FBSDE with Poisson Process and Its Application to Linear Quadratic Stochastic Optimal Control Problem with Random Jumps[1)]

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Abstract One kind of existence and uniqueness result of forward-backward stochastic differential equations with Brownian motion and Poisson process is given. The result is applied to get the explicit form of the optimal control for linear quadratic stochastic optimal control problem with random jumps. The optimal control can be proved to be unique. One kind of generalized Riccati equation system is introduced and its solvability is discussed. The linear feedback regulator for the optimal control problem with random jump is given by the solution of the generalized Riccati equation system

Key words Stochastic differential equations, poisson process, stochastic optimal control, Riccati equation

1 Introduction

The backward stochastic differential equations with Poisson process (BSDEP in short) were first discussed by Tang and Li[1]. The stochastic process in the equation is discontinuous with random jump. After then Situ Rong[2] obtained an existence and uniqueness result with non-Lipschitz coefficients for BSDEP. Fully coupled forward-backward stochastic differential equations with Brownian motion can be encountered in the optimization problem when applying stochastic maximum principle and in mathematics finance when considering large investor in security market. Under some monotone assumptions, Wu[3] obtained the existence and uniqueness of the solution to forward-backward stochastic differential equations with Brownian motion and Poisson process (FBSDEP in short) in an arbitrarily fixed time duration. In next section, we will give another existence and uniqueness result of FBSDEP under some monotone assumptions suitable for our optimal control problems. The result can be used to study the linear quadratic optimal control problems with random jump in the following section.

Stochastic linear quadratic optimal control problems have been first studied by Wonham[4,5] and then developed by Bensoussan[6], Davis[7], Peng[8], Zhong[9], Chen, Li and Zhou[10], Yong and Zhou[11]. The optimal control problem with random jump was first considered by Boel and Varaiya[12] and by Rishel[13] also. In this case the control system is disturbed by random jump and the optimal solution is a discontinuous stochastic process. This kind of optimal control problem has a practical background in engineering and financial market. In the general frame i. e., the noise source is the general square integrable martingale, Bismut[14] studied this kind of problem systematically. He proved the existence and uniqueness of the optimal control using the functional analysis method. He also introduced the adjoint linear backward stochastic differential equation using the dual method and discussed the solvability of one kind of Riccati equation. And then he got the feedback form of the optimal control. In Section 3, using the solution of FBSDEP discussed in

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Section 2, we can give the explicit form of the optimal control for stochastic linear quadratic problem with random jump, and we also can prove the optimal control is unique using the classical method. It is natural to study the associated Riccati equation for this kind of optimal control problem. We introduce a kind of generalized Riccati equation system formed by one matrix-value ordinary differential equation and two algebra equations which has three matrix-value variables. The equation form is different with that in [10,14]. The similar equation form for two matrix-value variables can be seen in Peng[15]. Using the solution of this kind of Riccati equation system, we can give the linear feedback regulator of the linear quadratic optimal control problem with random jump.

The above kind of generalized Riccati equation system is novel and complicated. In Section 4, we will discuss its solvability and give a simple example of this kind of equation which has a unique solution.

2 Existence and uniqueness of FBSDEP

We consider the following kind of FBSDEP

$$\begin{cases} dx_t = b(t,x_t,Bp_t,Cq_t,Dk_t)dt + \sigma(t,x_t,B\,p_t,Cq_t,Dk_t)dB_t + \\ \int_Z g(t_-,x_{t\_},Bp_{t\_},Cq_{t\_},Dk_{t\_}(z))\tilde{N}(dzdt) - dp_t = \\ f(t,x_t,p_t,q_t,k_t)dt - q_t dB_t - \int_Z k_{t\_}(z)\tilde{N}(dzds) \\ x_0 = a,\ p_T = \Phi(x_T) \end{cases} \tag{1}$$

For notational simplification, we assume Brownian motion is one dimensional and we use the notations in [3]. Here (x,p,q,k) take value in $\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n$, B, C and D are $k\times n$ bounded matrices, We introduce the notations

$$\boldsymbol{u} = \begin{pmatrix} x \\ p \\ q \\ k \end{pmatrix},\quad \boldsymbol{A}(t,\boldsymbol{u}) = \begin{pmatrix} -f \\ b \\ \sigma \\ g \end{pmatrix}(t,\boldsymbol{u})$$

and assume that

$$\begin{cases} \langle \boldsymbol{A}(t,\boldsymbol{u}) - \boldsymbol{A}(t,\bar{\boldsymbol{u}}), \boldsymbol{u}-\bar{\boldsymbol{u}}\rangle \le -\upsilon_1 |\hat{x}|^2 - \upsilon_2 |B\hat{p}+C\hat{q}+D\hat{k}|^2 \\ \langle \Phi(x) - \Phi(\bar{x}), x-\bar{x}\rangle \ge 0, \\ \forall \hat{\boldsymbol{u}} = (\boldsymbol{u}-\bar{\boldsymbol{u}}) = (\hat{x},\hat{p},\hat{q},\hat{k}) = (x-\bar{x},p-\bar{p},q-\bar{q},k-\bar{k}) \end{cases} \tag{2}$$

where $\upsilon_1 \ge 0$, $\upsilon_2 > 0$. We also assume that

$$\begin{cases} \text{i) } \boldsymbol{A}(t,\boldsymbol{u}) \text{ is uniformly Lipschitz with respect to } \boldsymbol{u}; \\ \text{ii) } \Phi(x) \text{ is uniformly Lipschitz with respect to } x\in\mathbb{R}^n; \\ \text{iii) for each } x,\ \Phi(x) \text{ is in } L^2(\Omega,\mathcal{F}_T,\mathbf{P}). \\ \text{iv) } l(\omega,t,0,0,0,0)\in M^2(0,T),\ l=b,\sigma,f, \text{ respectively,} \\ \quad \text{and } g(\omega,t,0,0,0,0)\in F_N^2(0,T) \text{ for } (\omega,t)\in\Omega\times[0,T]; \\ \text{v) } \forall x,\ |l(t,x,Bp,Cq,Dk) - l(t,x,B\bar{p},C\bar{q},D\bar{k})| \le K(|B\hat{p}+C\hat{q}+D\hat{k}|), \\ \quad K>0,\ l=b,\sigma,g. \end{cases} \tag{3}$$

So we have

Theorem 1. Under the assumption (2) and (3), FBSDEP (1) has a unique adapted solution $(x(\cdot),p(\cdot),q(\cdot),k(\cdot))\in M^2(0,T;\mathbb{R}^{n+n+n})\times F_N^2(0,T;\mathbb{R}^n)$.

The proof is similar to that of Theorem 3.1 in [3].

3 Linear quadratic stochastic optimal control problem with random jump

We consider the following linear stochastic control system with random jump

$$\begin{cases} dx_t = (Ax_t + Bv_t)dt + (Cx_t + Dv_t)dB_t + \int_Z (Ex_{t\_} + Fv_t)\tilde{N}(dzdt), \\ x_0 = a \end{cases} \tag{4}$$

where A, C and E are $n\times n$ bounded matrices, v_t, $t\in[0,T]$, is an admissible control process i. e., an $\mathcal{F}_t$-adapted square-integrable process taking values in a given subset U of $\mathbb{R}^k$. B, D and F are $n\times k$ bounded matrices. We also assume that there is no constraint imposed on the control processes: $U=\mathbb{R}^k$. A classical quadratic optimal control problem is to minimize the cost function

$$J(v(\cdot)) = \frac{1}{2}\mathbb{E}\Big[\int_0^T(\langle Rx_t,x_t\rangle + \langle Nv_t,v_t\rangle)dt + \langle Qx_T,x_T\rangle\Big] \tag{5}$$

over the set of admissible controls, where Q and R are $n\times n$ nonnegative symmetric bounded matrices, N is a $k\times k$ positive symmetric bounded matrix and the inverse N^{-1} is also bounded. In this section we will prove and give the explicit form of the optimal control using the solution of FBSDEP. By the solution of one kind of generalized matrix Riccati equation system, we also can give the optimal linear feedback regulator for the above linear quadratic optimal problem with random jump. We first have the following results.

Theorem 2. The function $u_t=-N^{-1}(B^\tau p_t+D^\tau q_t+F^\tau k_t)$, $t\in[0,T]$, is the unique optimal control for the linear quadratic control problem with random jump, where (x_t,p_t,q_t,k_t) is the solution of the following FBSDEP:

$$\begin{cases} dx_t = [Ax_t - BN^{-1}B^\tau p_t - BN^{-1}D^\tau q_t - BN^{-1}F^\tau k_t]dt + \\ \quad [Cx_t - DN^{-1}B^\tau p_t - DN^{-1}D^\tau q_t - DN^{-1}F^\tau k_t]dB_t + \\ \quad \int_Z (Ex_{t\_} - FN^{-1}B^\tau p_t - FN^{-1}D^\tau q_t - FN^{-1}F^\tau k_t)\tilde{N}(dzdt) \\ -dp_t = [A^\tau p_t + C^\tau q_t + E^\tau k_t + Rx_t]dt - q_t dB_t - \int_Z k_{t\_}(z)\tilde{N}(dzdt) \\ x_0 = a, \quad p_T = Q(\omega)x_T \end{cases} \tag{6}$$

Proof. It is easy to verify that (6) satisfies (2) and (3). So from Theorem 1, FBSDEP (6) has unique solution (x_t, p_t,q_t,k_t). We denote for $\forall\ v(\cdot)\in\mathbb{R}^k$, x_t^v is the corresponding trajectory of system (4), then we have

$$J(v(\cdot)) - J(u(\cdot)) = \frac{1}{2}\mathbb{E}\Big[\int_0^T(\langle Rx_t^v,x_t^v\rangle - \langle Rx_t,x_t\rangle + \langle Nv_t,v_t\rangle - \langle Nu_t,u_t\rangle)dt +$$

$$\langle Qx_T^v,x_T^v\rangle - \langle Qx_T,x_T\rangle\Big] = \frac{1}{2}\mathbb{E}\Big[\int_0^T(\langle R(x_t^v-x_t),x_t^v-x_t\rangle + \langle N(v_t-u_t),v_t-u_t\rangle +$$

$$2\langle Rx_t,x_t^v-x_t\rangle + 2\langle Nu_t,v_t-u_t\rangle)dt + \langle Q(x_T^v-x_T),x_T^v-x_T\rangle + 2\langle Qx_T,x_T^v-x_T\rangle\Big]$$

From $Qx_T=p_T$, we use Itô's formula for $\langle x_T^v-x_T,p_T\rangle$, since R , Q are nonnegative, and N is positive, we have

$$J(v(\cdot)) - J(u(\cdot)) \geq \mathbb{E}\int_0^T(\langle -NN^{-1}(B^\tau p_t + D^\tau q_t + F^\tau k_t),\ v_t - u_t\rangle +$$

$$\langle B^\tau p_t,v_t-u_t\rangle + \langle D^\tau q_t,v_t-u_t\rangle + \langle F^\tau k_t,v_t-u_t\rangle)dt = 0$$

So $u_t=-N^{-1}(B^\tau p_t+D^\tau q_t+F^\tau k_t)$ is the optimal control.

To prove the uniqueness of the optimal control, we use the classical parallelogram rule which can be seen in [6] for the case without jump. We omit it. □

Now we introduce the following generalized $n\times n$ marix Riccati equation system of $(Y(t),M(t),L(t))$, $t\in[0,T]$,

$$\begin{cases} -\dot{Y}(t) = A^\tau Y(t) + Y(t)A + C^\tau M(t) + E^\tau L(t) - Y(t)BN^{-1}B^\tau Y(t) - \\ \quad Y(t)BN^{-1}D^\tau M(t) - Y(t)BN^{-1}F^\tau L(t) + R \\ M(t) = Y(t)C - Y(t)DN^{-1}B^\tau Y(t) - Y(t)DN^{-1}D^\tau M(t) - Y(t)DN^{-1}F^\tau L(t) \\ L(t) = Y(t)E - Y(t)FN^{-1}B^\tau Y(t) - Y(t)FN^{-1}D^\tau M(t) - Y(t)FN^{-1}F^\tau L(t) \\ Y(T) = Q,\ t\in[0,T] \end{cases} \tag{7}$$

The above equation system is complex, it is one kind of generalized matrix — value Riccati equation system. We will discuss its solvability in next section. Now we have

Theorem 3. Suppose that there exist matrices $(Y(t), M(t), L(t))$, $t\in[0,T]$, satisfying the generalized matrix Riccati equation system (7); then the optimal linear feedback regulator for linear quadratic optimal problem with random jump is $u_t=-N^{-1}[B^\tau Y(t)+D^\tau M(t)+F^\tau L(t)]x_t$, and the optimal value function is $J(u(\cdot))=\frac{1}{2}\langle Y(0)a,a\rangle$.

Proof. Let $(Y(t), M(t), L(t))$ be the solution of (7). It is easy to show that the solution (x_t, p_t, q_t, k_t) of (6) satisfies $p_t=Y(t)x_t$, $q_t=M(t)x_t$, $k_t=L(t)x_t$. So the optimal control is $u_t=-N^{-1}[B^\tau Y(t)+D^\tau M(t)+F^\tau L(t)]x_t$, $t\in[0,T]$. Applying Itô's formula to $\langle x_T, p_T\rangle$ in $J(u(\cdot))$, we easily get $J(u(\cdot))$. □

4 Solvability of the generalized Riccati equation

In Section 3, the linear optimal feedback regulator for the linear quadratic optimal problem with random jump can be obtained by the solution of the generalized Riccati equation (7). However, (7) is so complicated that we can not prove the existence and uniqueness for the general case at this moment. In this section, we discuss the solvability for a special case of (7) when $D=0$. Now we study the following equations

$$\begin{cases} -\dot{Y}(t) = A^\tau Y(t) + Y(t)A + E^\tau(I_n + Y(t)FN^{-1}F^\tau)^{-1}(Y(t)E - Y(t)FN^{-1}B^\tau Y(t)) - \\ \quad Y(t)BN^{-1}B^\tau Y(t) + C^\tau Y(t)C + R - Y(t)BN^{-1}F^\tau(I_n + Y(t)FN^{-1}F^\tau)^{-1} \times \\ \quad Y(t)E + Y(t)BN^{-1}F^\tau(I_n + Y(t)FN^{-1}F^\tau)^{-1}Y(t)FN^{-1}B^\tau Y(t) \\ Y(T) = Q,\ I_n + Y(t)FN^{-1}F^\tau > 0,\ t\in[0,T] \end{cases} \tag{8}$$

If we get the solution $Y(t)$ for (8), then we can let

$$M(t) = Y(t)C,\quad L(t) = (I_n + Y(t)FN^{-1}F^\tau)^{-1}(Y(t)E - Y(t)FN^{-1}B^\tau Y(t))$$

to get the solution of (7) when $D=0$.

At first, applying the Gronwall's inequality we have the following uniqueness result.

Theorem 4. Riccati equation (8) has at most one solution $Y(\cdot)\in C(0,T;S^n_+)$. Here S^n_+ represents the space of all $n\times n$ nonnegative definite symmetric matrices. Now let us discuss the existence of (8) step by step. If we let $K(t)=F(Y)=(I_n+YFN^{-1}F^\tau)^{-1}Y$, then for any $K(\cdot)\in C(0,T;S^n_+)$, the following conventional Riccati equations

$$\begin{cases} -\dot{Y}(t) = (A - BN^{-1}F^\tau K(t)E)^\tau Y(t) + Y(t)(A - BN^{-1}F^\tau K(t)E) + \\ \quad C^\tau Y(t)C + E^\tau K(t)E + R - Y(t)[BN^{-1}B^\tau - BN^{-1}F^\tau K(t)FN^{-1}B^\tau]Y(t) \\ Y(T) = Q,\quad t\in[0,T] \end{cases} \tag{9}$$

has a unique solution $Y(\cdot)\in C(0,T;S^n_+)$ when

$$[BN^{-1}B^\tau - BN^{-1}F^\tau K(t)FN^{-1}B^\tau] \in C(0,T;S^n_+) \tag{10}$$

We denote by S^n_s the subspace of S^n_+ formed by the symmetric matrices satisfying (10). Obviously, $K\equiv 0\in S^n_s$, so this definition is reasonable. Thus we can define a mapping Ψ: $C(0,T;S^n_s)\to C(0,T;S^n_+)$ as $Y=\Psi(K)$, and have

Lemma 5. The operator $F(Y)$ is monotonously increasing when $Y>0$; The operator Ψ is monotonously increasing and continuous.

Proof. We notice , when $Y>0$,

$$F(Y) = (I_n + YFN^{-1}F^\tau)^{-1}Y = [Y^{-1}(I_n + YFN^{-1}F^\tau)]^{-1} = (Y^{-1} + FN^{-1}F^\tau)^{-1}$$

So, if $Y_1\geq Y_2$, then $F(Y_1)\geq F(Y_2)$.

Let $Y=\Psi(K)$ and $\bar{Y}=\Psi(\bar{K})$, and denote $\hat{K}=K-\bar{K}$. We rewrite (9) as

$$\begin{cases} -\dot{Y}(t) = A^\tau Y(t) + Y(t)A - Y(t)BN^{-1}B^\tau Y(t) + R + C^\tau Y(t)C + \\ \qquad (E - FN^{-1}B^\tau Y(t))^\tau K(t)(E - FN^{-1}B^\tau Y(t)) \\ Y(T) = Q,\quad t\in[0,T]. \end{cases}$$

From the first conclusion in this Lemma and Lemma 8.2 in [15], we know that if $K\geq\bar{K}$, then $Y \geq \bar{Y}$. This proves the monotonicity of Ψ. On the other hand, applying Gronwall inequality, we easily see that if $K \to \bar{K}$, $Y - \bar{Y} = \hat{Y} \to 0$. This yields the continuity of Ψ and the proof is completed. □

Looking back at (9), it is easy to know that

Thoerem 6. If there exists $K \in C(0, T; S_s^n)$ such that

$$K = (I_n + \Psi(K) F N^{-1} F^\tau)^{-1} \Psi(K) \tag{11}$$

then Riccati Equations (8) has a unique solution.

The following task is to find the suitable $K \in C(0, T; S_s^n)$ satisfying (11). We need the following result.

Lemma 7. If there exist K^+, $K^- \in C(0, T; S_s^n)$ which satisfy

$$K^+ \geq (I_n + \Psi(K^+) F N^{-1} F^\tau)^{-1} \Psi(K^+) \geq (I_n + \Psi(K^-) F N^{-1} F^\tau)^{-1} \Psi(K^-) \geq K^- \tag{12}$$

then (8) admits a solution.

Proof. Let K^+ and K^- be given and satisfy (12). We define the sequence K_i^+, K_i^-, Y_i^+, Y_i^- as following:

$$K_0^+ = K^+ \in S_s^n, \quad K_0^- = K^- \in S_s^n, \quad Y_0^+ = \Psi(K_0^+), \quad Y_0^- = \Psi(K_0^-),$$
$$K_{i+1}^+ = (I_n + Y_i^+ F N^{-1} F^\tau)^{-1} Y_i^+, \quad K_{i+1}^- = (I_n + Y_i^- F N^{-1} F^\tau)^{-1} Y_i^-$$
$$Y_{i+1}^+ = \Psi(K_{i+1}^+), \quad Y_{i+1}^- = \Psi(K_{i+1}^-), \quad i = 0, 1, 2, \cdots, s$$

From (12) and Lemma 5, by induction, we obtain

$$Y_0^+ \geq Y_i^+ \geq Y_{i+1}^+ \geq Y_{i+1}^- \geq Y_i^- \geq Y_0^- \geq 0$$
$$K_0^+ \geq K_i^+ \geq K_{i+1}^+ \geq K_{i+1}^- \geq K_i^- \geq K_0^- \geq 0$$

and K_i^+, $K_i^- \in S_s^n$. So we have

$$\lim_{i \to \infty} K_i^+ = K^+ \in S_s^n, \quad \lim_{i \to \infty} Y_i^+ = Y^+ \in S_+^n$$
$$Y^+ = \lim_{i \to \infty} Y_i^+ = \lim_{i \to \infty} \Psi(K_i^+) = \Psi(\lim_{i \to \infty} K_i^+) = \Psi(K^+)$$

and Y^+ is a solution of Riccati Equations (9) corresponding to $K = K^+$. Then $K^+ = (I_n + Y^+ F N^{-1} F^\tau)^{-1} Y^+$. By Theorem 6, Y^+ is one solution of (8). By the same step, we also can get $Y^- = \lim_{i \to \infty} Y_i^-$ and $K^- = \lim_{i \to \infty} K_i^-$, so Y^- is also a solution of (8). By the uniqueness result Theorem 4, $Y^+ = Y^-$. □

We only need to find K^+ and K^- satisfying (12). The existence of K^- is obvious. We can let $K^- = 0$, which by the conventional Riccati equation theory, satisfies (12). To find $K^+ \in S_s^n$ satisfying (12) and ensure the existence of (8), we assume that

$$\begin{cases} \text{There exists } \bar{K} \in S_s^n, \text{ such that} \\ \quad F^\tau \bar{K} F = N \text{ and } (I_n + \bar{Y} F N^{-1} F^\tau)^{-1} \bar{Y} \leq \bar{K}, \\ \text{here } \bar{Y} \text{ is the unique solution of the following equation} \\ -\dot{\bar{Y}}(t) = (A - BN^{-1} F^\tau \bar{K}(t) E)^\tau \bar{Y}(t) + \bar{Y}(t)(A - BN^{-1} F^\tau \bar{K}(t) E) + \\ \quad C^\tau \bar{Y}(t) C + E^\tau \bar{K}(t) E + R \\ \bar{Y}(T) = Q \end{cases} \tag{13}$$

It is easy to know that when $k = n$ and matrix F is invertible, the assumption (13) is satisfied. Hence, we have the main result of this section.

Theorem 8. We assume (13) and $D = 0$, the generalized Riccati Equations (7) has a unique solution $(Y, M, L) \in C^1(0, T; S_+^n) \times L^\infty(0, T; \mathbb{R}^{n \times n}) \times L^\infty(0, T; \mathbb{R}^{n \times n})$.

At last, we can give a simple example of the generalized Riccati equation which has a unique solution.

Example 9. We assume the dimensions of the state and control in control system (4) are the same i. e., $k = n$, and assume $D = 0$, $F = I_n$. Now we can let $\bar{K} = N$ and then check (13). $\bar{K} = N \geq 0$, so $\bar{K} F + \bar{Y} F N^{-1} F^\tau \bar{K} F \geq \bar{Y} F$, and $\bar{Y}$ is the solution of the equation in (13). $\bar{K} + \bar{Y} F N^{-1} F^\tau \bar{K} \geq \bar{Y}$, $\bar{K} \geq (I_n + \bar{Y} F N^{-1} F^\tau)^{-1} \bar{Y}$. So from Theorem 8, the Riccati Equations (7), when $k = n$, $D = 0$, $F = I_n$, has a unique solution.

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带有随机跳跃干扰的线性二次随机最优控制问题

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摘 要 给出一类布朗运动和泊松过程混合驱动的正倒向随机微分方程解的存在唯一性结果，应用这一结果研究带有随机跳跃干扰的线性二次随机最优控制问题，并得到最优控制的显式形式，可以证明最优控制是唯一的. 然后，引入和研究一类推广的黎卡提方程系统，讨论该方程系统的可解性并由该方程的解得到带有随机跳跃干扰的线性二次随机最优控制问题最优的线性反馈.

关键词 随机微分方程，泊松过程，随机最优控制，黎卡提方程

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