第 27 卷 第 4 期 自 动 化 学 报 Vol.27,No.4
2001 年 7 月 ACTA AUTOMATICA SINICA July, 2001

H_∞ Output Feedback Control Design of Fuzzy Dynamic Systems via LMI

ZHANG Ning[1] FENG Gang[2]

[1](School of Electrical Engineering and Telecommunications, University of New South Wales, Sydney, NSW 2052, Australia)

(E-mail: ningz@alpha2.ee.unsw.edu.au, Tel:(612)93855701, Fax: (612) 93855993)

[2](Dept. MEEM, City University of Hong Kong Tat Chee Ave., Hong Kong)

(E-mail: megfeng@cityu.edu.hk)

Abstract This paper presents H_∞ output feedback control design of complex nonlinear systems which can be represented by a fuzzy dynamic model. Based on a common Lyapunov function and a piecewise differentiable Lyapunov function respectively, two kinds of new H_∞ output feedback fuzzy control design methods are developed. The H_∞ output feedback controller can be obtained by solving a set of suitable linear matrix inequalities. An example is given to demonstrate the application of the proposed design methods.

Key words Fuzzy dynamic systems, H_∞ output feedback control, LMI.

1 Introduction

Fuzzy logic control (FLC) based on the conception of fuzzy sets proposed in [1] has been an appealing control approach[2~6]. During the last few years, a number of new methods have been proposed for the systematic analysis and design of fuzzy logic controllers based on the so-called fuzzy dynamic model, which consists of a family of local linear models smoothly connected through fuzzy membership functions[7~19].

H_∞ control has been an attractive research topic during the last decade[20~22]. So far a couple of papers[10,15] have discussed the H_∞ feedback control for fuzzy dynamic systems. However, those papers dealt with a state feedback control design that requires all states of systems to be measured. In many cases, this requirement is too restrictive. Therefore, an H_∞ output feedback control design for fuzzy dynamic systems is warranted. It should be noted that the state feedback control design methods can not be easily modified for the output feedback control design for fuzzy dynamic systems. Recently, there appeared a few results of output feedback control design for fuzzy dynamic systems[17,19]. However the result in [17] is not constructive, many trials might be needed before an acceptable controller is found; the other discusses H_2 output feedback control design[19]. In this paper, we will develop two new constructive H_∞ output feedback control design methods for fuzzy dynamic systems by using LMI technique[23].

The rest of this paper is organized as follows. Section 2 introduces the fuzzy dynamic model. Based on a common Lyapunov function and a piecewise differentiable Lyapunov function, Section 3 and Section 4 present two new H_∞ output feedback control design methods respectively. An example is given in Section 5. Finally, Section 6 concludes with some remarks.

Notation. The notation used is fairly standard. We denote by $\boldsymbol{z}^{\mathrm{T}}\boldsymbol{z}$ or $\|\boldsymbol{z}\|^2$ the square norm of a vector $\boldsymbol{z}\in R^k$. The notation $L_2(0,T)$ will also be used for vector-valued functions, i.e. we say that $\boldsymbol{z}:(0,T)\to R^k$ is in $L_2(0,T)$ if $\int_0^T\|\boldsymbol{z}(t)\|^2\mathrm{d}t<\infty$.

Received 18 May, 2000; in revised form 23 October, 2000
收稿日期 2000-05-18 收修改稿日期 2000-10-23

2 Fuzzy Dynamic Model

For a nonlinear system, it is difficult, if not impossible, to design a satisfactory control in general. However, it is known that many nonlinear systems can be expressed as a set of linear systems in local operating regions. Therefore, in this paper, we will consider the following class of nonlinear systems, which can be represented by the socalled fuzzy dynamic model, that is,

$$\begin{aligned} R^l: \text{ IF } \quad & q_1 \text{ is } F_1^l \text{ and } \cdots q_s \text{ is } F_s^l \\ \text{THEN } \quad & \dot{\boldsymbol{x}}(t) = A_l\boldsymbol{x}(t) + B_l\boldsymbol{u}(t) + B_{wl}\boldsymbol{w}(t), \\ & \boldsymbol{y}(t) = C_l\boldsymbol{x}(t), \\ & \boldsymbol{z}(t) = C_{zl}\boldsymbol{x}(t), \quad l = 1, 2, \cdots m, \end{aligned} \tag{1}$$

where $\boldsymbol{x}(t) \in R^n$ are the state variables, $\boldsymbol{u}(t) \in R$ is the input variable, $\boldsymbol{y}(t) \in R$ is the output variable, and $\boldsymbol{w}(t) \in R$ is the disturbance variable which is deterministic signal that belong to $L_2(0, \infty)$, $\boldsymbol{z}(t) \in R^v$ are the controlled output variables, R^l denotes the l-th approximation inference rule, m is the number of approximation inference rules, $F_j^l (j = 1, 2 \cdots s)$ are fuzzy sets, $(A_l, B_l, B_{wl}, C_l, C_{zl})$ represents the l-th local model of the nonlinear system, and $q(t) := [q_1, q_2, \cdots, q_s]$ are some measurable system variables.

By using the fuzzy inference methods described in [7 ~ 19], that is using a center-average defuzzifer, product inference and singleton fuzzifier, the dynamic fuzzy model (1) can be expressed by the following global model

$$\begin{cases} \dot{\boldsymbol{x}}(t) = A(\boldsymbol{\mu}(t))\boldsymbol{x}(t) + B(\boldsymbol{\mu}(t))\boldsymbol{u}(t) + B_w(\boldsymbol{\mu}(t))\boldsymbol{w}(t), \\ \boldsymbol{y}(t) = C(\boldsymbol{\mu}(t))\boldsymbol{x}(t), \\ \boldsymbol{z}(t) = C_z(\boldsymbol{\mu}(t))\boldsymbol{x}(t), \end{cases} \tag{2}$$

$$A(\boldsymbol{\mu}(t)) = \sum_{l=1}^m \mu_l A_l, \quad B(\boldsymbol{\mu}(t)) = \sum_{l=1}^m \mu_l B_l, \quad B_w(\boldsymbol{\mu}(t)) = \sum_{l=1}^m \mu_l B_{wl},$$

$$C(\boldsymbol{\mu}(t)) = \sum_{l=1}^m \mu_l C_l, \quad C_z(\boldsymbol{\mu}(t)) = \sum_{l=1}^m \mu_l C_{zl},$$

$$\boldsymbol{\mu}(q(t)) := \boldsymbol{\mu}(t) = [\mu_1(t), \mu_2(t), \cdots, \mu_m(t)],$$

where $\mu_l(q(t))$ is the normalized membership function of the inferred fuzzy set F^l where

$$F^l = \prod_{i=1}^n F_i^l \tag{3a}$$

and

$$\sum_{l=1}^m \mu_l = 1. \tag{3b}$$

Define M as a set of membership functions satisfying (3).

This paper focuses on the H_∞ output feedback control design for fuzzy dynamic systems. The H_∞ performance is defined as follows.

Definition 1. Given a constant $\gamma > 0$, a closed loop system is stable with the H_∞ performance bound γ if the following conditions are satisfied:

1) the closed loop system is asymptotically stable,

2) with zero initial condition, the controlled output satisfies $\|\boldsymbol{z}\| \leqslant \gamma\|\boldsymbol{w}\|$.

3 H_∞ Output Feedback Control Design based on a Common Lyapunov Function

We will present one method based on a common Lyapunov function in this section. For the fuzzy system (2), a smooth output feedback controller can be chosen as follows

$$\begin{cases} \dot{\boldsymbol{x}}_c = \sum_{l=1}^{m} \mu_l A_{cl} \boldsymbol{x}_c(t) + \sum_{l=1}^{m} \mu_l B_{cl} \boldsymbol{y}(t), \\ \boldsymbol{u} = \sum_{l=1}^{m} \mu_l C_{cl} \boldsymbol{x}_c(t). \end{cases} \tag{4}$$

With the output feedback controller (4), the closed loop fuzzy control system can be described as

$$\begin{cases} \dot{\tilde{\boldsymbol{x}}}(t) = \tilde{A}(\boldsymbol{\mu})\tilde{\boldsymbol{x}}(t) + \tilde{B}(\boldsymbol{\mu})\boldsymbol{w}(t), \\ \boldsymbol{z}(t) = \tilde{C}(\boldsymbol{\mu})\tilde{\boldsymbol{x}}(t), \end{cases} \tag{5}$$

$$\tilde{A}(\boldsymbol{\mu}) = \begin{bmatrix} \sum_{l=1}^{m} \mu_l A_l & \sum_{l=1}^{m} \mu_l B_l \sum_{i=1}^{m} \mu_i C_{ci} \\ \sum_{i=1}^{m} \mu_i B_{ci} \sum_{l=1}^{m} \mu_l C_l & \sum_{i=1}^{m} \mu_i A_{ci} \end{bmatrix}, \quad \tilde{B}(\boldsymbol{\mu}) = \begin{bmatrix} \sum_{l=1}^{m} \mu_l B_{wl} \\ 0 \end{bmatrix},$$

$$\tilde{C}(\boldsymbol{\mu}) = \begin{bmatrix} \sum_{l=1}^{m} \mu_l C_{zl} & 0 \end{bmatrix}, \quad \tilde{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{x}_c \end{bmatrix}, \quad \forall \boldsymbol{\mu} \in M.$$

For the closed loop system (5), a common Lyapunov function is considered

$$V = \tilde{\boldsymbol{x}}^{\mathrm{T}} P \tilde{\boldsymbol{x}}, \tag{6}$$

where P is a $(n+k) \times (n+k)$ fixed symmetric positive-definite matrix. n is the number of states of the plant, and k is the order of the controller.

Recently it is shown that the output feedback control problem can be changed into an LMI problem[23]. Here we will follows the idea to solve our H_∞ output feedback fuzzy control problem. For using LMI techniques, partition the common Lyapunov function P and P^{-1} as

$$P = \begin{bmatrix} Y & N \\ N^{\mathrm{T}} & * \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} X & M \\ M^{\mathrm{T}} & * \end{bmatrix}, \tag{7}$$

where X and Y have dimensions $n \times n$ and are symmetric. From $PP^{-1} = I$, we infer $P \begin{bmatrix} X \\ M^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}$ which leads to

$$P\Pi_1 = \Pi_2 \text{ with } \Pi_1 = \begin{bmatrix} X & I \\ M^{\mathrm{T}} & 0 \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} I & Y \\ 0 & N^{\mathrm{T}} \end{bmatrix}. \tag{8}$$

Then we have the main result in this section.

Theorem 1. Given a fuzzy dynamic system described as in (2), an output feedback fuzzy controller described as in (4) and a constant $\gamma > 0$, if there exists a set of variables $\bar{A}_{li}, \bar{B}_i, \bar{C}_i, X$ and Y satisfying the following inequalities,

$$\begin{bmatrix} \Omega_{li11} & \Omega_{li21}^{\mathrm{T}} & B_{wl} & (C_{zl}X)^{\mathrm{T}} \\ \Omega_{li21} & \Omega_{li22} & YB_{wl} & C_{zl}^{\mathrm{T}} \\ B_{wl}^{\mathrm{T}} & B_{wl}^{\mathrm{T}}Y & -\gamma^2 I & 0 \\ C_{zl}X & C_{zl} & 0 & -I \end{bmatrix} < 0, \tag{9}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0, \tag{10}$$

where

$$\Omega_{li11} = A_l X + X A_l^{\mathrm{T}} + B_l \bar{C}_i + (B_l \bar{C}_i)^{\mathrm{T}}, \quad \Omega_{li21} = \bar{A}_{li} + A_l^{\mathrm{T}},$$
$$\Omega_{li22} = A_l^{\mathrm{T}} Y + Y A_l + \bar{B}_i C_l + (\bar{B}_i C_l)^{\mathrm{T}}; \quad l, i = 1, 2 \cdots m,$$

then the closed loop system (5) is asymptotically stable with H_∞ performance bound γ.

Moreover, determining M, N by

$$MN^{\mathrm{T}} = I - XY, \tag{11}$$

the parameters of the H_∞ output feedback controller are given by

$$C_{cl} = \bar{C}_l M^{\mathrm{T}}, B_{cl} = N^{-1}\bar{B}_l, A_{cl} = N^{-1}[\bar{A}_{ll} - NB_{cl}C_l X - YB_l C_{cl} M^{\mathrm{T}} - YA_l X]M^{\mathrm{T}}. \tag{12}$$

Proof. Firstly, it will be shown that (9) implies the following inequality

$$\tilde{A}^{\mathrm{T}}(\boldsymbol{\mu})P + P\tilde{A}(\boldsymbol{\mu}) + \gamma^{-2}P\tilde{B}(\boldsymbol{\mu})\tilde{B}^{\mathrm{T}}(\boldsymbol{\mu})P + \tilde{C}^{\mathrm{T}}(\boldsymbol{\mu})\tilde{C}(\boldsymbol{\mu}) < 0. \tag{13}$$

Since

$$\tilde{A}^{\mathrm{T}}(\boldsymbol{\mu})P + P\tilde{A}(\boldsymbol{\mu}) + \gamma^{-2}P\tilde{B}(\boldsymbol{\mu})\tilde{B}^{\mathrm{T}}(\boldsymbol{\mu})P + \tilde{C}^{\mathrm{T}}(\boldsymbol{\mu})\tilde{C}(\boldsymbol{\mu}) =$$
$$\begin{bmatrix} \sum_{l=1}^m \mu_l A_l & \sum_{l=1}^m \mu_l B_l \sum_{i=1}^m \mu_i C_{ci} \\ \sum_{i=1}^m \mu_i B_{ci} \sum_{l=1}^m \mu_l C_l & \sum_{i=1}^m \mu_i A_{ci} \end{bmatrix}^{\mathrm{T}} P + P \begin{bmatrix} \sum_{l=1}^m \mu_l A_l & \sum_{l=1}^m \mu_l B_l \sum_{i=1}^m \mu_i C_{ci} \\ \sum_{i=1}^m \mu_i B_{ci} \sum_{l=1}^m \mu_l C_l & \sum_{i=1}^m \mu_i A_{ci} \end{bmatrix} +$$
$$\gamma^{-2} P \begin{bmatrix} \sum_{l=1}^m \mu_l B_{wl} \\ 0 \end{bmatrix} \begin{bmatrix} \sum_{l=1}^m \mu_l B_{wl} \\ 0 \end{bmatrix}^{\mathrm{T}} P + \begin{bmatrix} \sum_{l=1}^m \mu_l C_{zl} & 0 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \sum_{l=1}^m \mu_l C_{zl} & 0 \end{bmatrix}, \tag{14}$$

it is easy to see that if the following set of inequalities is satisfied, then (13) is established,

$$\begin{bmatrix} A_l & B_l C_{ci} \\ B_{ci} C_l & A_{ci} \end{bmatrix}^{\mathrm{T}} P + P \begin{bmatrix} A_l & B_l C_{ci} \\ B_{ci} C_l & A_{ci} \end{bmatrix} + \gamma^{-2} P \begin{bmatrix} B_{wl} \\ 0 \end{bmatrix} \begin{bmatrix} B_{wl} \\ 0 \end{bmatrix}^{\mathrm{T}} P + [C_{zl} \quad 0]^{\mathrm{T}} [C_{zl} \quad 0] < 0,$$
$$l, i = 1, 2, \cdots, m, \tag{15}$$

where the membership function property (3b) has been used.

Using Schur complement, (15) is equivalent to

$$\begin{bmatrix} \hat{A}_{li}^{\mathrm{T}} P + P\hat{A}_{li} & P\hat{B}_l & \hat{C}_l^{\mathrm{T}} \\ \hat{B}_l^{\mathrm{T}} P & -\gamma^2 I & 0 \\ \hat{C}_l & 0 & -I \end{bmatrix} < 0, \quad l, i = 1, 2 \cdots m, \tag{16}$$

where

$$\hat{A}_{li} = \begin{bmatrix} A_l & B_l C_{ci} \\ B_{ci} C_l & A_{ci} \end{bmatrix}, \quad \hat{B}_l = \begin{bmatrix} B_{wl} \\ 0 \end{bmatrix}, \quad \hat{C}_l = [C_{zl} \quad 0].$$

Multiplying diag $[\Pi_1 \quad I \quad I]$ on both sides of (16), we have

$$\begin{bmatrix} \Pi_1^{\mathrm{T}}(\hat{A}_{li}^{\mathrm{T}} P + P\hat{A}_{li})\Pi_1 & \Pi_1^{\mathrm{T}} P\hat{B}_l & \Pi_1^{\mathrm{T}}\hat{C}_l^{\mathrm{T}} \\ \hat{B}_l^{\mathrm{T}} P\Pi_1 & -\gamma^2 I & 0 \\ \hat{C}_l \Pi_1 & 0 & -I \end{bmatrix} = \begin{bmatrix} \bar{\Omega}_{li11} & \bar{\Omega}_{li21}^{\mathrm{T}} & B_{wl} & (C_{zl}X)^{\mathrm{T}} \\ \bar{\Omega}_{li21} & \bar{\Omega}_{li22} & YB_{wl} & C_{zl}^{\mathrm{T}} \\ B_{wl}^{\mathrm{T}} & B_{wl}^{\mathrm{T}} Y & -\gamma^2 I & 0 \\ C_{zl}X & C_{zl} & 0 & -I \end{bmatrix} < 0, \tag{17}$$

where

$$\bar{\Omega}_{li11} = A_l X + X A_l^{\mathrm{T}} + B_l C_{ci} M^{\mathrm{T}} + [B_l C_{ci} M^{\mathrm{T}}]^{\mathrm{T}},$$
$$\bar{\Omega}_{li21} = N A_{ci} M^{\mathrm{T}} + N B_{ci} C_l X + Y B_l C_{ci} M^{\mathrm{T}} + Y A_l X + A_l^{\mathrm{T}},$$
$$\bar{\Omega}_{li22} = A_l^{\mathrm{T}} Y + Y A_l + N B_{ci} C_l + [N B_{ci} C_l]^{\mathrm{T}}.$$

Now we can define the change formula of the output feedback controller variables for fuzzy dynamic systems,

$$\bar{A}_{li} = N A_{ci} M^{\mathrm{T}} + N B_{ci} C_l X + Y B_l C_{ci} M^{\mathrm{T}} + Y A_l X, \bar{B}_i = N B_{ci}, \bar{C}_i = C_{ci} M^{\mathrm{T}}. \tag{18}$$

Then substituting the variable change formula (18) into (17), we obtain (9).

Furthermore, multiply Π_1 on both sides of $P > 0$, that is,

$$\Pi_1^{\mathrm{T}} P \Pi_1 = \begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0,$$

the condition (10) is equivalent to $P > 0$. Then, it follows from (13) that the closed loop fuzzy control system is asymptotically stable, since (13) implies $\tilde{A}^{\mathrm{T}}(\boldsymbol{\mu})P + P\tilde{A}(\boldsymbol{\mu}) < 0$.

Now let us prove that the closed loop fuzzy control system satisfies the H_∞ performance. Differentiating the Lyapunov function (6),

$$\dot{V}(t) = \left(\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\boldsymbol{x}}^{\mathrm{T}}\right) P\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{x}}^{\mathrm{T}} P \left(\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\boldsymbol{x}}\right) = \tilde{\boldsymbol{x}}^{\mathrm{T}}\tilde{A}^{\mathrm{T}} P\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{x}}^{\mathrm{T}} P\tilde{A}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{x}}^{\mathrm{T}} P\tilde{B}\boldsymbol{w} + \boldsymbol{w}^{\mathrm{T}}\tilde{B}P\tilde{\boldsymbol{x}}.$$

Integrating the above function,

$$\begin{aligned} V(\infty) - V(0) = & \int_0^\infty [\tilde{\boldsymbol{x}}^{\mathrm{T}}(\tilde{A}^{\mathrm{T}}P + P\tilde{A})\tilde{\boldsymbol{x}} + \boldsymbol{w}^{\mathrm{T}}\tilde{B}^{\mathrm{T}}P\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{x}}^{\mathrm{T}}P\tilde{B}\boldsymbol{w}\tilde{\boldsymbol{x}}]\mathrm{d}t < \\ & \int_0^\infty [\tilde{\boldsymbol{x}}^{\mathrm{T}}(-\gamma^{-2}P\tilde{B}\tilde{B}^{\mathrm{T}}P - \tilde{C}^{\mathrm{T}}\tilde{C})\tilde{\boldsymbol{x}} + \boldsymbol{w}^{\mathrm{T}}\tilde{B}^{\mathrm{T}}P\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{x}}^{\mathrm{T}}P\tilde{B}\boldsymbol{w}\tilde{\boldsymbol{x}}]\mathrm{d}t = \\ & \int_0^\infty [-\boldsymbol{z}^{\mathrm{T}}\boldsymbol{z} + \gamma^2\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w} - (\boldsymbol{w}^{\mathrm{T}} - \gamma^{-2}\tilde{\boldsymbol{x}}^{\mathrm{T}}P\tilde{B})\gamma^2(\boldsymbol{w} - \gamma^{-2}\tilde{B}^{\mathrm{T}}P\tilde{\boldsymbol{x}})]\mathrm{d}t \leqslant \\ & \int_0^\infty [-\boldsymbol{z}^{\mathrm{T}}\boldsymbol{z} + \gamma^2\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w}]\mathrm{d}t \end{aligned}$$

where (13) has been used.

Therefore, with zero initial condition, it follows that the stable global closed loop system (5) satisfies the H_∞ performance, that is, $\|\boldsymbol{z}\| \leqslant \gamma\|\boldsymbol{w}\|$. □

In light of the above theorem, the following procedure to evaluate an H_∞ output feedback control law is developed.

Algorithm 1 of the H_∞ fuzzy output feedback control design:

Step 1. Choose an H_∞ performance bound $\gamma > 0$.

Step 2. Use the LMI toolbox in Matlab to solve the inequalities (9) and (10). If the solutions to $\bar{A}_{li}, \bar{B}_i, \bar{C}_i, X$ and $Y, l, i = 1, 2 \cdots m$, are found, then determine M, N to satisfy (11), and an output feedback controller can be obtained by (12), and stop. Otherwise declare that the proposed algorithm fails.

It is noted that the above H_∞ output feedback fuzzy control design is based on the assumption that there exists a common Lyapunov function for fuzzy dynamic systems. In many cases, it is very hard to find such a common Lyapunov function for fuzzy dynamic systems or such a Lyaponov function does not exist at all. In these cases, the above fuzzy control design method can not be applied. Thus a new H_∞ output feedback fuzzy control design method will be presented in next section.

4 H_∞ Output Feedback Control Design based on a Piecewise Differential Lyapunov Function

We will develop a new H_∞ output feedback fuzzy control design method based on a piecewise differential Lyapunov function. For the control design purpose, we decompose the state space into m independent regions which are defined as follows

$$D_l = \{q|\mu_l(q) \geqslant \mu_j(q) \quad j=1,2,\cdots,m, j\neq l\} \quad \forall \boldsymbol{\mu}\in M, l=1,2,\cdots,m. \tag{19}$$

The boundary of D_l is denoted by

$$\partial D_l = \{q|\mu_l(q)=\mu_j(q),\ \ D_l\cap D_j\neq 0,\ j=1,2,\cdots,m, j\neq l\}, \forall\mu\in M, l=1,2,\cdots,m. \tag{20}$$

Then, on every region D_l the fuzzy system (2) can be expressed by

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= (A_l+\Delta A_l(\boldsymbol{\mu}))\boldsymbol{x}(t) + (B_l+\Delta B_l(\boldsymbol{\mu}))\boldsymbol{u}(t) + (B_{wl}+\Delta B_{wl}(\boldsymbol{\mu}))\boldsymbol{w}(t),\\ \boldsymbol{y}(t) &= (C_l+\Delta C_l(\boldsymbol{\mu}))\boldsymbol{x}(t),\\ \boldsymbol{z}(t) &= (C_{zl}+\Delta C_{zl}(\boldsymbol{\mu}))\boldsymbol{x}(t),\end{aligned} \tag{21}$$

where

$$\begin{aligned}&\Delta A_l(\boldsymbol{\mu}) = \sum_{i=1,i\neq l}^{m}\mu_i\Delta A_{li},\quad \Delta B_l(\boldsymbol{\mu}) = \sum_{i=1,i\neq l}^{m}\mu_i\Delta B_{li},\quad \Delta B_{wl}(\boldsymbol{\mu}) = \sum_{i=1,i\neq l}^{m}\mu_i\Delta B_{wli},\\ &\Delta C_l(\boldsymbol{\mu}) = \sum_{i=1,i\neq l}^{m}\mu_i\Delta C_{li},\quad \Delta C_{zl}(\boldsymbol{\mu}) = \sum_{i=1,i\neq l}^{m}\mu_i\Delta C_{zli},\quad \Delta A_{li}=A_i-A_l,\\ &\Delta B_{li}=B_i-B_l,\quad \Delta B_{wli}=B_{wi}-B_{wl},\quad \Delta C_{li}=C_i-C_l,\quad \Delta C_{zli}=C_{zi}-C_{zl},\\ &\forall\boldsymbol{\mu}\in M,\quad q(t)\in D_l,\quad l=1,2,\cdots,m.\end{aligned}$$

Here the l-th local system is different from the fuzzy dynamic local model in (2) because it considers all interactions among the local models. We assume that if the system is in the l-th sate region. it will stay there for a period of time $\bar{t}$ where $\bar{t}=\tau_{i+1}-\tau_i, i=1,2,\cdots,\bar{T}.\tau_i$ is the i-th time instant at which the $q(t)$ is on a boundary $\partial D_l.\bar{T}$ is the total number of transits among regions.

Consider a piecewise continuous output feedback controller below,

$$K_l:\begin{cases}\dot{\boldsymbol{x}}_c = A_{cl}\boldsymbol{x}_c(t)+B_{cl}\boldsymbol{y}(t),\\ \boldsymbol{u} = C_{cl}\boldsymbol{x}_c(t).\end{cases}\quad q(t)\in D_l, t\neq\tau_i, l=1,2,\cdots,m. \tag{22}$$

With the output feedback controller (22), on every region D_l the closed loop fuzzy control system can be described as

$$\begin{aligned}\dot{\tilde{\boldsymbol{x}}}(t) &= \tilde{A}_l(\boldsymbol{\mu})\tilde{\boldsymbol{x}}(t)+\tilde{B}_l(\boldsymbol{\mu})\boldsymbol{w}(t) = (\hat{A}_l+\Delta\hat{A}_l(\boldsymbol{\mu}))\tilde{\boldsymbol{x}}(t)+(\hat{B}_l+\Delta\hat{B}_l(\boldsymbol{\mu}))\boldsymbol{w}(t),\\ \boldsymbol{z}(t) &= \tilde{C}_l(\boldsymbol{\mu})\tilde{\boldsymbol{x}}(t) = (\hat{C}_{zl}+\Delta\hat{C}_{zl}(\boldsymbol{\mu}))\tilde{\boldsymbol{x}}(t),\end{aligned} \tag{23}$$

where

$$\hat{A}_l=\begin{bmatrix}A_l & B_lC_{cl},\\ B_{cl}C_l & A_{cl}\end{bmatrix},\quad \Delta\hat{A}_l(\boldsymbol{\mu})=\begin{bmatrix}\Delta A_l & \Delta B_lC_{cl}\\ B_{cl}\Delta C_l & 0\end{bmatrix},\quad \hat{B}_l=\begin{bmatrix}B_{wl}\\ 0\end{bmatrix},\quad \Delta\hat{B}_l(\boldsymbol{\mu})=\begin{bmatrix}\Delta B_{wl}\\ 0\end{bmatrix},$$

$$\hat{C}_{zl}=[C_{zl}\quad 0],\quad \Delta\hat{C}_{zl}(\boldsymbol{\mu})=[\Delta C_{zl}\quad 0],\quad \tilde{\boldsymbol{x}}=\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{x}_c\end{bmatrix},\ q(t)\in D_l,\quad \forall\boldsymbol{\mu}\in M,\quad l=1,2,\cdots,m.$$

We assume that the state $\tilde{\boldsymbol{x}}(t) \in R^{n+k}$ does not jump at the time instant τ_i, that is,

$$\tilde{\boldsymbol{x}}(\tau_i^-) = \tilde{\boldsymbol{x}}(\tau_i) = \tilde{\boldsymbol{x}}(\tau_i^+), \quad i = 1, 2, \cdots, \bar{T}. \tag{24}$$

Consider the following piecewise differentiable Lyapunov function

$$V(t) = \tilde{\boldsymbol{x}}^{\mathrm{T}} P(t) \tilde{\boldsymbol{x}}, \quad P(t) = P_l, \quad q(t) \in D_l, t \neq \tau_i, \tag{25}$$

$$P(\tau_i^-) = P_l, \quad q(\tau_i^-) \in \partial D_l, \quad P(\tau_i^+) = P_j, \quad q(\tau_i^+) \in D_j, \quad D_l \cap D_j \neq 0, t = \tau_i, \tag{26}$$

$$j, l = 1, 2, \cdots, m, j \neq l, i = 1, 2, \cdots, \bar{T},$$

where $(P_1, P_2, \cdots, P_m)$ is a set of $(n+k) \times (n+k)$ fixed symmetric positive-definite matrices and $P(t)$ is a symmetric positive-definite piecewise differentiable matrix function. The following notations are used for $P(t)$.

The left limit of $P(t)$ at $t \in R$ is defined as $P(t^-) = \lim\limits_{\varepsilon \to 0} P(t - \varepsilon)$ if the limit exists; and the right limit of $P(t)$ at $t \in R$ is defined as $P(t^+) = \lim\limits_{\varepsilon \to 0} P(t + \varepsilon)$ if the limit exists. Then we refer to a lemma from [10] about stabilization of fuzzy dynamic control systems.

Lemma 1. The closed loop fuzzy system is globally asymptotically stable with H_∞ performance bound γ, if there exist a controller and a positive-definite symmetric matrix function $P(t)$ such that

$$\tilde{A}_l^{\mathrm{T}}(\boldsymbol{\mu}) P(t) + P(t) \tilde{A}_l(\boldsymbol{\mu}) + \gamma^{-2} P(t) \tilde{B}_l(\boldsymbol{\mu}) \tilde{B}_l^{\mathrm{T}}(\boldsymbol{\mu}) P(t) + \tilde{C}_l^{\mathrm{T}}(\boldsymbol{\mu}) \tilde{C}_l(\boldsymbol{\mu}) < 0, \tag{27}$$

$$P(\tau_i^+) \leqslant P(\tau_i^-), \quad i = 1, 2, \cdots, \bar{T}, \quad \forall \boldsymbol{\mu} \in M \text{ and } t \in [0, \infty). \tag{28}$$

For the control design purpose, the following upper bounds for the uncertainties of the fuzzy local systems (21) are defined, which can be found as in [7,9]

$$\Delta \bar{A}_l^{\mathrm{T}}(\boldsymbol{\mu}) \Delta \bar{A}_l(\boldsymbol{\mu}) \leqslant E_l^{\mathrm{T}} E_l, \Delta B_{wl} \Delta B_{wl}^{\mathrm{T}} \leqslant E_{Bwl} E_{Bwl}^{\mathrm{T}}, \Delta C_{zl}^{\mathrm{T}}(\boldsymbol{\mu}) \Delta C_{zl}(\boldsymbol{\mu}) \leqslant E_{Czl}^{\mathrm{T}} E_{Czl}, \tag{29}$$

where

$$\Delta \bar{A}_l(\boldsymbol{\mu}) = [\Delta A_l, \Delta B_l], E_l = [E_{Al} \quad E_{Bl}], \quad l = 1, 2, \cdots, m.$$

For using LMI techniques, partition the piecewise differentiable Lyapunov function P_l and P_l^{-1} as

$$P_l = \begin{bmatrix} Y_l & N_l \\ N_l^{\mathrm{T}} & * \end{bmatrix}, \; P_l^{-1} = \begin{bmatrix} X_l & M_l \\ M_l^{\mathrm{T}} & * \end{bmatrix}, \quad l = 1, 2, \cdots, m, \tag{30}$$

where X_l and Y_l have dimensions $n \times n$ and are symmetric. From $P_l P_l^{-1} = I$, we infer $P_l \begin{bmatrix} X_l \\ M_l^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}$ which leads to

$$P_l \Pi_{l1} = \Pi_{l2} \text{ with } \Pi_{l1} = \begin{bmatrix} X_l & I \\ M_l^{\mathrm{T}} & 0 \end{bmatrix}, \; \Pi_{l2} = \begin{bmatrix} I & Y_l \\ 0 & N_l^{\mathrm{T}} \end{bmatrix}, \quad l = 1, 2, \cdots, m. \tag{31}$$

Here we refer to a lemma in [24] for the later use.

Lemma 2. For any constant $\varepsilon > 0$ and any matrices X and Y with appropriate dimensions, we have $X^{\mathrm{T}} Y + Y^{\mathrm{T}} X \leqslant \varepsilon X^{\mathrm{T}} X + \varepsilon^{-1} Y^{\mathrm{T}} Y$.

Then we have the following result in this section.

Theorem 2. Given a fuzzy dynamic system described as in (21), an output feedback controller described as in (22), the approximate upper bounds as in (29) and a constant $\gamma > 0$, if there exist a set of constants $\varepsilon_l > 0$, and a set of variables $\bar{A}_l, \bar{B}_l, \bar{C}_l, \bar{E}_{l0}, \bar{E}_{l1}, \bar{E}_{l2}, X_l$ and Y_l

satisfying the following inequalities,

$$\begin{bmatrix} \Omega_{l11} & \Omega_{l21}^{\mathrm{T}} & X_l & I & \Omega_{l51}^{\mathrm{T}} & \bar{E}_{l0}^{\mathrm{T}} & B_{wl} & E_{Bwl} & (C_{zl}X_l)^{\mathrm{T}} & (E_{Czl}X_l)^{\mathrm{T}} \\ \Omega_{l21} & \Omega_{l22} & I & Y_l & E_{Al}^{\mathrm{T}} & (Y_lE_{Al})^{\mathrm{T}} & Y_lB_{wl} & Y_lE_{Bwl} & C_{zl}^{\mathrm{T}} & E_{Czl}^{\mathrm{T}} \\ X_l & I & -\varepsilon_l^{-1}\bar{E}_{l1} & -\varepsilon_l^{-1}X_l & 0 & 0 & 0 & 0 & 0 & 0 \\ I & Y_l & -\varepsilon_l^{-1}X_l & -\varepsilon_l^{-1}I & 0 & 0 & 0 & 0 & 0 & 0 \\ \Omega_{l51} & E_{Al} & 0 & 0 & -\varepsilon_lI & -\varepsilon_lY_l & 0 & 0 & 0 & 0 \\ \bar{E}_{l0} & Y_lE_{Al} & 0 & 0 & -\varepsilon_lY_l & -\varepsilon_l\bar{E}_{l2} & 0 & 0 & 0 & 0 \\ B_{wl}^{\mathrm{T}} & B_{wl}^{\mathrm{T}}Y_l & 0 & 0 & 0 & 0 & -2^{-1}\gamma^2I & 0 & 0 & 0 \\ E_{Bwl}^{\mathrm{T}} & E_{Bwl}^{\mathrm{T}}Y_l & 0 & 0 & 0 & 0 & 0 & -2^{-1}\gamma^2I & 0 & 0 \\ C_{zl}X_l & C_{zl} & 0 & 0 & 0 & 0 & 0 & 0 & -2^{-1}I & 0 \\ E_{Czl}X_l & E_{Czl} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2^{-1}I \end{bmatrix} < 0, \tag{32}$$

$$\begin{bmatrix} X_l & I \\ I & Y_l \end{bmatrix} > 0, \tag{33}$$

and the boundary condition

$$P(\tau_i^+) \leqslant P(\tau_i^-), \quad i = 1, 2, \cdots, \bar{T}, \tag{34}$$

where

$$\Omega_{l11} = A_lX_l + X_lA_l^{\mathrm{T}} + B_l\bar{C}_l + (B_l\bar{C}_l)^{\mathrm{T}}, \quad \Omega_{l21} = \bar{A}_l + A_l^{\mathrm{T}},$$
$$\Omega_{l22} = A_l^{\mathrm{T}}Y_l + Y_lA_l + \bar{B}_lC_l + (\bar{B}_lC_l)^{\mathrm{T}}, \quad \Omega_{l51} = E_{Al}X_l + E_{Bl}\bar{C}_l, \quad l = 1, 2, \cdots, m,$$

then the closed loop system (23) is asymptotically stable with H_∞ performance bound γ. And determining M_l, N_l by

$$M_lN_l^{\mathrm{T}} = I - X_lY_l, \tag{35}$$

the parameters of the H_∞ output feedback controller are given by

$$C_{cl} = \bar{C}_lM_l^{-\mathrm{T}}, B_{cl} = N_l^{-1}\bar{B}_l, A_{cl} = N_l^{-1}\left[\bar{A}_l - N_lB_{cl}C_lX_l - Y_lB_lC_{cl}M_l^{\mathrm{T}} - Y_lA_lX_l\right]M_l^{-\mathrm{T}}. \tag{36}$$

Proof. Firstly, it will be shown that (32) implies the inequality (27) in Lemma 1.

$$\begin{aligned} &\tilde{A}_l^{\mathrm{T}}(\boldsymbol{\mu})P(t) + P(t)\tilde{A}_l(\boldsymbol{\mu}) + \gamma^{-2}P(t)\tilde{B}_l(\boldsymbol{\mu})\tilde{B}_l^{\mathrm{T}}(\boldsymbol{\mu})P(t) + \tilde{C}_l^{\mathrm{T}}(\boldsymbol{\mu})\tilde{C}_l(\boldsymbol{\mu}) = (\hat{A}_l^{\mathrm{T}} + \Delta\hat{A}_l^{\mathrm{T}})P_l + \\ &\quad P_l(\hat{A}_l + \Delta\hat{A}_l) + \gamma^{-2}P_l(\hat{B}_{wl} + \Delta\hat{B}_{wl})\cdot(\hat{B}_{wl} + \Delta\hat{B}_{wl})^{\mathrm{T}}P_l + (\hat{C}_l + \Delta\hat{C}_l)^{\mathrm{T}}(\hat{C}_l + \Delta\hat{C}_l) \leqslant \\ &\quad \hat{A}_l^{\mathrm{T}}P_l + P_l\hat{A}_l + \varepsilon_lP_lP_l + \varepsilon_l^{-1}\Delta\hat{A}_l^{\mathrm{T}}\Delta\hat{A}_l + 2\gamma^{-2}P_l\hat{B}_{wl}\hat{B}_{wl}^{\mathrm{T}}P_l + \\ &\quad 2\gamma^{-2}P_l\Delta\hat{B}_{wl}\Delta\hat{B}_{wl}^{\mathrm{T}}P_l + 2\hat{C}_l^{\mathrm{T}}\hat{C}_l + 2\Delta\hat{C}_l^{\mathrm{T}}\Delta C_l = \\ &\quad \hat{A}_l^{T}P_l + P_l\hat{A}_l + \varepsilon_lP_lP_l + 2\gamma^{-2}P_l\hat{B}_{wl}\hat{B}_{wl}^{\mathrm{T}}P_l + 2\hat{C}_l^{\mathrm{T}}\hat{C}_l + 2[\Delta C_{zl} \quad 0]^{\mathrm{T}}[\Delta C_{zl} \quad 0] + \\ &\quad \varepsilon_l^{-1}\begin{bmatrix} I & 0 \\ 0 & C_{cl} \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} \Delta A_l & \Delta B_l \\ 0 & 0 \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} I & 0 \\ 0 & B_{cl} \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} I & 0 \\ 0 & B_{cl} \end{bmatrix}\begin{bmatrix} \Delta A_l & \Delta B_l \\ 0 & 0 \end{bmatrix}\begin{bmatrix} I & 0 \\ 0 & C_{cl} \end{bmatrix} + \\ &\quad 2\gamma^{-2}P_l\begin{bmatrix} \Delta B_{wl} \\ 0 \end{bmatrix}\begin{bmatrix} \Delta B_{wl} \\ 0 \end{bmatrix}^{\mathrm{T}}P_l \leqslant \\ &\quad \hat{A}_l^{\mathrm{T}}P_l + P_l\hat{A}_l + \varepsilon_lP_lP_l + 2\gamma^{-2}P_l\hat{B}_{wl}\hat{B}_{wl}^{\mathrm{T}}P_l + 2\hat{C}_l^{\mathrm{T}}\hat{C}_l + 2[E_{Czl} \quad 0]^{\mathrm{T}}[E_{Czl} \quad 0] + \\ &\quad \varepsilon_l^{-1}\begin{bmatrix} I & 0 \\ 0 & C_{cl} \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} E_{Al} & E_{Bl} \\ 0 & 0 \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} I & 0 \\ 0 & B_{cl} \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} I & 0 \\ 0 & B_{cl} \end{bmatrix}\begin{bmatrix} E_{Al} & E_{Bl} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} I & 0 \\ 0 & C_{cl} \end{bmatrix} + \\ &\quad 2\gamma^{-2}P_l\begin{bmatrix} E_{Bwl} \\ 0 \end{bmatrix}\begin{bmatrix} E_{Bwl} \\ 0 \end{bmatrix}^{\mathrm{T}}P_l, \end{aligned} \tag{37}$$

where the lemma 2 has been used.

Thus if the right hand side of (37) is less than zero,we can Show (27). Using Schur complement, the right hand side of (37) being less than zero is equivalent to

$$\begin{bmatrix} \hat{A}_l^{\mathrm{T}}P_l + P_l\hat{A}_l & P_l & \Sigma_{Al}^{\mathrm{T}} & P_l\hat{B}_{wl} & P_l\Sigma_{Bwl} & \hat{C}_l^{\mathrm{T}} & \Sigma_{Cl}^{\mathrm{T}} \\ P_l & -\varepsilon^{-1}I & 0 & 0 & 0 & 0 & 0 \\ \Sigma_{Al} & 0 & -\varepsilon I & 0 & 0 & 0 & 0 \\ \hat{B}_{wl}^{\mathrm{T}}P_l & 0 & 0 & -2^{-1}\gamma^2 I & 0 & 0 & 0 \\ \Sigma_{Bwl}^{\mathrm{T}}P_l & 0 & 0 & 0 & -2^{-1}\gamma^2 I & 0 & 0 \\ \hat{C}_l & 0 & 0 & 0 & 0 & -2^{-1}I & 0 \\ \Sigma_{Cl} & 0 & 0 & 0 & 0 & 0 & -2^{-1}I \end{bmatrix} < 0, \tag{38}$$

where

$$\Sigma_{Al} = \begin{bmatrix} E_{Al} & E_{Bl}C_{cl} \\ 0 & 0 \end{bmatrix}, \quad \Sigma_{Bwl} = \begin{bmatrix} E_{Bwl} \\ 0 \end{bmatrix}, \quad \Sigma_{Cl} = [E_{Czl} \quad 0].$$

Multiplying diag$[\Pi_{l1} \quad \Pi_{l1} \quad \Pi_{l2} \quad I \quad I \quad I \quad I]$ on both sides of (38), we have

$$\begin{bmatrix} \bar{\Omega}_{l11} & \bar{\Omega}_{l21}^{\mathrm{T}} & X_l & I & \bar{\Omega}_{l51}^{\mathrm{T}} & \bar{\Omega}_{l61}^{\mathrm{T}} & B_{wl} & E_{Bwl} & (C_{zl}X_l)^{\mathrm{T}} & (E_{Czl}X_l)^{\mathrm{T}} \\ \bar{\Omega}_{l21} & \bar{\Omega}_{l22} & I & Y_l & E_{Al}^{\mathrm{T}} & \bar{\Omega}_{l62}^{\mathrm{T}} & Y_lB_{wl} & Y_lE_{Bwl} & C_{zl}^{\mathrm{T}} & E_{Czl}^{\mathrm{T}} \\ X_l & I & \bar{\Omega}_{l33} & -\varepsilon_l^{-1}X_l & 0 & 0 & 0 & 0 & 0 & 0 \\ I & Y_l & -\varepsilon_l^{-1}X_l & -\varepsilon_l^{-1}I & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{\Omega}_{l51} & E_{Al} & 0 & 0 & -\varepsilon_l I_l & -\varepsilon_l Y_l & 0 & 0 & 0 & 0 \\ \bar{\Omega}_{l61} & \bar{\Omega}_{l62} & 0 & 0 & -\varepsilon_l Y_l & \bar{\Omega}_{l66} & 0 & 0 & 0 & 0 \\ B_{wl}^{\mathrm{T}} & B_{wl}^{\mathrm{T}}Y_l & 0 & 0 & 0 & 0 & -2^{-1}\gamma^2 I & 0 & 0 & 0 \\ E_{Bwl}^{\mathrm{T}} & E_{Bwl}^{\mathrm{T}}Y_l & 0 & 0 & 0 & 0 & 0 & -2^{-1}\gamma^2 I & 0 & 0 \\ C_{zl}X_l & C_{zl} & 0 & 0 & 0 & 0 & 0 & 0 & -2^{-1}I & 0 \\ E_{Czl}X_l & E_{Czl} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2^{-1}I \end{bmatrix}$$
$$< 0, \tag{39}$$

where

$$\bar{\Omega}_{l11} = A_lX_l + X_lA_l^{\mathrm{T}} + B_lC_{cl}M_l^{\mathrm{T}} + [B_lC_{cl}M_l^{\mathrm{T}}]^{\mathrm{T}}, \bar{\Omega}_{l22} = A_l^{\mathrm{T}}Y_l + Y_lA_l + N_lB_{cl}C_l + [N_lB_{cl}C_l]^{\mathrm{T}},$$
$$\bar{\Omega}_{l21} = N_lA_{cl}M_l^{\mathrm{T}} + N_lB_{cl}C_lX_l + Y_lB_lC_{cl}M_l^{\mathrm{T}} + Y_lA_lX_l + A_l^{\mathrm{T}}, \bar{\Omega}_{l33} = -\varepsilon_l^{-1}(X_lX_l + M_lM_l^{\mathrm{T}}),$$
$$\bar{\Omega}_{l51} = E_{Al}X_l + E_{Bl}(C_{cl}M_l^{\mathrm{T}}), \bar{\Omega}_{l61} = Y_lE_{Bl}C_{cl}M_l^{\mathrm{T}} + Y_lE_{Al}X_l,$$
$$\bar{\Omega}_{l62} = Y_lE_{Al}, \bar{\Omega}_{l66} = -\varepsilon_l(Y_lY_l + N_lN_l^{\mathrm{T}}).$$

Now we can define the change formula of the output feedback controller variables for fuzzy dynamic systems,

$$\bar{A}_l = N_lA_{cl}M_l^{\mathrm{T}} + N_lB_{cl}C_lX_l + Y_lB_lC_{cl}M_l^{\mathrm{T}} + Y_lA_lX_l, \bar{B}_l = N_lB_{cl}, \bar{C}_l = C_{cl}M_l^{\mathrm{T}},$$
$$\bar{E}_{l0} = Y_lE_{Bl}C_{cl}M_l^{\mathrm{T}} + Y_lE_{Al}X_l, \bar{E}_1 = X_lX_l + M_lM_l^{\mathrm{T}}, \bar{E}_2 = Y_lY_l + N_lN_l^{\mathrm{T}}. \tag{40}$$

Then substituting the variable change formula (40) into (39), we obtain (32). Furthermore, multiply Π_{l1} on both sides of $P_l > 0$, that is,

$$\Pi_{l1}^{\mathrm{T}}P_l\Pi_{l1} = \begin{bmatrix} X_l & I \\ I & Y_l \end{bmatrix} > 0,$$

the condition (33) is equivalent to $P_l > 0$.

Then it follows from Lemma 1 that the closed loop fuzzy control system is asymptotically stable.

Now let us show H_∞ performance. Differentiating $\tilde{\boldsymbol{x}}^{\mathrm{T}}P\tilde{\boldsymbol{x}}$ and then integrating form τ_i^+ to τ_{i+1}^- we have

$$\int_{\tau_i^+}^{\tau_{i+1}^-}\left(\frac{\mathrm{d}}{\mathrm{d}t}(\tilde{\boldsymbol{x}}^{\mathrm{T}}P\tilde{\boldsymbol{x}})\right)\mathrm{d}t =$$

$$\int_{\tau_i^+}^{\tau_{i+1}^-}[\tilde{\boldsymbol{x}}^{\mathrm{T}}(\tilde{A}_l^{\mathrm{T}}P+P\tilde{A}_l)\tilde{\boldsymbol{x}}+\boldsymbol{w}^{\mathrm{T}}\tilde{B}_l^{\mathrm{T}}P\tilde{\boldsymbol{x}}+\tilde{\boldsymbol{x}}^{\mathrm{T}}P\tilde{B}_l\boldsymbol{w}\tilde{\boldsymbol{x}}]\mathrm{d}t <$$

$$\int_{\tau_i^+}^{\tau_{i+1}^-}[\tilde{\boldsymbol{x}}^{\mathrm{T}}(-\gamma^{-2}P\tilde{B}_l\tilde{B}_l^{\mathrm{T}}P-\tilde{C}_l^{\mathrm{T}}\tilde{C}_l)\tilde{\boldsymbol{x}}+\boldsymbol{w}^{\mathrm{T}}\tilde{B}_l^{\mathrm{T}}P\tilde{\boldsymbol{x}}+\tilde{\boldsymbol{x}}^{\mathrm{T}}P\tilde{B}_l\boldsymbol{w}\tilde{\boldsymbol{x}}]\mathrm{d}t =$$

$$\int_{\tau_i^+}^{\tau_{i+1}^-}[-\boldsymbol{z}^{\mathrm{T}}\boldsymbol{z}+\gamma^2\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w}-(\boldsymbol{w}^{\mathrm{T}}-\gamma^{-2}\tilde{\boldsymbol{x}}^{\mathrm{T}}P\tilde{B}_l)\gamma^2(\boldsymbol{w}-\gamma^{-2}\tilde{B}_l^{\mathrm{T}}P\tilde{\boldsymbol{x}})]\mathrm{d}t,$$

where the proved condition (27) has been used. Thus we obtain

$$\begin{aligned}&\tilde{\boldsymbol{x}}^{\mathrm{T}}P(\infty)\tilde{\boldsymbol{x}}-\tilde{\boldsymbol{x}}^{\mathrm{T}}P(0)\tilde{\boldsymbol{x}} <\\ &\int_0^{\infty}\left[-\boldsymbol{z}^{\mathrm{T}}\boldsymbol{z}+\gamma^2\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w}-(\boldsymbol{w}^{\mathrm{T}}-\gamma^{-2}\tilde{\boldsymbol{x}}^{\mathrm{T}}P\tilde{B}(t))\gamma^2(\boldsymbol{w}-\gamma^{-2}\tilde{B}^{\mathrm{T}}(t)P\tilde{\boldsymbol{x}})\right]\mathrm{d}t+\\ &\sum_i^{\bar{T}}[\tilde{\boldsymbol{x}}^{\mathrm{T}}P(\tau_i^+)\tilde{\boldsymbol{x}}-\tilde{\boldsymbol{x}}^{\mathrm{T}}P(\tau_i^-)\tilde{\boldsymbol{x}}]\leqslant\\ &\int_0^{\infty}[-\boldsymbol{z}^{\mathrm{T}}\boldsymbol{z}+\gamma^2\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w}]\mathrm{d}t+\sum_i^{\bar{T}}[\tilde{\boldsymbol{x}}^{\mathrm{T}}P(\tau_i^+)\tilde{\boldsymbol{x}}-\tilde{\boldsymbol{x}}^{\mathrm{T}}P(\tau_i^-)\tilde{\boldsymbol{x}}].\end{aligned}\tag{41}$$

From (34) we have $\sum_{i=0}^{\bar{T}}[\tilde{\boldsymbol{x}}^{\mathrm{T}}P(\tau_i^+)\tilde{\boldsymbol{x}}-\tilde{\boldsymbol{x}}^{\mathrm{T}}P(\tau_i^-)\tilde{\boldsymbol{x}}]\leqslant 0$.

Therefore, with zero initial condition, it follows that the stable global closed loop system (23) satisfies the H_∞ performance, that is, $\|\boldsymbol{z}\|\leqslant\gamma\|\boldsymbol{w}\|$. □

Based on the above theorem, an algorithm to obtain an H_∞ output feedback controller is as follows.

Algorithm 2 of the H_∞ fuzzy output feedback control design:

Step 1. Choose the approximate upper bounds (29) by using nonlinear programming algorithm or experience in [7] and a constant γ.

Step 2. Set $\varepsilon_l = 1$, for $l = 1, 2, \cdots, m$.

Step 3. Use the LMI toolbox in Matlab to solve the inequalities (34) and (35). If the solutions to $\bar{A}_l, \bar{B}_l, \bar{C}_l, \bar{E}_{l0}, \bar{E}_{l1}, \bar{E}_{l2}, X_l$ and Y_l are found, then determine M_l, N_l to satisfy (37), and an H_∞ output feedback controller can be obtained by (38), and stop. Otherwise go to the next step.

Step 4. Set $\varepsilon_l = \varepsilon_l/2$. If ε_l is less than some prespecified computational threshold, then stop and declare that the proposed algorithm fails. Otherwise, go to Step 3.

5 An Example

To illustrate the H_∞ fuzzy output feedback control design algorithms, we consider the following problem of balancing an inverted pendulum on a cart. The equations of motion for the pendulum are

$$\dot{x}_1 = x_2,\quad \dot{x}_2 = \frac{g\sin(x_1) - amlx_2^2\sin(2x_1)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)} + w,\tag{42}$$

where x_1 denotes the angle of the pendulum from the vertical, x_2 is the angular velocity, $g = 9.8\text{m/s}^2$ is the gravity constant, w is the external disturbance variable which is a sinusoidal signal, $w = \sin(2\pi t).m$ is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pendulum, and u is the force applied to the cart. $a = 1/(m+M)$. We choose m =2.0kg, M=8.0kg, $2l$=1.0m in the simulation. The following fuzzy model is used to design H_∞ output feedback fuzzy controllers.

$$\begin{aligned} R^l:\ \text{IF}\quad & x_1(t) \text{ is about } 0, x_2(t) \text{ is about } 0,\\ \text{THEN}\quad & \dot{\boldsymbol{x}}(t) = A_1\boldsymbol{x}(t) + B_1u(t) + B_{w1}w(t),\\ & y(t) = C_1\boldsymbol{x}(t),\\ & z(t) = C_{z1}\boldsymbol{x}(t), \end{aligned} \tag{43a}$$

$$\begin{aligned} R^2:\ \text{IF}\quad & x_1(t) \text{ is about } \pm\pi/3, x_2(t) \text{ is about } 0,\\ \text{THEN}\quad & \dot{\boldsymbol{x}}(t) = A_2\boldsymbol{x}(t) + B_2u(t) + B_{w2}w(t),\\ & y(t) = C_2\boldsymbol{x}(t),\\ & z(t) = C_{z2}\boldsymbol{x}(t), \end{aligned} \tag{43b}$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ 17.2941 & 0 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix},\ B_{w1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C_1 = [1\quad 0],\ C_{z1} = [1\quad 0];$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ 12.6305 & 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix},\ B_{w2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C_2 = [1\quad 0],\ C_{z2} = [1\quad 0].$$

We use the following membership functions

$$\begin{aligned} &\mu_1(x_1(t)) = (1 - 1/(1+\exp(-7(x_1(t)-\pi/4)))) \cdot (1/(1+\exp(-7(x_1(t)+\pi/4)))),\\ &\mu_2(x_1(t)) = 1 - \mu_1(x_1(t)). \end{aligned} \tag{44}$$

Application of Algorithm 1.

Case 1 ($\gamma = \infty$). Let the H_∞ performance bound $\gamma = \infty$. According to the first algorithm, the following result has been obtained

$$\bar{A}_{11} = \begin{bmatrix} -90.1320 & -.7910 \\ -.7910 & 0.5188 \end{bmatrix},\ \bar{B}_1 = \begin{bmatrix} -302.2 \\ 31 \end{bmatrix},\ \bar{C}_1 = [302.8\quad 726.5],$$

$$\bar{A}_{22} = \begin{bmatrix} -11.9122 & -.4066 \\ -.4066 & 0.4810 \end{bmatrix},\ \bar{B}_2 = \begin{bmatrix} -302.2 \\ 31 \end{bmatrix},\ \bar{C}_2 = [302.8\quad 726.5],$$

$$X = \begin{bmatrix} 1.2716 & -3.7912 \\ -3.7912 & 28.5691 \end{bmatrix},\ Y = \begin{bmatrix} 100.9821 & -2.6440 \\ -2.6440 & 0.9071 \end{bmatrix}.$$

Choosing $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and computing $N = \begin{bmatrix} -137.4 & 458.4 \\ 6.8 & -34.9 \end{bmatrix}$ by (11). For the first local system and the second local system respectively, the local output feedback controllers based on (12) have been obtained as follows

$$A_{c1} = \begin{bmatrix} -521.0 & 679.3 \\ -92.15 & 402.1 \end{bmatrix},\ B_{c1} = \begin{bmatrix} 498.2041 \\ 217.2123 \end{bmatrix},\ C_{c1} = [231.1\quad 486.3];$$

$$A_{c2} = \begin{bmatrix} -582.9 & 786.2 \\ -84.60 & 390.4 \end{bmatrix},\ B_{c2} = \begin{bmatrix} 498.2041 \\ 217.2123 \end{bmatrix},\ C_{c2} = [231.1\quad 486.3].$$

Fig.1 sows the angle response of the system (42) for an initial condition $x_1 = 70°, x_2 = 0.1t$ can be observed that the performance of disturbance attenuation is not satisfactory.

Case 2 ($\gamma = 1$). Now we choose the bound $\gamma = 1$. Similar to the procedure of Case 1, the following result has been also obtained

$$\bar{A}_{11} = \begin{bmatrix} -110.0881 & -.6168 \\ -.6168 & -.9160 \end{bmatrix}, \quad \bar{B}_1 = \begin{bmatrix} -30256 \\ 26 \end{bmatrix}, \quad \bar{C}_1 = [275.9 \quad 1392.4],$$

$$\bar{A}_{22} = \begin{bmatrix} -83.0442 & -.4807 \\ -.4807 & -.3013 \end{bmatrix}, \quad \bar{B}_2 = \begin{bmatrix} -30256 \\ 26 \end{bmatrix}, \quad \bar{C}_2 = [275.9 \quad 1392.4],$$

$$X = \begin{bmatrix} 1.1705 & -4.2171 \\ -4.2171 & 39.4660 \end{bmatrix}, \quad Y = \begin{bmatrix} 215.5097 & -2.3192 \\ -2.3192 & .5490 \end{bmatrix}.$$

With $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $N = \begin{bmatrix} -261.0 & 1000.4 \\ -5.0 & -30.4 \end{bmatrix}$, the first and the second local controllers have been obtained respectively

$$A_{c1} = \begin{bmatrix} -1604.9 & 3071.7 \\ -233.9 & 1014.0 \end{bmatrix}, \quad B_{c1} = \begin{bmatrix} 1020.9 \\ 324.1 \end{bmatrix}, \quad C_{c1} = [205.2 \quad 1188.3];$$

$$A_{c2} = \begin{bmatrix} -2249.5 & 2928.6 \\ -269.7 & 984.8 \end{bmatrix}, \quad B_{c2} = \begin{bmatrix} 1020.9 \\ 324.1 \end{bmatrix}, \quad C_{c2} = [205.2 \quad 1188.3].$$

Fig.2 shows the angle response for the same initial conditon. The performance of disturbance attenuation has been greatly improved.

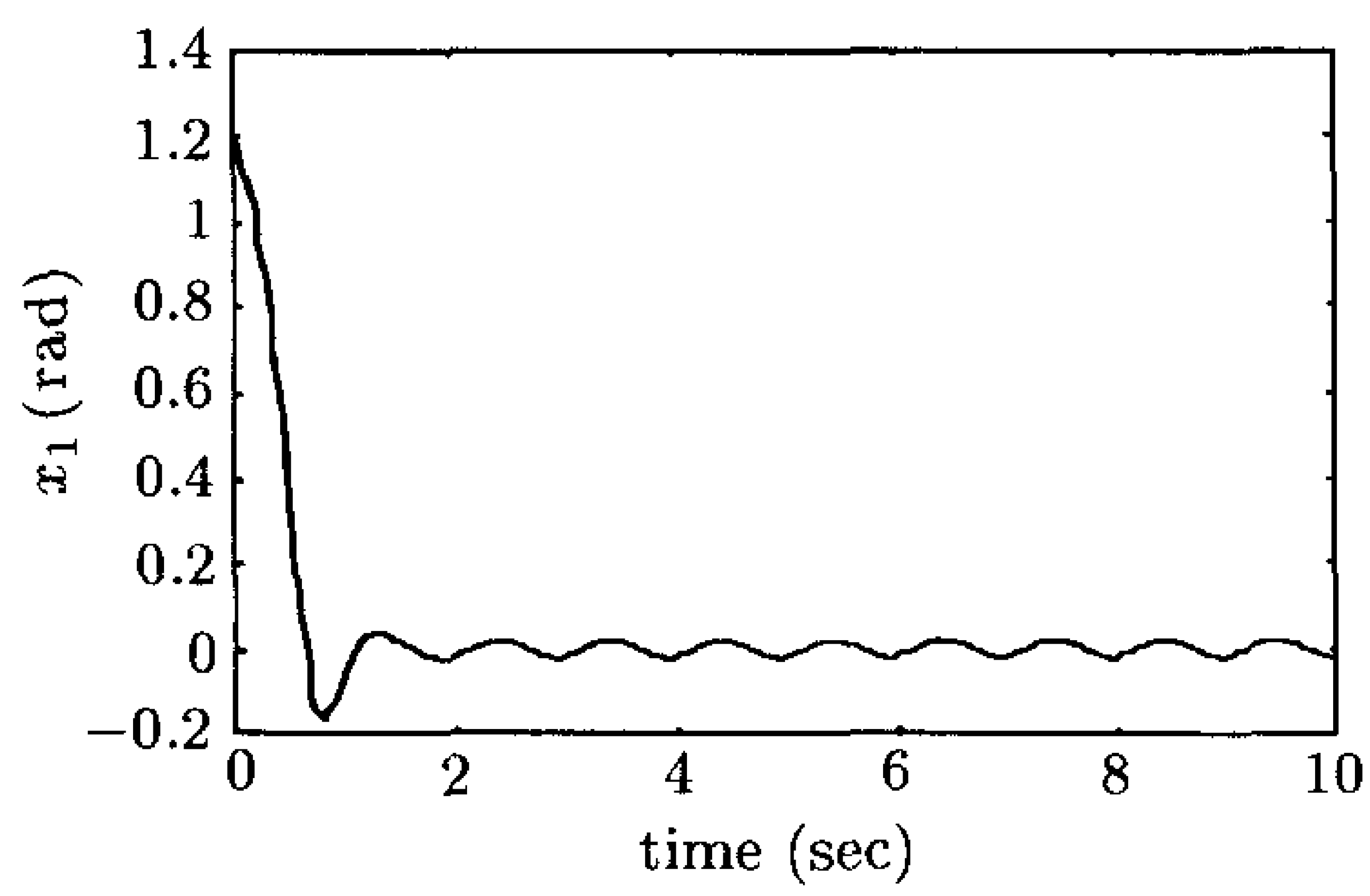

Fig.1 Angle response for Algorithm 1 ($\gamma = \infty$)

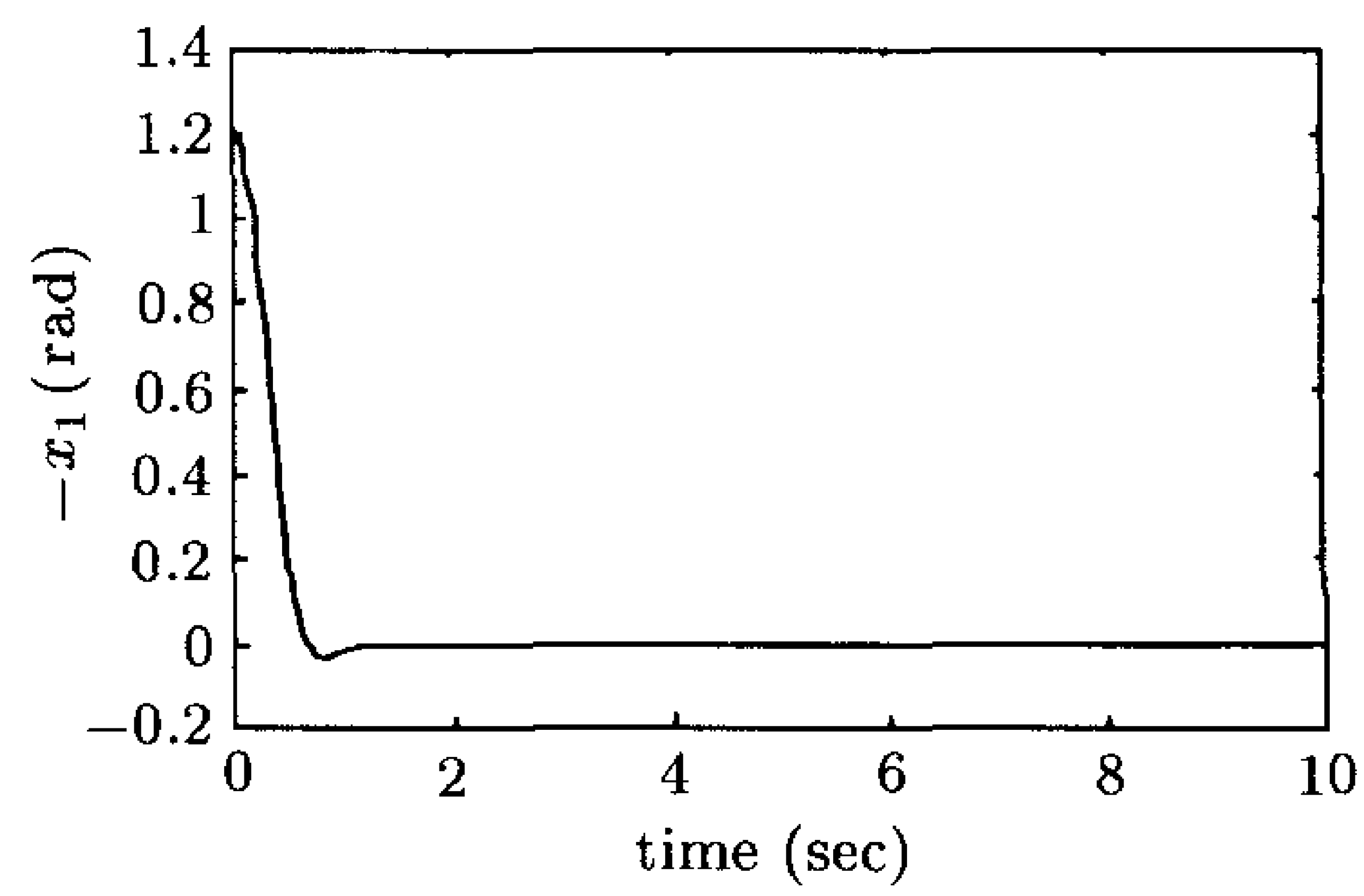

Fig.2 Angle response for Algorithm 1($\gamma = 1$)

Application of Algorithm 2.

Using the approximate upper bound searching algorithm in [7], we can find the upper bounds for the first local system and the second local system respectively,

$$E_1 = [E_{A1} \quad E_{B1}]; \quad E_{A1} = 0.1(A_2 - A_1), \quad E_{B1} = 0.01(B_2 - B_1). \tag{45a}$$

$$E_2 = [E_{A2} \quad E_{B2}]; \quad E_{A2} = 0.1(A_1 - A_2), \quad E_{B2} = 0.01(B_1 - B_2). \tag{45b}$$

Case 3 ($\gamma = \infty$). Let the H_∞ performance bound $\gamma = \infty$. For the first local system, according to Algorithm 2, for $\varepsilon_1 = 0.1$, the following result has been obtained

$$\bar{A}_1 = \begin{bmatrix} -52.6031 & -.5922 \\ -.5922 & 0.3003 \end{bmatrix}, \quad \bar{B}_1 = \begin{bmatrix} -1809 \\ 392.1 \end{bmatrix}, \quad \bar{C}_1 = [270.8 \quad 659.1],$$

$$X_1 = \begin{bmatrix} 0.8211 & -2.3202 \\ -2.3202 & 35.1244 \end{bmatrix}, \quad Y_1 = \begin{bmatrix} 196.4483 & -3.6804 \\ -3.6804 & 0.5611 \end{bmatrix}.$$

Choosing $M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and computing $N_1 = \begin{bmatrix} -168.8430 & 585.0712 \\ 4.3238 & -27.2476 \end{bmatrix}$ by (35), we can obtain the first local output feedback controller based on (36) as follows

$$A_{c1} = \begin{bmatrix} -366.4508 & 866.2116 \\ -22.3366 & 93.2886 \end{bmatrix}, \quad B_{c1} = \begin{bmatrix} 378.9967 \\ 52.0554 \end{bmatrix}, \quad C_{c1} = [270.8 \quad 659.1].$$

For the second local system, for $\varepsilon_2 = 0.1$, the following result has been obtained

$$\bar{A}_2 = \begin{bmatrix} -29.0099 & -.8601 \\ -.8601 & -.8092 \end{bmatrix}, \quad \bar{B}_2 = \begin{bmatrix} -497.8 \\ 6.3 \end{bmatrix}, \quad \bar{C}_2 = [235.8507 \quad 799.0445],$$

$$X_2 = \begin{bmatrix} .9132 & -2.1156 \\ -2.1156 & 18.5530 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} 88.2243 & -3.3092 \\ -3.3092 & 1.0091 \end{bmatrix}.$$

Selecting $M_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and finding $N_2 = \begin{bmatrix} -86.5764 & 248.0429 \\ 5.1568 & -24.7228 \end{bmatrix}$ by (35), we can obtain the second local output feedback controller based on (36) as follows

$$A_{c2} = \begin{bmatrix} -181.2207 & 328.0026 \\ -36.1109 & 53.0203 \end{bmatrix}, \quad B_{c2} = \begin{bmatrix} 280.2263 \\ 57.0036 \end{bmatrix}, \quad C_{c2} = [235.8507 \quad 799.0445].$$

Fig.3 shows the angle response of the system (42) for the initial conditon $x_1 = 70°, x_2 = 0$. It can be observed that the performance of disturbance attenuation is not satisfactory.

Case 4 ($\gamma = 1$). Now we choose the bound $\gamma = 1$. Similar to Case 3, for the first local system, for $\varepsilon_1 = 0.1$, the following result has been obtained

$$\bar{A}_1 = \begin{bmatrix} -25.4997 & -.5922 \\ -.5922 & .1824 \end{bmatrix}, \quad \bar{B}_1 = \begin{bmatrix} -4908.3 \\ -11.2 \end{bmatrix}, \quad \bar{C}_1 = [231.0 \quad 1970.5],$$

$$X_1 = \begin{bmatrix} .6008 & -1.9942 \\ -1.9942 & 19.8816 \end{bmatrix}, \quad Y_1 = \begin{bmatrix} 112.0088 & -2.0147 \\ -2.0147 & .8447 \end{bmatrix}.$$

With $M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $N_1 = \begin{bmatrix} -70.3126 & 263.4234 \\ 2.8949 & -19.8117 \end{bmatrix}$ the first local controller as follows

$$A_{c1} = \begin{bmatrix} -155.0308 & 354.3088 \\ -26.3354 & 28.0917 \end{bmatrix}, \quad B_{c1} = \begin{bmatrix} 278.7704 \\ 46.0198 \end{bmatrix}, \quad C_{c1} = [231.0 \quad 1970.5].$$

For the second local system, for $\varepsilon_2 = 0.1$, result, has been found

$$\bar{A}_2 = \begin{bmatrix} -20.6622 & -.8038 \\ -.8038 & 1.8507 \end{bmatrix}, \quad \bar{B}_2 = \begin{bmatrix} -3946.6 \\ -8.3 \end{bmatrix}, \quad \bar{C}_2 = [258.9 \quad 920.2],$$

$$X_2 = \begin{bmatrix} .5455 & -1.2942 \\ -1.2942 & 17.8494 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} 88.6051 & -3.2240 \\ -3.2240 & 0.5047 \end{bmatrix}.$$

With $M_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $N_2 = \begin{bmatrix} -51.5066 & 172.2192 \\ 2.4119 & -12.1811 \end{bmatrix}$, the second local controller is

$$A_{c2} = \begin{bmatrix} -110.7732 & 323.8752 \\ -26.2210 & 40.9022 \end{bmatrix}, \quad B_{c2} = \begin{bmatrix} 224.4703 \\ 31.6650 \end{bmatrix}, \quad C_{c2} = [258.9 \quad 920.2].$$

Fig.4 shows the angle response for the same initial condition. The performance of disturbance attenuation has been greatly improved.

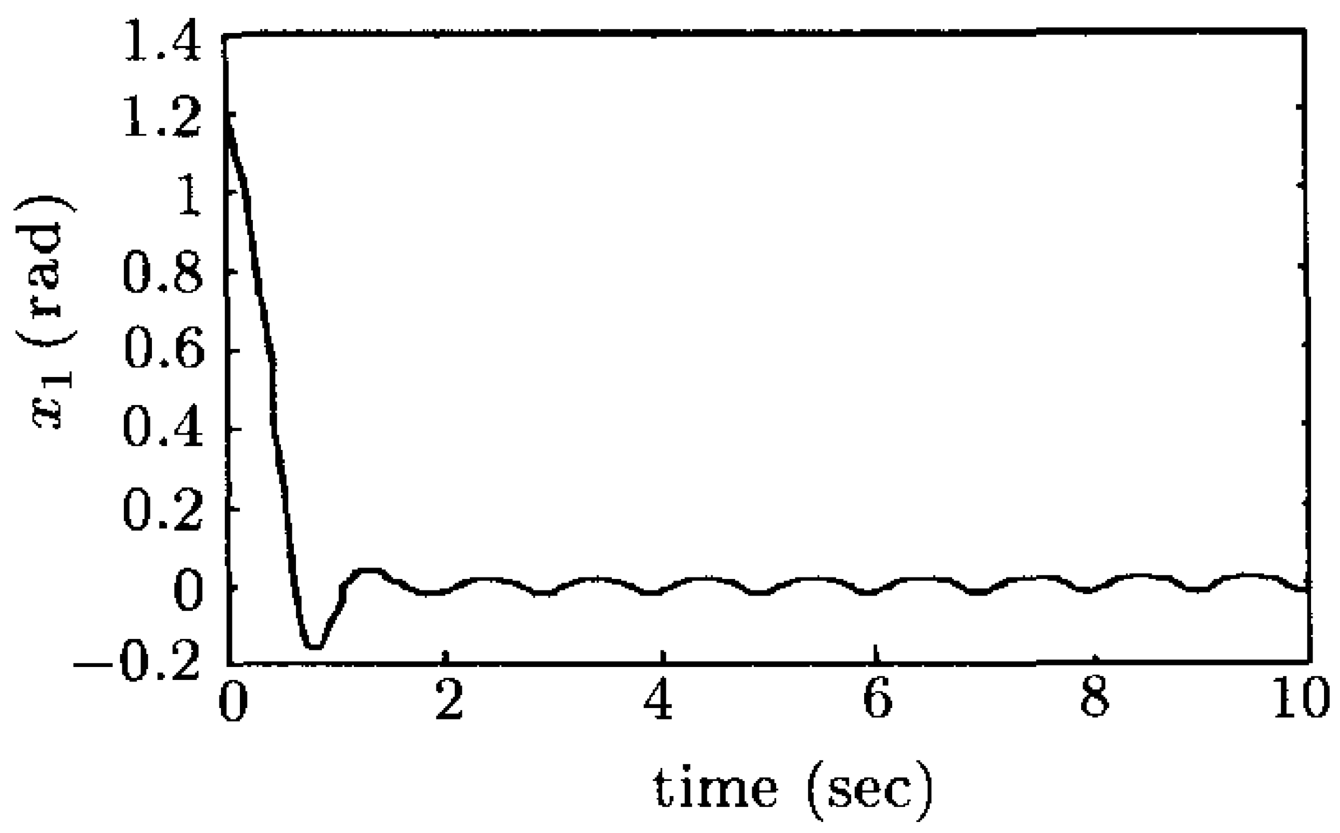

Fig.3 Angle response for Algorithm 2($\gamma = \infty$)

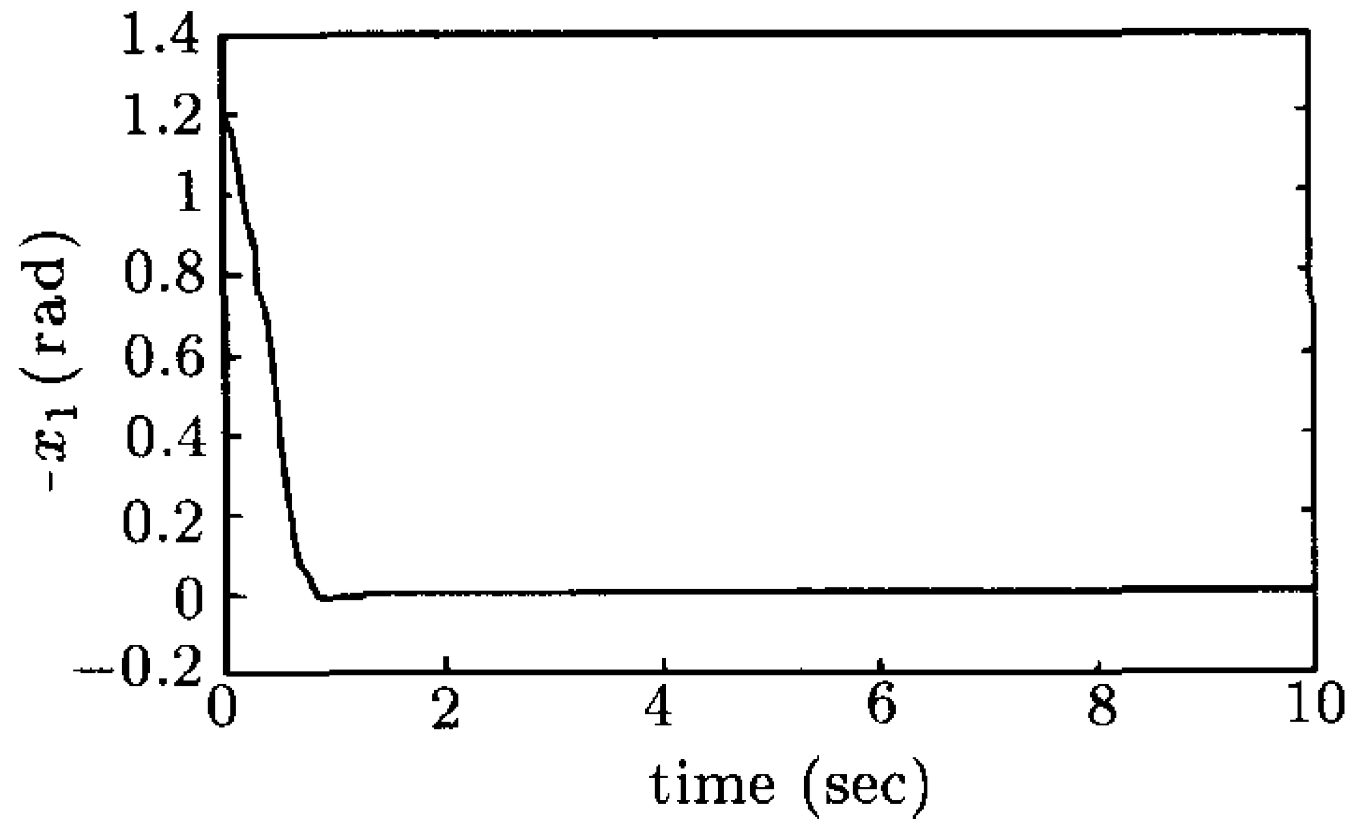

Fig.4 Angle response for Algorithm 2($\gamma = 1$)

6 Conclusions

This paper addresses two new kinds of output feedback control design methods with H_∞ performance for fuzzy dynamic systems. The H_∞ fuzzy output feedback controllers can be obtained by using LMI techniques. However, due to using the piecewise differentiable Lyapunov function, one boundary condition has been imposed in the second method, which is the topic of further study.

References

1 Zadeh L A. Outline of a new approach to the analysis of complex systems and decision processes. *IEEE Trans. Systems, Man & Cybernetics*, 1973, **3**: 28~44

2 Chen M S, Wang S W. Fuzzy clustering analysis for optimizing fuzzy membership functions. *Fuzzy Sets & Systems*, 1999, **103**: 239~254

3 Huwendiek O, Brockmann W. Function approximation with decomposed fuzzy systems. *Fuzzy Sets & Systems*, 1999, **101**: 273~286

4 Margaliot M, Langholz G. Hyperbolic optimal control and fuzzy control. *IEEE Trans. System, Man & Cybernetics*, 1999, **29**: 1~10

5 Takagi T, Sugeno M. Fuzzy identification of systems and its application to modeling and control. *IEEE Trans. System, Man & Cybernetics*, 1985, **15**: 116~132

6 Tanaka K, Ikeda T, Wang H O. Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs. *IEEE Trans. Fuzzy, Systems*, 1998, **6**: 250~265

7 Cao S G, Rees N W, Feng G. Stability analysis and design for a class of continuous time fuzzy control system. *Int. J. Control*, 1996, **64**(6): 1069~ 1089

8 Cao S G, Rees N W, Feng G. Analysis and design for a class of complex control system, Part I: fuzzy modeling and identification. *Automatica*, 1997, **33**(6): 1017~1028

9 Cao S G, Rees N W, Feng G. Analysis and design for a class of complex control system, Part II: fuzzy controller design. *Automatica*, 1997, **33**(6): 1028~1039

10 Cao S G, Rees N W, Feng G. H_∞ control of nonlinear continuous-time systems based on dynamic fuzzy models. *Int. J. Systems Science*, 1997, **27**(9): 821~830

11 Cao S G, Rees N W, Feng G. Quadratic stability analysis and design of a class of continuous time fuzzy control systems. *Int. J. Systems Science*, 1997, **27**: 193~203

12 Cao S G, Rees N W, Feng G. Further results about quadratic stability of continuous time fuzzy control systems. *Int. J. Systems Science*, 1997, **28**: 397~404

13 Cao S G, Rees N W, Feng G. Lyapunov-like stability theorems for continuous-time fuzzy control systems. *Int. J. Systems Science*, 1998, **69**(1): 49~64

14 Feng G, Cao S G, Rees N W, Chak C K. Design of fuzzy control systems based on state feedback. *J. Intelligent & Fuzzy Systems*, 1995, **3**: 295~304

15 Feng G, Cao S G, Rees N W, Chak C K. An approach to H_∞ control of a class of nonlinear system. *Automatica*, 1996, **32**: 1469~1474

16 Feng G, Cao S G, Rees N W, Chak C K. Design of fuzzy control systems with guaranteed stability. *J. Fuzzy Sets & Systems*, 1997, **85**: 1~10

17 Feng G, Cao S G, Rees N W, Ma J. Quadratic stabilization of fuzzy control systems using output feedback. *J. Intelligent & Fuzzy Systems*, 1997, **5**: 219~227
18 Zhang N, Feng G. Mixed H_2/H_∞ control design for fuzzy dynamic systems via LMI. In: Proc. 6th Int. Conf. Neural Information Processing, Perth, Australia, 1999. 298~303
19 Zhang N, Feng G. Output feedback fuzzy control design using LMI with H_2 constraint. In: Proc. Asia Control Conf., Shanghai, China. 2000. 509~514
20 Doyle J C, Glover K, Khargonekar P P, Francis B A. State space solution to standard H_2 and H_∞ control problem. *IEEE Trans. Automatica Control*, 1989, **34**: 831~846
21 Isidori A, Astofi A. Disturbance attenuation and H_∞ control via measurement feedback in nonlinear systems. *IEEE Trans. Automatica Control*, 1992, **37**: 1283~1293
22 Khargoneker P P, Petersen I R, Zhou K M. Robust stabilization of uncertain linear systems: quadratic stability and H_∞ control theory. *IEEE Trans. Automatica Control*, 1990, **29**: 1~10
23 Scherer C, Gahinet P, Chilali M. Multiobjective output-feedback control via LMI optimization. *IEEE Trans. Automatica Control*, 1997, **42**: 896~911
24 Petersen I R. A stabilization algorithm for a class of uncertain linear systems. *Systems and Control Letters*, 1987, **8**: 351~357

模糊动态系统的 H_∞ 输出反馈控制设计——应用线性矩阵不等式的算法

张 宁 冯 刚

(New South Wales 大学电力工程和电讯学院 澳大利亚悉尼)

(E-mail: ningz@alpha2.ee.unsw.edu.au)

摘 要 讨论了一类能用模糊动态系统模型表示的非线性系统的 H_∞ 输出反馈控制的设计. 分别基于一个共同的李亚普诺夫方程和一个分段微分的李亚普诺夫方程，给出了两种 H_∞ 输出反馈模糊控制的新设计方法. 通过解一组线性矩阵不等式，可以得到 H_∞ 输出反馈控制器. 最后，举例说明了这两种新的设计方法.

关键词 模糊动态系统，H_∞ 输出反馈控制，线性矩阵不等式.

张 宁 1984 年于清华大学自动化系获工程学士学位. 现在澳大利亚 New South Wales 大学攻读博士学位. 目前研究领域是：模糊控制，智能控制和模糊控制系统.

冯 刚 于 1982 年和 1984 年分别获得南京航空学院的工程学士和工程硕士学位. 1992 年获得澳大利亚 Melbourne 大学工程博士学位. 1991 年在澳大利亚 Royal Melbourne Institute of Technology 任教. 1992 年至 1999 年在澳大利亚 The University of New South Wales, the School of Electrical Engineering 任教. 现任 City University of Hong Kong 副教授. 曾作为高级访问学者在 National University of Singapore (1996~1997) 和德国 Aachen Technology University (1997~1998) 工作过. IEEE 资深会员，IEEE Trans. on Fuzzy Systems of IEEE Neural Network Council 助理编辑，并曾是 Conference Editorial Board of IEEE Control System Society (IEEE 控制系统协会的学术会议编辑委员会) 的助理编辑. 1997 年至 1998 年曾得到 Alexander von Humboldt Fellowship. 目前研究领域：鲁棒自适应控制，模糊控制，机器人控制，智能控制系统和混合控制系统.