第 29 卷 第 6 期 自 动 化 学 报 Vol. 29, No. 6
2003 年 11 月 ACTA AUTOMATICA SINICA Nov., 2003

Adaptive Robust Tracking Scheme for Uncertainty Systems[1)]

LI Sheng-Ping
(*Department of Mechatronics Engineering, Shantou University, Shantou* 515063)
(E-mail: spli@stu.edu.cn)

Abstract An adaptive robust tracking problem is investigated when a discrete-time plant is subject to both unmodelled dynamics and unknown external disturbances. Firstly, combining the ℓ_1 optimization and deadbeat control scheme we present a procedure for designing the optimal robust steady tracking controller. Then, based on the idea of set-membership identification, we propose a recursive estimation for extended parameters which include the parameters of nominal model and the bound of unmodelled dynamics and disturbances. Finally, we propose a novel adaptive robust tracking scheme, and prove the overall convergence of the adaptive algorithm. For this scheme a computable tight upper bound on robust tracking performance is also provided. The adaptive scheme proposed in this paper has non-conservative robust stability and asymptotically optimal robust performance.

Key words Unknown disturbances, unmodeled dynamics, adaptive control, robust tracking

1 Introduction

The ℓ_1 design methodology, formulated in the mid of 1980's, is concerned with designing a feedback controller that reduces the effect of uncertainty on system[1]. If this uncertainty is in the input/output, that is, the plant is subject to unknown external persistent disturbances, ℓ_1 controller can be designed to reduce the effect of disturbances on output to the greatest extent. If the uncertainty is in the plant, that is, the plant is subject to unmodelled dynamics, ℓ_1 controllers can be designed to robustly stabilize the plant. Therefore, the ℓ_1 optimization is recognized as a kind of practical robust design methodology[2].

There are many advantages using the ℓ_1 optimal design method to deal with the uncertainty system. For this reason, application of the ℓ_1 design method to develop adaptive robust control has become an important and significant research subject. In [3], the ℓ_1 optimal design method is firstly used for the design of adaptive control scheme for the optimal rejection of unknown persistent disturbances. In [4], the adaptive robust stabilization of the plant with unmodelled dynamics has been investigated, and an adaptive control scheme was provided based on the certainty equivalence principle. This work is extended to systems with coprime factor perturbations and external disturbances in [5]. Up to this point, the existing research only addresses the problem of adaptive robust stabilization, and the conditions of robust stability are very conservative as compared with non-adaptive control. Moreover, it is difficult to consider the adaptive robust performance, such as robust tracking performance, in the framework available.

This paper investigates an adaptive robust tracking problem when a discrete-time plant is subject to both unmodelled dynamics and unknown external disturbances. First, combining the ℓ_1 optimization and deadbeat control scheme we present an exact formula for computing the optimal robust steady tracking error. Using this formula we show that the robust tracking performance optimization is equivalent to robust stability optimization, and both are reduced to a standard linear programming problem. Then, based on the idea of

1) Supported by National Natural Science Foundation of P. R. China (60374009, 59975056) and Natural Science Foundation of Guangdong Province (990795)
Received December 30, 2002; in revised form May 14, 2003
收稿日期 2002-12-30; 收修改稿件日期 2003-05-14

set-membership identification, we propose a recursive estimation for extended parameters which include the parameters of nominal model and the bound of unmodelled dynamics and disturbances. Finally, we propose a novel adaptive robust tracking scheme, a tight bound for robust performance and prove the overall convergence of the adaptive algorithm. The adaptive scheme proposed in this paper has non-conservative robust stability and asymptotically optimal robust performance.

2 Notations

$$\|x\|_\infty \stackrel{def}{=} \max_i |x(i)|, \quad \ell_\infty \stackrel{def}{=} \{x: \|x\|_\infty < \infty\}, \quad \|x\|_1 \stackrel{def}{=} \sum_{i=0}^{\infty} |x(i)|,$$
$$\ell_1 \stackrel{def}{=} \{x: \|x\|_1 < \infty\}, \quad c_0 \stackrel{def}{=} \{x \in \ell_\infty \mid \lim_{k\to\infty} x(k) = 0\};$$

Let $\hat{x} \stackrel{def}{=} \sum_{i=0}^{\infty} x(i) z^i$ denote the z-transform of sequence x. It is clear that z is equivalent to the forward shift operator, and that the normed space A is isomorphic to ℓ_1 space. So x and $\hat{x}$ can be viewed as the same where no confusion occurs.

Let $R[z]$ denote all the real coefficient polynomials of complex variant z, and $\partial(\hat{x})$ denote the degree of polynomial $\hat{x}$. Clearly, $R(z) \in A$.

Let S_d denote the right shift operator with d steps, i. e., $S_2(x(0), x(1), x(2), \cdots) = (0, 0, x(0), x(1), x(2), \cdots)$.

3 Optimally robust steady tracking control

Consider the class of uncertainty SISO discrete-time systems described by

$$(\hat{a}(z) - \Delta) y(t) = z^d \hat{b}(z) u(t) + v(t) \tag{1}$$

where $y(t)$, $u(t)$ and $v(t)$ denote the output, input and external disturbance, respectively, t is the discrete time, d is the controlled delay.

$$\hat{a}(z) = 1 + a(1) z^1 + \cdots + a(n) z^n, \ \hat{b}(z) = b(0) + b(1) z^1 + \cdots + b(m) z^m$$

where Δ denotes the unmodelled dynamics.

Assumption 1. The structural parameters n, m, d are known a priori. The polynomials $\hat{a}(z)$ and $\hat{b}(z)$ are coprime, that is, they do not have the same unstable zeros.

Assumption 2. The unmodelled dynamics $\Delta \in D_{\infty, FM}(w_\Delta)$, where $D_{\infty, FM}(w_\Delta) = \{\Delta: \Delta$ is the casual time-varying operator with finite-memory, and satisfies $\|\Delta\|_1 = \left\{\sup_{x \in \ell_\infty} \frac{\|\Delta x\|_\infty}{\|x\|_\infty} \leqslant w_\Delta\right\}$, which implies that there exists $\mu > 0$ such that $|\Delta x(t)| \leqslant w_\Delta \{\max_{0 \leqslant k \leqslant \mu} |x(t-k)|\}$, where $\Delta x(t) = \sum_{k=t-\mu}^{t} \Delta(t-k) x(k)$. The external disturbances satisfy $\sup_t |v(t)| \leqslant w_v$.

Consider the feedback system depicted in Fig. 1, where $\hat{c}$ is the controller, r is the eference signal.

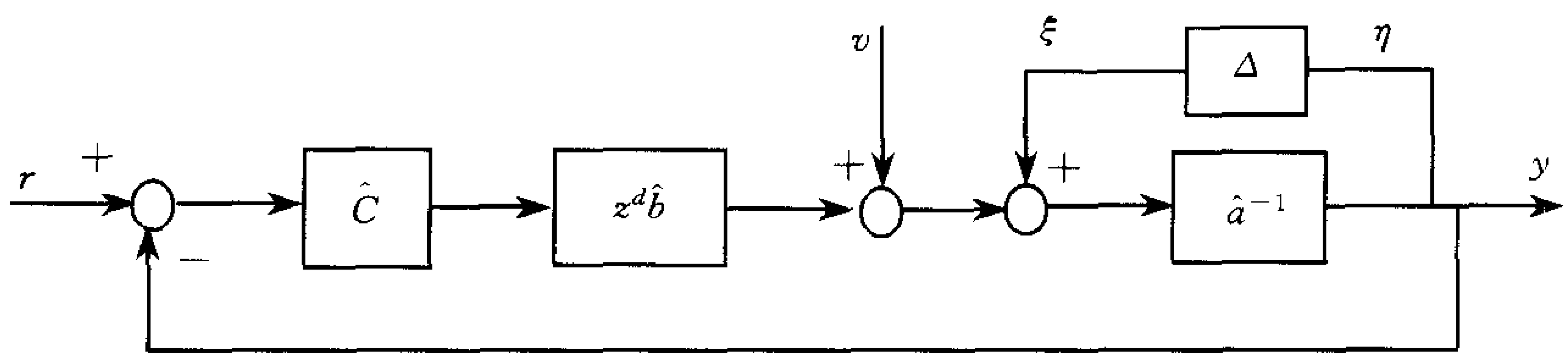

Fig. 1 Feedback system

Assumption 3. $\hat{r} \in D_r(\hat{a}_r)$, where $D_r(\hat{a}_r) = \{\hat{r} = \hat{b}_r \hat{a}_r^{-1}: \hat{a}_r \in R[z]$ is known, but $\hat{b}_r \in R[z]$ is unknown). The polynomials $\hat{a}_r$ and $\hat{b}$ are coprime.

Definition 1. The feedback system depicted in Fig. 1 is called robust ℓ_∞ stable if the system is inner-stable for any $\Delta\in D_{\infty,FM}(w_\Delta)$. Denote by $\hat{\varepsilon}=\hat{r}-\hat{y}$ the tracking error. If the plant is robust ℓ_∞ stable, and the tracking error is bounded. Then $\|\hat{\varepsilon}\|_{ss}=\lim_{t\to\infty}\sup|\varepsilon(t)|$ is defined as the steady state tracking error.

In this section, we provide, under Assumptions 1~3, a method for designing controllers such that feedback system depicted in Fig. 1 satisfies the following specifications:

1) For any given reference input $\hat{r}\in D_r(\hat{d}_r)$, the tracking error settles down to zero in a finite number of control steps in the case that $\Delta=0$ and $\hat{v}=0$;

2) The feedback system has optimally robust ℓ_∞ stability, and the worst-case steady state tracking errors

$$J_{\text{trac}}=\sup_{\Delta\in D_{\infty,FM}(w_\Delta)}\ \sup_{\|v\|_\infty\leqslant w_v}\|\varepsilon\|_{ss}$$

is minimized.

3.1 The formula of worst-case steady state tracking performance

In [6], The computation of formula of worst-case steady state tracking performance is firstly provided for the systems free from external disturbances. The following theorem, aiming at the systems with both unmodelled dynamics and external disturbances, provides the robust stable condition and the formula of worst-case steady state tracking performance in the same way.

Theorem 1. If the reference signals satisfy $\hat{G}_{yr}r\in\ell_\infty$ and $\hat{G}_{\eta r}r\in\ell_\infty$, where $\hat{G}_{yr}$ is the transfer function from r to y in the nominal system in Fig. 1, then

1) For the given $\Delta\in D_{\infty,FM}(w_\Delta)$ and external disturbance v, the closed-loop system depicted in Fig. 1 is robust stable if and only if $w_\Delta\|\hat{G}_{\eta\xi}\|_A\leqslant 1$;

2) If the closed-loop system is robust stable for $\Delta\in D_{\infty,FM}(w_\Delta)$, then

$$J_{\text{trac}}=\|(1-\hat{G}_{yr})r\|_{ss}+w_v\|\hat{G}_{yv}\|_A+w_\Delta\|\hat{G}_{y\xi}\|_A(1-w_\Delta\|\hat{G}_{\eta\xi}\|_A)^{-1}(\|\hat{G}_{\eta r}r\|_{ss}+w_v\|\hat{G}_{\eta v}\|_A).$$

Proof. Since the robust stability is not affected by the external disturbances, conclusion 1) follows from [6].

We shall prove conclusion 2) the same way as in [7,8]. The only difference here is to consider external disturbances. According to the Theorem 2 in [7], the closed-loop system which subject to $\Delta\in D_{\infty,FM}(w_\Delta)$ and disturbance $v(\|v\|_\infty\leqslant w_v)$ satisfies $\|y\|_{ss}\leqslant 1$ if and only if $\rho\begin{bmatrix}\|\hat{G}_{yr}r+\hat{G}_{yv}v\|_{ss} & w_\Delta\|\hat{G}_{y\xi}\|_A\\ \|\hat{G}_{\eta r}r+\hat{G}_{\eta v}v\|_{ss} & w_\Delta\|\hat{G}_{\eta\xi}\|_A\end{bmatrix}<1$

From $\|\hat{G}_{yr}r+\hat{G}_{yv}v\|_{ss}\leqslant\|\hat{G}_{yr}r\|_{ss}+w_v\|\hat{G}_{yv}\|_A$ and $\|\hat{G}_{\eta r}r+\hat{G}_{\eta v}v\|_{ss}\leqslant\|\hat{G}_{\eta r}r\|_{ss}+w_v\|\hat{G}_{\eta v}\|_A$, it follows that

$$\rho\begin{bmatrix}\|\hat{G}_{yr}r+\hat{G}_{yv}v\|_{ss} & w_\Delta\|\hat{G}_{y\xi}\|_A\\ \|\hat{G}_{\eta r}r+\hat{G}_{\eta v}v\|_{ss} & w_\Delta\|\hat{G}_{\eta\xi}\|_A\end{bmatrix}\leqslant\rho\begin{bmatrix}\|\hat{G}_{yr}r\|_{ss}+w_v\|\hat{G}_{yv}\|_A & w_\Delta\|\hat{G}_{y\xi}\|_A\\ \|\hat{G}_{\eta r}r\|_{ss}+w_v\|\hat{G}_{\eta v}\|_A & w_\Delta\|\hat{G}_{\eta\xi}\|_A\end{bmatrix}$$

Also, in the same argument as the Theorem 3 in [7], we can construct the signal v such that $\|\hat{G}_{yv}v\|_{ss}=w_v\|\hat{G}_{yv}\|_A$ and $\|\hat{G}_{\eta v}v\|_{ss}=w_v\|\hat{G}_{\eta v}\|_A$. This means that there exists v such that the equality in the above inequality is true. Therefore, Theorem 2 in [7] can be extended to the following conclusion: The closed-loop plant depicted in Fig1, subject to $\Delta\in D_{\infty,FM}(w_\Delta)$ and disturbance v ($\|v\|_\infty\leqslant w_v$), satisfies $\|y\|_{ss}\leqslant 1$ if and only if $\rho\begin{bmatrix}\|\hat{G}_{yr}r\|_{ss}+w_v\|\hat{G}_{yv}\|_A & w_\Delta\|\hat{G}_{y\xi}\|_A\\ \|\hat{G}_{\eta r}r\|_{ss}+w_v\|\hat{G}_{\eta v}\|_A & w_\Delta\|\hat{G}_{\eta\xi}\|_A\end{bmatrix}<1$. Also according to Lemma 2 in [8], we have

$$\sup_{\Delta\in D_{\infty,FM}(w_\Delta)}\ \sup_{\|v\|_\infty\leqslant w_v}\|y\|_{ss}=$$

$$\|\hat{G}_{yr}r\|_{ss}+w_v\|\hat{G}_{yv}\|_A+w_\Delta\|\hat{G}_{y\xi}\|_A(1-w_\Delta\|\hat{G}_{\eta\xi}\|_A)^{-1}(\|\hat{G}_{\eta r}r\|_{ss}+w_v\|\hat{G}_{\eta v}\|_A)$$

In the above equation, substituting $\|\hat{G}_{yr}r\|_{ss}$ with $\|(1-\hat{G}_{yr})r\|_{ss}$ we obtain conclusion 2). This completes the proof. □

3.2 The optimal steady state tracking controller

The stabilizing compensators of nominal system can be parameterized in the form [1]

$$\hat{C} = \hat{d}^{-1}\hat{n} = (\hat{y} - z^d\hat{b}\hat{q})^{-1}(\hat{x} + \hat{a}\hat{q}) \quad (2)$$

where $z^d\hat{b}\hat{x}+\hat{a}\hat{y}=1$, $\hat{y}-z^d\hat{b}\hat{q}\neq 0$, $\hat{q}$ is the free parameter satisfying $\hat{q}\in A$.

Since $G_{\varepsilon r}=1-G_{yr}$, it follows from Fig. 1 and (2) that

$$\hat{G}_{\varepsilon r}r = \hat{a}(\hat{y} - z^d\hat{b}\hat{q})\hat{b}_r\hat{a}_r^{-1} \quad (3)$$

Now we determine the free parameter $\hat{q}$ such that the system satisfies the specification 1), which implies that $\hat{G}_{\varepsilon r}r$ is a polynomial. Let $\hat{a}=\hat{a}_s\hat{a}_i$ and $\hat{b}=\hat{b}_s\hat{b}_i$, where $\hat{a}_i$ and $\hat{b}_i$ are unstable polynomials of $\hat{a}$ and $\hat{b}$, respectively. $\hat{a}_s$ and $\hat{b}_s$ are the rest unstable polynomials. Clearly, both $\hat{a}_s$ and $\hat{b}_s$ are stable polynomials. Define $\hat{g}=\hat{a}_i(\hat{y}-z^d\hat{b}\hat{q})\hat{a}_r^{-1}$, we have

$$\hat{q} = \frac{\hat{a}_i\hat{y} - \hat{a}_r\hat{g}}{z^d\hat{a}_i\hat{b}} \quad (4)$$

It follows from (4) that $\hat{q}_2\in A$ if and only if $\frac{\hat{a}_i\hat{y}-\hat{a}_r\hat{g}}{z^d\hat{a}_i\hat{b}_i}\in R[z]$. Let $\frac{\hat{a}_i\hat{y}-\hat{a}_r\hat{g}}{z^d\hat{a}_i\hat{b}_i}=\hat{h}$, $\hat{a}_i=\hat{a}_i'\hat{\beta}$, $\hat{a}_r=\hat{a}_r'\hat{\beta}$, where $\hat{\beta}$ is the greatest common divisor of $\hat{a}_i$ and $\hat{a}_r$. Then (4) can be rewritten as the following Diophantine equation:

$$z^d\hat{a}_i'\hat{b}_i\hat{h} + \hat{a}_r'\hat{g} = \hat{a}_i'\hat{y} \quad (5)$$

From Assumption 3, (5) has solutions. Denoting by $\hat{g}_0$ and $\hat{h}_0$ the smallest degree solution, it is clear that all solutions of (5) can be represented as

$$\hat{h} = \hat{h}_0 + \hat{a}_r'\hat{k} \quad (6)$$

$$\hat{g} = \hat{g}_0 - z^d\hat{a}_i'\hat{b}_i\hat{k} \quad (7)$$

where the free parameter $\hat{k}\in R[z]$ is an arbitrary chosen polynomial. From (4)~(6), the controllers that satisfy the specification 1) can be represented as (2) with the free parameter

$$\hat{q} = \frac{\hat{h}_0 + \hat{a}_r'\hat{k}}{\hat{b}_s} \quad (8)$$

where $\hat{k}\in R[z]$. With this controller, it is clear that $\hat{g}$ is a polynomial, thus

$$\hat{G}_{\varepsilon r}r = \hat{a}_s\hat{g}\hat{b}_r \in R[z] \quad (9)$$

From (6) and (8), it follows that $\hat{q}\in A$. Thus, (9) implies that with any controllers of (2) with $\hat{q}=\frac{\hat{h}_0+\hat{a}_r'\hat{k}}{\hat{b}_s}$, the tracking error settles down to zero in a finite number of control steps for any given reference input $\hat{r}\in D_r(\hat{d}_r)$. That is, with this controller the specification 1) is satisfied.

Therefore, for a given $\hat{r}\in D_r(\hat{d}_r)$, the problem of designing optimal robust tracking controller can be reduced to that of choosing $\hat{k}\in R[z]$ such that $J_{\text{trac}} = \sup_{\Delta\in D_{\infty,FM}(w_\Delta)} \sup_{\|v\|_\infty\leqslant w_v} \|\varepsilon\|_{ss}$ is minimized. From (2), (8) and Fig. 1, we have the following transfer functions:

$$\hat{G}_{yv} = \hat{G}_{y\xi} = \hat{G}_{\eta\xi} = \hat{G}_{\eta v} = \hat{y} - z^d\hat{b}\hat{q} \quad (10)$$

$$\hat{G}_{\varepsilon r} = \hat{a}(\hat{y} - z^d\hat{b}\hat{q}) \quad (11)$$

$$\hat{G}_{\eta r} = z^d\hat{b}(\hat{x} + \hat{a}\hat{q}) \quad (12)$$

It follows from (9) that $\|(1-\hat{G}_{yr})r\|_{ss} = \|\hat{G}_{\varepsilon r}r\|_{ss} = \|\hat{a}_s\hat{g}\hat{b}_r\|_{ss} = 0$. Also, from (11) and (12), we have $\hat{G}_{\varepsilon r}+\hat{G}_{\eta r}=1$, thus $\|G_{\eta r}r\|_{ss} = \|r-\hat{G}_{\varepsilon r}r\|_{ss} = \|r\|_{ss}$. Then, from (10), the worst-case steady state tracking error can be written as:

$$J_{\text{trac}} = w_v\|\hat{G}_{\eta\xi}\|_A + w_\Delta\|\hat{G}_{\eta\xi}\|_A(1 - w_\Delta\|\hat{G}_{\eta\xi}\|_A)^{-1}(\|r\|_{ss} + w_v\|\hat{G}_{\eta\xi}\|_A) \quad (13)$$

Thus, from (13) and Theorem 1, it follows that the problem of minimization of J_{trac} is equivalent to the one of optimization of robust stability, both are reduced to the same ℓ_1 optimization problem

$$\mu = \inf_{\hat{q}\in A}\|\hat{G}_{\eta\xi}\|_A = \inf_{\hat{q}\in A}\|\hat{y} - z^d\hat{b}\hat{q}\|_A \quad (14)$$

Substituting (8) into (14), we have $\mu=\inf_{\hat{k}\in R[z]}\|(\hat{y}-z^d\hat{b}_i\hat{h}_0)-z^d\hat{a}_r'\hat{k}\|_A$. Let $\hat{\alpha}=\hat{y}-z^d\hat{b}_i\hat{h}_0$, $\hat{\beta}=z^d\hat{a}_r'$, $n_\alpha=\partial(\hat{\alpha})$, $n_\beta=\partial(\hat{\beta})$, the problem (14) can be rewritten as

$$\mu = \inf_{k \in R^p} \|\alpha + \beta \times k\|_1 \tag{15}$$

where p is an positive integer not less than zero. From (8)~(10) and (15), we can conclude that the degree of controller and the settling time increase but μ decreases as p increases.

Let $M=\max\{n_\alpha, n_\beta + p\}$, and the optimization problem (15) is equivalent to

$$\mu = \inf_{k \in R^p} \left\{ \sum_{t=0}^{M} \left| \alpha(t) + \sum_{j=0}^{t} \beta(t-j)k(j) \right| \right\} \tag{16}$$

Let $\alpha(t) + \sum_{j=0}^{t}[\beta(t-j)k(j)] = \phi(t)^+ - \phi(t)^-$, where $\phi(t)^+ \geqslant 0$, $\phi(t)^- \geqslant 0$. From [9], it follows that $\sum_{t=0}^{M} \left| \alpha(t) + \sum_{j=0}^{t} \beta(t-j)k(j) \right| = \sum_{t=0}^{M}[\phi(t)^+ + \phi(t)^-]$. Thus the problem (16) is equivalent to the following linear programming (LP) problem:

$$\mu = \min \gamma \tag{17}$$

$$\text{s.t.} \quad \gamma - \sum_{t=0}^{M}[\phi(t)^+ + \phi(t)^-] \geqslant 0$$

$$\phi(t)^+ - \phi(t)^- - \sum_{j=0}^{t}[\beta(t-j)k(j)] = \alpha(t)$$

$$\phi(t)^+ \geqslant 0,\ \phi(t)^- \geqslant 0,\ t = 0, 1, \cdots, M$$

Let $\hat{k}^*$ denote the solution of the above LP. Substituting it into (8), we can get the optimal parameter $\hat{q}$.

From the above argument, we obtain the following theorem.

Theorem 2. If the controller of (2) is designed according to (5)~(8), then the optimization of robust steady state tracking performance is equivalent to the optimization of robust stability. The problem of designing controller is reduced to solving the standard LP (17). For a given p, the minimized worst-case steady state tracking error is represented as

$$J_{\text{trac}} = w_v \mu + \frac{w_\Delta \mu}{1 - w_\Delta \mu}(\|r\|_{ss} + w_v \mu)$$

where μ is the solution of LP (17).

4 Adaptive robust steady state tracking control

4.1 Adaptive control algorithm

We will consider the system described by (1), which can be rewritten as

$$y(t) = \phi(t-1)^{\mathrm{T}}\theta + (-\Delta y)(t) + u(t) + v(t) \tag{18}$$

where $\theta^{\mathrm{T}} = (a(1), \cdots, a(n), b(0), \cdots, b(m))$ and $\phi(t-1)^{\mathrm{T}} = (-y(t-1), \cdots, -y(t-n), u(t-d), \cdots, u(t-m-d))$.

Assumption 4. $\Delta \in D_{\infty, FM}(w_\Delta)$, $\sup_t |v(t)| \leqslant w_v$, and parameter θ, w_Δ and w_v are unknown, but $0 \leqslant w_\Delta \leqslant \overline{w}_\Delta$, $0 \leqslant w_v \leqslant \overline{w}_v$, where $\overline{w}_\Delta$ and $\overline{w}_v$ are known.

In the following, we will present a recursive parameter estimation based on set-membership identification. Denote the present and past observations of the system output and input by $\{y(0), y(1), \cdots, y(t)\}$ and $\{u(0), u(1), \cdots, u(t-d)\}$, respectively. Denote the extended parameter vector by $\bar{\theta} = (\theta^{\mathrm{T}}, w_\Delta, w_v)^{\mathrm{T}}$. From (18) and Assumption 4, all extended parameter vectors that are compatible with the prior information and observations satisfy the following inequality

$$|y(k) - \phi(k-1)^{\mathrm{T}}\theta| \leqslant |\Delta y(k)| + |v(k)| \leqslant w_\Delta\{\max_{k-\mu \leqslant s \leqslant k} |y(s)|\} + w_v,\ 0 \leqslant k \leqslant t \tag{19}$$

Let $\phi_y(t-1)^{\mathrm{T}} = (-y(t-1), \cdots, -y(t-n))$, $\phi_u(t-1)^{\mathrm{T}} = (u(t-d), \cdots u(t-m-d))$, $s(t) = \mathrm{Sign}\{y(t) - \phi(t-1)^{\mathrm{T}}\theta(t-1)\}$, $\psi_1(t-1)^{\mathrm{T}} = [s(t)\phi_y(t-1)^{\mathrm{T}}, \max_{t-\mu \leqslant s \leqslant t} |y(s)|, 1]^{\mathrm{T}}$, $\psi(t-1)^{\mathrm{T}}$

$=[s(t)\phi(t-1)^{\mathrm{T}}, \max_{t-\mu\leqslant s\leqslant t}|y(s)|, 1]^{\mathrm{T}}$. Denote the parameter estimation at time t by $\bar{\theta}(t)=[\theta(t)^{\mathrm{T}}, w_{\Delta}^{t}, w_{v}^{t}]^{\mathrm{T}}$, where $\theta(t)=(a^{t}(1),\cdots,a^{t}(n),b^{t}(0),\cdots,b^{t}(m))^{\mathrm{T}}$ is an estimation of parameter θ at time t. From inequality (19), it is clear that $\bar{\theta}(t)$ satisfies

$$s(t)y(t)\leqslant \psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1) \tag{20}$$

Now, we present an estimation algorithm for extended parameter vector as follows

$$\hat{\bar{\theta}}(t)=\bar{\theta}(t-1)+\frac{\eta(t-1)\psi(t-1)}{\psi(t-1)^{\mathrm{T}}\psi(t-1)}[s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)] \tag{21}$$

where $\hat{\bar{\theta}}(t)=[\theta(t)^{\mathrm{T}}, \hat{w}_{\Delta}^{t}, \hat{w}_{v}^{t}]^{\mathrm{T}}$, and

$$\eta(t-1)=\begin{cases}1, & s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)>\delta\sqrt{\psi_1(t-1)^{\mathrm{T}}\psi_1(t-1)}\\ 0, & \text{otherwise}\end{cases} \tag{22}$$

where $\delta>0$ is a given weighting that specifies the dimension of dead zone. If the estimations $\hat{w}_{\Delta}^{t}$ and $\hat{w}_{v}^{t}$ obtained by algorithm (21) and (22) do not satisfy Assumption 4, then w_{Δ}^{t} and w_{v}^{t} are chosen as follows

$$\text{if } \hat{w}_{\Delta}^{t}\leqslant 0,\text{ then } w_{\Delta}^{t}=0;\ \text{if } \hat{w}_{\Delta}^{t}\geqslant \bar{w}_{\Delta},\text{ then } w_{\Delta}^{t}=\bar{w}_{\Delta} \tag{23}$$

$$\text{if } \hat{w}_{v}^{t}\leqslant 0,\text{ then } w_{v}^{t}=0;\ \text{if } \hat{w}_{v}^{t}\geqslant \bar{w}_{v},\text{ then } w_{v}^{t}=\bar{w}_{v} \tag{24}$$

Lemma 1. The algorithm (21)~(24) has the following properties:

1) $\|\bar{\theta}(t)-\bar{\theta}_0\|^2\leqslant\|\bar{\theta}(t-1)-\bar{\theta}_0\|^2\leqslant\|\bar{\theta}(0)-\bar{\theta}_0\|^2$

2) $\lim\limits_{t\to\infty}\dfrac{[s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)]^2}{\psi(t-1)^{\mathrm{T}}\psi(t-1)}=0$

3) $\lim\limits_{t\to\infty}\|\bar{\theta}(t)-\bar{\theta}(t-1)\|=0$

If there exists a positive number c such that $\|\phi_u(t-1)\|\leqslant c$, then the algorithm (21)~(24) also has the following properties

4) There exists a positive integer N^* such that $\bar{\theta}(t)=\lim\limits_{t\to\infty}\bar{\theta}(t)=:\bar{\theta}(\infty)$ as $t\geqslant N^*$

5) For the above given N^*,

$$|y(t)-\phi(t-1)^{\mathrm{T}}\theta(\infty)|\leqslant(\delta\sqrt{n+1}+w_{\Delta}^{\infty})\{\max_{t-\mu\leqslant s\leqslant t}|y(s)|\}+(w_{v}^{\infty}+\delta) \tag{25}$$

as $t\geqslant N^*$

Proof. It results from [10] that the convergence of estimation algorithm will be improved by parameter constraint. Thus we need only to prove the above properties in the non-constraint case. From (22), we have $\bar{\theta}(t)\neq\bar{\theta}(t-1)$ when $s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)>\delta\sqrt{\psi_1(t-1)^{\mathrm{T}}\psi_1(t-1)}$. Taking $\hat{\bar{\theta}}(t)=\bar{\theta}(t)$ and subtracting $\bar{\theta}_0$ from both sides of (21) give

$$\|\bar{\theta}(t)-\bar{\theta}_0\|^2=\|\bar{\theta}(t-1)-\bar{\theta}_0\|^2+2[\bar{\theta}(t-1)-\bar{\theta}_0]^{\mathrm{T}}\frac{\psi(t-1)[s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)]}{\psi(t-1)^{\mathrm{T}}\psi(t-1)}+\frac{\psi(t-1)^{\mathrm{T}}\psi(t-1)[s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)]^2}{[\psi(t-1)^{\mathrm{T}}\psi(t-1)]^2}$$

Since $s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}_0=0$, it follows that $[\bar{\theta}(t-1)-\bar{\theta}_0]^{\mathrm{T}}\psi(t-1)=\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)-s(t)y(t)$. Substituting this equation into the above equation, we have

$$\|\bar{\theta}(t)-\bar{\theta}_0\|^2=\|\bar{\theta}(t-1)-\bar{\theta}_0\|^2-\frac{[s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)]^2}{\psi(t-1)^{\mathrm{T}}\psi(t-1)} \tag{26}$$

It is clear from eqns. (21) and (22) that $\|\bar{\theta}(t)-\bar{\theta}_0\|^2\leqslant\|\bar{\theta}(t-1)-\bar{\theta}_0\|^2$. This establishes 1).

Summing up both sides of (26) with respect t, we get $\sum\limits_{t=1}^{N}\dfrac{[s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)]^2}{\psi(t-1)^{\mathrm{T}}\psi(t-1)}=\|\bar{\theta}(0)-\bar{\theta}_0\|^2-\|\bar{\theta}(N)-\bar{\theta}_0\|^2\leqslant\|\bar{\theta}(0)-\bar{\theta}_0\|^2$, which implies that $\lim\limits_{N\to\infty}\sum\limits_{t=1}^{N}\dfrac{[s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)]^2}{\psi(t-1)^{\mathrm{T}}\psi(t-1)}<\infty$. Then 2) follows. From (21) and the property 2), we can easily obtain 3).

When $s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)>\delta\sqrt{\psi_1(t-1)^{\mathrm{T}}\psi_1(t-1)}$, it follows from (26) that

$\|\bar{\theta}(t)-\bar{\theta}_0\|^2<\|\bar{\theta}(t-1)-\bar{\theta}_0\|^2-\frac{\delta^2\psi_1(t-1)^{\mathrm{T}}\psi_1(t-1)}{\psi(t-1)^{\mathrm{T}}\psi(t-1)}$. Noting that $\psi(t-1)^{\mathrm{T}}\psi(t-1)=\psi_1(t-1)^{\mathrm{T}}\psi_1(t-1)+\phi_u(t-1)^{\mathrm{T}}\phi_u(t-1)$, and combining with $\|\phi_u(t-1)\|\leqslant c$, we obtain $\|\bar{\theta}(t)-\bar{\theta}_0\|^2<\|\bar{\theta}(t-1)-\bar{\theta}_0\|^2-\frac{\delta^2}{1+\frac{\phi_u(t-1)^{\mathrm{T}}\phi_u(t-1)}{\psi_1(t-1)^{\mathrm{T}}\psi_1(t-1)}}<\|\bar{\theta}(t-1)-\bar{\theta}_0\|^2-\frac{\delta^2}{1+c}$. Taking $N^*=(1+c)\times\frac{\|\bar{\theta}(0)-\bar{\theta}_0\|^2}{\delta^2}$, the property 3) is established.

When $t\geqslant N^*$, it follows from (21) and (22) that $s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)\leqslant\delta\sqrt{\psi_1(t-1)^{\mathrm{T}}\psi_1(t-1)}$. Using the property 1), and expanding the above inequation, we have $|y(t)-\phi(t-1)^{\mathrm{T}}\theta(\infty)|\leqslant\delta\sqrt{\psi_1(t-1)^{\mathrm{T}}\psi_1(t-1)}+w_\Delta^\infty\{\max_{t-\mu\leqslant s\leqslant t}|y(s)|\}+w_v^\infty$. Noting that $\sqrt{\psi_1(t-1)^{\mathrm{T}}\psi_1(t-1)}=\sqrt{\sum_{s=1}^{n}y(t-s)^2+\{\max_{t-\mu\leqslant s\leqslant t}|y(s)|\}^2+1}\leqslant\sqrt{\sum_{s=1}^{n}y(t-s)^2+\{\max_{t-\mu\leqslant s\leqslant t}|y(s)|\}^2+1}\leqslant\sqrt{n+1}\{\max_{t-\mu\leqslant s\leqslant t}|y(s)|\}+1$, property 2) follows. This completes the proof. □

Suppose that $G_{t-1}=Z^d\hat{b}^{t-1}(\hat{a}^{t-1})^{-1}$ is the system model generated by the above estimation algorithm at time t, and the input /output sequences are related by

$$\hat{a}^{t-1}y(t)=Z^d\hat{b}^{t-1}u(t)+e(t) \tag{27}$$

where $e(t)=y(t)-\phi(t-1)^{\mathrm{T}}\theta(t-1)$.

Suppose the controller is given by

$$\hat{d}^t u(t)=\hat{n}^t(r-y)(t) \tag{28}$$

Combining the parameter estimation and the ℓ_1 optimal controller design gives the following indirect adaptive law

1) Estimate a^t, b^t using algorithm (21)~(24)

2) Design the optimal controller a^t, b^t using the method given in Section 3.2

3) Compute and implement control according to (28)

4) Back to 1).

4.2 Analysis of global stability and robust tracking performance of the adaptive system

Lemma 2. For the above adaptive law, if sequences $\{e(t)\}$ and $\{\psi(t-1)\}$ satisfy the following conditions:

1) $\lim_{t\to\infty}\frac{[s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)]^2}{\psi(t-1)^{\mathrm{T}}\psi(t-1)}=0$

2) There exist constants $0\leqslant c_1<\infty$, $0<c_2<\infty$, such that $\|\psi_{(t-1)}\|\leqslant c_1+c_2\max_{0\leqslant\tau\leqslant t}|e(\tau)|$

3) There exists constant $0<c_3<1$, such that $\bar{w}_\Delta\max_{0\leqslant s\leqslant t}|y(s)|\leqslant c_3\max_{0\leqslant s\leqslant t}|e(s)|$

then, sequences $\{e(t)\}$ and $\{\psi(t-1)\}$ are bounded.

Proof. If sequence $\{e(t)\}$ is bounded, the condition 2) implies that sequence $\{\psi(t-1)\}$ is bounded. Now assume that $\{e(t)\}$ is unbounded. Then there exists a subsequence $\{e(t_k)\}$ such that $\lim_{k\to\infty}|e(t_k)|=\infty$, and $|e(t)|\leqslant|e(t_k)|$ as $t<t_k$. Since $s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)=s(t)y(t)-s(t)\phi(t-1)^{\mathrm{T}}\theta(t-1)-w_\Delta^t\max_{t-\mu\leqslant s\leqslant t}|y(s)|-w_v^t=|y(t)-\phi(t-1)^{\mathrm{T}}\theta(t-1)|-w_\Delta^t\max_{t-\mu\leqslant s\leqslant t}|y(s)|-w_v^t=|e(t)|-w_\Delta^t\max_{t-\mu\leqslant s\leqslant t}|y(s)|-w_v^t$, it follows from condition 3) that $s(t)y(t)-\psi(t-1)^{\mathrm{T}}\bar{\theta}(t-1)\geqslant|e(t)|-c_3\max_{0\leqslant s\leqslant t}|e(s)|-w_v^t$. Since sequence $\{e(t_k)\}$ is mono-increase, $s(t_k)y(t_k)-\psi(t_k-1)^{\mathrm{T}}\bar{\theta}(t_k-1)\geqslant(1-c_3)|e(t_k)|-w_v^{t_k}\geqslant 0$. Also, from condition 2), it follows that

$$\frac{[s(t)y(t_k)-\psi(t_k-1)^{\mathrm{T}}\bar{\theta}(t_k-1)]^2}{\psi(t_k-1)^{\mathrm{T}}\psi(t_k-1)}\geqslant\frac{[(1-c_3)|e(t_k)|-w_v^{t_k}]^2}{[c_1+c_2\max_{0\leqslant\tau\leqslant t_k}|e(\tau)|]^2}=\frac{[(1-c_3)|e(t_k)|-w_v^{t_k}]^2}{[c_1+c_2|e(t_k)|]^2}$$

Since sequence $\{e(t_k)\}$ is mono-increase and w_Δ^t is bounded, we have

$$\lim_{t_k\to\infty}\frac{[s(t)y(t_k)-\psi(t_k-1)^{\mathrm T}\bar{\theta}(t_k-1)]^2}{\psi(t_k-1)^{\mathrm T}\psi(t_k-1)} \geqslant \lim_{t_k\to\infty}\frac{[(1-c_3)\,|e(t_k)|-w_v^{t_k}]^2}{[c_1+c_2\,|e(t_k)|\,]^2} = \frac{1-c_3}{c_2}>0,$$

which contradicts condition 1). The lemma follows. □

Theorem 3. Suppose the plant described by (1) satisfies Assumption 4, and that the adaptive system depicted by Fig. 2 satisfies robust stability condition that is represented as $\sup_t\|G_{\eta\xi}^t\|_1<\frac{1}{\overline{w}_\Delta}$, where $G_{\eta\xi}^t$ is transfer function from ξ to η at time t. Then sequences $\{e(t)\}$, $\{u(t)\}$ and $\{y(t)\}$ are bounded. For a given reference input r that satisfies the Assumption 3, the worst-case steady state tracking error satisfies:

$$\limsup_{t\to\infty} J_{\mathrm{trac}}^t \leqslant (w_v^\infty+\delta)\mu^\infty + \frac{(w_\Delta^\infty+\delta\sqrt{n+1})\mu^\infty}{1-(w_\Delta^\infty+\delta\sqrt{n+1})\mu^\infty}[\,\|r\|_{ss}+(w_v^\infty+\delta)\mu^\infty\,]$$

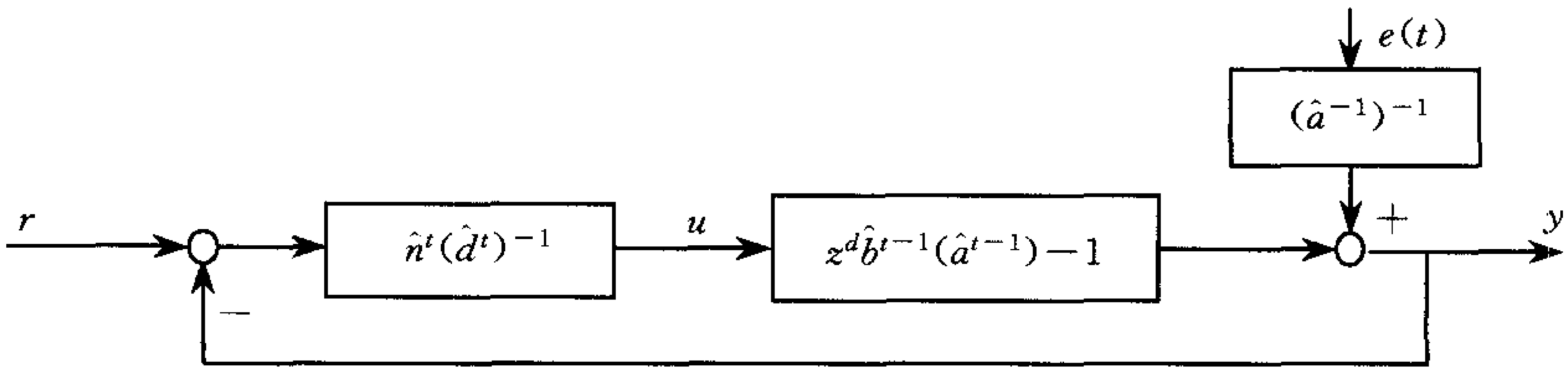

Fig. 2 The adaptive control plant

where J_{trac}^t is the worst-case steady state tracking error at time t, and μ^t is the solution of the corresponding ℓ_1 optimal problem (14) at time t, that is, $\mu^t=\inf_{\hat{q}\in A}\|G_{\eta\xi}^t\|_A$.

Proof. First, we show the adaptive scheme is globally convergent. We first obtain a closed-loop model for the adaptive system depicted by Fig. 2. For simplicity, denote $\hat{a}^{t-1}$ by $\hat{a}$ and $z^d\hat{b}^{t-1}$ by $\hat{b}_1$, so as to other time-varying polynomials. Define the following:

$$\hat{a}\hat{b} = \sum_i\sum_j a^t(i)b^t(j)z^{-i-j} \tag{29}$$

$$\hat{a}\circ\hat{b} = \sum_i\sum_j a^t(i)b^{t-i}(j)z^{-i-j} \tag{30}$$

The closed-loop polynomial at time t is

$$\hat{a}_* = \hat{a}\hat{d}+\hat{b}_1\hat{n} \tag{31}$$

(27) is rewritten as

$$e(t) = \hat{a}y(t)-\hat{b}_1u(t) \tag{32}$$

Define $p(t)=\hat{a}\circ\hat{n}r(t)$ and $z(t)=\hat{b}_1\circ\hat{n}r(t)$. From eqns. (28), (31)~(32), it follows that

$$\begin{aligned}p(t)= {} & \hat{a}\circ\hat{d}u(t)+\hat{a}\circ\hat{n}y(t)=\hat{a}\hat{d}u(t)+[\hat{a}\circ\hat{d}-\hat{a}\hat{d}]u(t)+\hat{a}\hat{n}y(t)+[\hat{a}\circ\hat{n}-\hat{a}\hat{n}]y(t) = \\ & \hat{a}\hat{d}u(t)+\hat{b}_1\hat{n}u(t)+\hat{n}e(t)+[\hat{a}\circ\hat{d}-\hat{a}\hat{d}]u(t)+[\hat{a}\circ\hat{n}-\hat{a}\hat{n}]y(t)+ \\ & [\hat{n}\circ\hat{b}_1-\hat{n}\hat{b}_1]u(t)-[\hat{n}\circ\hat{a}-\hat{n}\hat{a}]y(t) = \hat{a}_*u(t)+\hat{n}e(t)+[\hat{a}\circ\hat{d}-\hat{a}\hat{d}]u(t)+ \\ & [\hat{a}\circ\hat{n}-\hat{a}\hat{n}]y(t)+[\hat{n}\circ\hat{b}_1-\hat{n}\hat{b}_1]u(t)-[\hat{n}\circ\hat{a}-\hat{n}\hat{a}]y(t)\end{aligned} \tag{33}$$

Using an argument similar to the one above, we have

$$\begin{aligned}z(t) = {} & \hat{a}_*y(t)-\hat{n}e(t)+[\hat{b}_1\circ\hat{d}-\hat{b}_1\hat{d}]u(t)+[\hat{b}_1\circ\hat{n}-\hat{b}_1\hat{n}]y(t)- \\ & [\hat{d}\circ\hat{b}_1-\hat{d}\hat{b}_1]u(t)+[\hat{d}\circ\hat{a}-\hat{d}\hat{a}]y(t)\end{aligned} \tag{34}$$

Combining (33) and (34), we obtain the following closed-loop model:

$$\begin{bmatrix}\hat{a}_*+[\hat{a}\circ\hat{d}-\hat{a}\hat{d}]+[\hat{n}\circ\hat{b}_1-\hat{n}\hat{b}_1] & [\hat{a}\circ\hat{n}-\hat{a}\hat{n}]-[\hat{n}\circ\hat{a}-\hat{n}\hat{a}] \\ [\hat{b}_1\circ\hat{d}-\hat{b}_1\hat{d}]-[\hat{d}\circ\hat{b}_1-\hat{d}\hat{b}_1] & \hat{a}_*+[\hat{b}_1\circ\hat{n}-\hat{b}_1\hat{n}]+[\hat{d}\circ\hat{a}-\hat{d}\hat{a}]\end{bmatrix}\begin{bmatrix}u(t)\\ y(t)\end{bmatrix}= \begin{bmatrix}p(t)-\hat{n}e(t)\\ z(t)+\hat{n}e(t)\end{bmatrix} \tag{35}$$

(35) can be regarded as a linear time-varying dynamic system, where the terms in square brackets arise from the time-varying nature of the parameters estimates. All these

terms approach zero as t tends to infinity.

Now, we use closed-loop model (35) to prove the global convergence of the adaptive system. It follows from property 1) of Lemma 1 that the coefficients of $\hat{a}^t$ and $\hat{b}^t$ are bounded for all t. Then the coefficients of $\hat{d}^t$ and $\hat{n}^t$ are also bounded. Also, it follows from property 3) of Lemma 1 that the coefficients of $\hat{a}^t$ and $\hat{b}^t$ are close to asymptotical time-invariety. From the continuity of Diophantine equation and the continuity of ℓ_1 optimal design[3, 11], it follows that the coefficients of $\hat{d}^t$ and $\hat{n}^t$ are close to asymptotical time-invariety. Then, all terms in square brackets of (35) approach zero as t tends to infinity, which implies that the closed-loop system (35) approach a robust stable system. Thus the condition 2) of Lemma 2 is satisfied. From $\sup_t \|G^t_{\eta\xi}\|_1 \leqslant \frac{c_3}{\bar{w}_\Delta}$ and $G^t_{\eta\xi} = G^t_{ye}$, $\max_{0\leqslant s\leqslant t} |y(s)| \leqslant \{\sup_t \|G^t_{ye}\|_1\} \max_{0\leqslant s\leqslant t} |e(s)| \leqslant \frac{c_3}{\bar{w}_\Delta} \max_{0\leqslant s\leqslant t} |e(s)|$, the condition 3) of Lemma 2 is satisfied. It follows from 2) of Lemma 1 that 1) of Lemma 2 is satisfied. Thus we are able to apply Lemma 2 to conclude that the sequences $\{e(t)\}$ and $\{\psi(t-1)\}$ are bounded, and hence the adaptive system is globally convergent.

Finally, we establish the worst-case steady state tracking error. Since sequence $\{\psi(t-1)\}$ is bounded, it follows that $\{\|\phi_u(t-1)\|\}$ is a bounded sequence, and hence there exists a positive c such that $\|\phi_u(t-1)\| \leqslant c$. Thus, from the properties 4) and 5) of Lemma 1, we can conclude that under Assumption 4 the real system can be represented by inequality (19) with $(\theta(\infty)^{\mathrm{T}}, w^\infty_\Delta + \delta\sqrt{n+1}, w^\infty_v + \delta)^{\mathrm{T}}$ as $t \geqslant N^*$. Then the theorem follows from Theorem 2. This completes the proof. □

Remark 1. From Theorem 3, the unmodelled dynamics permitted by the adaptive system are $\bar{w}_\Delta < \frac{1}{\sup_t \|G^t_{\eta\xi}\|_1}$. According to the Small Gain Theorem, the maximal unmodelled dynamics that the corresponding non-adaptive system can permit are $\bar{w}_\Delta < \frac{1}{\|G_{\eta\xi}\|_1}$, which implies that the adaptive system has the same robust stability as the non-adaptive control system. On the other hand, in the adaptive system the controller is designed on the basis of ℓ_1 optimization at all time t, that is, $\sup_t \|G^t_{\eta\xi}\|_1$ is minimal. Hence, we conclude from the above argument that the adaptive system proposed in this paper has non-conservative and optimal robust stability, which greatly improves the results of [3~5].

Remark 2. The upper bound for the asymptotical worst-case steady state tracking error provided in Theorem 3 is tight. Its conservation depends on the precision of parameter estimation, which further depends on the weighting δ in (22).

5 Conclusion

This paper investigates an adaptive robust tracking problem when a discrete-time plant is subject to both unmodelled dynamics and unknown external disturbances. The main contribution of this paper is as follows.

1) By combining of ℓ_1 optimization and deadbeat control scheme, an optimal robust steady state tracking scheme is provided. For the scheme, the robust tracking performance optimization is shown to be equivalent to robust stability optimization, and both are reduced to a standard linear programming.

2) A recursive extended parameters estimation is provided based on the idea of set-membership identification.

3) Using the certainty equivalence principle an adaptive robust tracking scheme is presented. A tight bound on robust performance is provided. The adaptive scheme has

non-conservative and optimally robust stability, and asymptotically optimal robust steady state tracking performance.

References

1 Dahleh M A, Diaz-Bobilllo I J. Control of Uncertain Systems: A Linear Programming Approach. Englewood Cliffs: Prentice-Hall, 1995

2 Ljung L. Control Theory 1984～1986. Progress Report from IFAC's Technical Committee on Theory

3 Dahleh M, Dahleh M A. Optimal rejection of persistent and bounded disturbances: Continuity properties and adaptation. *IEEE Transactions on Automatic Control*, 1990, **35**: 687～696

4 Li Sheng-Ping, Chai Tian-You. A new adaptive control scheme and analysis of its closed loop stability. *Chinese Journal of Automation*, 1999, **11**(4): 277～284

5 Li Sheng-Ping, Zhang Xian-Min. Optimal ℓ_1 robust control for plants with coprime factor perturbations: Continuity properties and adaptation. *Acta Automatica Sinica*, 2003, **29**(2):

6 Khammash M, Pearson J B. Analysis and design for robust performance with structured uncertainty. *Systems and Control Letters*, 1993, **20**(3): 179～187

7 Khammash M. Robust steady tracking. *IEEE Transactions on Automatic Control*, 1995, **40**: 1872～1880

8 Khammash M. Robust performance: Unknown disturbances and known fixed inputs. *IEEE Transactions on Automatic Control*, 1997, **42**: 1730～1734

9 Mendlovitz M A. A simple solution to the ℓ_1 optimization problem. *Systems and Control Letters*, 1989, **12**(5): 461～463

10 Goodwin G C, Sin K S. Adaptive Filtering, Prediction and Control. Englewood Cliffs: Prentice-Hall, 1984

11 Li Sheng-Ping. On continuity properties of two-block ℓ_1 optimal design. *Systems and Control Letters*, 2002, **47**(1): 47～55

LI Sheng-Ping Received his Ph. D. degree from Huazhong University of Science and Technology in 1995. He held a Post-Doctoral position at the Research Center of Automation, Northeastern University from 1995 to 1997. Since 1998, he has been with the Department of Mechatronic Engineering, Shantou University, where he is currently an associate professor and the director of the department. His research interests include robust control and identification, adaptive control, and applications of advanced control technology in mechanical engineering.

不确定性系统的自适应鲁棒跟踪控制

李昇平

(汕头大学机械电子工程系 汕头 515063)

(E-mail: spli@stu.edu.cn)

摘 要 针对存在未知干扰和未建模动态等不确定性的系统的自适应鲁棒跟踪控制问题进行了探讨. 首选将 ℓ_1 优化控制器的有限拍设计方法结合给出了最优鲁棒稳态跟踪控制器的设计方法. 然后利用集员辨识的思想,将名义模型的参数和未建模动态及干扰的大小作为未知参数,提出了一种递推参数估计方法. 最后将上述研究结果结合起来提出了一种自适应鲁棒跟踪控制策略,证明了自适应算法的全局收敛性并给出了鲁棒跟踪性能指标的一下较紧的上界. 与现有的结果相比,本文提出的自适应控制具有非保守的鲁棒稳定性,具有渐近最优的鲁棒跟踪性能.

关键词 未建模动态,未知干扰,自适应控制,鲁棒跟踪

中图分类号 TP273