METHOD OF EQUIVALENT WORKSTATION FOR MODELING AND ANALYSIS OF MULTISTAGE UNRELIABLE TRANSFER LINES WITH RANDOM PROCESSING TIMES

SHU Songgui JIANG Changjun

(*Institute of Automation, Chinese Academy of Sciences, Beijing* 100080, *China*)

Abstract This paper presents an efficient method of "equivalent workstation" for modeling and analysis of multistage transfer lines with unreliable machines and finite buffers. The random processing times for discrete parts and random failure and repair times for machines are assumed. The unrelible machines and their random processing times lead to blockage and stavation in operation due to limited storage capacities. This makes the problem of modeling and analysis very difficult to treat as they require large state spaces and can't be decomposed exactly. In this paper a single buffer between two reliable workstations is analysed first. Then an equivalent workstation without starvation and blockage is constructed. Thereafter connecting all the equivalent workstations in series we get the "equivalent transfer line." A set of performance measures such as the production rates, efficiencies and average inventory levels are derived in explicit analysical expressions. Finally two numerical examples are given for comparing the calculated results with those of Y.F. Choong & S.B. Gershwin, Ref. [10].

Key words Flexible manufacturing system(FMS), computer integrated manufacturing system(CIMS), buffer inventoy levers, throughput (poduction rate), reliability engineering, operations research.

1 INTRODUCTION

The unreliable multistage transfer line with finite buffer is a special large system. It is subject to interruptions in operation due to finite buffers and unreliable machines with random processing times. The performance characteristic of such a system is very difficult to solve analytically because it requires large state spaces and can not be decomposed exactly.

The literature shows that a lot of good work about this problem has been done by some researchers for different conditions in different ways. As the character of processing times is concerned, the related literature can be divided into two major classes. The first consists of papers with deterministic processing times, but the failure and repair times are random, such as Ref.[1-7]. In which [2,3] were solved by discrete approach and others by continuous approach. A second class of the papers includes those with random processing times. Moreover the failure and repair times are also random. Ref.[8-12] belong to this class in which only [8] was solved by discrete approach and others by ontinuous method.

As far as the authors are aware, the exact results have been obtained only in a few papers for a very limited stage lines, such as Ref.[1] for two stage production line and [2] for three stage line. For more than three stages only the approximate and simulation methods have been studied.

The research work in this paper falls into the second class. An equivalent workstation method is proposd for analysizing the n-stage transfer line with random processing times. A finite intermediate buffer between two reliable workstations is solved first by Markov process into $k+1$ states, but only two states (unfull and unempty) are used for constructing the "equivalent workstation". Then connect all the equivalent workstations in series to form an "equivalent transfer line". This new model can be treated as a continuous operating production system, thus removing all the interrupting operations out of the equivalent transfer line. Finally, two examples are given for illustrating the application of this new method in engineering analysis and design, and the results are compared with Ref.[10].

2 ASSUMPTION

Consider a n-stage transfer line as shown in Fig.1.

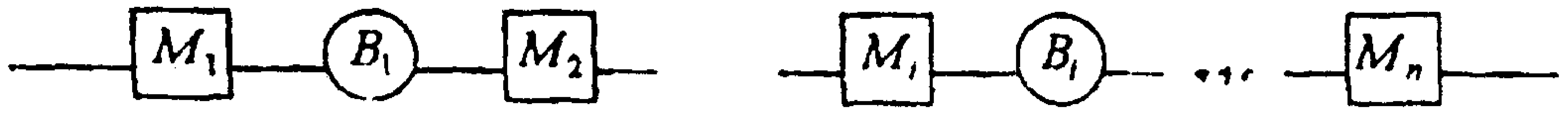

Fig.1 n-stage tansfer line

Here M_i indicates the ith machine and B_i indicates the ith buffer, $i=1,2,\cdots,n$.

Assumptions listed below are used for formulating the mathematical model:

1) The processing times for any discrete parts at the individual workstations are random variables. let ω_i denote the rated production rate of the ith machine in pieces per time unit. Transportation takes negligible time compared to machining times.

2) Any failured machines are repaired without delay. The life times and repair times are distributed exponentially with parameters λ_i and μ_i respectively There are no two or more failures occured simultaneously.

3) If the buffer B_{i-1} is empty and M_{i-1} (or another before it) in failure condition (under repair), then M_i is starved. If the buffer B_i is full and M_{i+1} (or another after it) in failure condition (under repair), then M_i is blocked.

4) The first workstation is never starved and the last workstation is never blocked.

5) Machines fail only while processing parts (operation or state dependent failures). It means that the machine in starvation or blockage state can not fail.

3 ANALYSIS OF INTERMEDIATE SYSTEM BUFFERS

Consider the ith intermediate buffer between two reliable workstations as an isolated system shown in Fig.2.

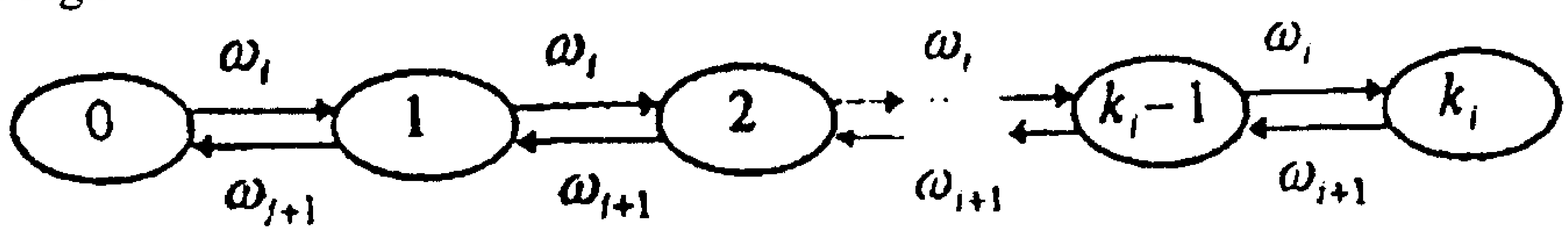

Fig.2 Diagram of buffer state transition

For random processing times, the buffer states have been analyzed by Markov process [11,12] and the steady state results obtained as follows:

The probability for the jth state of buffer,

$$P_j = \frac{\rho_i^j(1-\rho_i)}{1-\rho^{k_i+1}} \qquad \left[\sum_{j=0}^{k_i} P_j = 1\right] \tag{1}$$

where $\rho_i = \omega_i / \omega_{i+1}, k_i$ is the capacity of the buffer storage (including one unit in workstation). Obviously, when $\rho_i = \omega_i / \omega_{i+1} = 1$.

$$P_j = \frac{1}{k_i + 1} \tag{1a}$$

For constructing the equivalent workstation in the next section, we need only the following two states:

$$\left.\begin{aligned} \overline{P}_{k_i} &= 1 - P_{k_i} = \frac{1-\rho_i^{k_i}}{1-\rho_i^{k_i+1}} \quad \text{(unfull)} \\ \overline{P}_{0i} &= 1 - P_{0i} = \frac{\rho_i(1-\rho_i^{k_i})}{1-\rho_i^{k_i+1}} \quad \text{(unempty)} \end{aligned}\right\} \tag{2}$$

At $\rho_i = 1$, then

$$\overline{P}_{k_i} = \overline{P}_{0i} = \frac{k_i}{k_i + 1} \tag{2a}$$

The average invetory level in buffer,

$$M_{k_i} = \sum_{j=1}^{k_i} jP_j = \frac{\rho_i - (k_i+1)\rho_i^{k_i+1} + k_i\rho_i^{k_i+2}}{1 - \rho_i - \rho_i^{k_i+1} + \rho_i^{k_i+2}} \tag{3}$$

At $\rho_i = \omega_i / \omega_{i+1} = 1$, then

$$M_{k_i} = \frac{k_i}{2} \tag{3a}$$

4 EQUIVALENT WORKSTATION

The equivalent workstation is defined as an isolated machine with neither starvation nor blockage.

Let P_{ji} represent the probability of the jth state for ith workstation in the transfer line. Where j ranging from 1 to 5 denotes the states of normal working, blockage, starvation, both blockge and starvation and repair respectively. Then the state transition diagram of the ith workstation is shown as Fig.3.

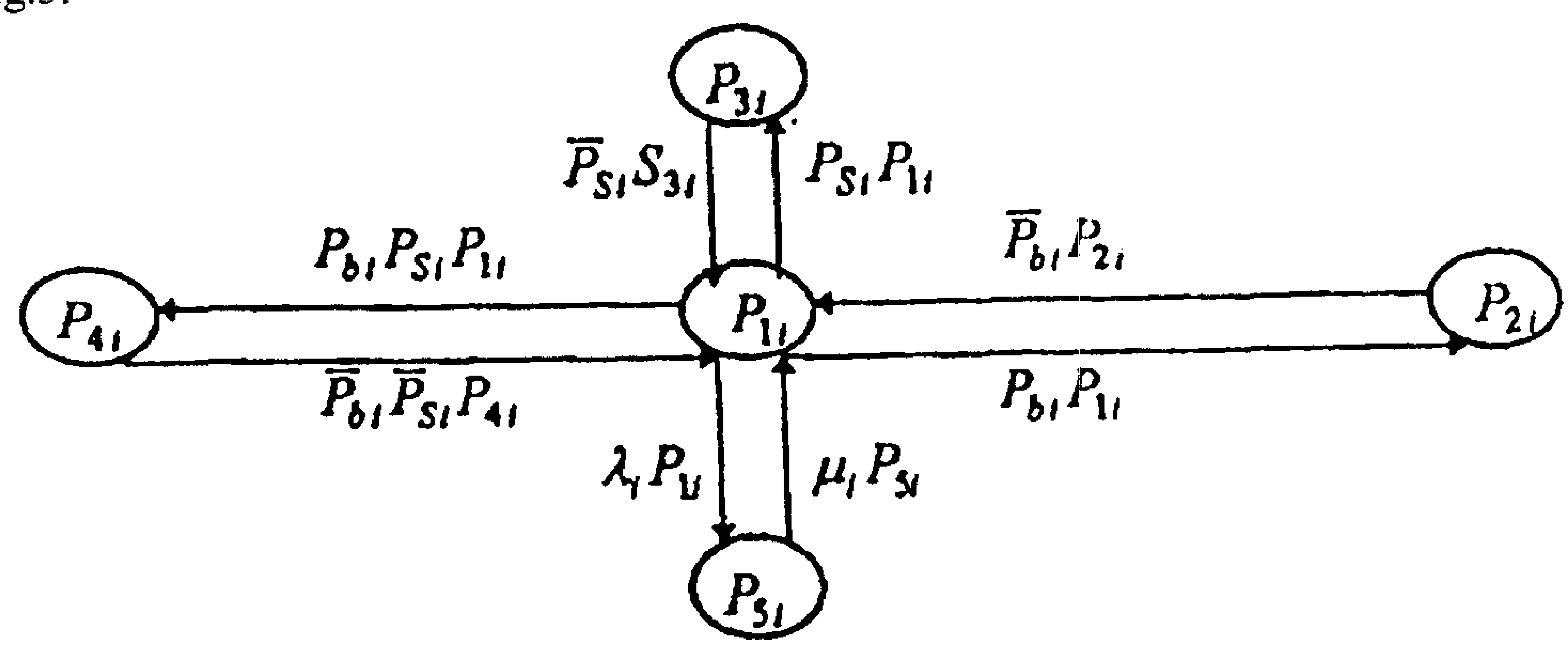

Fig.3 Transition diagram of machine M_i

According to definitions of blockage (P_b) and starvation (P_s) as given in assumption (3) (neglecting the influences of failures before M_{i-1} and after M_{i+1}), we have

$$\left.\begin{aligned} P_{bi} &= P(\overline{P}_{0(i-1)}, P_{k_i}, \overline{P}_{a(i+1)}) = \overline{P}_{0(i-1)} P_{k_i} \overline{P}_{a(i+1)} \\ P_{Si} &= P(P_{0(i-1)}, \overline{P}_{k_i}, \overline{P}_{a(i-1)}) = P_{0(i-1)} \overline{P}_{k_i} \overline{P}_{a(i-1)} \end{aligned}\right\} \tag{4}$$

where $P_{ai} = \dfrac{\mu_i}{\mu_i + \lambda_i} = A_i$ availability of isolated M_i. (4a)

and define $B_i = \overline{P}_{bi}\overline{P}_{Si}$, availability of buffers related to M_i.

then

$$\left.\begin{aligned} \dot{P}_{1i} &= \overline{P}_{b1} P_{2i} + \overline{P}_{Si} P_{3i} + \overline{P}_{bi}\overline{P}_{Si} P_{4i} + \mu_i P_{5i} \\ &\quad -(P_{bi} + P_{Si} + P_{bi}P_{Si} + \lambda_i) P_{1i} \\ \dot{P}_{2i} &= P_{bi} P_{1i} - \overline{P}_{bi} P_{2i} \\ \overline{P}_{3i} &= P_{Si} P_{1i} - \overline{P}_{Si} P_{3i} \\ \overline{P}_{4i} &= P_{bi} P_{Si} P_{1i} - \overline{P}_{bi}\overline{P}_{Si} P_{4i} \\ \overline{P}_{5i} &= \lambda_i P_{1i} - \mu_i P_{5i} \end{aligned}\right\} \tag{5}$$

For the steady state, the left sides of (5) should be equal to zero. Then solve the equations simultaneously, we get.

$$\left.\begin{aligned} &P_{2i}=\frac{P_{bi}}{\bar{P}_{bi}}P_{1i}, \quad P_{4i}=\frac{P_{bi}P_{Si}}{B_i}P_{1i} \\ &P_{3i}=\frac{P_{Si}}{\bar{P}_{Si}}P_{1i}, \quad P_{5i}=\frac{\lambda_i}{\mu_i}P_{1i} \\ &\sum_{j=1}^{5}P_{ji}=1=(1+\frac{P_{bi}}{\bar{P}_{bi}}+\frac{P_{Si}}{\bar{P}_{Si}}+\frac{P_{bi}P_{Si}}{B_i}+\frac{\lambda_i}{\mu_i})P_{1i}=(\frac{1}{B_i}+\frac{\lambda_i}{\mu_i})P_{1i} \end{aligned}\right\} \tag{6}$$

$$P_{1i}=\frac{1}{1/B_i+\lambda_i/\mu_i}=\frac{\mu_i B_i}{\mu_i+\lambda_i B_i}=A_i'=E_i$$ probability of normal working, or availability (also efficiency) of equivalent workstation

$$P_{2i}=\frac{P_{bi}}{\bar{P}_{bi}}A_i' \text{ (probability of blockage for } M_i\text{)}$$

$$P_{3i}=\frac{P_{Si}}{\bar{P}_{Si}}A_i' \text{ (probability of starvation for } M_i\text{)} \tag{6a}$$

$$P_{4i}=\frac{P_{bi}P_{Si}}{B_i}A_i' \text{ (probability of both blockage and starvation for } M_i\text{)}$$

$$P_{5i}=\frac{\lambda_i}{\mu_i}A_i'=\frac{\lambda_i B_i}{\mu_i+\lambda_i B_i} \text{ (probability of repair for } M_i\text{)}$$

The above five exprssions in eqn.(6) or (6a) are also the steady state time proportions of the respective states (see calculated results in Section VII-B-(3)).

The idle time proportion due to blockage and starvation of M_i is

$$P_{bS_i}=P_{2i}+P_{3i}+P_{4i}=\frac{\mu_i(1-B_i)}{\mu_i+\lambda_i B_i} \tag{6b}$$

The production rate (throughput) of the equivalent workstation for M_i,

$$W_i=P_{1i}\omega_i=\frac{\omega_i\mu_i B_i}{\mu_i+\lambda_i B_i} \tag{7}$$

The isolated production rate of M_i,

$$W_i'=\omega_i A_i=\frac{\omega_i\mu_i}{\mu_i+\lambda_i} \tag{7a}$$

5 EQUIVALENT TRANSFER LINE

Connecting all the equivalent workstations in series we get an equivalent transfer line as shown in Fig.4.

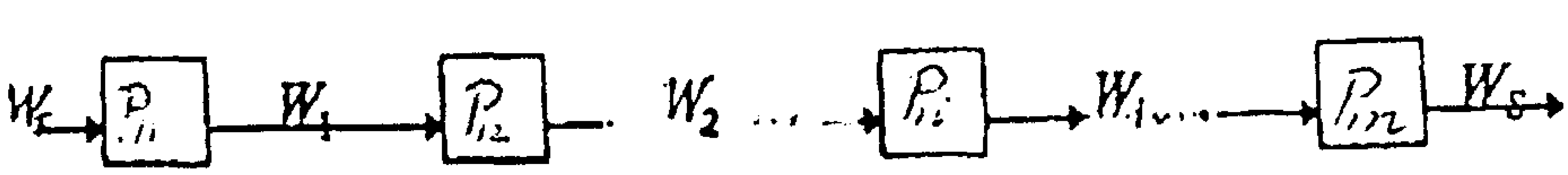

Fig.4 Equivalent transfer line

From the principle of conservation of the processing piece flow in the transfer line, the steady state manufacturing rate for each workstation as eqn.(7) should equal to the system production rate namely

$$\left.\begin{aligned} W_S = W_i = \frac{\omega_i \mu_i B_i}{\mu_i + \lambda_i B_i} = \frac{\omega_i}{\left(\frac{1}{B_i} + \frac{\lambda_i}{\mu_i}\right)} = A'\omega_i \\ \text{or } W_S = P_{1i}\omega_i = A_i'\omega_i, i = 1,2,\cdots,n \end{aligned}\right\} \tag{8}$$

The above eqution is also the ncessary and sufficient condition for multistage repaiable transfer line without any creation or missing of processing piece along the line.

The efficiency of the system is defined as the probability of production at full rate, or the ratio of actual processing time for all the individual workstations to the total rated system production time.

$$E_s = \sum_{i=1}^{n} \frac{1}{\omega_i} \Big/ \sum_{i=1}^{n} \frac{1}{W_i} = \frac{W_s}{n} \sum_{i=1}^{n} \frac{1}{\omega_i} = \frac{1}{n} \sum_{i=1}^{n} E_i \tag{9}$$

The above equation may be used as the objective function for optimal design.

If equation (8) can not hold, then the system production rate should be the bottleneck throughput,

$$W_s = \min_i W_i = \min_i \frac{\omega_i \mu_i B_i}{\mu_i + \lambda_i B_i} \tag{10}$$

For the homogeneous transfer line (e.i: $\rho = \rho_i = \omega_i / \omega_{i+1} = 1$, and let $k_i = k_{i+1} = k$, then by eqn.(4) and (2a), we have

$$\left.\begin{aligned} P_{bi} &= \overline{P}_{0(i-1)} P_{ki} \overline{P}_{a(i+1)} = \frac{k}{k+1} \times \frac{1}{k+1} \times \frac{\lambda}{\mu+\lambda} = \frac{k}{(k+1)^2} \times \frac{\lambda}{\mu+\lambda} \\ P_{Si} &= P_{0(i-1)} \overline{P}_{ki} \overline{P}_{a(i-1)} = \frac{1}{k+1} \times \frac{k}{k+1} \times \frac{\lambda}{\mu+\lambda} = \frac{k}{(k+1)^2} \times \frac{\lambda}{\mu+\lambda} \\ B_i &= \overline{P}_{bi} \overline{P}_{Si} = (1 - P_{bi})(1 - P_{Si}) = [1 - \frac{k}{(k+1)^2} \times \frac{\lambda}{\mu+\lambda}]^2 \end{aligned}\right\} \tag{4b}$$

Then substituting B_i in Eqn.(8) and (9) we get W_S and E_S.

For nonhomogeneous transfer line, the bottleneck production rate will be the system throughout. Moreover, if the device of intelligent control is used for adjusting the production rates of each workstation on line, then eqn.(8) can hold for the whole system and the system will operate as a homogeneous transfer line with bottleneck throughput.

6 AVERAGE INVENTORY LEVELS IN THE INTERMEDIATE BUFFERS OF MULTISTAGE TRANSFER LINE

Eqn.(3) represens the average inventory level of a buffer between two reliable workstations. For the homogeneous transfer line, $\rho_i = \omega_i \div \omega_{i+1} = 1$, then Eqn.(3) becomes (3a).

However, for either homogeneous or nonhomogeneous unreliable multistage transfer line, we can derive the average inventory levels for each intermediate buffer and whole line as follows.

Based upon assumption (2), the individual inventory levels at different failure cases in steady state can be determined separately according to the following procedures.

1) For the case of no failure, the homogeneous transfer line or the line with intellent control will operate as a homogeneous line with bottleneck throughput. The inventory levels will be uniformly distributed for each buffer in the line as shown by Eqn.(3a). The probability in this case is

$$P(A_S)=\prod_{i=1}^{n} A_i=\prod_{i=1}^{n} \frac{\mu_i}{\mu_i+\lambda_i} \tag{11}$$

2) If the failure occures at the ith workstation only, then all the buffers before it will be full and the others after it will be empty. The probability for this case is

$$P(\overline{A}_s)=\frac{\overline{A}_i}{A_i}\prod_{i=1}^{n} A_i=I_i\prod_{i=1}^{n} A_i \tag{12}$$

where $I_i=\overline{A}_i/A_i=\dfrac{\lambda_i/(\mu_i+\lambda_i)}{\mu_i/(\mu_i+\lambda_i)}=\dfrac{\lambda_i}{\mu_i}, i=1,2,\cdots,n$.

3) For the simplification of calculations, we take the weight of the probability for the case of no failure (Eqn.(11)) as unit, then the weight for the ith workstation failure (eqn(12)) will be I_i.

4) Finally, the average inventory level of the ith buffer can be calculated as listed in table 1.

Table 1 Average inventory Levels for homogeneous line

Workstations \ Buffers		Average Inventory Levels, M_{k_i}					Remarks
Condition	Weight	1	2	…	$n-2$	$n-1$	
no failure	1	$k_1/2$	$k_2/2$	…	$k_{n-2}/2$	$k_{n-1}/2$	Eqn.(3a)
1st failure	I_1	0	0	…	0	0	
2nd failure	I_2	k_1I_2	0	…	0	0	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	
$(n-1)$th	I_{n-1}	k_1I_{n-1}	k_2I_{n-1}	…	$k_{n-2}I_{n-1}$	0	
nth	I_n	k_1I_n	k_2I_n	…	$k_{n-2}I_n$	$k_{n-1}I_n$	
Sum	$1+\sum_{i=1}^{n}I_i$	$\frac{k_1}{2}\left[1+2\sum_{i=2}^{n}I_i\right]$	$\frac{k_2}{2}\left[1+2\sum_{i=3}^{n}I_i\right]$	…	$\frac{k_{n-2}}{2}\left[1+2\sum_{i=n-1}^{n}I_i\right]$	$\frac{k_{n-1}}{2}\left[1+2I_n\right]$	

In general, the average inventory level of the ith buffer for homogeneous line is

$$M_{k_i}=\frac{\dfrac{k_i}{2}\left[1+2\sum_{i+1}^{n} I_i\right]}{1+\sum_{i=1}^{n} I_i},\quad \left(I_i=\frac{\lambda_i}{\mu_i}\right),\quad i=1,2,\cdots,n-1 \tag{13}$$

Moreover, for nonnomogeneoous line without intelligent control, in the case of no filure the inventory level for the ith buffer, M'_{ki}, should be calculated by eqn.(3). Then eqn.(13) becomes

$$M_{k_i}=\frac{M'_{k_i}+k_i\sum_{i=1}^{n} I_i}{1+\sum_{i=1}^{n} I_i} \tag{13a}$$

The average inventory level for either kind of whole line is

$$M_{k_i} = \sum_{i=1}^{n-1} M_{k_i} + 1 \tag{13b}$$

The last one added in the above equation is the one storage unit in the workstation M_n.

7 NUMERICAL RESULTS AND DISCUSSION

7.1 Numerical Results

Example 1 Take a three stage nonhomogeneous transfer line with parameters as given in Table 1 of Ref.[10] (p. 156) as an example.

Find: 1) System throughput (production rate), W_S.

2) Individual buffer inventory levels (M_{k1} and M_{k2}) and whole line inventory level (M_{ks}).

Solution

Given Parameters (From Table 1, Ref [10])

i	$\mu_i(r)$	$\lambda_i(p)$	$A_i(I_i)$	$\omega_i(\mu_i)$	$W'_i(\rho_i)$	$k_i(c+1)$	Remarks
1	0.05	0.03	0.625	0.5	0.3125	9	Symbols in brackets used
2	0.06	0.04	0.600	~	~	9	in [10] W'_i
3	0.05	0.03	0.625	0.5	0.3125	-	by eqn.(7a)

The varying values of ω_2 and the corresponding results calculated are listed in the following table:

Results (Numerical answers in brackets given in [10])

ω_2	$W'_2 = \omega_2 A_2$	$\rho_1 = \frac{\omega_1}{\omega_2}$	$\rho_2 = \frac{\omega_2}{\omega_3}$	$\bar{P}_b$	$\bar{P}_S$	B	W_S	M_{k1}	M_{k2}	Remarks
0	0	∞	0	1	1	1	0	9.0	0	P_b, P_S
0.1	0.06	5.0	0.2	1	1	1	0.0714 (0.060)	7.029 (9.618)	1.953 (0.382)	by eqn.(4)
0.2	0.12	2.5	0.4	0.99994	0.99994	0.9999	0.1428 (0.115)	6.884 (8.836)	2.116 (1.164)	B by eqn.(4a)
0.5	0.30	1.0	1.0	0.9663	0.933	0.9336	0.2877 (0.209)	5.546 (6.203)	3.454 (3.736)	
0.8	0.48	0.625	1.6	1	0.9978	0.9978	0.3121 (0.238)	4.526 (4.841)	4.474 (5.159)	
1.0	0.60	0.5	2.0	1	0.9996	0.9996	0.3124 (0.246)	4.322 (4.365)	4.678 (5.635)	
1.6	0.96	0.3125	3.2	1	1	1	0.3125 (0.258)	4.135 (3.669)	4.865 (6.331)	
3.0	1.80	0.1667	6.0	1	1	1	0.3125 (0.266)	3.977 (3.153)	4.953 (6.849)	
10.0	6.0	0.05	20.0	1	1	1	0.3125	3.977	5.021	
100.0	60.0	0.005	200	1	1	1	0.3125	3.977	5.021	
∞	∞	0	∞	1	1	1	0.3125	3.977	5.023	

Notation:

1) W_S are determined by eqn.(10) at the bottleneck of the transfer line. A_s ω_2 increases gradually until W_2 reachs W_3 (corresponding to $\omega_2 = 5 \div 9$), the bottleneck shifts from M_2 to M_3.

2) M_{ki} are calculated by eqn.(13a) directly. Then by (13b), we get $M_{ks} = M_{k1} + M_{k2} + 1 =$ 9+1=10 for all ω_2 from 0 to ∞ (also Ref.[10]: $\tilde{n} + \bar{n} = 10$).

3) From the results of this example, we can see that our method may extend to the whole range of ω_2 from 0 to ∞. Similarly it is true for all other parameters, such as λ, μ, k.

Example 2. Take a seven stage transfer line with paramters given in Table 3 of Ref.[10] (p.157) as a second example. Answer the same questions as example 2.

Solution

Given paramaters (From Table 3, Ref.[10])

i	$\mu_i(r)$	$\lambda_i(p)$	$A_i(l_i)$	$\omega_i(\mu_i)$	$W'_i(\rho_i)$	$k_i(c+1)$	Remarks
1	0.3	0.02	0.9375	0.20	0.1875	3	symbols in
2	0.4	0.05	0.8889	0.23	0.2044	3	brackets used
3	0.1	0.01	0.9091	0.30	0.2727	5	in Ref.10]
4	0.4	0.07	0.8511	0.26	0.2213	3	W'_i by
5	0.3	0.03	0.9091	0.21	0.1909	3	eqn.(7a)
6	0.1	0.03	0.7692	0.27	0.2077	5	
7	0.4	0.06	0.8696	0.26	0.2261	--	

The results calculated are listed in the following table;

Results

i	1	2	3	4	5	6	7
$\rho_i = \omega_i / \omega_{i+1}$	2/2.3	2.3/3	3/2.6	2.6/2.1	2.1/2.7	2.7/2.6	
W_i by eqn.(7)	0.1836	0.1982	0.2596	0.2111	0.1672	0.2011	0.2192
M_{k_i} by eqn.(13a)	2.0709	1 8095	3.2915	1.6927	1.3566	1.6661	
$\bar{n}_i$ from [10]	2.3460	1.9121	3.8464	2.4337	1.6054	1.4906	

Notation:

1) The line throughput by eqn.(10), $W_S = \min W_i = W_5 = 0.1672$, which is larger than 0.1333 in Ref.[10].

2) The individual buffer inventory levels calculated in the above table as M_{ki}. Then by eqn.(13b) we get $\sum_i M_{ki} = 11.8873 + 1 = 12.8873$, while $\sum_i \bar{n}_i = 13.6242$.

B. Discussion

1) The storage capacity of B_i between M_i and M_{i+1} is denoted by $k_i = c_i + 1$, which includes one unit in M_i. Another unit in M_{i+1} should be accounted as an additional unit in the next buffer B_{i+1}. While c_i is the actual storage capacility of the external buffer as defined in literatures [5] and [12], this definition of k_i agrees with those of Ref. [4] and [5].

From the above two examples, the answers of the whole line inventory levels M_{ks} are almost the same as those of Ref.[10]. However, the answers of individual inventory levels M_{ki} are somewhat different.

2) For state dependent failures (assumotion 5), the exact expression of system throughput has been derived as shown by formula (8).

$$W_S = \frac{\omega_i \mu_i B_i}{\mu_i - \lambda_i B_i} = \frac{\omega_i}{\frac{1}{B_i} + \frac{\lambda_i}{\mu_i}} \tag{8}$$

Then from eqn.(6), we have

$$P_{1i} + P_{Si} = 1 - P_{2i} - P_{3i} - P_{4i} = P_{1i}\left(1 + \frac{\lambda_i}{\mu_i}\right)$$

$$P_{1i} = \frac{\mu_i}{\mu_i + \lambda_i}(1 - P_{2i} - P_{3i} - P_{4i})$$

$$\therefore W_S = W_i = \frac{\omega_i \mu_i}{\mu_i + \lambda_i}(1 - P_{2i} - P_{3i} - P_{4i}) \tag{8a}$$

The above formula is another exact expression of system throughput This can be proved by substituting eqn.(6a) for eqn.(8a). Then eqn.(8a) reduces to eqn.(8).

Compare eqn.(8a) with eqn.(22) of Ref. [10], which may be written as

$$W_S'(P) = \text{-----}(1 - P_{0(i-1)} - P_{ki}) \tag{14}$$

The magnitude of eqn.(14) is less then that of eqn.(8a) in two respects:

i) Neglect the effect of the probability of both starvation and blocking simutaneously, since $P_{0(i-1)}$ and P_{ki} in eqn.(14) are not exclusible.

ii) P_{2i}, P_{3i} and P_{4i} in eqn.(8a) are the probability of blockage, starvation, both blockage and starvation for M_i as defined in eqn.(6), while $P_{0(i-1)}$ and P_{ki} are the probability for the empty and full state of the $(i-1)$ th and the ith buffer as defined in eqn.(2). Here $P_{0(i-1)}$ is larger then P_{3i} and P_{ki} is larrger then P_{2i} respectivelsy. So the calculated results of system throughput by eqn.(8) or (8a) are larger than those by eqn.(14) as shown in the above two examples.

Morreover, equation (8) is also true for the whole range of parameters (λ_i, μ_i, k_i). For instance, if $\lambda_i \div \mu_i = 0$ (reliable machine), then by eqn.(8) we have $W_i = \omega_i B_i$. On the other hand, if $\lambda_i \div \mu_i = \infty$ (unrepairable machine), then $W_S = 0$. If $k_i = k_{i+1} = \infty$, then by eqn.(2) we get $P_{0(i-1)} = P_{ki} = 1$ and $B_i = P_{0(i-1)} P_{ki} = 1 * 1 = 1$. thus from eqn.(8), $W_S = \omega_i \mu_i \div (\mu_i + \lambda_i)$. That means all the stations working as isolated ones. In this case eqn.(14) is also exact, but it is only an approximate formula in general.

iii) In addition, the state probabilities, namely the time proportions of machine M_i can be calculated by eqn.(6). The results for the above two examples have been obtained as listed in the following table:

Example	M_i	P_{1i}	P_{2i}	P_{3i}	P_{4i}	P_{5i}	$\sum_{j=1}^{5} P_{ji}$	Remarks
1	M_2	0.5754	0.0201	0.0201	0.0007	0.3836	1.00	$\omega_2 = 0.5$
2	M_5	0.8376	0.0625	0.0149	0.0011	0.0838	1.00	

8 CONCLUSION

This paper presents a new method of "equivalent workstation" for the modeling and analysis of multistage capacitated tansfer lines with unreliable machies and random prcessing times. The main contributions are listed as follows:

a) The buffer between two reliable machines is analyzed for (k_i+1) storage states, but only two states (unfull and unempty) are used for constructing an equivalent workstation. Thus reduce the state spaces very much for solving the problem analytically.

b) This equivalent workstation is decomposed into five independent state probabilites (or time proportions) as expressed exactly in eqn.(6).

c) An interrupting operation transfer line is converted into a continuous production line by connecting all the equivalent workstations in series.

d) The important performance measures, such as system throughput (production rate) (eq.8), system efficiency (eqn.9), and the state probabilities namely the time proportions of machine M_i (eqn.6a) are all derived in exact and explicit exressions.

e) Based upon assumption (2), the average inventory levels for the individual buffers and whole line are also obtained as shown in (eqn.13, 13a, 13b).

f) Using P_{bi} and P_{Si} (eqn.4) instead of P_{ki} and P_{0i} (eqn.2), so the accuracy of performance measures is improved.

g) The numerical results of two examples (obtained simply by TI-36 solar calculator) are discussed and compared with those of Ref. [10].

h) This proposed method can be applied to practical engineering design directly.

REFERENCES

1 Wijingaard, J., The Effect of Interstage Buffer Storage on the Output of Two Unreliable Production Units in Series, with Different Production Rates, *AIIE Trans*, **11** (1): 42-47, 1979.

2. Gershwin, S.B., and I.C.Schick, Modeling and Analysis of Three-Stage Transfer Lines with Unreliable Machines and Finite Buffers, *Operations Research*, **31** (2): 354-380, 1983.

3. Gershwin, S.B., An Efficient Decomposition Method for The Approximate Evaluation of Tandem Queues with Finite Space and Blocking, *Oper, Res.*, **35** (2): 291-305, 1987.

4. Glassey, C.R., and Y.Hong, Analysis of Behaviour of an Unreliable n-stage Transfer Line with (n — 1) Inter-stage storage Buffers, *Int. J Prod. Res.* **31** (3): 519-530, 1993.

5. Dallery, Y., R.David, and X.L. Xie, Approximate Analysis of Transfer Lines with Unreliable Machines and Finite Buffer, *IEEE Trans. on Automatic Control*, **34** (9): 943-953, 1989.

6. D.Koster, Estimation of Line Efficiency by Aggregation, *Int. J. of Prod. Res.*, **25** (4): 615-626, 1987.

7. S.G. Shu, and C.J. Jiang, Modeling and Analysis of Capacitated Transfer Lines with Unreliable Machines and Deterministic Processing Times. *J. Syst. Sci. & Syst. Eng.*, **6** (1)· 53-60, 1997

8. Gershwin S.B., and O.Berman, Analysis of Transfer Lines Consisting of Two Unreliable Machines with Random Processing Times and Finite Storage Buffers, *AIIE Transactions*, **13** (1): 2-11, 1981
9. Koster, D., An Improved Algorithm to Approximate the Behavior of Flow Lines, *Int., J. Prod. Res.*, **26** (4): 691-700, 1988.
10. Choong, Y.F., and S.B. Gershwin, A Decomposition Method for the Approximate Evaluation of Capacitated Transfer Lines with Unreliable Machines and Random Processing Times. *IIE Trans.*, **19** (2): 1987.
11. Shu, S.G., An Analysis of The Repairable Computer Integrated Manufacturing System (CIMS) with Buffers, *Acta Automatica, Sinica*, **18** (1): 1992 (in Chinese).
12. Shu, S.G., and Z.Q.Yao, The Modeling and Analysis of the Multiple Stage Repairable CIMS with Finite Buffers, Preprints of IFAC/IFORS/IMACS Symposium Large Scale System; Theory and Applications, August, 1992.

(上接 192 页)

5) In the case of the resource being constrained, the generalized optimizing design model is derived and its optimum solution is given;

6) The application of this method is illustrated with an example;

7) The optimizing design method given by this paper can also be applied to other similar systems (such as IMS, DEDS, and CPMS).

REFERENCES

1. Kraemer, S.A. and Lovem R.F., 1970. A model for optimizing the buffer inventory storage size in a sequential production system. *AIIE Trans.*, 2, pp.64-69.
2. Anderson, D.R., *et al.*, Optimal buffer storage capacity in production line systems. *Int. J. Prod Res.*, 7, 1968.
3. Songgui, S., Reliability and productivity of the computer integrated manufacturing system (CIMS). Selected Papers of Chinese Annual Meeting on Control Theory and Its Application, Xian (in Chinese) 1989.
4. Shu Songgui, S., An analysis of the repairable CIM system with buffers and a study of the system reliability. *Acta Automatica Sinica*, 1992, 18, pp.15-22 (in Chinese).
5. Shu Songgui, S., A study of optimal design of CIM system. Selected Papers of the Annual Meeting of the Theory Committee of the Chinese Association of Automation, Wuhan, Ocean Press, Beijing, China (in Chinese) 1993.