of binary SS with threshold  $t$ . If the SS is genuine or comes from a forgery seal very similar to the original one, then under the ideal conditions, the numbers of foreground points of these two binary seal imprints should be the closest. Therefore, the optimal threshold should be the optimal solution of the following mathematical programming problem:

$$(P_1) \quad \min\{|sum(t) - sum_0| : t^* \leq t \leq T\}$$

To make it simple,  $sum(t)$  can approximately be a quadratic function between  $t^*$  and  $T$ :  $f(t) = at^2 + bt + c_0$ . Let  $c = c_0 - sum_0$ ,  $q(t) = |at^2 + bt + c|$ , then  $(P_1)$  is changed into the above (QP) in Theorem 1. From Theorem 1, we obtain the optimal threshold  $t_{opt}$ .

The above  $t_{opt}$  is the globally optimal threshold. However, it is not the best one. In the following section, the verification image thresholding method is considered. Let  $g_M(p)$  be the value of point  $p$  on  $MS$  and  $g_M(p) \in \{0, 1\}$ , and  $g_M(p) = 1$  if  $p$  is the foreground point of  $MS$ , otherwise  $g_M(p) = 0$ . Similarly, we can define the function  $g_s(q)$  of  $RBS$ .

### 3.2 Registration

**Definition 8.** Let  $J(\cdot)$  be the cost function,  $J(f) = \sum_{p \in MS} g_M(p) \oplus g_s(f(p))$  where  $f: MS \rightarrow RBS$  is a mapping. We call  $f^*: MS \rightarrow RBS$  to be registration mapping if  $f^*$  is the optimal solution of problem  $\min(J(f) : f: MS \rightarrow RBS \text{ is a mapping})$ .

We need to set up a searching algorithm to get the mapping between  $MS$  and  $RBS$   $f: MS \rightarrow RBS$ . The binary seal imprint image would lose some local information compared

to the original grey level image no matter how the thresholding works. In general, the image registration process needs some stable reference points. By searching and matching according to the shapes and directions of these stable reference points, the congruent mapping  $f$  for all pixels can be obtained. Therefore, the main difficulty of former registration methods is that stable reference points are unavailable. The topological structure of seal imprint is used as a basis in our registration procedure. The reasons are that "stroke skeleton shows extremely relative stability, i. e., it is not disturbed by stamping conditions, stroke thickness or thinness, etc." A lot of experimental results show that it is feasible to get the registration mapping  $f$  by matching the feature points selected from the skeleton image. The additional merit of this method is that it is not restricted by geometry shape of seals. Next we are going to introduce the registration procedure. The procedure includes two sub-procedures, i. e., coarse and fine registration procedures. The binary images  $MS$  and  $RBS$  are thinned<sup>[10]</sup> and the end points and branch points of their stroke skeleton are extracted as feature points. Let  $p$  be a feature point.

**Definition 9.** The number of edges starting from feature point  $p$  is called branch number of feature point  $p$ , denoted by  $n(p)$ . Generally, we assume that  $n(p) \leq 4$ .

**Definition 10.** The feature point which can be directly reached from feature point  $p$  through its branch is called a neighbor feature point of  $p$ . The set of all neighbor feature points of  $p$  is denoted by  $N(p) = \{p_1, p_2, \dots, p_{n(p)}\}$ .

**Definition 11.** Let  $p_c$  be the center of gravity of a seal imprint image,  $p$  be a feature point and  $N(p) = \{p_1, p_2, \dots, p_{n(p)}\}$  be the set of all neighbor feature points of  $p$ ,  $p_c$  and  $p_k$  belonging to  $N(p)$ . The angle between direction  $\overrightarrow{pp_c}$  and  $\overrightarrow{pp_k}$  is called a branch direction angle of feature point  $p$ , denoted by  $\varphi_k(p)$ ,  $0^\circ \leq \varphi_k(p) \leq 360^\circ (k=0, 1, 2, \dots, n(p))$ .

For convenience, the branch direction angles of feature point  $p$  are arranged by:  $\varphi_1(p) \leq \varphi_2(p) \leq \dots \leq \varphi_{n(p)}(p)$ . Let  $d_k(p)$  denote the distance between feature point  $p$  and its neighbor feature point  $p_k$ .

**Definition 12.** Let  $p_c$  and  $q_c$  be the centers of gravity of  $MS$  and  $RBS$ , respectively,  $p$  and  $q$  be a couple of feature points of  $MS$  and  $RBS$ , respectively,  $\delta_1, \delta_2, \delta_3 > 0$  be the three given thresholds,  $k \in \{1, 2, \dots, n(p)\}$ . We believe  $p$  can match  $q$  if they satisfy the following conditions: 1)  $n(p) = n(q)$ , 2)  $|dist(p, p_c) - dist(q, q_c)| < \delta_1$ , 3)  $|\varphi_k(p) - \varphi_k(q)| < \delta_2$  and 4)  $|d_k(p) - d_k(q)| < \delta_2$ , where  $dist(p_1, p_2)$  denotes the distance between point  $p_1$  and point  $p_2$ .

Let the mapping between  $MS$  and  $RBS$   $f: MS \rightarrow RBS((x, y) \rightarrow (x', y'))$  be the rotation and translation transformation; then  $f$  can be determined by the following equation:

$$\begin{pmatrix} x' & -x'_0 \\ y' & -y'_0 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} x & -x_0 \\ y & -y_0 \end{pmatrix} \quad (2)$$

where  $\theta \in [-\pi, \pi]$  is rotation angle,  $(x_0, y_0)$  and  $(x'_0, y'_0)$  denote the coordinates of  $p_c$  and  $q_c$ , respectively. Now, we give the coarse registration algorithm based on skeleton feature points.

### Registration Algorithm.

Step 0. Divide the interval  $[-\pi, \pi]$  into  $n$  equal portions.

$$[\alpha_0, \alpha_1], [\alpha_1, \alpha_2], \dots, [\alpha_{n-1}, \alpha_n], \quad -\pi = \alpha_0 < \alpha_1 < \dots < \alpha_{n-1} < \alpha_n = \pi$$

Step 1. Select a couple of matchable feature points  $p$  and  $q$  with coordinates  $(x, y)$  and  $(x', y')$ , respectively. Angle  $\theta$  is computed by Equation (2). There may be several couples of matchable feature points, thus there will be several angles.

Step 2. Compute the number of points  $h_i$ , which can be matched for  $\theta \in [\alpha_{i-1}, \alpha_i]$  ( $1 \leq i \leq n$ ).

Step 3. Solve  $h_k = \max_{1 \leq i \leq n} h_i$  to get integer  $k$ , compute the average value of angles  $\theta$  in  $[\alpha_{k-1}, \alpha_k]$ , and then  $f(\theta_0, x_0, y_0)$  can be determined.

Step 4. Compute  $\theta^*$ ,  $\Delta x^*$ ,  $\Delta y^*$  to make the number of matchable feature points exceed the given threshold  $T(MS)$ . Thus the best coarse registration mapping  $f^*$  is got.

After the coarse registration mentioned above, the registration error between  $MS$  and  $RBS$  is very small. The translation errors in horizontal and vertical directions are smaller than 5 pixels, and the error of rotation angle is smaller than  $3^\circ$ . But we still cannot identify seal imprint exactly and must make further fine registration to achieve the best. This procedure is called minor adjustment. Let  $f(\Delta\theta, \Delta x, \Delta y)$  be the minor adjustment mapping; then the translation scales in two directions and rotation angle of compound mapping  $F = f \circ f^*$  are  $\Delta x^* + \Delta x$ ,  $\Delta y^* + \Delta y$  and  $\theta^* + \Delta\theta$ , respectively. The best mapping can be obtained by solving the following optimal problem:

$$\min\{J(F) : -3 \leq \Delta\theta \leq 3, -5 \leq \Delta x \leq 5, -5 \leq \Delta y \leq 5\}$$

#### 4 Comparison

The following Fig. 2 is the basic procedure of our comparison method.

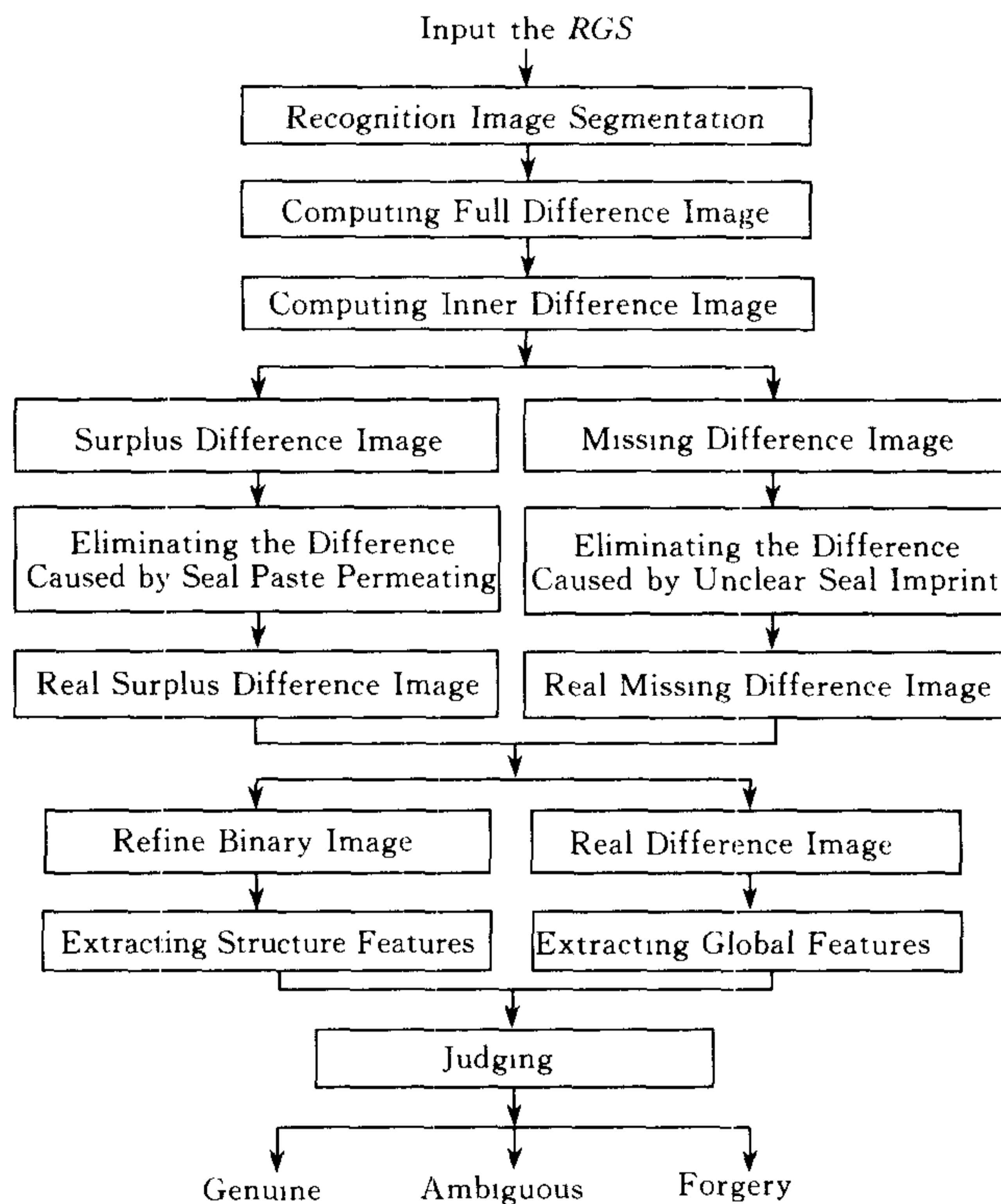


Fig. 2 Comparison procedure

##### 4.1 Verification image thresholding

Great progress has been made in the seal imprint verification methods based on difference image. However, these methods particularly depend on image thresholding. The image thresholding results will have a direct impact on the accuracy of verification. In practice, the unstable quality of seal imprint image is caused by uneven stamping or seal paste. The traditional thresholding algorithm cannot yield anticipated results. Therefore, it is very hard to discriminate real differences from false differences on the difference image. To identify seal imprints correctly, we must avoid all kinds of errors caused by various factors. The image thresholding is an important step. Its results will directly influence the verification effects and practical applicability. The former automatic identification algo-

rithm of seal imprints only contains one step of image thresholding. In the very beginning, the SS is segmented by a threshold, then the registration and identification are processed according to the binary image. However, stamping conditions are so complicated that it is very hard to meet the requirement no matter what thresholding methods are adopted. In this paper, we segment the SS by a thresholding method for registration, obtain translation scales in two directions and rotation angle, and translate and rotate grey level SS instead of binary SS. Then we segment the SS once again with the information of MS.

In the extended Otsu method mentioned above, an initial threshold could be obtained for any image. In the following, we further consider the image thresholding procedure with the information of MS. Experimental results show that, in many cases, the background grey level is not constant and the contrast of seals varies within the image. In such cases, a threshold that works well in one area of the image might work poorly in other areas. Therefore, it is convenient to use a threshold that is a slowly varying function of position in the image. The adaptive segmentation technique is implemented as a two-pass technique. Before the first pass, the image is divided into  $N$  ( $N = 8$  or  $16$  in general case) sectors. From these sectors, we set up an  $N$ -dimension optimal problem, then solve this problem and obtain an  $N$ -dimension optimal threshold vector. The following assumption is reasonable in practice.

**Assumption 1.** The SS comes from the corresponding MS or seals made to imitate the MS.

Let RGS be registration grey level image of GSS to MS. The RGS is divided into  $N$  sectors:  $RGS_1, RGS_2, \dots, RGS_N$ , correspondingly, MS is also divided into  $N$  sectors:  $MS_1, MS_2, \dots, MS_N$ , and there are some background points and foreground points in every sector. Let  $t_i^*$  ( $i = 1, 2, \dots, N$ ) be the initial threshold of sector  $RGS_i$ , which is obtained by the extended Otsu method. Denote  $\mathbf{t}^* = (t_1^*, t_2^*, \dots, t_N^*)^T \in R^N$ . Let  $\mathbf{t} = (t_1, t_2, \dots, t_N)^T \in R^N$ , the threshold of  $RGS_i$  be  $t_i$ ,  $RBS_i, (i = 1, 2, \dots, N)$  be the image after thresholding image by threshold  $t_i$ ,  $RBS = \bigcup_{i=1}^N RBS_i$ , the full deference image between RGS and MS

is  $Fdif(\mathbf{t}, RBS, MS) = \sum_{i=1}^N (MS_i \oplus RBS_i)$ , where  $MS_i \oplus RBS_i = \sum_{p \in X_i} (MS_i(p) \oplus RBS_i(p))$ ,

$X_i$  denotes the area of  $MS_i$  in the image plane. The optimal threshold vector is the optimal solution of the following  $N$ -dimension optimal problem:

$$(P) \quad \min \{ Fdif(\mathbf{t}, RBS, MS) : \mathbf{t} \in R^N \}$$

Our experimental results show that the solution  $\bar{\mathbf{t}}$  of (P) always satisfies  $\bar{\mathbf{t}} \geq \mathbf{t}^*$ . Therefore, problem (P) is equivalent to the following problem (P\*);

$$(P^*) \quad \min \{ Fdif(\mathbf{t}, RBS, MS) : \mathbf{t} \geq \mathbf{t}^* \}$$

Since function  $Fdif(\mathbf{t}, RBS, MS)$  has no analytic expression, it is difficult to solve problem (P\*). Therefore, we propose another optimal problem, whose solution is the approximately optimal threshold vector.

Let  $\sum_{p \in X_i} (MS_i(p) \oplus RBS_i(p)) \approx \frac{1}{2} a_i t_i^2 + b_i t_i + c_i = q(t_i)$ , ( $a_i > 0$ ). Then  $Fdif(\mathbf{t}, RBS, MS) \approx \sum_{i=1}^N \left( \frac{1}{2} a_i t_i^2 + b_i t_i + c_i \right) = \frac{1}{2} \mathbf{t}^T \mathbf{A} \mathbf{t} + \mathbf{b}^T \mathbf{t} + c = Q(\mathbf{t})$ , where

$$\mathbf{A} = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_N \end{pmatrix}, \quad \mathbf{b} = (b_1, b_2, \dots, b_N)^T, \quad c = \sum_{i=1}^N c_i$$

Then problem (P\*) can be rewritten as:

$$(QP) \quad \min \left\{ Q(\mathbf{t}) = \frac{1}{2} \mathbf{t}^T \mathbf{A} \mathbf{t} + \mathbf{b}^T \mathbf{t} + c : \mathbf{t}^* \leq \mathbf{t} \leq \mathbf{T} \right\}$$

It is clear that (QP) is a separable convex quadratic programming problem. The following theorem gives the solution of (QP).

**Theorem 2.** The separable convex quadratic programming problem (QP) has optimal solution  $\bar{\mathbf{t}}$ , i. e.,  $\bar{\mathbf{t}} = (\bar{t}_1, \bar{t}_2, \dots, \bar{t}_N)^T$ , where

$$\bar{t}_i = \arg \min \left\{ q_i \left( -\frac{b_i}{a_i} \right), q_i(t_i^*), q_i(T_i) \right\} \quad (i = 1, 2, \dots, N) \quad (3)$$

**Proof.** Since (QP) is a separable convex quadratic programming problem,  $\bar{\mathbf{t}} = (\bar{t}_1, \bar{t}_2, \dots, \bar{t}_N)^T$  is the optimal solution of (QP) if and only if  $\bar{t}_i$  is the optimal solution of the following problem:

$$(QP_i) \quad \min \{ q_i(t_i) : t_i^* \leq t_i \leq T_i \} \quad (i = 1, 2, \dots, N)$$

The optimal solution of problem (QP<sub>i</sub>) is as shown in (3). In fact, from  $q'_i(t_i) = a_i t_i + b_i$ , we know that the solution of equation  $q'_i(t_i) = 0$  is  $\tilde{t}_i = -b_i/a_i$ . If  $\tilde{t}_i \in [t_i^*, T_i]$ , then  $\tilde{t}_i$  is the optimal solution of (QP<sub>i</sub>); if  $\tilde{t}_i \notin [t_i^*, T_i]$ , then  $t_i^*$  or  $T_i$  is the optimal solution of (QP<sub>i</sub>). Therefore,  $\bar{t}_i$  is the optimal solution of (QP<sub>i</sub>), and  $\bar{\mathbf{t}} = (\bar{t}_1, \bar{t}_2, \dots, \bar{t}_N)^T$  is the solution of (QP). The proof is ended.  $\square$

## 4.2 The computing and processing of various difference images

### 4.2.1 The computing and processing of full difference image

The full difference image is the full difference between MS and SS. The difference may come from the real difference between the two seal imprints or stamping conditions. However, in addition to the background noise, there exists significant visual difference between the difference image which results from a forgery seal imprint and the difference image which is caused by a genuine seal imprint. That is, the difference image which results from a genuine seal imprint, mainly shows points or thin lines, while the difference image which is caused by a forgery seal imprint, may show a lot of blocks. In order to reduce the background noise, we propose the inner difference image, as mentioned in Definition 2 in Section 2.

### 4.2.2 The computing and processing of surplus difference image

Surplus difference image is the surplus part of SS compared with MS, as mentioned in Definition 4 in Section 2. We also consider it in grey-level and propose the grey level surplus difference image.

**Definition 13.** Let GSS be the grey level image after registration, GSdif is called the grey level surplus difference image, where

$$GSdif(i, j) = \begin{cases} GSS(i, j), & \text{if } Sdif(i, j) = 1 \\ 255, & \text{if } Sdif(i, j) = 0 \end{cases}$$

**Definition 14.** The thresholding image of grey level surplus difference image GSdif is called grey level surplus difference binary image, denoted by GSBdif.

Based on a great deal of experiments and observation, we find that the difference, resulting from hard stamping or seal paste permeating in the grey level surplus difference image, has a large grey level compared with main strokes of seal, and the difference resulting from forging should be approximately equal to the grey level of main strokes of seal. Therefore, the grey level surplus difference image processing procedure includes image segmentation based on main strokes threshold and the elimination of the difference caused by seal paste permeating. Then the real surplus difference image is obtained.

1) Segment image based on main strokes threshold: the average grey level  $T_m$  of main strokes of grey level SS is computed, then the grey level surplus difference image GSdif is segmented by the above  $T_m$  to obtain grey level surplus difference binary image GSBdif:

$$GSBdif(i, j) = \begin{cases} 1, & \text{if } GSdif(i, j) \leq T_m \\ 0, & \text{otherwise} \end{cases}$$

2) Eliminate the difference caused by seal paste permeating: the great majority of



foreground points in the grey level surplus difference image  $GSdif$  result from seal paste permeating. If some of them caused by forging, their grey level should approximately equal the grey level of the average value of main strokes. Therefore,  $GSBdif$ , the grey level surplus difference binary image after the above processing, should be the approximately real surplus difference image.

#### 4.2.3 The computing and processing of missing difference image

The missing difference image is the missing part in  $SS$  compared to  $MS$ . The missing part results from light stamping or short of seal paste if the  $SS$  is genuine. The missing difference may be caused by stroke missing if the sample seal is forgery. We propose the concept of grey level missing difference image.

**Definition 15.** Let  $GSS$  be the grey level image after registration,  $GMdif$  is called grey level missing difference image, where

$$GMdif(i,j) = \begin{cases} GSS(i,j), & \text{if } Mdif(i,j) = 1 \\ 255, & \text{if } Mdif(i,j) = 0 \end{cases}$$

**Definition 16.** The thresholding image of grey level missing difference image  $GMdif$  is called grey level missing difference binary image, denoted by  $GMBdif$ .

We will re-pick out the strokes from the grey level missing difference image, which have been "lost" because of the stamping conditions. A great deal of experimental results and observations show that the missing part in the grey level missing difference image caused by light stamping or shortness of seal paste is generally the larger grey level area. This area is easily handled as background. The missing part in the grey level missing difference image caused by forgery is generally as the same grey level as the background. Therefore, these two types of missing parts cannot be distinguished by usual processing methods. In order to re-pick out the strokes resulting from stamping conditions, we segment the grey level missing difference image by thresholding, and re-pick out strokes from the missing difference image. The real missing difference image is obtained. Now, we will introduce the processing steps.

1) Segment image based on "the threshold of the strokes to be re-picked out": the average grey level  $T_b$  of background of grey level  $SS$  is computed, then the grey level missing difference image  $GMdif$  is segmented by the above  $T_b$  to obtain grey level missing difference binary image  $GMBdif$ :

$$GMBdif(i,j) = \begin{cases} 1, & \text{if } GMdif(i,j) \leq T_b \\ 0, & \text{otherwise} \end{cases}$$

2) Re-pick out the strokes that have been "lost" because of the stamping conditions: some light foreground parts in  $GMdif$  are caused by stamping conditions. Therefore, if some strokes in  $GMdif$  result from the stamping conditions, their grey level should be slightly different from the average grey level of background although they may not be very clear. On the other hand, if some strokes in  $GMdif$  are caused by forgery, their grey level should be as same as the average grey level of background. Therefore, strokes re-picked out from missing difference image are subtracted from  $Mdif$ . The real missing difference image should be the approximately real missing difference image.

Fig. 3(a), (c), (e) are full difference image, real difference image and the final difference image corresponding to a genuine  $SS$ , respectively; Fig. 3(b), (d), and (f) are full difference image, real difference image and the final difference image corresponding to a forgery  $SS$ , respectively; Fig. 3(g) is an example of full difference image and Fig. 3(h) is its corresponding inner difference image.

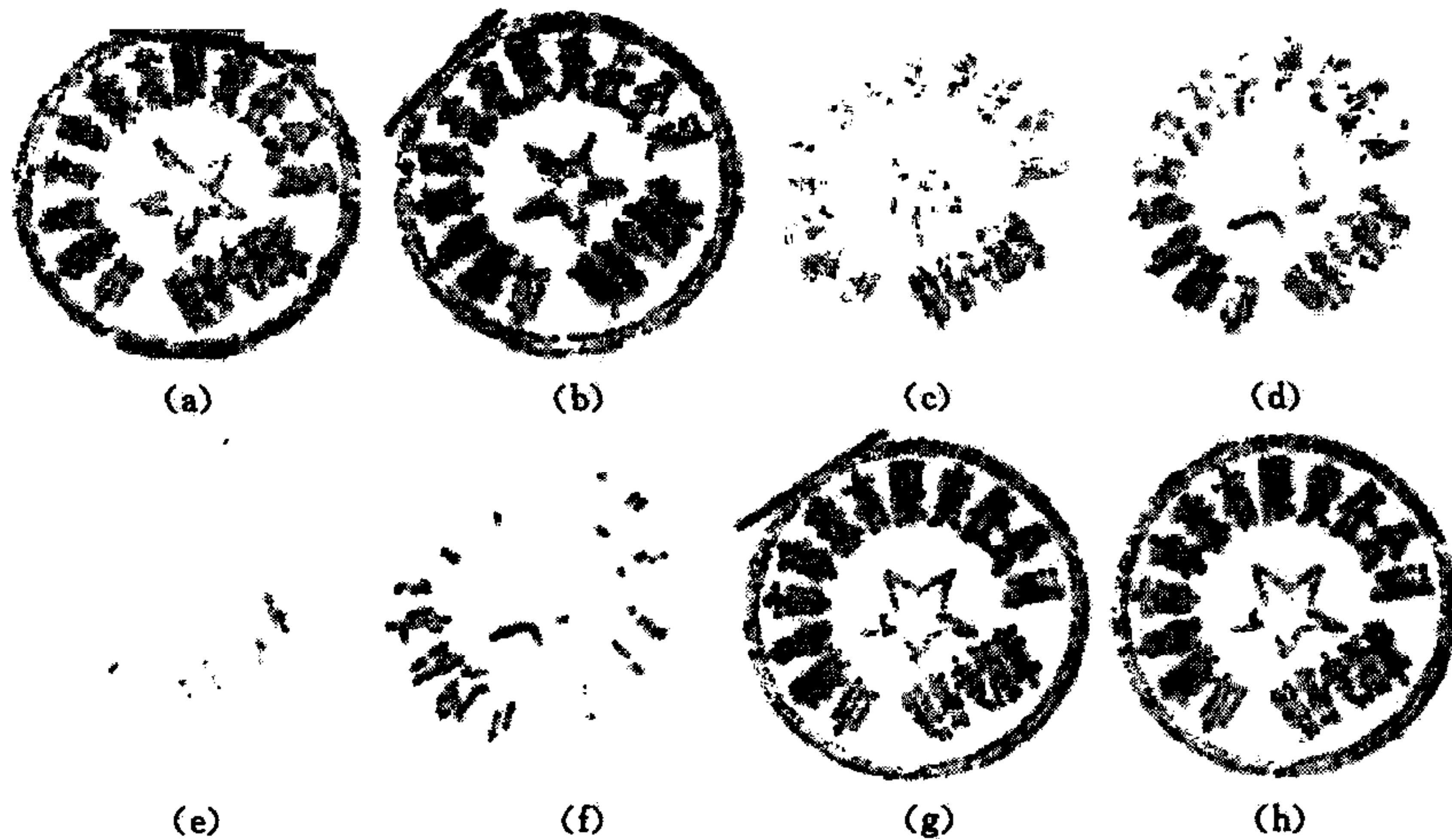


Fig. 3. The examples of difference images

### 4.3 Feature extractions and verification

After the refined processing of difference image, the “real surplus difference image” and “real missing difference image” are obtained. To combine these two difference images, we get the “real difference image”  $Rdif$ , see (c) and (d) in Fig. 3. Furthermore, we also obtain the “refined verification binary  $SS$ ”  $RVBS$ , i. e.,

$$RVBS(i, j) = MS(i, j) \vee Rdif(i, j)$$

The lines and points in the real difference image are not the real difference between  $MS$  and  $SS$  and should be eliminated. By eliminating them, we obtain the last difference image, see (e) and (f) in Fig. 3. Based on the final difference image, the verification features are extracted. They include three types: structure features, global features and auxiliary features. Structure features contain mean square length error (MSLE), average shifting deviation (ASD), etc. Global features include area ratio, block features, etc. Auxiliary features contain direct stamping conditions and indirect stamping conditions. Feature extractions and verification are not described in details due to the length of this paper. Please see [4~8, 11].

## 5 Experimental results

We have asked six seal engravers to engrave six groups of different seals for a special financial purpose to verify our series of computation. Each engraver engraves one seal for a special financial purpose. Then the same engraver tries to engrave four other seals by modeling after the seal he has already engraved. Although the engraver tries his best to model after the first seal, even the five seals engraved by the same person cannot be exactly the same in theory. There should exist some tiny difference. Nevertheless, since the five seals are from the same engraver, the differences are sometimes so tiny that it is very hard to discriminate them by eyes. The experimental goal is to check whether the tiny difference can be found through the method mentioned above. If differences are verified in such seals, we believe that most forgery seals should be detected. Each of the 30 seals is stamped 10 times on different Chinese bank checks only for account. The total of 300 checks correspond to six groups of seals. The banks include the Industrial and Commercial Bank of China, the Construction Bank of China, the Beijing Branch of Guangdong Development Bank and so on. In each group of seals, suppose one seal is the standard seal once a time, its seal imprint on the white paper is the standard seal imprint, while its seal imprints on the 10 checks and the seal imprints on the other 40 checks stamped by the other

four seals are sampling seal imprints. Compare the one standard seal imprint with the 50 sampling seal imprints. Each seal is used as a standard seal once in each group, and each standard seal imprint is compared 50 times with sampling seal imprints on the checks. Therefore, every group of standard seal imprints are compared 250 times with sampling seal imprints on the checks. The total six groups of standard seal imprints are compared 1,500 times with sampling seal imprints. It takes an average of about 0.7 seconds to compare once a time by Pentium 4 computer. The comparison results are shown in Table1.

The experimental results show that verification correct ratio is about 99%, the ambiguous percentage is 0.8%, and error ratio is 0.02%. There are three errors in the 1,500 times of comparison. The error in the fourth group is made because of bad quality of seal imprints, while the other two errors are caused by the high similarity of two seals.

## 6 Conclusions

This paper mainly discusses the identification of seal imprints and each phrase of verification, provides registration image thresholding and verification image thresholding, and proposes several types of difference images and image processing methods, i. e., full difference image, inner difference image, surplus difference image and missing difference image, so as to make the identification of seal imprints based on difference images more practical.

Table 1 Experimental results

Seals	Group1	Group2	Group3	Group4	Group5	Group6	Totals
No. of Comparison	250	250	250	250	250	250	1500
No. of Genuine	50	50	50	50	50	50	300
No. of Forgery	200	200	200	200	200	200	1200
No. of Genuine As Genuine	50	50	49	48	50	49	296
No. of Forgery As Forgery	196	199	199	198	197	199	1188
No. of Ambiguous	3	1	2	3	2	1	12
No. of False Judgments	1	0	0	1	0	3	3

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## 印鉴自动识别算法研究

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**摘要** 印鉴在许多东方国家被广泛使用了多年. 印鉴自动识别是模式识别中的一项非常困难的课题. 文中对计算机印鉴自动识别的各个阶段进行了仔细研究, 并给出一系列相关算法. 为了把“伪造印鉴多出的笔划部分”与“由于印泥渗出而多出的部分”区别开来以及将“伪造印鉴少出的笔划部分”与“由于印泥少而盖印不清晰的部分”区别开来, 文中提出了全差图、内差图、多差图、少差图等一系列差图的概念, 并对这些差图进行了处理. 在此基础上, 完成了一套完整的计算机印鉴自动识别方法. 实验结果表明, 该方法可能在实际应用中是可行的.

**关键词** 模式识别, 印鉴自动识别, 特征提取

**中图分类号** TP391