Impulse Response Identification Based on Varying Scale Orthogonal Wavelet Packet Transform[1)]

LI He-Sheng[1] MAO Jian-Qin[2] ZHAO Ming-Sheng[3]

[1](*The Institute of Electronic Engineering, China Academy of Engineering Physics, Mianyang* 621900)

[2](*The Seventh Research Division, Beijing University of Aeronautics and Astronautics, Beijing* 100083)

[3](*Department of Electronic Engineering, Tsinghua University, Beijing* 100084)

(E-mail: liheshengphd@sina.com, liheshengphd@163.com)

Abstract In this paper, by applying a group of specific orthogonal wavelet packet to Eykhoff algorithm, a new impulse response identification algorithm based on varying scale orthogonal WPT is provided. In comparison to Eykhoff algorithm, the new algorithm has better practicability and wider application range. Simulation results show that the proposed impulse response identification algorithm can be applied to both deterministic and random systems, and is of higher identification precision, stronger anti-noise interference ability and better system dynamic tracking property.

Key words Wavelet packet transform (WPT), time-frequency analysis, Eykhoff algorithm, impulse response identification

1 Introduction

Impulse response identification is a kind of imparametrization identification algorithm. The traditional identification algorithms mainly include Eykhoff algorithm and correlation analysis. Correlation analysis is suitable for random system, and is based on Wiener-Hopf equation given below

$$R_{uz}(\tau) = \int_0^{+\infty} \hat{g}(t) R_u(t-\tau)\mathrm{d}t \tag{1}$$

where the impulse response function is calculated using both correlation function $R_{uz}(\tau)$ of system input-output, and self-correlation function $R_u(\tau)$ of system input, respectively. However, the analytic expression of the impulse response $\hat{g}(t)$ is difficult to be obtained by using the algorithm. Only if the input signal $u(t)$ is limited to be white Gaussian noise with zero-mean, we can obtain

$$\hat{g}(\tau) = \frac{1}{\sigma_u^2} R_{uz}(\tau) \tag{2}$$

where σ_u^2 is the variance of the input white noise signal. To obtain more accurate estimates of $R_{uz}(\tau)$ and $R_u(\tau)$, the system input and output are required to be stationary random signals for a long term. Thus, in general, correlation analysis does not trace the change of the system behavior.

In contrast to correlation analysis, Eykhoff algorithm is suitable for impulse response identification of deterministic systems. The main idea of Eykhoff algorithm is to express the impulse response function $g(t)$ as a linear combination of a group of orthogonal function bases:

$$g(t) = \sum_{i=1}^{N} f_i(t)\xi_i = \boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{\xi} \tag{3}$$

where $f_i(t)$, for $i = 1, 2, \cdots, N$, are the base functions, and $\boldsymbol{\xi} = [\xi_1, \xi_2, \cdots, \xi_N]^{\mathrm{T}}$, which can be obtained by using grads descent algorithm, is projection coefficients of $g(t)$ on the orthogonal function bases $\boldsymbol{F}(t)$. In Eykhoff algorithm, the choice of a group of proper orthogonal function bases is important since the performance of the algorithm is closely related to the localization of the orthogonal function basis in both time and frequency domains. However, though the resolving power of the traditional orthogonal function bases is better in time domain, it is worse in frequency domain[1].

Being developed from wavelet transform, WPT can be used as an efficient tool for precisely analyzing the localization information of a signal at arbitrary time and frequency point[2]. Actually,

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by applying the advantages of WPT, we have successfully developed an impulse response identification algorithm based on fixed scale orthogonal WPT[3]. In this paper, a new impulse response identification algorithm based on varying scale orthogonal WPT is proposed, which has better practicability and wider application range.

2 Basic idea of impulse response identification based on varying scale orthogonal WPT

In WPT, for a given orthonormal scale function $\varphi(t)$, from the double scale difference equation

$$\begin{cases} w_{2n}(t) = \sqrt{2}\sum_{k\in Z} h_k w_n(2t-k) \\ w_{2n+1}(t) = \sqrt{2}\sum_{k\in Z} g_k w_n(2t-k) \end{cases} \tag{4}$$

the orthogonal wavelet packet for $\varphi(t)$ can be generated as follows.

$$\{w_{n,j,k}(t) := 2^{-j/2} w_n(2^{-j}t-k),\ n \in Z/Z^-,\ j \in Z, k \in Z\}$$

where $w_0(t) = \varphi(t)$, $\{h_k\}_{k\in Z}$ and $\{g_k\}_{k\in Z}$ are a pair of conjugate orthogonal filter coefficients deduced from $\varphi(t)$, respectively.

The impulse response identification based on varying scale orthogonal WPT is that the impulse response function $g(t) \in V_{j_0}$ is projected on some chosen orthogonal wavelet packet spaces at different decomposition scales, and by identifying the projection coefficients, namely, WPT coefficients of $g(t)$, impulse response function can be identified indirectly, where the chosen orthogonal wavelet packet spaces are not overlapped with each other, and the initial scale space must be covered with the chosen orthogonal wavelet packet spaces. Given the initial scale v_{j_0} and decomposition layer m, for impulse response identification algorithm based on varying scale orthogonal WPT, the chosen orthogonal wavelet packet spaces satisfy

$$\begin{cases} U^{\bullet}_{j_0+k} \cap U^{\bullet}_{j_0+l} = \varPhi, & \text{for arbitary } k \neq l \\ V_{j_0} = \oplus_{l=1}^{m}(\oplus_{n=n_{l_1}}^{n_{l_k}} U^n_{j_0+l}) \end{cases} \tag{5}$$

where $\oplus$ denotes direct sum operation, $U^{\bullet}_{j_0+k}$ and $U^{\bullet}_{j_0+l}$ denote, respectively, an arbitrary chosen orthogonal wavelet packet spaces at scales $j = j_0 + k$ and $j = j_0 + l$, and $U^n_{j_0+l}$ $(n = n_{l_1}, \cdots, n_{l_k}$ denotes the chosen orthogonal wavelet packet spaces at scale $j = j_0 + l$.

As an illustrative example, given the initial scale j_0, let the biggest decomposition layer $l = 4$. Then the structure for original signal space V_{j_0} being divided by orthogonal wavelet packet spaces $U^n_{j_0+l}$ $(l = 1, 2, 3, 4;\ 0 \leqslant n \leqslant 2^l - 1)$ at different scale $j = j_0 + l$ is shown in Fig. 1, where the shadowed spaces are the orthogonal wavelet packet spaces chosen by impulse response identification algorithm based on varying scale WPT. Of course, according to engineering requirement, the other orthogonal wavelet packet spaces satisfying (5) can be chosen also.

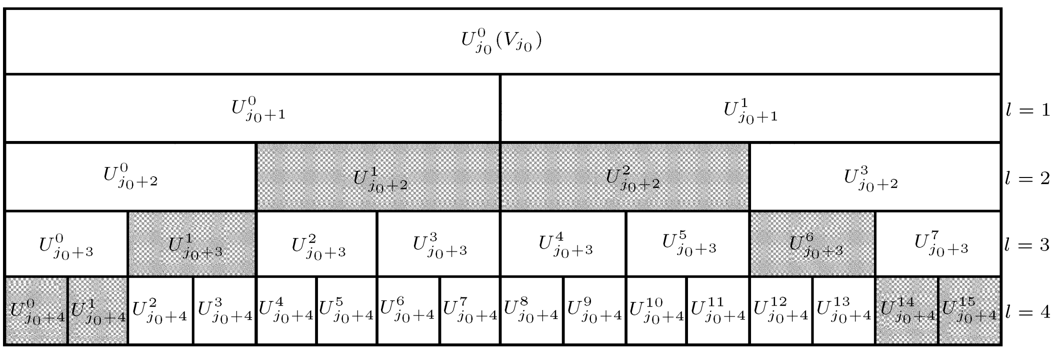


Fig. 1 An example of orthogonal wavelet packet spaces selection for impulse response identification based on varying scale orthogonal WPT (namely shadowed spaces)

Remark. In the following, we will take the chosen orthogonal wavelet packet spaces in Fig. 1 as an example.

According to multiresolution theory of wavelet transform[2], if initial scale $j_0 \in Z$ is small enough, the scale space V_{j_0} can approach square integral space $L^2(R)$, *i.e.*,

$$V_{j_0} \approx L^2(R) \tag{6}$$

Hence, for orthogonal wavelet packet spaces, we have

$$L^2(R) \approx U^0_{j_0+4} \oplus U^1_{j_0+4} \oplus U^1_{j_0+3} \oplus U^1_{j_0+2} \oplus U^2_{j_0+2} \oplus U^6_{j_0+3} \oplus U^{14}_{j_0+4} \oplus U^{15}_{j_0+4} \tag{7}$$

In (7), the orthogonal wavelet packet spaces $U^0_{j_0+4}$, $U^1_{j_0+4}$, $U^1_{j_0+3}$, $U^1_{j_0+2}$, $U^2_{j_0+2}$, $U^6_{j_0+3}$, $U^{14}_{j_0+4}$, and $U^{15}_{j_0+4}$, whose direct sum is the original signal space V_{j_0}, are just the chosen orthogonal wavelet packet spaces for impulse response identification based on varying scale orthogonal WPT in Fig. 1. According to the definition of wavelet packet space, $\{w_{0,j_0+4,k}(t)\}_{k\in Z}$, $\{w_{1,j_0+4,k}(t)\}_{k\in Z}$, $\{w_{1,j_0+3,k}(t)\}_{k\in Z}$, $\{w_{1,j_0+2,k}(t)\}_{k\in Z}$, $\{w_{2,j_0+2,k}(t)\}_{k\in Z}$, $\{w_{6,j_0+3,k}(t)\}_{k\in Z}$, $\{w_{14,j_0+4,k}(t)\}_{k\in Z}$, and $\{w_{15,j_0+4,k}(t)\}_{k\in Z}$ just compose a group of orthogonal bases of $L^2(R)$.

If the impulse response function $g(t)$ is approached by its projection on the above-mentioned orthogonal bases of space $L^2(R)$, then $g(t)$ can be written as

$$\begin{aligned} g(t) = &\sum_{k=0}^{k_0^{j_0+4}-1} p_g(0, j_0+4, k) w_{0,j_0+4,k}(t) + \sum_{k=0}^{k_1^{j_0+4}-1} p_g(1, j_0+4, k) w_{1,j_0+4,k}(t) + \\ &\sum_{k=0}^{k_1^{j_0+3}-1} p_g(1, j_0+3, k) w_{1,j_0+3,k}(t) + \sum_{k=0}^{k_1^{j_0+2}-1} p_g(1, j_0+2, k) w_{1,j_0+2,k}(t) + \\ &\sum_{k=0}^{k_2^{j_0+2}-1} p_g(2, j_0+2, k) w_{2,j_0+2,k}(t) + \sum_{k=0}^{k_6^{j_0+3}-1} p_g(6, j_0+3, k) w_{6,j_0+3,k}(t) + \\ &\sum_{k=0}^{k_{14}^{j_0+4}-1} p_g(14, j_0+4, k) w_{14,j_0+4,k}(t) + \sum_{k=0}^{k_{15}^{j_0+4}-1} p_g(15, j_0+4, k) w_{15,j_0+4,k}(t) \end{aligned} \tag{8}$$

where projection coefficients $\{p_g(0, j_0+4, k)\}_{k\in Z}$, $\{p_g(1, j_0+4, k)\}_{k\in Z}$, $\{p_g(1, j_0+3, k)\}_{k\in Z}$, $\{p_g(1, j_0+2, k)\}_{k\in Z}$, $\{p_g(2, j_0+2, k)\}_{k\in Z}$, $\{p_g(6, j_0+3, k)\}_{k\in Z}$, $\{p_g(14, j_0+4, k)\}_{k\in Z}$, and $\{p_g(15, j_0+4, k)\}_{k\in Z}$ are the WPT coefficients for $g(t)$ in the wavelet packet spaces $U^0_{j_0+4}$, $U^1_{j_0+4}$, $U^1_{j_0+3}$, $U^1_{j_0+2}$, $U^2_{j_0+2}$, $U^6_{j_0+3}$, $U^{14}_{j_0+4}$, and $U^{15}_{j_0+4}$, respectively. Since the WPT coefficients characterize the time-frequency properties of $g(t)$ in a specific frequency band, respectively, we can learn about the frequency components of $g(t)$ by using the WPT coefficients, and we can even learn about the frequency components of $g(t)$ in a specific frequency band.

Consider a single input single output linear system

$$y(t) = \int g(t) u(t-\tau) \mathrm{d}\tau = g(t) \otimes u(t) \tag{9}$$

where $u(t), y(t)$ and $g(t)$ are system input, output and impulse response function, respectively, the sign $\otimes$ denotes convolution operation.

Substituting (8) into (9), and letting

$$y(t) = \int g(t) u(t-\tau) \mathrm{d}\tau = g(t) \otimes u(t) \tag{10}$$

we have

$$y(t) = \sum_{k=0}^{k_0^{j_0+4}-1} p_g(0, j_0+4, k) \zeta_{0,j_0+4,k}(t) + \sum_{k=0}^{k_1^{j_0+4}-1} p_g(1, j_0+4, k) \zeta_{1,j_0+4,k}(t) +$$

$$\sum_{k=0}^{k_1^{j_0+3}-1} p_g(1, j_0+3, k)\zeta_{1,j_0+3,k}(t) + \sum_{k=0}^{k_1^{j_0+2}-1} p_g(1, j_0+2, k)\zeta_{1,j_0+2,k}(t) +$$
$$\sum_{k=0}^{k_2^{j_0+2}-1} p_g(2, j_0+2, k)\zeta_{2,j_0+2,k}(t) + \sum_{k=0}^{k_6^{j_0+3}-1} p_g(6, j_0+3, k)\zeta_{6,j_0+3,k}(t) +$$
$$\sum_{k=0}^{k_{14}^{j_0+4}-1} p_g(14, j_0+4, k)\zeta_{14,j_0+4,k}(t) + \sum_{k=0}^{k_{15}^{j_0+4}-1} p_g(15, j_0+4, k)\zeta_{15,j_0+4,k}(t) \tag{11}$$

Since system input $u(t)$, output $y(t)$ and the chosen orthogonal wavelet packet bases are known, $\zeta_{n,j_0+m,k}(t)$ can be calculated. From (11) and by using the least-square method, the projection coefficients for impulse response function $g(t)$ in orthogonal wavelet packet spaces $U^0_{j_0+4}$, $U^1_{j_0+4}$, $U^1_{j_0+3}$, $U^1_{j_0+2}$, $U^2_{j_0+2}$, $U^6_{j_0+3}$, $U^{14}_{j_0+4}$ and $U^{15}_{j_0+4}$ can be identified. Thus from (8) the impulse response function $g(t)$ is identified indirectly, and by using reconstruction algorithm of orthogonal WPT we can obtain the impulse response function $\hat{g}(t)$ in the time domain.

3 Theory algorithm and fast binary tree recursion algorithm for $\zeta_{n,j_0+m,k}(t)$

Definition 1[4]**.** The projection operator from space $l^2(Z)$ to space $l^2(2Z)$ is defined to be F_0 and F_1, *i.e.*,

$$\begin{cases} F_0(S_k)(l) = \sum_{k\in Z} h_{k-2l} S_k \\ F_1(S_k)(l) = \sum_{k\in Z} g_{k_2 l} S_k \end{cases} \tag{12}$$

where $\{h_k\}$ and $g\{g_k\}$ are low pass filter coefficients and high pass filter coefficients in the double scale difference equation (4), respectively.

In the following, a lemma and a theoretical algorithm for $\zeta_{n,j_0+m,k}(t)$ proposed in [3] by us will be quoted.

Lemma 1[3]**.** For WPT, let initial scale be j_0, and decomposition layer $m \in Z^+$. For arbitrary $n \in \{0, 1, \cdots, 2^m - 1\}$ at the scale $j = j_0 + m$, the wavelet packet basis function $w_{n,j,l}(t)$ of wavelet packet space $U^n_{j_0+m}$ can be expressed as follows.

$$w_{n,j,l}(t) = F_{e_1}\{F_{e_2}\{\cdots\{F_{e_{n_i}}\{w_{0,j-n_i,k}(t)\}(k_{n_i})\}\cdots\}(k_2)\}(l) \tag{13}$$

which can be simplified as

$$w_{n,j,l}(t) = F_{e_1}F_{e_2}\cdots F_{e_{n_i}}\{w_{0,j-n_i,k}(t)\}(l) \tag{14}$$

Here $\{w_{0,j-n_i,k}(t)\}_{k\in Z}$ are orthonormal bases of wavelet packet space $U^0_{j-n_i}$ (namely scale space V_{j-n_i}), and e_i is the ith coefficient of binary expression of n, *i.e.*,

$$n = \sum_{i=1}^{n_i} e_i 2^{i-1} \tag{15}$$

where n_i is the maximal index for binary expression of n.

Proof. Omitted.

Theorem 1. (Theoretical algorithm for $\zeta_{n,j_0+m,k}(t)$)[3]: Assume that for WPT, its initial scale is j_0. For arbitrary decomposition layer $m \in Z^+$, arbitrary integer $n \in \{0, 1, \cdots, 2^m - 1\}$ and $k \in Z$, at decomposition scale $j = j_0 + m$, $\zeta_{n,j_0+m,k}(t)$ defined as (10), namely $\zeta_{n,j,k}(t)$, satisfys that

$$\zeta_{n,j,k}(t) = F_{e_1}F_{e_2}\cdots F_{e_{n_i}}\{\zeta_{0,j-n_i,l}(t)\}(k) \tag{16}$$

where e_i is the ith coefficient for binary expression of n, and n_i is the maximal index of binary expression of n, *i.e.*,

$$n = \sum_{i=1}^{n_i} e_i 2^{i-1} \tag{17}$$

Proof. Omitted.

To conveniently calculate $\zeta_{n,j_0+m,k}(t)$, it is necessary to propose a discrete fast algorithm. Owning to the specific form of orthogonal wavelet packet bases, in this paper the sampling interval of time variant t is set[1]

$$T_s = 2^{j_0} \tag{18}$$

Thus time variable t can be written as

$$t = i \cdot T_s = 2^{j_0} i, \quad i \in Z/Z^- \tag{19}$$

Substituting (19) to (11), we get

$$\begin{aligned} y(2^{j_0} i) = &\sum_{k=0}^{k_0^{j_0+4}-1} p_g(0, j_0+4, k)\zeta_{0,j_0+4,k}(2^{j_0} i) + \sum_{k=0}^{k_1^{j_0+4}-1} p_g(1, j_0+4, k)\zeta_{1,j_0+4,k}(2^{j_0} i) + \\ &\sum_{k=0}^{k_1^{j_0+3}-1} p_g(1, j_0+3, k)\zeta_{1,j_0+3,k}(2^{j_0} i) + \sum_{k=0}^{k_1^{j_1+2}-1} p_g(1, j_0+2, k)\zeta_{1,j_0+2,k}(2^{j_0} i) + \\ &\sum_{k=0}^{k_2^{j_0+2}-1} p_g(2, j_0+2, k)\zeta_{2,j_0+2,k}(2^{j_0} i) + \sum_{k=0}^{k_6^{j_0+3}-1} p_g(6, j_0+3, k)\zeta_{6,j_0+3,k}(2^{j_0} i) + \\ &\sum_{k=0}^{k_{14}^{j_0+4}-1} p_g(14, j_0+4, k)\zeta_{14,j_0+4,k}(2^{j_0} i) + \sum_{k=0}^{k_{15}^{j_0+4}-1} p_g(15, j_0+4, k)\zeta_{15,j_0+4,k}(2^{j_0} i) \end{aligned} \tag{20}$$

It must be pointed out that (20) is not a kind of approximation for (11). Since (11) for arbitrary time variable t holds, (11) necessarily holds for t taking $2^{j_0} i$. Therefore, the discrete form (20) of (11) does not cause any error for $p_g(n, j_0+m, k)$ identification. In order to simplify the expression, here we use $*(i)$ to substitute for $*(2^{j_0} i)$, which is defined in [1], *i.e.*,

$$*(i) = *(2^{j_0} i) \tag{21}$$

Thus (20) can be simplified as

$$\begin{aligned} y(i) = &\sum_{k=0}^{k_0^{j_0+4}-1} p_g(0, j_0+4, k)\zeta_{0,j_0+4,k}(i) + \sum_{k=0}^{k_1^{j_0+4}-1} p_g(1, j_0+4, k)\zeta_{1,j_0+4,k}(i) + \\ &\sum_{k=0}^{k_1^{j_0+3}-1} p_g(1, j_0+3, k)\zeta_{1,j_0+3,k}(i) + \sum_{k=0}^{k_1^{j_0+2}-1} p_g(1, j_0+2, k)\zeta_{1,j_0+2,k}(i) + \\ &\sum_{k=0}^{k_2^{j_0+2}-1} p_g(2, j_0+2, k)\zeta_{2,j_0+2,k}(i) + \sum_{k=0}^{k_6^{j_0+3}-1} p_g(6, j_0+3, k)\zeta_{6,j_0+3,k}(i) + \\ &\sum_{k=0}^{k_{14}^{j_0+4}-1} p_g(14, j_0+4, k)\zeta_{14,j_0+4,k}(i) + \sum_{k=0}^{k_{15}^{j_0+4}-1} p_g(15, j_0+4, k)\zeta_{15,j_0+4,k}(i) \end{aligned} \tag{22}$$

Based on the generating mode of orthogonal wavelet packet bases in multiresolution structure, a discrete fast binary tree recursive algorithm for $\zeta_{n,j_0+m,k}(i)$ computation, proposed in [3] by us, will be quoted in the following.

Theorem 2. (Discrete fast binary tree recursive algorithm for $\zeta_{n,j_0+m,k}(t)$)[3]: Assume that for WPT, its initial scale is j_0. For arbitrary decomposition layer $m \in Z^+$ and arbitrary integer $n \in \{0, 1, \cdots, 2^m - 1\}$, the discrete algorithm for $\zeta_{n,j_0+m,k}(t)$ is given by

1) For $j = j_0$, $\zeta_{0,j_0,0}(i) \approx 2^{j_0/2} u(i)$

2) From $\zeta_{n,j,0}(i)$, $\zeta_{n,j,k}(i)$ can be calculated as

$$\zeta_{n,j,k}(i) = \zeta_{n,j,0}(i - 2^{j-j_0} k) \tag{23}$$

where $j = j_0+1, j_0+2, \cdots, j_0+m$, $0 \leqslant n \leqslant 2^{j-j_0}-1$.

3) According to Fig. 2, $\zeta_{n,j,0}(i)$ can be calculated step by step as follows

$$\begin{cases} \zeta_{2n,j,0}(i) = \sum\limits_{k\in Z} h_k \zeta_{n,j-1,0}(i - 2^{j-1-j_0}k) \\ \zeta_{2n+1,j,0}(i) = \sum\limits_{k\in Z} g_k \zeta_{n,j-1,0}(i - 2^{j-1-j_0}k) \end{cases} \tag{24}$$

where $j = j_0+1, j_0+2, \cdots, j_0+m$, $0 \leqslant n \leqslant 2^{j-j_0}-1$.

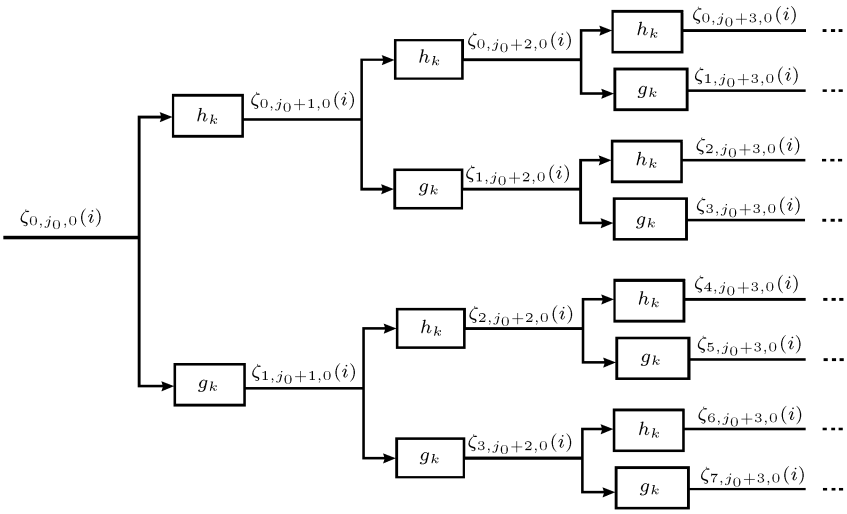


Fig. 2 The structure of discrete fast binary tree recursive algorithm for $\zeta_{n,j_0+m,k}(i)$

Proof. Omitted.

According to Theorem 2, all $\zeta_{n,j_0+m,k}(i)$ can be calculated step by step. Since the lengths of high pass filter group and low pass filter group of wavelet are limited, the burden of recursive computation in Theorem 2 is very low, and the speed of computing $\zeta_{n,j_0+m,k}(i)$ will be improved very much. Hence, the discrete fast binary tree recursive algorithm for $\zeta_{n,j_0+m,k}(t)$ in Theorem 2 is suitable for practical application.

4 Problems discussion

In the proposed impulse response identification algorithm, some problems such as orthogonal wavelet packet bases, initial scale j_0, decomposition layer m of orthogonal WPT, and $k_n^{j_0+m}$ in (8) must be considered.

1) Orthogonal wavelet packet bases selection

The selected orthogonal wavelet packet bases are generated by initial scale function of Daubechies II (DB2)[5].

2) Initial scale j_0 selection

The selected j_0 directly affects the precision of the orthogonal wavelet packet series of $g(t)$ in (8). At present, there is no widely accepted criterion to select j_0, and in general, j_0 takes a less negative integer.

3) Decomposition layer m selection

For given initial scale space V_{j_0}, decomposition layer doesn′t affect the precision of the proposed impulse response identification algorithm, but finally determines the number ($i.e., = 2^m$) of frequency bands of $g(t)$ divided by wavelet packet decomposition.

4) $k_n^{j_0+l}$ computation in (8)

In (8), $k_n^{j_0+l}$ $(l = 1, 2, \cdots, m)$ virtually denotes the number of WPT coefficients for $g(t)$ in the wavelet packet space $U_{j_0+l}^n$ at decomposition scale $j = j_0 + l$. Suppose that the number of WPT coefficients for $g(t)$ in the initial scale space V_{j_0} (*i.e.*, $= U_{j_0}^0$) is $k_0^{j_0}$. We then have

$$k_n^{j_0+l} = 2^{-l} k_0^{j_0} \quad n \in \{0, 1, \cdots, 2^l - 1\} \tag{25}$$

5 Impulse response identification algorithm based on varying scale orthogonal WTP

Impulse response identification algorithm based on varying scale orthogonal WPT is proposed as follows.

Step 1. Determine the initial scale j_0 and decomposition layer m of orthogonal WPT.

Step 2. Estimate the valid action time t_g of impulse response function $g(t)$. From (25) for $g(t)$ in the wavelet packet space $U_{j_0+l}^n$ $(0 \leqslant n \leqslant 2^l - 1)$ at the given decomposition scale $j = j_0$ $(l = 1, 2 \cdots, m)$, the number of WPT coefficients, can be computed.

Step 3. By adopting orthogonal wavelet packet bases function of Daubechies II (*i.e.*, DB2), and according to discrete fast binary tree recursion algorithm in Theorem 2, all $\zeta_{n,j_0+l,k}(i)$ required in (22) can be computed step by step, where $0 \leqslant l \leqslant m$, $0 \leqslant n \leqslant 2^l - 1$, $0 \leqslant k \leqslant 2^{-1} k_0^{j_0} - 1$.

Step 4. Construct the identifying parameter vector $\boldsymbol{X}$.

Now, take the chosen orthogonal wavelet packet spaces in Fig. 1 as an example, and let $\boldsymbol{x}_s$ $(s = 1, 2, \cdots, 8)$ denote the row vector composed of all WPT coefficients $p_g(n, j_0 + l, k)$ $(l = 1, 2, \cdots, m(= 4)$, $0 \leqslant k \leqslant 2^{-l} k_0^{j_0} - 1)$ for $g(t)$ in the wavelet packet spaces $U_{j_0+4}^0$, $U_{j_0+4}^1$, $U_{j_0+3}^1$, $U_{j_0+2}^1$, $U_{j_0+2}^2$, $U_{j_0+3}^6$, $U_{j_0+4}^{14}$, and $U_{j_0+4}^{15}$, respectively. The identifying parameter vector is constructed as

$$\boldsymbol{X} = [x_1, x_2, \cdots, x_8]^{\mathrm{T}} \tag{26}$$

where sign T denotes matrix transpose operation.

Step 5. Construct data vector $\boldsymbol{A}(i)$.

Let $\boldsymbol{a}_s(i)$ $(s = 1, 2, \cdots, 8)$ denote the row vector composed of all $\zeta_{n,j_0+l,k}(i)$ $(0 \leqslant k \leqslant 2^{-l} k_0^{j_0} - 1)$ generated by convolution in the wavelet packet spaces $\boldsymbol{U}_{j_0+4}^0$, $U_{j_0+4}^1$, $U_{j_0+3}^1$, $U_{j_0+2}^1$, $U_{j_0+2}^2$, $U_{j_0+3}^6$, $U_{j_0+4}^{14}$, and $U_{j_0+4}^{15}$, respectively. The data vector $\boldsymbol{A}(i)$ is constructed as

$$\boldsymbol{A}(i) = [a_1(i), a_2(i), \cdots, a_8(i)]^{\mathrm{T}} \tag{27}$$

Step 6. Identify WPT coefficient sequences.

According to the definition of $\boldsymbol{X}$ in (26) and $\boldsymbol{A}(i)$ in (27), (22) can be rewritten as

$$\boldsymbol{A}^{\mathrm{T}}(i)\boldsymbol{X} = y(i) \tag{28}$$

For $(i = 1, 2, \cdots, L)$ taking all sampling points, (28) can be rewritten as

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{X} = \boldsymbol{Y} \tag{29}$$

where $\boldsymbol{A} = [\boldsymbol{A}(1), \boldsymbol{A}(2), \cdots, \boldsymbol{A}(L)]$ and $\boldsymbol{Y} = [y(1), y(2), \cdots, y(L)]^{\mathrm{T}}$.

By using the least square method, WPT coefficient sequences $\boldsymbol{X}$, for $g(t)$ onto the chosen orthogonal wavelet packet spaces, can be identified easily.

Step 7. Process the WPT coefficient sequences $\boldsymbol{X}$.

Consider that WPT coefficient consequences $\boldsymbol{X}$ for $g(t)$ onto the chosen orthogonal wavelet packet spaces are of specific physics meaning. So $\boldsymbol{X}$ can be directly used to analyze the properties for $g(t)$ in some specific frequency bands. Furthermore, according to the identified WPT coefficients, by using reconstruction algorithm of WPT, the original impulse response function $\hat{g}(t)$ in time domain can be reconstructed easily.

6 Simulation example

To demonstrate the validity of the proposed impulse response identification algorithm, in this section two numerical simulation examples with different conditions are given. In numerical simulation, identification error $e(i)$, relative error RE and variance $\hat{\sigma}$ are defined by

$$e(i) = g(i) - \hat{g}(i) \tag{30}$$

$$RE = \frac{\|g(t) - \hat{g}(t)\|}{\|g(t)\|} \cdot 100\% \tag{31}$$

$$\hat{\sigma} = \frac{1}{L-1} \sum_{i=1}^{L} [e(i)]^2 \tag{32}$$

where $\| \bullet \|$ is norm operation defined by $\|X(t)\| = \int_{-\infty}^{+\infty} |X(t)| \mathrm{d}t$.

Simulation examples adopt the following transfer function in [6].

$$G(s) = \frac{1.2}{(8.3s+1)(6.2s+1)} \tag{33}$$

Example 1. For system output not existing noise, consider the identifying object

$$Y(s) = \frac{1.2}{(8.3s+1)(6.2s+1)} U(s) \tag{34}$$

where input $u(t)$ is Gaussian white noise with 0 mean and variance 1.

Set valid action time of impulse response function $t_g = 50$ seconds. To satisfy data length requirement of WPT at the initial scale, the valid action time virtually takes $t'_g = 56$ seconds. Let initial scale $j_0 = -1$, decomposition layer $m = 4$, and sampling time of system input and output $t = 500$ seconds. Then sampling interval $t_s = 2^{-1} = 0.5$ second, the length of discrete sequences for $g(t)$ $k_0^{j_0} = 112$, and sampling data length $L = 1000$. By using the proposed impulse response identification algorithm, identification result $\hat{g}(i)$ for impulse response function $g(i)$ is showed in Fig. 3.

Let system output $u(t)$ be stochastic bar signal with its amplitude subjected to normal distribution and its period taking 40 sampling intervals. By using the proposed impulse response identification algorithm, impulse response identification results are showed in Fig. 4.

In Fig. 3, the variance of identification error $e(i)$ is $1.1676e-6$, and the relative error $RE = 2.66\%$, showing that the identification precision of the proposed algorithm for deterministic system is high. In Fig. 4, the variance of identification error $e(i)$ is $2.3173e-6$, and the relative error $RE = 3.10\%$, showing that the identification precision of the proposed algorithm for system input being deterministic signal is also high.

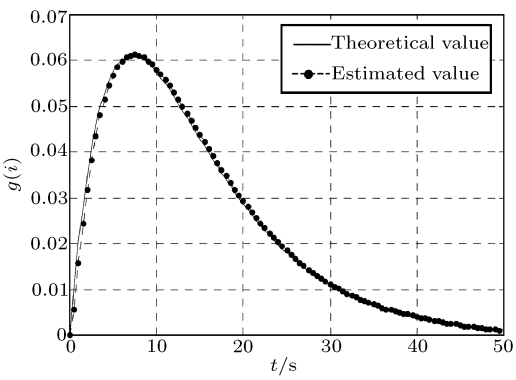


Fig. 3 Impulse response identification results for input being Gaussian white noise and output not existing noise

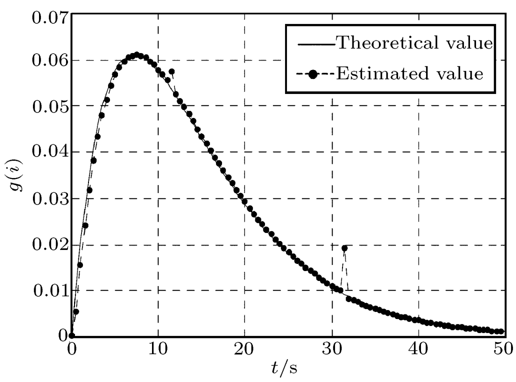


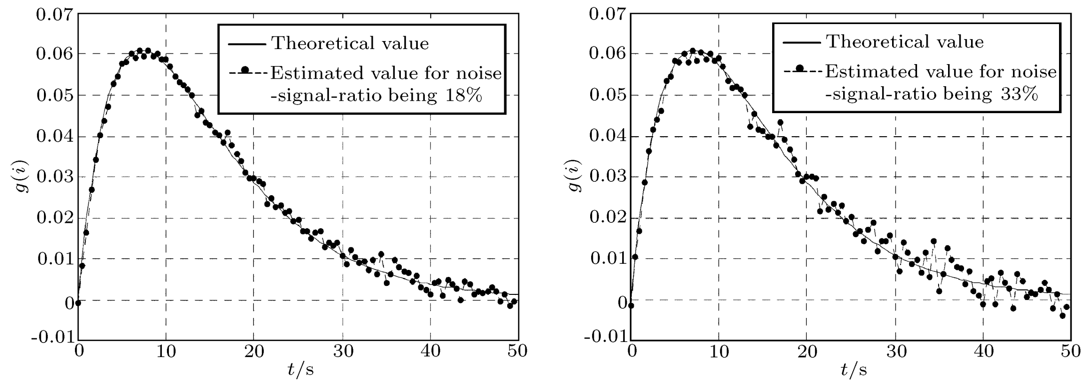
Fig. 4 Impulse response identification results for input being stochastic bar signal and output not existing noise

Example 2. For system output existing noise, consider the identifying object

$$Y(s) = \frac{1.2}{(8.3s+1)(6.2s+1)} U(s) + \lambda N(s) \tag{35}$$

where $N(s)$ denotes Gaussian white noise with 0 mean and variance 1, $u(t)$ and $n(t)$ are all white noise, $u(t)$ is not correlative with $n(t)$, λ is a parameter which limits noise. Let noise-signal-ratio be 18% and 33%, respectively.

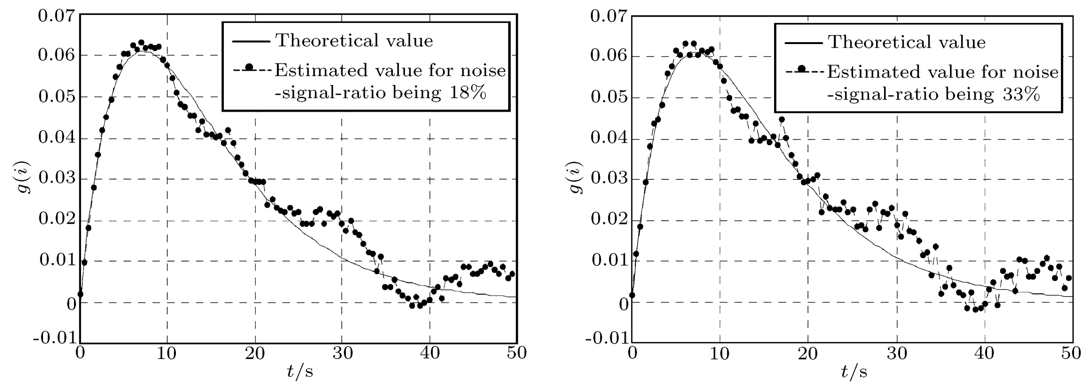
Parameters selection is the same as Example 1. Impulse response identification results for noise-signal-ration being 18% and 33% are showed in Fig. 5, where variances for identification error $e(i)$ are $\hat{\sigma}_{18} = 2.4798e-6$ and $\hat{\sigma}_{33} = 6.2803e-6$, respectively, and their relative errors are $RE_{18} = 5.17\%$ and $RE_{33} = 8.32\%$, respectively. The proposed impulse response identification algorithm for system output existing strong noise is still of high identification precision.



(a) Noise-signal-ration being 18% (b) Noise-signal-ration being 33%

Fig. 5 Impulse response identification results for system output existing noise

To compared with other impulse response identification algorithm, impulse response identification results based on correlation analysis algorithm for noise-signal-ratio being 18% and 33% are showed in Fig. 6, where variances for identification error $e(i)$ are $\hat{\sigma}_{18} = 1.8038e-5$ and $\hat{\sigma}_{33} = 2.2320e-5$, and their relative errors are $RE_{18} = 14.31\%$ and $RE_{33} = 15.97\%$. Compared with correlation analysis algorithm, it is easily seen that the proposed impulse response identification algorithm is of higher identification precision.



(a) Noise-signal-ration being 18% (b) Noise-signal-ration being 33%

Fig. 6 Impulse response identification based on correlation analysis algorithm for system output existing noise

Now, let sampling data length $L = 5600$. For Example 2, we adopt correlation analysis algorithm again. The corresponding impulse response identification results are that, variance $\hat{\sigma}_{18} = 7.0481e-6$ for identification error $e(i)$ when noise-signal-ratio is 18%, variance $\hat{\sigma}_{33} = 8.2515e-6$ for identification error $e(i)$ when noise-signal-ratio is 33%, and their relative errors are $RE_{18} = 9.25\%$ and $RE_{33} = 9.84\%$. The general impulse response identification result comparison of the proposed impulse response identification algorithm in this paper and correlation analysis algorithm is listed in Table 1 and Table 2, where initial scale $j_0 = -1$. Table 1 shows that, to obtain the same identification precision, the sampling data length required by varying scale orthogonal WPT algorithm is much shorter than that required by correlation analysis algorithm.

Table 1 The relative error comparison of the two algorithms

Noise-signal-ratio	Fixed scale orthogonal wavelet packet transform algorithm ($L = 1000$)	Correlation analysis algorithm ($L = 1000$)	Correlation analysis algorithm ($L = 5600$)
18%	5.17%	14.31%	9.25%
33%	8.32%	15.97%	9.84%

Table 2 The variance comparison of identification error of the two algorithms

Noise-signal-ratio	Fixed scale orthogonal wavelet packet transform algorithm ($L = 1000$)	Correlation analysis algorithm ($L = 1000$)	Correlation analysis algorithm ($L = 5600$)
18%	$2.4798e-6$	$1.8038e-5$	$7.0481e-6$
33%	$6.2803e-6$	$2.2320e-5$	$8.2515e-6$

Furthermore, to know the relationship between identification precision and initial scale j_0 selection, in this paper some numerical simulations, for $j_0 = 0, -2, -3$, and noise-signal-ratio taking 0%, 18% and 33%, are finished. Limited by the space of this paper, all impulse response identification result curves and their identification error curves are not listed. Only all variances of identification errors and their relative errors are listed in Table 3 and Table 4, respectively. It indicates that, the identification precision gets higher as initial scale j_0 gets less, and the proposed impulse response identification algorithm, in general, has a higher identification precision and better capacity of anti-noise interference.

Table 3 The relative error comparison of impulse response identification for different simulation conditions

	0%	18%	33%
0	4.79%	5.80%	8.38%
-1	2.66%	5.17%	8.32%
-2	1.70%	4.85%	8.13%
-3	0.83%	4.01%	6.52%

Table 4 The variance comparison of impulse response identification errors for different simulation conditions

	0%	18%	33%
0	$3.8179e-6$	$4.5357e-6$	$6.3415e-6$
-1	$1.1676e-6$	$2.4798e-6$	$6.2803e-6$
-2	$3.3043e-7$	$2.0589e-6$	$6.2389e-6$
-3	$1.4173e-7$	$1.9440e-6$	$5.8378e-6$

By the way, compared with the impulse response identification algorithm based on fixed scale orthogonal WPT presented in [3], the impulse response identification algorithm proposed in this paper has the almost same identification precision as the algorithm presented in [3], however, the most virtues of the algorithm proposed in this paper lie in that, according to the identified orthogonal WPT coefficients, we can directly obtain the frequency distribution attributes of the identifying impulse response function in some specific frequency bands.

7 Conclusion

A new impulse response identification algorithm based on the varying scale orthogonal WPT is proposed successfully. Simulation results show that the proposed impulse response identification algorithm has better capacity for trailing the changes of system dynamic behavior and for anti-noise interference, and that compared with correlation analysis algorithm, for same identification precision, the sampling data length required by varying scale orthogonal WPT algorithm is much shorter than that required by correlation analysis algorithm. Furthermore, the identified WPT coefficients of impulse response $g(t)$ have specific physical meaning, and can be directly used to accurately analyze system frequency attributes in some specific frequency bands.

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LI He-Sheng Received his master degree from Southwest Jiaotong University in 1997. He received his Ph. D. degree from Beijing University of Aeronautics and Astronautics in 2002. From 2002 May to 2004 April, as a postdoctor he worked in the Department of Electronic Engineering, Tsinghua University. He is currently working for Institute of Electronic Engineering, China Academy of Engineering Physics. His research interests include the signal and information processing, radar signal detection, signal modeling and spectrum analysis, pattern recognition, and wavelet theory and application.

MAO Jian-Qin Professor in the Seventh Research division at Beijing University of Aeronautics and Astronautics (BUAA). Her research interests include control theory and control engineer, intelligent control and application, robust control and application, intelligent identification and modeling, and computer assistant design in control system.

ZHAO Ming-Sheng Received his Ph. D. degree from Tsinghua University in 1995. He is now a associate professor in the Department of Electronic Engineering at Tsinghua University. His research interests include the neural network, pattern recognition, and signal and information processing.

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