A Personified Annealing Algorithm for Circles Packing Problem[1)]

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Abstract Circles packing problem is an NP-hard problem and is difficult to solve. In this paper, a hybrid search strategy for circles packing problem is discussed. A way of generating new configuration is presented by simulating the moving of elastic objects, which can avoid the blindness of simulated annealing search and make iteration process converge fast. Inspired by the life experiences of people, an effective personified strategy to jump out of local minima is given. Based on the simulated annealing idea and personification strategy, an effective personified annealing algorithm for circles packing problem is developed. Numerical experiments on benchmark problem instances show that the proposed algorithm outperforms the best algorithm in the literature.

Key words Packing problem, simulated annealing algorithm, personification

1 Introduction

Packing problems have found many industrial applications. For example, in wood or glass industries, rectangular components have to be cut from large sheets of material. In warehousing context, goods have to be placed on shelves, and a bunch of optical fibers have to be accommodated in a pipe with perimeter as small as possible. In VLSI floor planning, VLSI has to be laid. These applications can be formalized as packing problems[1]. The circles packing problem has been shown to be NP-hard[2,3], it is unlikely that there exists a polynomial time algorithm to solve it optimally. Hence people turn to nature for wisdom, hoping to obtain heuristic algorithm[4] that is not absolutely rigorous but is of high speed, high reliability and high efficiency.

Inspired by biology evolution, physical process, social life *etc.*, many good algorithms, especially, meta-heuristics have been found. So far, genetic algorithm and simulated annealing algorithm (SA)[5,6] have been widely applied to combinatorial optimization. Especially, some heuristic algorithms were presented to solve the circles packing problem and some valuable results have been obtained, for example, the quasi-physical method[7], quasi-physical and quasi-human method (QuasiPQuasiH)[8,9], SA[10], genetic algorithm[1,11], expansion algorithm[12] and hybrid algorithm[13]. Based on the idea of SA and some personification strategies, a personified annealing algorithm is developed.

2 Mathematical formulation of the problem

Given an empty round plate and N disks of different sizes, where N is a positive integer, we shall ask if these disks can be packed into the empty round plate without overlapping one another. This problem is stated more formally as follows.

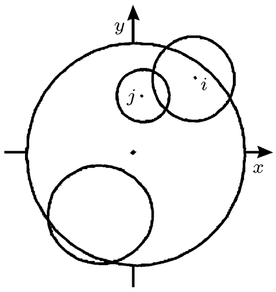

Fig. 1 A simple coordinate system

Taking the central point of the round plate of radius R_0 as the origin of the two-dimensional Cartesian coordinate system and (x_i, y_i) as the coordinates of the center of the i-th disk of R_i, one asks if there exist a set of real numbers $(x_1, y_1, \cdots, x_N, y_N)$, such that $\begin{cases} \sqrt{x_i^2+y_i^2} \leqslant R_0 - R_i \\ \sqrt{(x_i-x_j)^2+(y_i-y_j)^2} \geqslant R_i + R_j \end{cases}$ (see Fig. 1). If there exist such real numbers, then please give them. Here, $i, j = 1, 2, \cdots, N$, and $i \neq j$.

Imagine all the N disks to be smooth elastic solids, and the round plate to be the remaining infinite part after a disk of radius R_0 is hollowed out of the whole two-dimensional infinite elastic solid. Imagine that the N disks are squeezed into the round plate. As each object has the tendency to restore its shape and size, there occurs the interaction of

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extrusion elastic forces between the solids. Driven by such extrusion elastic forces, a series of movements will take place. It is possible that the result of the motion is the establishment of a solution to the problem, with each disk in an appropriate position with respect to another, with no two solids embedded into each other. By these approaches[7~9], the circles packing problem can be transformed into an optimization problem of the potential energy function:

$$U(X) = U(x_1, y_1, \cdots, x_N, y_N) = \sum_{j=1}^{N} U_j \tag{1}$$

where

$$U_j = \sum_{i=0, j\neq i}^{N} u_{ij},\ j = 1, 2, \cdots, N,\ u_{ij} = d_{ij}^2,\ i, j = 0, 1, \cdots, N,\ i \neq j$$

$$d_{0i} = \begin{cases} R_i - R_0 + \sqrt{x_i^2 + y_i^2}, & \text{if } \sqrt{x_i^2 + y_i^2} > R_0 - R_i \\ 0, & \text{else} \end{cases}$$

$$d_{ij} = \begin{cases} R_i + R_j - \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}, & \text{if } \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} < R_i + R_j \\ 0, & \text{else} \end{cases}$$

where U_j denotes the extrusion elastic potential energy possessed by disk j, u_{ij} denotes the extrusion elastic potential energy between two smooth elastic objects, d_{0i} denotes the embedding depth between disk i and the round plate, d_{ij} denotes the embedding depth between disk i and disk j. Obviously, $U \geqslant 0$, $d_{ij} \geqslant 0$; $d_{ij} > 0$ signifies that disk i and disk j embed each other, and $d_{ij} = 0$ signifies that two disks do not embed each other. Find the minimum configuration $X^* = (x_1^*, y_1^*, x_2^*, y_2^*, \cdots, x_N^*, y_N^*)$ of the potential energy function (1). If $U(X^*) = 0$, then X^* is one solution to the circles packing problem, whereas if $U(X^*) > 0$, then the problem has no solution.

3 Personified annealing algorithm

3.1 The idea of SA

SA introduced by Kirkpatrick[14] is based on the analogy between the annealing of solids and the solving of large-scale optimization problems. Solutions in a combinatorial optimization problem are equivalent to the states of a physical system, and the cost of a solution is equivalent to the energy of a state. In the process of search, SA accepts not only better but also worse neighbor solutions with a certain probability. The temperature determines the probability of accepting worse solutions. The probability of accepting a worse solution is large at higher temperatures. As the value of the temperature declines, the probability of accepting worse solutions also decreases as well. This feature implies that SA, in contrast to other local search algorithms, has more opportunities to escape from a local minimum trap. The annealing process first raises the temperature to a sufficiently high level so that the system can be transferred to all possible states. The temperature is then maintained for a certain time at each level and is gradually decreased until the desired state is attained.

Generally, one can use the annealing procedure as follows to obtain a solution for a optimization problem[14~17].

1) Generate an initial configuration X, obtain an initial temperature T_0
2) Generate a new configuration X', let $\Delta U = U(X') - U(X)$
3) If $\Delta U < 0$, go to 4); otherwise, if $\exp(-\Delta U/T_0) \leqslant random(0, 1)$, go to 2)
4) $X = X'$, $U(X) = U(X')$
5) Check whether energy U is in equilibrium at temperature T_0, if it is not equilibrium, go to 2)
6) $T_0 = \alpha T_0$, if the annealing process is over, then stop; otherwise, go to 2).

The above procedure is a formal statement of SA, it must consider actual problem when SA is applied.

3.2 The way of generating a new configuration

Due to the initial configuration is generated randomly, there must exist the extrusion among the disks. According to the idea of quasi-physical, these disks are looked as elastic objects. If there exist extrusion between different objects, they must have elastic forces acted on them, so they will move with the action of elastic forces. As shown in Fig. 2 disk i and the plate embed each other, disk i and disk j embed each other (see Fig. 2). Disk j will move along the direction of from i to j with the reaction of elastic forces. How far on earth it moves depends on its embedding depth d_{ij}. If disk i keeps actionless, then disk j moves its depth d_{ij} until it does not embed with disk i. If disk j keeps actionless, then the plate makes disk i move d_{0i} along the direction of from i to 0 and disk j makes disk i move its depth d_{ij} along the direction of from j to i. If the embedding depth is considered as a vector, then the placement of disk i is the sum of all its embedding depth vectors. The placement size of disk i is the module of the placement of disk i, its direction is the direction of the vector sum of all elastic forces acted on it. The moving distance can be calculated by the projection of all embedding depth vectors in axes x and y. In particular, it can be calculated as follows.

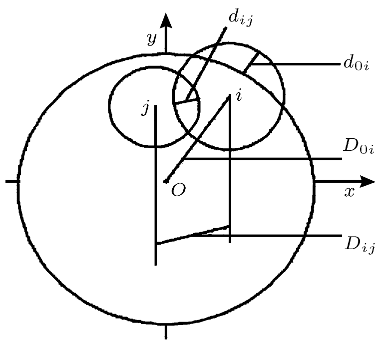


Fig. 2 An example for generation configuration

Firstly, disk j makes disk i move the following distance

$$\frac{x_i - x_j}{\mathrm{d}x_j} = \frac{D_{ij}}{d_{ij}} \Rightarrow \mathrm{d}x_j = \frac{x_i - x_j}{D_{ij}} d_{ij}, \qquad \frac{y_i - y_j}{\mathrm{d}y_j} = \frac{D_{ij}}{d_{ij}} \Rightarrow \mathrm{d}y_j = \frac{y_i - y_j}{D_{ij}} d_{ij}$$

where D_{ij} denotes the distance from the center of disk i to that of disk j, d_{ij} is the same as the previous definition, dx_{ij} is the projection of d_{ij} in the horizontal axis x, dy_{ij} is the projection of d_{ij} in the vertical axis y. It is noted that dx_{ij} and dy_{ij} are not distance, so they may be positive or negative. Similarly,

$$dx_0 = -\frac{x_i}{D_{0i}} d_{0i}, \qquad dy_0 = \frac{-y_i}{D_{0i}} d_{0i}$$

where D_{0i} denotes the distance from the center of the plate to the center of disk i, d_{0i} is the same as the previous definition, $\mathrm{d}x_{i0}$ is the projection of d_{0i} in the horizontal axis, $\mathrm{d}y_{i0}$ is the projection of d_{0i} in the vertical axis. Therefore, the next position of disk i is

$$x_i' = x_i + dx_j + dx_0, \qquad y_i' = y_i + dy_j + dy_0$$

Namely, if the previous configuration is $X = (x_1, y_1, \cdots, x_i, y_i, \cdots, x_N, y_N)$, then the new configuration is $X' = (x_1, y_1, \cdots, x_i', y_i', \cdots, x_N, y_N)$. Although the example of Fig. 2 is simple, it gives the way of calculating a new configuration. For complex cases, it can be calculated similarly. For example, if the number of disks embedding one another is more than two, as long as disk i is selected, its new configuration can be calculated by the above way. As to how to choose disk i, one can begin with small radius of disk and choose the disks in return. It is noted that other disks and the plate do not move when calculating the new position of disk i. With the help of the configuration X, a configuration X' is obtained by simulating the physical moving process of circle i. It is noted that whether disk i can get to a new position depends on the probably mechanism of SA. According to above statements, new configuration is generated consciously, the range of search is significantly reduced. Therefore, this way enlightened by physical process can avoid blind search to some extent, and it allows the iterative process to converge fast and enhances the efficiency of computing.

3.3 Personification strategy

Since SA costs too much time in order to obtain a high quality solution, the running time beyond endurance makes SA infeasible as the problem scale increases. The performance of SA is significantly impacted by the choice of parameters, so these parameters should be selected rationally by referring to [6,10]. In this paper, $T_0 = \frac{N}{20}$, $\alpha = 0.92$ the initial temperature can be adjusted with the change of problem scale.

For SA, when the temperature T_0 tends to zero at the end of the process, the probability of accepting worse neighboring configurations is approximately zero. In that case, SA loses its feature to accept worse configurations, thereby becoming identical to other local search algorithms. For example, from Fig. 3, two small disks crush together, when the temperature decreases to some extent, it is very difficult to jump out of local minimum. In addition, the way of generating X' may often lead SA to getting stuck in a local optimum during the course of execution. Under this circumstance, the promising approach is to put forward some good heuristic strategies for jumping out of the local minimum trap by taking the calculating point out of the local minimum and place it in a position with better prospects. Then a new SA process can be carried out.

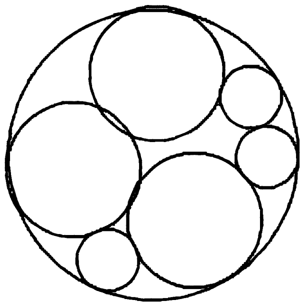


Fig. 3 An example of the trap

This strategy of "jumping out of the trap" can be obtained by observing and learning from the social and nature phenomena and is therefore called personification strategy.

In daily life, the main conflict is often solved with priority in certain problems. When some objects are packed into a trunk, the large ones are packed first, and then small ones. Otherwise, it is possible that the large objects cannot be packed into the trunk, because small ones have occupied the needed space of the large objects. Under this circumstance, the positions of small ones have to be adjusted to find a better layout. Namely, one small object is moved away from among small objects huddled together. As in Fig. 3, disks which embed with others are considered to have conflicts with others. Disks with the maximum energy function value have maximum conflict, so they are given priority, for example, the two big disks and the two small disks. Intuitively, if one small disk is selected from the two small ones, and put into the left-top position of the plate, then the conflict can be solved quickly. Maybe the chance that solves conflict successfully does not always occur, however, choosing a small disk from the two small disks has more chance to jump out of local minimum than choosing a big disk from the two big disks. All of these inspire us to obtain the following heuristic strategies:

When getting stuck, the disk with maximum relative potential energy ($RU(i)$) can be picked out and randomly placed in the plate, where $RU(i) = U_i/R_i$. According to this formulation, for disks with same maximum potential energy, the smaller the radius of the disk, the bigger its relative potential energy is. Thus the smaller disks with the same maximum potential energy have more chances to jump out of a local minimum.

Combining the idea of SA and personification strategy, a personified annealing algorithm is developed:

1) Running SA(), if $U < 10^{-6}$, then go to 3)

2) Under current configuration, the disk with maximum relative potential energy $RU(i)$ is picked out and randomly placed in the plate, go to 1)

3) Stop.

It is noted that the temperature is very low when the search process gets stuck, the strategy of enhancing temperature[14] must be adopted in order to jump out of a local minimum because when the temperature is high, the search process has more chances to jump out of a local minimum.

4 Computational results

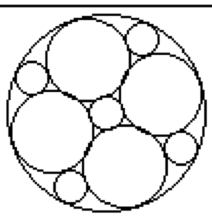
In order to verify the performance of the personified annealing (PA), PA is compared with QuasiPQuasiH. Both algorithms have been implemented with C language on a Pentium 4 to perform large amounts of calculation with nine problem instances taken from [8, 9, 15]. The nine problem instances (P) below, which include packing equal disks and unequal disks, hard and easy problem instances, are typical representatives. For each instance, 5 times of trial calculation have been executed with each algorithm because the initial solution of problem instance is generated randomly. In addition, each problem instance has the optimization solution objectively, so the running time is recorded when the optimization solution is found.

The computation results of both algorithms are shown in Table 1, and the geometric morphologies of the solutions are shown in Table 2 (the geometric morphologies of the solutions may be different for different calculation). From the results shown in Table 1, the speed of PA is about eight times that of QuasiPQuasiH except for problem instances 1,2,6. For rather difficult problem instances, *e.g.* instance 4, such an increase in speed is especially notable, so PA is feasible and high efficient.

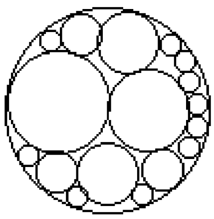
Table 1 Comparisons of PA and QuasiPQuasiH

P	N	R_0	$R_1, \cdots, R_N$	PA	QuasiPQuasiH
				The running time and $\bar{t}(s)$	The running time and $\bar{t}(s)$
1	6	100	$R_1 = R_2 = R_3 = 22.4$ $R_4 = R_5 = R_6 = 46.4$	$0.00, 0.00, 0.00$ $0.00, 0.00, \bar{t} = 0.00$	$0.00, 0.00, 0.00$ $0.00, 0.00, \bar{t}$=0.00
2	9	241.43	$R_1 = \cdots = R_3 = R_4 = 100$ $R_5 = R_6 = \cdots, R_9 = 41.415$	$0.00, 0.05, 0.00$ $0.05, 0.00, \bar{t}$=0.02	$0.11, 0.11, 0.00$ $0.00, 0.05, \bar{t}$=0.05
3	17	50	$R_1 = 25, R_2 = 20, R_3 = R_4 = 15$ $R_5 = R_6 = R_7 = 10, R_8 = \cdots = R_{17} = 5$	$0.33, 0.16, 0.22$ $0.16, 0.22, \bar{t}$=0.22	$4.56, 4.18, 2.97$ $0.27, 0.22, \bar{t}$=2.44
4	17	241.43	$R_1 = \cdots = R_4 = 100, R_5 = \cdots = R_9$ $= 41.415, R_{10} = \cdots = R_{17} = 20$	$0.55, 0.55, 1.43$ $0.33, 1.15, \bar{t} = 0.80$	$119.29, 169.67, 78.13$ $102.8, 0.99, \bar{t} = 110.18$
5	50	159.32	$R_1 = R_2 = \cdots = R_{50} = 20$	$7.24, 51.9, 5.46$ $57.21, 34.73, \bar{t} = 31.31$	$3.13, 93.46, 10.05$ $9.95, 120.99, \bar{t} = 47.52$
6	12	215.47	$R_1 = \cdots = R_6 = 23.72, R_7 \cdots R_9$ $= 48.26, R_{10} = R_{11} = R_{12} = 100$	$0.00, 0.05, 0.00$ $0.00, 0.01, \bar{t} = 0.012$	$1.70, 0.99, 0.93$ $1.32, 2.14, \bar{t} = 1.42$
7	15	39.37	$R_1 = 1, R_{i+1} = R_i + 1, i = 1, 2, \cdots, 14$	$10.42, 3.53, 7.62$ $31.38, 1.57, \bar{t} = 10.91$	$49.95, 47.91, 6.59$ $10.44, 9.29, \bar{t} = 24.84$
8	37	135.176	$R_1 = \cdots = R_{37} = 20$	$0.00, 0.05, 0.05$ $0.05, 0.05, \bar{t} = 0.04$	$0.27, 0.27, 0.11$ $0.11, 0.11, \bar{t} = 0.18$
9	61	173.226	$R_1 = \cdots = R_{61} = 20$	$0.22, 0.27, 0.27$ $0.22, 0.60, \bar{t} = 0.32$	$9.12, 5.99, 6.26$ $3.13, 5.77, \bar{t} = 6.05$

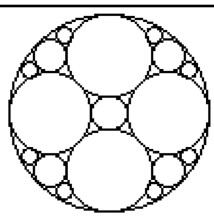
Table 2 The geometric morphologies of the solutions



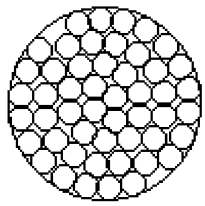
P_2



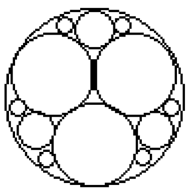
P_3



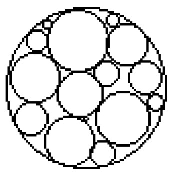
P_4



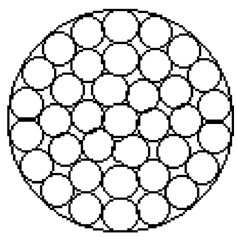
P_5



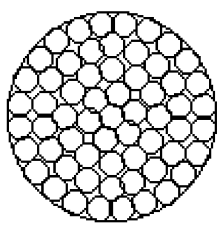
P_6



P_7



P_8



P_9

In this paper, the comparisons of PA, quasi-physical method and methods in [3,8] have been omitted because the running time of the later is longer than that of QuasiPQuasiH[8]. For the case of equal circles, some results can be found in [6,9], PA also found the optimized geometric morphologies of the solutions. Comparing PA with methods in [1,11] is not appropriate because their objectives are different. The algorithm in [10] was only for study on feasibility and efficiency. What is more, the way of generating a new configuration was different from PA. The paper tried their methods, but the results were bad. As to the expansion algorithm[12], its idea is more novel, but its running time is almost the same as QuasiPQuasiH, so PA is faster than it.

5 Conclusions

The numerical experiments have shown that PA outperforms one of the best algorithms, so PA is feasible and high efficient. In addition, PA is easily extendable to packing circles in other bounded space or other NP-hard problems. PA may be of practical value to the rational layout of the round objects in the engineering fields. The future work is to find highly efficient algorithm for other NP problem of even greater practical significance.

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