THE MODELING AND ANALYSIS OF THE MULTIPLE STAGE REPAIRABLE CIMS WITH FINITE BUFFERS*

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Abstract A new model is proposed for analyzing the repairable CIMS with finite buffers in this paper. These buffers lead to blockage and starvation in the transfer lines and make the problem of the system very difficult to solve. An equivalent workstation without starvation and blockage is constructed by means of queuing theory of Markovian process. Connecting all the equivalent workstations in series, we get the new model of the whole system. Then from the principle of equilibrium of the working piece flow in the transfer line an accurate formula for the system without any loss of the working pieces is obtained. Finally two examples are used for illustrating the application of the method.

Keywords Computer integrated manufacturing system (CIMS), Discrete event dynamic system (DEDS), reliability of large scale system.

INTRODUCTION

The computer integrated manufacturing system (CIMS) with storage buffers is a special kind of discrete event dynamic system (DEDS) which is subject to interruptions due to blockage and starvation in the transfer lines. This system is very complicate and very difficult to solve in the usual way. The finite intermediate buffers are solved first by Markovian process. Then an equivalent workstation is constructed by combination of the workstation with un-empty state of the up-stream buffer and un-full state of the down-stream one. Finally connect all the equivalent workstations with intermediate buffers in series and solve the problem of CIMS by the principle of equilibrium of the working piece flow in the transfer line.

THE MODELING AND ANALYSIS OF A REPAIRABLE CIMS

Let us consider a production line of n workstations M_i with intermediate buffers B_i

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connected in series respectively as shown in Fig.1.

Fig.1 A multistage CIMS with intermediate buffers

where M_1 represents the ith workstation and B_i the ith buffer $(i=1,2,\cdots,n)$ with a capacity k_i (including one piece within the corresponding station).

1 Assumptions

1) Any faults in M_i can be repaired without delay and the manufactuting time, lifetime and repair time are distributed exponentially with parameters ω_i, λ_i and μ_i respectively.

2) Buffers are always reliable.

3) The time to put or take the work pieces in or from the buffers is negligible.

4) The first workstation will not be starved and the last one will not be blocked.

5) The working machine will stop automatically either in starved or in blocked state.

6) No failures can occur in the stopped machine.

All the following work are based upon the above six assumptions.

2 Analysis of the Intermediate Storage Buffers

The ith buffer is considered as a closed system as shown in Fig.2.

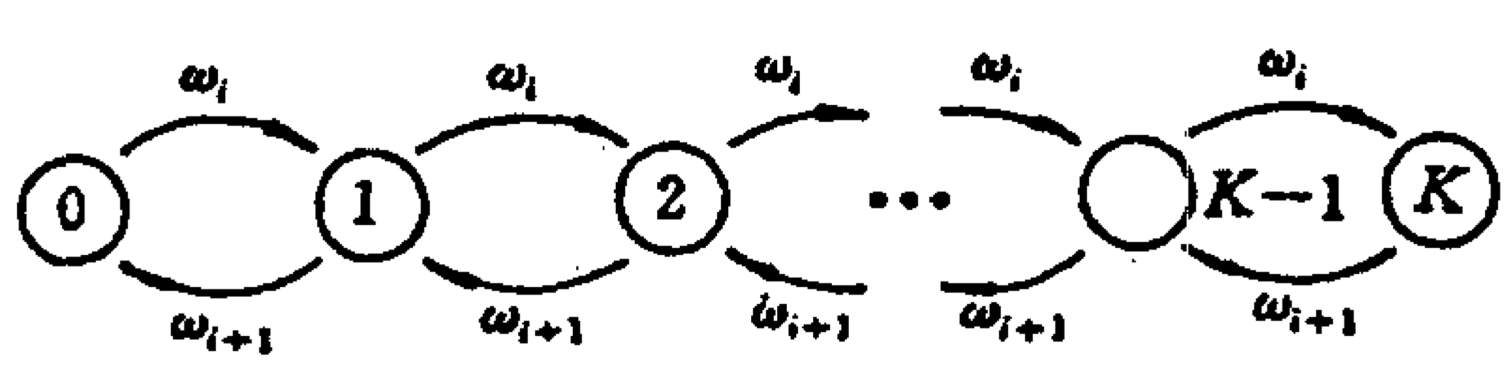

Fig.2 The translative state diagram of the ith buffer

There are $k+1$ states for the ith buffer with capacity k, where

0 represents the empty of the buffer;

1 represents one piece stored in the buffer;

2 represents two pieces stored in the buffer;

……

k represents full in the buffer.

The arrowhead of the manufacturing rate ω_i of M_i indicating the state shift from left to right and that ω_{i+1} of M_{i+1} indicating the state shift from right to left. Then we can obtain the following differential equations:

$$\left.\begin{aligned}
&\dot{p}_0 = -\omega_i p_0 + \omega_{i+1} p_1 \\
&p_1 = \omega_i p_0 - (\omega_i + \omega_{i+1}) p_1 + \omega_{i+1} p_2 \\
&\cdots\cdots \\
&\dot{p}_{k-1} = \omega_i p_{k-2} - (\omega_i + \omega_{i+1}) p_{k-1} + \omega_{i+1} p_k \\
&\dot{p}_k = \omega_i p_{k-1} - \omega_{k+1} + p_k \\
&\sum_{j=0}^{k} p_j = 1
\end{aligned}\right\} \tag{1}$$

For the steady state solution, setting all the differential terms of (1) equal to zero and let $\omega_i / \omega_{i+1} = \rho_i$, we obtain (for clear neglecting the station index of ρ)

$$\begin{aligned}
&p_1 = \rho p_0 \\
&p_2 = \rho^2 p_0 \\
&\cdots\cdots \\
&p_k = \rho^k p_0 \\
&\sum_{j=0}^{k} p_j = p_0(1-\rho^{k+1})/(1-\rho)
\end{aligned}$$

Then

$$p_j = \frac{\rho^j(1-\rho)}{(1-\rho^{k+1})}, \quad j = 0,1,2,\cdots,k \tag{2}$$

Obviously, if $\rho = 1$, then $p_j = 1/(k+1)$.

For construction of the equivalent workstation we define the following four state probabilities of the intermediate buffer as:

$$\left.\begin{aligned}
&\text{Full} && p_k = \frac{\rho^k(1-\rho)}{1-\rho^{k+1}} \\
&\text{Unfull} && p_{\bar{k}} = 1 - p_k = \frac{1-\rho^k}{1-\rho^{k+1}} \\
&\text{Empty} && p_0 = \frac{1-\rho}{1-\rho^{k+1}} \\
&\text{Unempty} && p_{\bar{0}} = 1 - p_0 = \frac{\rho(1-\rho^k)}{1-\rho^{k+1}}
\end{aligned}\right\} \tag{3}$$

When the buffer is full the workstation before it will be stopped by blockage and when the buffer is empty the workstation after it will be stopped by starvation. So the availability of the buffers relative to ith workstation can be defined as

$$A_{bi} = B_i = p_{\overline{0(i-1)}} p_{\overline{ki}} \tag{4}$$

Obviously the average number of the working pieces stored in the ith buffer

$$\begin{aligned}
Mk_i &= 1p_1 + 2p_2 + \cdots + kp_k \\
&= \frac{\rho - (1+k)\rho^{k+1} + k\rho^{k+2}}{1-\rho-\rho^{k+1}+\rho^{k+2}}
\end{aligned} \tag{5}$$

3 Construction and Analysis of the Equivalent Workstation

For the original workstation, we have

$$p_{ai} + p_{bi} = 1 \tag{6}$$

where p_{ai} and p_{bi} represent the probability in good state and failure state respectively. Then we get

$$(p_{ai} + p_{bi})(p_{0(i-1)} + p_{\overline{0(i-1)}}(p_{ki} + p_{\overline{ki}}) = 1$$

$$p_{ai}(p_{\overline{0(i-1)}}p_{\overline{ki}} + p_{0(i-1)}p_{\overline{ki}} + p_{\overline{0(i-1)}}p_{ki} + p_{0(i-1)}p_{ki}) + p_{bi} = 1 \tag{6a}$$

Comparing (6) and (6a), we know that the total value in the bracket of (6a) should be unit. The first term in left side represent the probability of M_i in the normal working state. The other three terms represent M_i stopped due to starvation, blockage and both respectively.

From the point view of the equivalent workstation, only the first term is necessary for construction. Let

$$p'_{ai} = p_{\overline{0(i-1)}}p_{\overline{ki}}p_{ai} = B_i p_{ai} \tag{7}$$

Based upon the above analysis and assumption (6) we can construct an equivalent repairable workstation as shown in Fig.3

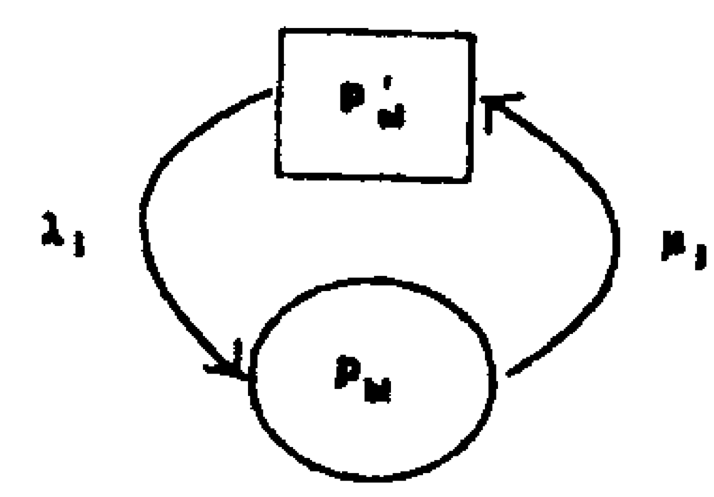

Fig.3 Equivalent repairable workstation

From Fig.3, we have

$$\left.\begin{aligned} \dot{p}'_{ai} &= -\lambda_i p'_{ai} + \mu_i p_{bi} \\ \dot{p}_{bi} &= \lambda_i p'_{ai} - \mu_i p_{bi} \end{aligned}\right\} \tag{8}$$

Let the left sides of the above equation equal to zero, and solve them simultaneously, we get the steady state solution as

$$p_{bi} = \frac{p'_{ai}\lambda_i}{\mu_i} = \frac{B_i(1-p_{bi})\lambda_i}{\mu_i}$$

$$p_{bi} = \frac{\lambda_i B_i}{\mu_i + \lambda_i B_i} \tag{9}$$

The availability of ith equivalent workstation,

$$A'_i = p'_{ai} = \frac{\mu_i B_i}{\mu_i + \lambda_i B_i} \tag{9a}$$

The manufacturing rate of ith equivalent workstation

$$W_i = \omega_i A'_i = \frac{\omega_i \mu_i B_i}{\mu_i + \lambda_i B_i} \tag{10}$$

The average manufacturing time for each piece in the ith workstation

$$T_i = 1/W_i \tag{11}$$

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4 The Modeling and Analysis of the Repairable CIMS with Finite Buffers

Connecting the n equivalent workstations in series we have the whole CIMS as shown in Fig.4.

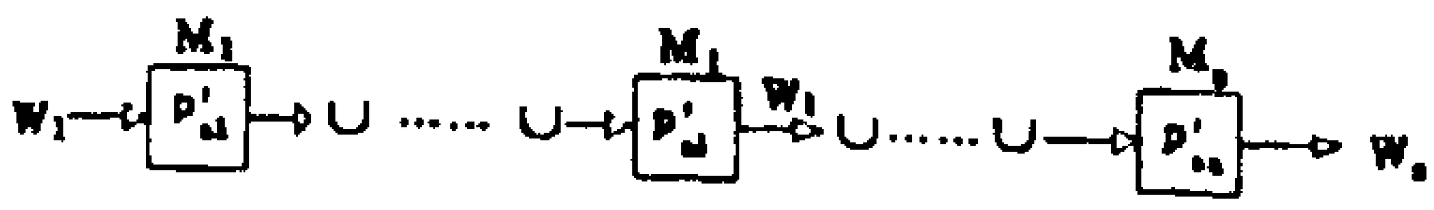

Fig.4 Logical diagram of the multistage CIMS

Actually the buffer transforms the rigid connection to flexible one, i.e., to change the series connection to parallel connection in redundancy, the symbol $\cup$ represents logical sum of the availabilities of the equivalent workstations. So the system availability,

$$A_s = A_1' \bigcup_{i=2}^{n-1} A_i' \bigcup A_n' = p_{a1}' \bigcup_{i=2}^{n-1} P_{an'}' \bigcup p_{an}' \tag{12}$$

From the principle of equilibrium of working pieces flow in the transfer line, the manufacturing rate for each equivalent workstation should be equal. By (10), we have

$$W_s = W_i = \frac{\omega_i \mu_i B_i}{\mu_i + \lambda_i B_i} \qquad i = 1,2,\cdots,n \tag{13}$$

The above equation is the necessary and sufficient conditions for the multistage repairable CIMS without any loss of the working pieces.

The total working time for each working piece in n working stations

$$T_p = \sum_{i=1}^{n} (1/W_i) = n/W_s \tag{14}$$

The total number of working pieces in the whole production line,

$$Mk = \sum_{i=1}^{n-1} Mk_i$$

$$= \sum_{i=1}^{n-1} \frac{\rho_i - (1+k_i)\rho_i^{k_i+1} + k_i \rho_i^{k_i+2}}{1-\rho_i - \rho_i^{k_i+1} + \rho_i^{k_i+2}} \tag{15}$$

The total store time in $(n-1)$ buffers

$$T_B = Mk / W_s \tag{16}$$

The total time for each working piece storing in the whole production line,

$$T_s = T_B + T_p = Mk/W_s + n/W_s \tag{17}$$

NUMERAL EXAMPLES

Example 1 Given a two-stage repairable production line with given parameters as shown in the following table

parameter \ i	1	2	remark
λ	0.001	0.002	n=2, k=5
μ	0.02	0.03	W_s=10pieces/Hr

Find the rated manufacturing rates of workstations, namely ω_1, ω_2, and the total number of working pieces in the production line Mk and the total time for each working piece storing in the production line.

Solution:

From (13), we have

$$W_s = \frac{\omega_1 \mu_1 B_1}{\mu_1 + \lambda_1 B_1} = \frac{\omega_2 \mu_2 B_2}{\mu_2 + \lambda_2 B_2}$$

where

$$B_1 = p_{\overline{0(l-1)}} p_{\overline{ki}} = p_{\overline{ki}} = \frac{1-\rho^k}{1-\rho^{k+1}}$$

$$B_2 = p_{\overline{0(2-1)}} p_{\overline{k2}} = p_{\overline{0(2-1)}} = \frac{\rho(1-\rho^k)}{1-\rho^{k+1}} = \rho B_1$$

Then by substituting $B_2 = \rho B_1$ in W_s, we get

$$\frac{\omega_1 \mu_1 B_1}{\mu_1 + \lambda_1 B_1} = \frac{\omega_2 \mu_2 B_2}{\mu_2 + \lambda_2 B_2}$$

Simplifying the above equation, we have

$$\rho = \frac{\lambda_1 \mu_2}{\lambda_2 \mu_1} = \frac{\omega_1}{\omega_2} = \frac{0.001 \times 0.03}{0.002 \times 0.02} = \frac{3}{4} = 0.75$$

$$B_1 = \frac{1-0.75^5}{1-0.75^6} = 0.9278289$$

$$B_2 = \rho B_1 = 0.6958717$$

$$\omega_1 = \frac{W_s(\mu_1 + \lambda_1 B_1)}{\mu_1 B_1} = 11.278$$

$$\omega_2 = \frac{W_s(\mu_2 + \lambda_2 B_2)}{\mu_2 B_2} = 15.037$$

$$Mk = \frac{0.75 - (1+5) \times 0.75^6 + 5 \times 0.75^7}{1 - 0.75 - 0.75^6 + 0.75^7} = 1.701 \text{ pieces}$$

$$T = Mk / W_s + n / W_s = 0.370 \text{ Hr}$$

Example 2 Given a three-stage repairable production line with the parameters as shown in the following table:

parameter \ i	1	2		remark
λ	0.002	0.001	0.002	n=3, k=5
μ	0.03	0.02	0.03	W_s=10pieces/Hr

Assume that the final storage space is also limited.

Solution:

The rated manufacturing rate of any stations must be larger than that of the system production rate required $W_s = 10$ pieces/hour. Then set $\omega_1 = 12$ and from eqn. (13) we have

$$B_1 = \frac{W_s \mu_1}{\omega_1 \mu_1 - \lambda_1 W_s} = \frac{10 \times 0.03}{12 \times 0.03 - 0.002 \times 10} = 0.8823529$$

For the first workstation, $B_1 = p_{\overline{k1}} = \dfrac{1-\rho_1^5}{1-\rho_1^6}$, and solving by interpolation method, we get

$$\rho_1 = 0.8784658$$

then

$$\omega_2 = \omega_1 / \rho_1 = 12 / 0.8784658 = 13.660179$$

and

$$B_2 = \frac{W_s \mu_2}{\omega_2 \mu_2 - \lambda_2 W_s} = \frac{10 \times 0.02}{13.660179 \times 0.02 - 0.001 \times 10} = 0.7598681 = p_{\overline{01}} p_{\overline{k2}}$$

where $p_{01} = \rho_1 p_{\overline{k1}} = 0.7751169$

$\therefore p_{\overline{k2}} = 0.7598681 / 0.7751169 = 0.9803271$

Calling $p_{k2} = \dfrac{1-\rho_2^5}{1-\rho_2^6}$ and by interpolation method we get $\rho_2 = 0.5195710$, then $\omega_3 = \omega_2 / \rho_2$ =26.291265, and

$$B_3 = \frac{W_s \mu_3}{\omega_3 \mu_3 - \lambda_3 W_3} = \frac{10 \times 0.03}{26.291265 \times 0.03 - 0.001 \times 10} = 0.3852387 = p_{\overline{02}} p_{\overline{k3}}$$

where $p_{\overline{02}} = \rho_2 p_{\overline{k2}} = 0.5195710 \times 0.9803271 = 0.5093495$, $\therefore p_{\overline{k3}} = 0.3852387 / 0.5093495$ =0.7563347

for the last storage space.

$$Mk = \frac{\rho_1 - (1+k)\rho_1^6 + k\rho_1^7}{1-\rho_1-\rho_1^6+\rho_1^7} + \frac{\rho_2 - (1+k)\rho_2^6 + k\rho_2^7}{1-\rho_2-\rho_2^6+\rho_2^7} = 2.125921 + 0.8742557 = 3.00 \text{ pieces}$$

$$T = \frac{Mk}{W_s} + \frac{n}{W_s} = \frac{3.00}{10} + \frac{3}{10} = 0.600 \text{ hour.}$$

The above answer for example 2 shows that this solution is only the available one but not the sole one.

CONCLUSION

The method of decomposition and combination has been used for solving the problem of repairable CIMS with finite buffers. The procedure and results can be outlined as follows:

1) The buffer is analyzed by Markov process and only two states $p_{\bar{k}}$ and $p_{\bar{0}}$ from $m+1$ ones are picked up for the next step.

2) The ith equivalent workstation is constructed by combining $p_{\overline{0(i-1)}}$ and $p_{\overline{ki}}$ with the ith workstation.

3) Connecting all the equivalent workstations in series, we get the whole CIMS model

4) From the principle of equilibrium of the workpiece flow an accurate formula for the necessary and sufficient condition has been obtained for the system design.

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