Robust Stabilization of a Class of Feedforward Systems[1)]

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Abstract Two kinds of saturated controllers are designed for a class of feedforward systems and the closed-loop resulted is locally input-to-state stable and input-to-state stable, respectively. By the word "locally", it is meant that there are restrictions on the amplitude of inputs. At first, under the guidance of suitable energy functions, two kinds of saturated controllers are designed as locally input-to-state stabilizers for a class of perturbed linear systems, from which explicit gain estimations can be obtained for the subsequent design. Then under the conditions that two subsystems of the feedforward system are respectively of locally input-to-state stability and input-to-state stability, the small gain theory is used to determine saturated degrees for corresponding robust stabilizers. The stability proofs are given by using a new characterization of input-to-state stability that is based on the concept of ultimate boundedness. As an application, saturated controllers are designed for the partial dynamics of a certain inverted pendulum.

Key words Input-to-state stability, input-to-state stability with restrictions on inputs, small gain theory, saturated control

1 Introduction

In [1] and [2] which are related to saturation designs, the robust stabilization problem of a class of feedforward systems has been investigated successfully by using notions of "small input small state stability" and "asymptotical boundedness of output with restriction on inputs", respectively. However, these two notions have no systematic characterizations, as a result, the conditions expressed by them are hard to test, and the results expressed are also hard to be exploited widely. By contrast, we will study the robust stabilization problem of the class of feedforward systems based on the notion of input-to-state stability that has quantity of dissipative characterizations, and has now been a central concept in nonlinear control and analysis[3]. It is deserved to point out that in this paper locally input-to-state stability is more frequently used, which involves explicit restrictions on the amplitude of inputs. As a matter of fact, just due to using the notion of locally input-to-state stability, we can deal conveniently with the interconnection term of the feedforward system in question, and use the small gain theory to determine saturation degrees for corresponding robust stabilizers. Under the precondition of non finite escape, we give our stability analysis based on a new characterization in [3] for input-to-state stability.

2 Preparation

In this section we introduce some concepts and lemmas.

Consider local Lipschitz system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}),\ \boldsymbol{f}(\boldsymbol{0}, \boldsymbol{0}) = \boldsymbol{0}, \quad \boldsymbol{x} \in R^n,\ \boldsymbol{u} \in R^m \tag{1}$$

Definition 1. System (1) is said to be input-to-state stable with restrictions on inputs, for short, locally input-to-state stable, if there is a $\Delta > 0$ such that, for all $\boldsymbol{x}$ and all $|\boldsymbol{u}| \leqslant \Delta$, system (1) is input-to-state stable with respect to $\boldsymbol{u}$.

In the followings, $\boldsymbol{u} = \boldsymbol{k}(\boldsymbol{x})$ is said to be an input-to-state stabilizer of system (1), if $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{k}(\boldsymbol{x}) + \boldsymbol{v})$ is input-to-state stable with respect to $\boldsymbol{v}$. Similarly, $\boldsymbol{u} = \boldsymbol{k}(\boldsymbol{x})$ is said to be a locally input-to-state stabilizer of system (1), if $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{k}(\boldsymbol{x}) + \boldsymbol{v})$ is locally input-to-state stable with respect to $\boldsymbol{v}$.

Lemma 1[3]**.** System (1) is input-to-state stable if and only if $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{0})$ is globally asymptotically stable (GAS), and it has the ultimate boundedness property, that is, there exists a non-decreasing function γ such that $|\boldsymbol{x}(t)|_a = \limsup_{t\to\infty} |\boldsymbol{x}(t)| \leqslant \gamma(|\boldsymbol{u}(t)|_\infty)$.

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The feedforward system in question is described as

$$\begin{aligned}&\dot{\boldsymbol{x}}_1 = A\boldsymbol{x}_1 + \boldsymbol{b}u + \boldsymbol{g}(x_2, u), \quad \boldsymbol{x}_1 \in R^n\\ &\dot{x}_2 = f(x_2, u), \quad x_2 \in R, u \in R\\ &\boldsymbol{g}(0,0) = \boldsymbol{0}, \quad f(0,0) = 0\end{aligned} \tag{2}$$

H1-1. For each $\varepsilon > 0$, there exists a $\Delta > 0$ such that $|(x_2, u)| \leqslant \Delta \Rightarrow |\boldsymbol{g}(x_2, u)| \leqslant \varepsilon|(x_2, u)|$.

H1-2. The x_2 subsystem is input-to-state stable, and holds $|x_2|_a \leqslant C|u|_a$.

H1-3. $\dot{\boldsymbol{x}} = A\boldsymbol{x}_1 + \boldsymbol{b}[\lambda \mathrm{sat}(F\boldsymbol{x}_1/\lambda) + v] + \boldsymbol{d}$ is input-to-state stable with restrictions on $(v, \boldsymbol{d})$, in particular, there are $\bar{\Delta} > 0$ and $\bar{C} > 0$ such that for any $\lambda > 0$, $|\boldsymbol{b}v + \boldsymbol{d}|_a \leqslant \lambda\bar{\Delta} \Rightarrow |\boldsymbol{x}_1|_a \leqslant \bar{C}\max\{|v|_a, |\boldsymbol{d}|_a\}$ holds.

H1-3*. $\dot{\boldsymbol{x}}_1 = A\boldsymbol{x}_1 + \boldsymbol{b}\lambda\mathrm{sat}((F\boldsymbol{x}_1 + v)/\lambda) + \boldsymbol{d}$ is input-to-state stable with restrictions on $\boldsymbol{d}$, in particular, there are $\bar{\Delta} > 0$ and $\bar{C} > 0$ such that for any $\lambda > 0$, $|\boldsymbol{d}|_a \leqslant \lambda\bar{\Delta} \Rightarrow |\boldsymbol{x}_1|_a \leqslant \bar{C}\max\{|v|_a, |\boldsymbol{d}|_a\}$ holds.

Hypotheses H1-3 and H1-3* actually show that two kinds of saturated controllers can be taken as locally input-to-state stabilizers for $\dot{\boldsymbol{x}}_1 = A\boldsymbol{x}_1 + \boldsymbol{b}u + \boldsymbol{d}$. Below are some discussions as for how to determine F and other parameters.

Consider a perturbed linear system

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + \boldsymbol{b}u + \boldsymbol{d}, \quad A \in R^{n\times n}, \boldsymbol{b} \in R^n \tag{3}$$

where $\boldsymbol{d} \in R^n$ is an external disturbance.

H2-1. $(A, \boldsymbol{b})$ is a controllable pair; there exists a matrix H, $H = H^{\mathrm{T}} > 0$ such that $A^{\mathrm{T}}H + HA \leqslant 0$.

In view of the passivity theory, H2-1 implies that $A - \boldsymbol{b}\boldsymbol{b}^{\mathrm{T}}H$ is a Hurwitz matrix, that is, there exists a matrix $P = P^{\mathrm{T}} > 0$ such that

$$[A - \boldsymbol{b}\boldsymbol{b}^{\mathrm{T}}H]^{\mathrm{T}}P + P[A - \boldsymbol{b}\boldsymbol{b}^{\mathrm{T}}H] = -\boldsymbol{I} \tag{4}$$

H2-2. $|\boldsymbol{b}v + \boldsymbol{d}|_a \leqslant \lambda\dfrac{\sqrt{\lambda_{\min}(H)}}{8(\lambda_{\max}(H))^{3/2}\|P\boldsymbol{b}\|}$. Here $\lambda_{\min}(H)$ and $\lambda_{\max}(H)$ denote the minimum eigenvalue and maximum eigenvalue of H, respectively; P is referred to (4), λ and v are referred to the following saturated control:

$$u = \lambda\mathrm{sat}\left(\frac{-\boldsymbol{b}^{\mathrm{T}}H\boldsymbol{x}}{\lambda}\right) + v, \ \lambda > 0; \quad \mathrm{sat}(s) = \begin{cases} 1, & s > 1\\ s, & -1 \leqslant s \leqslant 1\\ -1, & s < -1\end{cases} \tag{5}$$

Lemma 2. If H2-1and H2-2 hold, then system (3) with control (5) is locally input-to-state stable.

Proof. The closed loop is

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + \boldsymbol{b}\lambda\mathrm{sat}((-\boldsymbol{b}^{\mathrm{T}}H\boldsymbol{x})/\lambda) + \boldsymbol{b}v + d \tag{6}$$

From H2-2, there must exist a sufficient large time $T \geqslant 0$ such that $\sup\limits_{t\in[T,\infty)} |\boldsymbol{b}v + \boldsymbol{d}| \leqslant \dfrac{\lambda\sqrt{\lambda_{\min}(H)}}{4(\lambda_{\max}(H))^{3/2}\|P\boldsymbol{b}\|}$. Using the same argument in proving Lemma 3.2 of [1], it can be shown that the closed loop has no finite escape time. So, without loss of generality, we consider systems (3), (5) with $t \geqslant T$.

Define an energy function $U = \frac{1}{3}(\boldsymbol{x}^{\mathrm{T}}H\boldsymbol{x})^{3/2}$ and let $y := -\boldsymbol{b}^{\mathrm{T}}H\boldsymbol{x}/\lambda$, use H2-1and calculate the time differential of U along the solution to system (6). One has

$$\dot{U} \leqslant -\lambda^2\sqrt{\lambda_{\min}(H)}|\boldsymbol{x}|y\mathrm{sat}(y) + (\lambda_{\max}(H))^{3/2}|\boldsymbol{x}|^2|\boldsymbol{b}y + \boldsymbol{d}|$$

Rewrite system (6) as

$$\dot{\boldsymbol{x}} = [A - \boldsymbol{b}\boldsymbol{b}^{\mathrm{T}}H]\boldsymbol{x} - \lambda\boldsymbol{b}[y - \mathrm{sat}(y)] + \boldsymbol{b}y + \boldsymbol{d} \tag{7}$$

Define an energy function $V = \frac{1}{2}\boldsymbol{x}^{\mathrm{T}}P\boldsymbol{x}$ for system (7), P is referred to (4). Observing that $|y - \mathrm{sat}(y)| \leqslant y\mathrm{sat}(y)$, one has $\dot{V} \leqslant -\frac{1}{2}|\boldsymbol{x}|^2 + \lambda\|P\boldsymbol{b}\||\boldsymbol{x}|y\mathrm{sat}(y) + \|P\boldsymbol{b}\||\boldsymbol{x}||v| + \|P\||\boldsymbol{x}||\boldsymbol{d}|$.

Defining a total energy function $W = \dfrac{\|P\boldsymbol{b}\|U}{\lambda\sqrt{\lambda_{\min}(H)}} + V$, using $|\boldsymbol{b}v + \boldsymbol{v}| \leqslant \dfrac{\lambda\sqrt{\lambda_{\min}(H)}}{4(\lambda_{\max}(H))^{3/2}\|P\boldsymbol{b}\|}$ and letting $G = \max\{\|P\boldsymbol{b}\|, \|P\|\}$, one has

$$\dot{W} \leqslant -\frac{1}{4}|\boldsymbol{x}|^2 + G(|\boldsymbol{x}||v| + |\boldsymbol{x}||\boldsymbol{d}|) \leqslant -\frac{1}{8}|\boldsymbol{x}|^2 + 4G^2(|v|^2 + |\boldsymbol{d}|^2)$$

From Lemma 1 and observing that there is a restriction on inputs, the lemma is proven. □

Remark 1. From the last equation in the proof of Lemma 2, the following asymptotic bound can be obtained:

$$|\boldsymbol{b}v+\boldsymbol{d}|_a \leqslant \lambda\frac{\sqrt{\lambda_{\min}(H)}}{8(\lambda_{\max}(H))^{3/2}\|P\boldsymbol{b}\|} \Rightarrow \|\boldsymbol{x}|_a \leqslant 8\sqrt{2}G\max\{|v|_a,|\boldsymbol{d}|_a\}$$

H2-2*. $|\boldsymbol{d}|_a \leqslant \lambda\frac{\sqrt{\lambda_{\min}(H)}}{8(\lambda_{\max}(H))^{3/2}\|P\boldsymbol{b}\|}$. Using the same argument as that in proving Lemma 2, one can prove:

Lemma 3. If H2-1and H2-2* hold, then system (3) with $u=\lambda\mathrm{sat}((-\boldsymbol{b}^{\mathrm{T}}H\boldsymbol{x}+v)/\lambda),\lambda$, is locally input-to-state stable, and the following holds

$$|\boldsymbol{d}|_a \leqslant \lambda\frac{\sqrt{\lambda_{\min}(H)}}{8(\lambda_{\max}(H))^{3/2}\|P\boldsymbol{b}\|} \Rightarrow |\boldsymbol{x}|_a \leqslant 8\sqrt{2}\max\left\{\|P\boldsymbol{b}\|\left(1+\sqrt{\frac{\lambda_{\max}(H)}{\lambda_{\min}(H)}}\right),\|P\|\right\}\max\{|v|_a,|\boldsymbol{d}|_a\}$$

3 Main results

Now we come to design two kinds of saturated controllers to robustly stabilize system (2).

1) Set saturation degree λ based on H1-1, H1-2 and H1-3: Let $L_1=\bar{C}\|F\|$, choose L_2 such that $L_1L_2\leqslant 1/2$; for any $\varepsilon>0$ which satisfies $\varepsilon\sqrt{C^2+1}\leqslant L_2$ and $\varepsilon\sqrt{C^2+1}(1+\bar{\Delta}/(2\|\boldsymbol{b}\|))\leqslant\bar{\Delta}/2$, fix $\Delta>0$ according to H1-1 ; then choose $\lambda>0$ such that $\lambda\sqrt{C^2+1}(1+\bar{\Delta}/(2\|\boldsymbol{b}\|))\leqslant\Delta$.

Theorem 1. If H1-1, H1-2 and H1-3 hold and λ is chosen as above, then system (2) with the control $u=\lambda\mathrm{sat}(F\boldsymbol{x}_1/\lambda)+v$, $|v|_a\leqslant\lambda\bar{\Delta}/(2\|\boldsymbol{b}\|)$, is locally input-to-state stable.

Proof. System (2) with $u=\lambda\mathrm{sat}(F\boldsymbol{x}_1/\lambda)+v:=z+v$ becomes

$$\dot{\boldsymbol{x}}_1=A\boldsymbol{x}_1+\boldsymbol{b}\lambda\mathrm{sat}(F\boldsymbol{x}_1/\lambda)+\boldsymbol{b}v+\boldsymbol{g}(x_2,z+v),\quad \dot{x}_2=f(x_2,z+v)$$

Firstly, confirm that $|\boldsymbol{b}v+\boldsymbol{g}|_a\leqslant\lambda\bar{\Delta}$. From H1-1, H1-2 and $|z|_a\leqslant\lambda,|v|_a\leqslant\lambda\bar{\Delta}/(2\|\boldsymbol{b}\|)$, one has

$$|x_2|_a\leqslant C|z+v|_a\Rightarrow|(x_2,z+v)|_a\leqslant\sqrt{C^2+1}|z+v|_a\leqslant\lambda\sqrt{C^2+1}(1+\bar{\Delta}/(2\|\boldsymbol{b}\|))\leqslant\Delta$$
$$|\boldsymbol{b}v+\boldsymbol{g}|_a\leqslant|\boldsymbol{b}v|_a+|\boldsymbol{g}|_a\leqslant|\boldsymbol{b}v|_a+\varepsilon|(x_2,z+v)|_a\leqslant|\boldsymbol{b}v|_a+\varepsilon\lambda\sqrt{C^2+1}(1+\bar{\Delta}/(2\|\boldsymbol{b}\|))\leqslant\lambda\bar{\Delta}$$

Since $|\boldsymbol{b}v+\boldsymbol{g}|_a\leqslant\lambda\bar{\Delta}$, it follows from H1-3 that

$$|z|_a\leqslant\min\{\lambda,|F\boldsymbol{x}_1|_a\}\leqslant\min\{\lambda,\bar{C}\|F\|\max\{|v|_a,|\boldsymbol{g}|_a\}\}=$$
$$\min\{\lambda,L_1\max\{|v|_a,|\boldsymbol{g}|_a\}\}\leqslant\max\{\min\{\lambda,L_1|v|_a\},L_1|\boldsymbol{g}|_a\}$$

The last inequality is obtained by using "$\min\{a,\max\{b,c\}\}\leqslant\max\{\min\{a,b\},c\}\ \forall a,b,c\geqslant 0$".

Secondly, confirm that the closed loop has the ultimate boundedness property.

If $\min\{\lambda,L_1|v|_a\}\geqslant L_1|\boldsymbol{g}|_a$, then $|z|_a\leqslant\min\{\lambda,L_1|v|_a\}\leqslant L_1|v|_a$; if $\min\{\lambda,L_1|v|_a\}\leqslant L_1|\boldsymbol{g}|_a$, then $|z|_a\leqslant L_1|\boldsymbol{g}|_a\leqslant L_1L_2|z|_a+L_1\varepsilon\sqrt{C^2+1}|v|_a$. Again, $L_1L_2\leqslant 1/2$, so one has $|z|_a\leqslant 2L_1\varepsilon\sqrt{C^2+1}|v|_a$. Hence, $|z|_a\leqslant\max\{L_1,2L_1\varepsilon\sqrt{C^2+1}\}|v|_a:=H|v|_a$, $|x_2|_a\leqslant C|z+v|_a\leqslant C(H+1)|v|_a:=\Pi_1|v|_a$.

Observing that $|\boldsymbol{g}|_a\leqslant\varepsilon\sqrt{C^2+1}|z+v|_a\leqslant\varepsilon\sqrt{C^2+1}(|z|_a+|v|_a)\leqslant\varepsilon\sqrt{C^2+1}(H+1)|v|_a$, from H1-3 one has $|\boldsymbol{x}_1|_a\leqslant\bar{C}\max\{1,\varepsilon\sqrt{C^2+1}(H+1)\}|v|_a:=\Pi_2|v|_a$.

From the above estimations, there exists some constant $\Pi>0$ such that $|\boldsymbol{x}|_a\leqslant\Pi|v|_a\leqslant\Pi|v|_\infty$.

Next, one can claim that the closed loop is GAS with $v\equiv 0$, *i.e.*, the following system is GAS:

$$\dot{\boldsymbol{x}}_1=A\boldsymbol{x}_1+\boldsymbol{b}\lambda\mathrm{sat}(F\boldsymbol{x}_1/\lambda)+\boldsymbol{g}(x_2,\lambda\mathrm{sat}(F\boldsymbol{x}_1/\lambda)),\quad \dot{x}_2=f(x_2,\lambda\mathrm{sat}(F\boldsymbol{x}_1/\lambda))$$

In fact, it follows from H1-2, H1-3 and Lemma 3.5 of [2] that the closed loop has no finite escape time. But from the proof above it can be deduced that $|\boldsymbol{g}|_a=0$ when $v\equiv 0$. Once again, in view of H1-3 and H1-2, one can deduce that $\boldsymbol{x}$ is bounded and $\lim_{t\to\infty}x_i(t)=0$ $(i=1,2)$. Then according to the essential of input-to-state stability and locally input-to-state stability, the claim is proven

In view of Lemma 1 and observing that there is a restriction on inputs, the theorem is proven. □

Corollary 1. Consider system (2) with $\boldsymbol{g}(x_2,u)=\boldsymbol{g}(x_2)$. H2-1, H2-2 and H2-3 hold. Let $L_1=\bar{C}\|F\|$, choose L_2 such that $L_1L_2\leqslant 1/2$; for any $\varepsilon>0$ satisfying $\varepsilon C\leqslant L_2$ and $\varepsilon\sqrt{C^2+1}(1+\bar{\Delta}/(2\|\boldsymbol{b}\|))\leqslant\bar{\Delta}/2$, fix $\Delta>0$ according to H1-1; choose $\lambda>0$ such that $\lambda C(1+\bar{\Delta}/(2\|\boldsymbol{b}\|))\leqslant\Delta$. Then system (2) with the control $u=\lambda\mathrm{sat}(F\boldsymbol{x}_1/\lambda)+v$, $|v|_a\leqslant\lambda\bar{\Delta}/(2\|\boldsymbol{b}\|)$, is local input-to-state stable.

2) Set saturation degree λ based on H1-1, H1-2 and H1-3*: Let $L_1 = \bar{C}\|F\|$, choose L_2 such that $L_1 L_2 \leqslant 1/2$; for any $\varepsilon > 0$ which satisfies $\varepsilon\sqrt{C^2+1} \leqslant L_2$ and $\varepsilon\sqrt{C^2+1} \leqslant \bar{\Delta}$, fix $\Delta > 0$ according to H1-1; then choose $\lambda > 0$ such that $\lambda\sqrt{C^2+1} \leqslant \Delta$.

Theorem 2. If H1-1, H1-2 and H1-3* hold and λ is chosen as above, then system (2) with the control $u = \lambda \mathrm{sat}((F\boldsymbol{x}_1 + v)/\lambda)$ is input-to-state stable with respect to v.

The proof is omitted because it is basically the same as that of Theorem 1. By checking the meanings of input-to-state stabilization, it is not hard to find that the saturated controller in Theorem 2 is not a real input-to-state stabilizer, here we call it a quasi input-to-state stabilizer.

Corollary 2. Consider system (2) with $\boldsymbol{g}(x_2, u) = \boldsymbol{g}(x_2)$. H2-1, H2-2 and H2-3* hold. Let $L_1 = \bar{C}\|F\|$, choose L_2 such that $L_1 L_2 \leqslant 1/2$; for any $\varepsilon > 0$ satisfying $\varepsilon C \leqslant L_2$ and $\varepsilon C \leqslant \bar{\Delta}$, fix $\Delta > 0$ according to H1-1; choose $\lambda > 0$ such that $\lambda C \leqslant \Delta$. Then system (2) with the control $u = \lambda \mathrm{sat}((F\boldsymbol{x}_1 + v)/\lambda)$ is input-to-state stable.

Now we consider the partial dynamics of an inverted pendulum and design two kinds of saturated controllers as corresponding robust stabilizers. This model is described as[4]

$$\dot{z}_2 = \sin x_1, \quad \dot{x}_1 = x_3, \quad \dot{x}_3 = u \tag{8}$$

Design initially $u = -x_1 - x_3 + v$; one has $\dot{z}_2 = \sin x_1, \dot{x}_1 = x_3, \dot{x}_3 = -x_1 - x_3 + v$. For the z_2 subsystem to be controllable explicitly, let $\xi = z_2 + x_1 + x_3$. Then one has

$$\dot{\xi} = v - x_1 + \sin x_1, \quad \dot{x}_1 = x_3, \quad \dot{x}_3 = -x_1 - x_3 + v \tag{9}$$

1) Locally input-to-state stabilization. For the ξ subsystem, design $v = -\lambda \mathrm{sat}(\xi/\lambda) + \bar{v}$; using Lemma 2 one gets that $|\bar{v} - x_1 + \sin x_1|_a \leqslant \lambda \times (1/2) := \lambda\bar{\Delta} \Rightarrow |\xi|_a \leqslant \bar{C}\max\{|\bar{v}|_a, |-x_1 + \sin x_1|_a\}$, $\bar{C} = 4\sqrt{2}$. Here H and P in H2-1 are taken as $H = 1, P = 1/2$.

Consider the (x_1, x_3) subsystem, and define an energy function $V = x_1^2 + x_3^2 + x_1 x_3$. Calculation gives $\dot{V} \leqslant -(1/4)(x_1^2 + x_3^2) + 5|v|^2$, that is, the (x_1, x_3) subsystem is input-to-state stable with respect to v and one has $|x_1|_a \leqslant 2\sqrt{5}|v|_a := C|v|_a$.

Now one uses Corollary 1. Let $L_1 = \bar{C}\|F\| = \bar{C} = 4\sqrt{2}$, and choose $L_2 = 1/8\sqrt{2}$ so that $L_1 L_2 \leqslant 1/2$; given a constant $\varepsilon = 1/51$ which satisfies $\varepsilon C \leqslant L_2$ and $\varepsilon C(1 + \bar{\Delta}/(2\|b\|)) \leqslant \bar{\Delta}/2$, note that $-x_1 + \sin x_1 = -x_1(x_1^2/6 - x_1^4/120 + O(x_1^4))$, and choose $\Delta = 1/\sqrt{8.5}$ to ensure that $|x_1| \leqslant \Delta \Rightarrow |x_1 - \sin x_1| \leqslant (1/51)|x_1| := \varepsilon|x_1|$; finally, choose $\lambda = 1/17$ so that $\lambda C(1 + \bar{\Delta}/(2\|b\|)) \leqslant \Delta$. Then $v = -(1/17)\mathrm{sat}(17\xi) + \bar{v}, |\bar{v}|_a \leqslant \lambda\bar{\Delta}/(2\|b\|) = 1/68$ is a locally input-to-state stabilizer of system (9), consequently one has designed a locally input-to-state stabilizer for system (8): $u = -x_1 - x_3 - (1/17)\mathrm{sat}(17(z_2 + x_1 + x_3)) + \bar{v}, \quad |\bar{v}|_a \leqslant 1/68$.

2) Quasi input-to-state stabilization. Using Lemma 3 and Corollary 2 to set saturation degree, one can get the quasi input-to-state stabilizer of system (8): $u = -x_1 - x_3 - (1/20)\mathrm{sat}(20(z_2 + x_1 + x_3 + \bar{v}))$.

4 Conclusions

In this paper, two kinds of saturated controller have been designed for a feedforward system (2) and the closed loop resulted is locally input-to-state stable and input-to-state stable, respectively. The clue to our design is that non finite escape is guaranteed by the robust stability of subsystems and boundedness of saturated controllers, then with this precondition, small gain theory is used to determine saturation degrees to finish corresponding robust stabilization designs. The difference between our investigation and the previous is that our conditions and results are expressed with notions of locally input-to-state stability and input-to-state stability that have been characterized systematically.

It is not hard to find that using the standard function has simplified the parameter esitimations and led to simple saturated controllers. However, the non differentiability of function often constitutes an obstacle to further stabilization design, so our next work will be to use differentiable saturation function such as to design saturated controllers as locally input-to-state stabilizers of system (3), and consequently suggest differentiable robust stabilizer for system (2). These efforts might lead to some simple stabilizers for practical models such as the inverted pendulum, the ball-and-bean model and so on.

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