# Anti-noise Capability of Bidirectional Associative Memory[1)]

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**Abstract** This paper analyzes noise sensitivity of bidirectional association memory (BAM) and shows that the anti-noise capability of BAM relates not only to the minimum absolute value of net inputs(MAV), as some researchers found, but also to the variance of weights associated with synapse connections. In fact, it is determined by the quotient of these two factors. On this base, a novel learning algorithm—small variance leaning for BAM(SVBAM) is proposed, which is to decrease the variance of the weights of synapse matrix. Simulation experiments show that the algorithm can decrease the variance of weights efficiently, therefore, noise immunity of BAM is improved. At the same time, perfect recall of all training pattern pairs still can be guaranteed by the algorithm.

**Key words** Associative memory, neural network, noise sensitivity, digital pattern

## 1 Introduction

Association is one of the dramatic characters of human brain. Several associative memory models[1~7] have been proposed to simulate this character. Among these models, bidirectional associative memory (BAM) has been attracting much attention for its simplicity and similarity to human brain in the aspects of structure and working style. Since BAM was proposed by Kosko[4] in 1988, a great deal of related work has appeared, such as in terms of learning algorithm, storage, and stability *etc*. Many of them have obtained interesting results[8~11].

The Pseudo-relaxation method brings BAM with larger pattern storage and guarantees perfect recall of training pattern pairs, which is widely used in BAM, MAM, CBAM, CMAM *etc*. Motonobu *et al.*[11] concluded that quick learning algorithm for BAM (QLBAM)[10] is robust to noisy inputs and the minimum absolute value of net inputs (MAV) indexes a noise margin. However, it is demonstrated that even if the noise level is very low (for example, only one bit in input pattern flips), the BAM still can not guarantee 100% perfect recall of all training patterns. From this fact, the sensitivity of BAM to noise is analyzed. Unstable neurons (UN) and sensitive neurons (SN) are defined and found in a BAM. And it is concluded that higher noise immunity can be obtained by decreasing the number of sensitive neurons (NSN). And NSN is proved to be related not only to the minimum absolute value of net inputs (MAV) defined in [11], but also to the variance of weights associated with synapses. It decreases with the quotient of MAV divided by the variance. Following the conclusion, a novel learning algorithm (small variance learning algorithm for BAM, SVBAM) is proposed. Several experiments are conducted to confirm the performance of SVBAM.

## 2 Bidirectional associative memory

BAM is a full-connected full-feedback double-layer neural network. The states of neurons in the two layers are computed as the following.

$$y_m = \text{sign}\left(\sum_{n=1}^{N} W_{nm}x_n - \theta_{ym}\right), \text{ for } m = 1, \cdots, M \tag{1}$$

$$x_n = \text{sign}\left(\sum_{m=1}^{M} W_{nm}x_n - \theta_{ym}\right), \text{ for } n = 1, \cdots, N \tag{2}$$

where $M$ and $N$ are numbers of neurons in $X$-layer and $Y$-layer, respectively, $W_{nm}$ is the weight associated with the synapse between the $n$th neuron in $X$-layer and the $m$th neuron in $Y$-layer, $\theta_{xn}$ and $y_{ym}$ are the biases of the $n$th neuron in $X$-layer and the $m$th neuron in $Y$-layer, respectively.

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$\{(\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)})\}, k = 1, \cdots, K$ are training pattern pairs, $\boldsymbol{X}^{(k)} = [x_1^{(k)}, \cdots, x_N^{(k)}]$, $\boldsymbol{Y}^{(k)} = [y_1^{(k)}, \cdots, y_M^{(k)}]$. When associating, at the first iteration, pattern $\boldsymbol{X}^{(k)}$ is given to $X$-layer of BAM as initial state when recall is from $X$ pattern to $Y$ pattern. Else if the direction is inverse, $\boldsymbol{Y}^{(k)}$ is given to $Y$-layer.

### 2.1 Hebbian learning algorithm

In the original model, Kosko[4] used the correlation matrix of the vectors in $\boldsymbol{V} = [(\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)})], k = 1, \cdots, K$ to store the training pattern pairs

$$\boldsymbol{W} = \sum_{k=1}^{K} \boldsymbol{X}^{(k)^{\mathrm{T}}} \boldsymbol{Y}^{(k)} \tag{3}$$

After BAM learns all training pattern pairs, when $\boldsymbol{X}^{(1)}$ is given to $X$-layer, for example, the output in $Y$-layer appears as follows.

$$\boldsymbol{Y} = \mathrm{sign}(\boldsymbol{X}^{(1)} \sum_{k=1}^{K} \boldsymbol{X}^{(k)^{\mathrm{T}}} \boldsymbol{Y}^{(k)}) = \mathrm{sign}(\boldsymbol{X}^{(1)} \boldsymbol{X}^{(1)^{\mathrm{T}}} \boldsymbol{Y}^{(1)} + \boldsymbol{X}^{(1)} (\boldsymbol{X}^{(2)^{\mathrm{T}}} \boldsymbol{Y}^{(2)} + \cdots + \boldsymbol{X}^{(K)^{\mathrm{T}}} \boldsymbol{Y}^{(K)}))$$

If $\boldsymbol{X}^{(1)}$ is not orthogonal to $\boldsymbol{X}^{(2)}, \cdots, \boldsymbol{X}^{(K)}$ the output pattern will be superimposed with $\boldsymbol{Y}^{(1)}, \cdots, \boldsymbol{Y}^{(k)}$ so BAM can not guarantee perfect recall of training pattern pairs unless they are mutually orthogonal.

### 2.2 Pseudo-relaxation learning algorithm for BAM (PRLAB)

In fact, the vectors in $\boldsymbol{V}$ will be perfectly recalled if the following system of linear inequalities holds for all $k = 1, \cdots, K$.

$$\left( \sum_{n=1}^{N} W_{nm} x_n^{(k)} - \theta_{ym} \right) y_m^{(k)} > 0, \text{ for } m = 1, \cdots, M \tag{4}$$

$$\left( \sum_{m=1}^{M} W_{nm} y_m^{(k)} - \theta_{xn} \right) x_n^{(k)} > 0, \text{ for } n = 1, \cdots, N \tag{5}$$

Heekuck *et al.*[9] analyzed the BAM learning process, and concluded that the goal of learning in BAM is to find a feasible solution to satisfy (4) and (5). They used pseudo-relaxation method to solve these inequalities, and obtained the following iterative learning algorithm for BAM (PRLAB).

For $X$-layer:

$$\Delta W_{nm} = -\frac{\lambda}{1+M}[S_{xn}^{(k)} - \xi x_n^{(k)}] y_m^{(k)}, \text{ if } S_{xn}^{(k)} x_n^{(k)} \leqslant 0 \tag{6}$$

$$\Delta \theta_{xn} = +\frac{\lambda}{1+M}[S_{xn}^{(k)} - \xi x_n^{(k)}], \text{ if } S_{xn}^{(k)} x_n^{(k)} \leqslant 0 \tag{7}$$

$$W_{nm} = W_{nm} + \Delta W_{nm} \tag{8}$$

$$\theta_{xn} = \theta_{xn} + \Delta \theta_{xn} \tag{9}$$

For $Y$-layer it is similar.

In the above, $S_{xn}^{(k)} = \sum_{m=1}^{M} W_{nm} y_m^{(k)} - \theta_{xn}, S_{ym}^{(k)} = \sum_{n=1}^{N} W_{nm} x_n^{(k)} - \theta_{ym}$, $\lambda$ is a relaxation factor, which is a constant between 0 and 2, $\xi$ is a positive normalizing constant. PRLAB uses randomly generated weights as the initial system. Heekuck reported that PRLAB can always obtain a feasible solution if there exist weights satisfying (4) and (5). But this method does not include a stopping criterion, which means if it does not exist a feasible solution, PRLAB may come to a dead loop.

### 2.3 Quick learning algorithm for BAM (QLBAM)

Hattori *et al.*[10] used Hebbian correlation matrix (3) as the initial value of weights, and then employed PRLAB to get feasible solution to (4) and (5). This method includes two steps: firstly, using Hebbian correlation matrix as the initial weights, and then, in the second step, employing PRLAB to obtain feasible solution of (4) and (5). This method is reported to converge faster than PRLAB.

## 3 Anti-noise capability of BAM

Since BAM is a feedback neural network, when a noisy pattern is given to one of its layer, it will go through the two layers recurrently. Each of its two layers has some training patterns associated with stable state points which attract the noisy pattern to their attracting regions continuously. This feedback and attracting mechanism provide BAM with a capability of recovering stable pattern from noise. Motonobu *et al.*[11] reported that BAM obtained by QLBAM method is robust for noisy inputs and the minimum absolute value of net inputs (MAV) indexes a noise margin.

However, there is a puzzling phenomenon: although BAM is reported robust to noisy inputs, it can not always guarantee perfect recall of noisy patterns even the noise level is very low (for example, only a single bit flips in the input pattern). Failure association always occurs in experiments. The reason can be found in the followings.

**Definition 1.** Sensitive neurons

Denote $\boldsymbol{X}^{n(k)}$ as the noisy version of $\boldsymbol{X}^{(k)}$ with the $p$th ($p \in \{1, \cdots, N\}$) bit flipped. That is,

$$\boldsymbol{X}^{n(k)} = \boldsymbol{X}^{(k)}\boldsymbol{E}_p \tag{10}$$

$$\boldsymbol{E}_p = \begin{pmatrix} \boldsymbol{I}_{p-1} & & 0 \\ & -1 & \\ 0 & & \boldsymbol{I}_{N-p-1} \end{pmatrix} \tag{11}$$

Give $\boldsymbol{X}^{n(k)}$ to $X$-layer in BAM. The $p$th neuron is called the sensitive neuron in $X$-layer corresponding to pattern $\boldsymbol{X}^{(k)}$, if the states of $Y$-layer neurons are not consistent with $\boldsymbol{Y}^{(k)}$ after the first associating iteration. In this definition $(\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)})$ represents a training pattern pair.

**Definition 2.** Unstable neurons

Give $\boldsymbol{X}^{n(k)}$ to $X$-layer in BAM. The $p$th neuron is called the unstable neuron in $X$-layer corresponding to pattern $\boldsymbol{X}^{(k)}$, if the states of $Y$-layer neurons are not consistent with $\boldsymbol{Y}^{(k)}$ after enough associating iterations. In this definition $(\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)})$, $\boldsymbol{E}_p$ are the same to ones in Definition 1.

Both of these two kinds of neurons are easy to be observed in BAM. In Fig. 1, (a) shows the 1st neuron in $X$-layer is not sensitive to pattern "A", (b) shows the 7th neuron is sensitive since the neurons′ states in $Y$-layer are not consistent with $\boldsymbol{Y}^{(k)}$ (pattern "a") after the first associating iteration, and (c) shows the 15th neuron is unstable since the neurons′ states in $Y$-layer are not consistent to with $\boldsymbol{Y}^{(k)}$ (pattern "a") after BAM converges to a stable state. This experiment is based on QLBAM with $\xi = 0.4$ and $\lambda = 1.9$. As shown in Fig. 1, unstable neurons always invoke more and more neurons

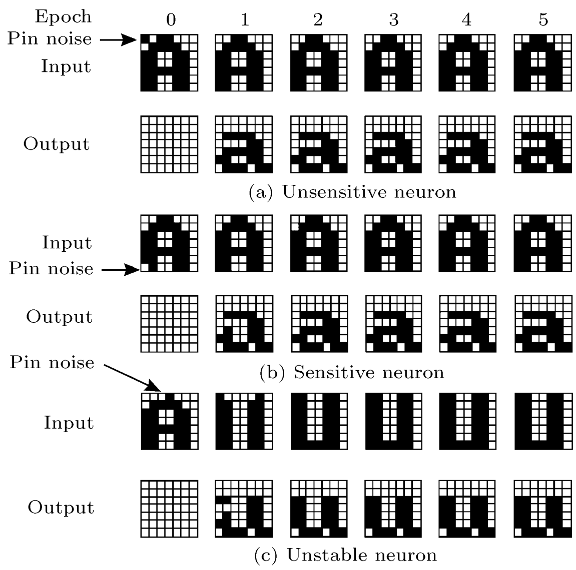


Fig. 1 The behavior of unsensitive neurons, sensitive neurons, and unstable neurons

error-flipping until these neurons attain to a stable loop which leads to an error recall. These unstable neurons explain the reason why failure association may occur even at very low noise level.

In this paper, anti-noise ability of BAM is found to be related to the number of sensitive neurons (NSN) and NSN relates to the weights statistic feature. In order to study this relation, the following hypothesis is proposed.

**Hypothesis 1.** The weights of BAM satisfy zero-mean normal distribution. That is,

$$H_0: \; W_{nm} \sim N(0, \sigma^2) \tag{12}$$

To study whether this hypothesis is over restrict, the following experiment is conducted. In the experiment, BAM has 21×21 neurons in each layer, the vector of 20 pattern pairs $\boldsymbol{V} = [(\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)})], k = 1, \cdots, 20$ are randomly generated. QLBAM is employed as the learning algorithm, and 20 weights matrices are gotten through 20 times of learning based on various $\xi$ values between 0.1 and 500. Lilliefors normal distribution test and t-test (for 0 mean value) are then applied to these 20 matrices. The test results (figures are omitted) illustrate that this assumption is feasible in most cases.

**Theorem 1.** The probability of a neuron being a sensitive neuron is not greater than $\frac{1}{2}P(\chi > \chi(\frac{a_{\min}}{2\sigma}))$, where $a_{\min}$ is the minimum absolute value of net inputs (MAV) defined in [11], $\sigma$ is the variance of weights in BAM.

**Proof.** The $p$-th neuron is sensitive to $\boldsymbol{X}^{(k)}$ means that there exists at least one neuron in $Y$-layer which satisfies the following inequality.

$$\left(\sum_{n=1}^{N} W_{nq} x_n^{n(k)} - \theta_{yq}\right) y_q^{(k)} \leqslant 0, \quad q \in \{1, \cdots, M\} \tag{13}$$

After QLBAM stops, connection weights in BAM must satisfy the following inequalities.

$$\left(\sum_{n=1}^{N} W_{nm} x_n^{(k)} - \theta_{ym}\right) y_m^{(k)} > 0, \text{ for } m = 1, \cdots, M, \; k = 1, \cdots, K \tag{14}$$

These inequalities can be written in equal form as follows.

$$\left(\sum_{n=1}^{N} W_{nm} x_n^{(k)} - \theta_{ym}\right) y_m^{(k)} = a_m \tag{15}$$

where $a_m$ is the absolute net input defined in [11]. Each $a_m$ is positive.

Firstly, assume $y_q^{(k)} > 0$ (this assumption also means $y_q^{(k)} = 1$, since each bit of binary patterns is $-1$ or 1). Then

$$\sum_{n=1}^{N} W_{nq} x_n^{(k)} - \theta_{yq} = a_q \tag{16}$$

Denote $a_q^n = \sum_{n=1}^{N} W_{nq} x_n^{n(k)} - \theta_{yq}$. It is clear that $a_q^n < 0$ means (13) holds. Since

$$a_q^n = \sum_{n=1}^{N} W_{nq} x_n^{n(k)} - \theta_{yq} = \left(\sum_{n \neq p} W_{nq} x_n^{n(k)} - \theta_{yq}\right) + W_{pq} x_p^{n(k)}$$

and $x_n^{n(k)} = x_n^{(k)} (n \neq p)$, $x_p^{n(k)} = -x_p^{(k)}$.

$$a_q^n = \left(\sum_{n \neq p} W_{nq} x_n^{n(k)} - \theta_{yq}\right) - W_{pq} x_p^{(k)} = \left(\sum_{n=1}^{N} W_{nq} x_n^{n(k)} - \theta_{yq}\right) - 2W_{pq} x_p^{(k)} = a_q - 2W_{pq} x_p^{(k)}$$

So, $P\{a_q^n \leqslant 0\} = P\{a_q - 2W_{pq} x_p^{(k)} \leqslant 0\} = P\{2W_{pq} x_p^{(k)} \geqslant a_q\} = P\{2W_{pq} x_p^{(k)} > 0\} P\{|2W_{pq} x_p^{(k)}| \geqslant a_q\}$.

Because $|x_p^{(k)}| = 1$, the above equation can be written as the following version

$$P\{a_q^n \leqslant 0\} = P\{W_{pq} x_p^{(k)} > 0\} P\left\{\left|\frac{W_{pq}}{\sigma}\right| \geqslant \frac{a_q}{2\sigma}\right\}$$

where $\sigma$ is the variance of $W$.

From the hypothesis of $W_{nm} \sim N(0, \sigma^2)$, the following conclusion can be drawn.

$$\frac{W_{pq}}{\sigma} \sim N(0,1) \tag{17}$$

so $\left|\frac{W_{pq}}{\sigma}\right|$ satisfies $\chi$-distribution and

$$P\{a_q^n \leqslant 0\} = \frac{1}{2} P\{\chi \geqslant \chi\left(\frac{a_q}{2\sigma}\right)\} \tag{18}$$

Denote $a_{\min}$ as the minimum of $a_m$. Then

$$P\{a_q^n \leqslant 0\} \leqslant \frac{1}{2} P\{\chi \geqslant \chi\left(\frac{a_{\min}}{2\sigma}\right)\} \tag{19}$$

In the case of $y_q^{(k)} < 0$ ($y_q^{(k)} = -1$), the proof is similar.

Now sensitivity to noise in BAM is proved to be related to the variance of the weights besides the minimum absolute value of net inputs (MAV).

Since $P\{\chi \geqslant \chi\left(\frac{a_{\min}}{2\sigma}\right)\}$ is a monotonically decreasing function of $\frac{a_{\min}}{\sigma}$, we can expect to decrease the probability of sensitive neuron through increasing $\frac{a_{\min}}{\sigma}$. □

## 4 Small variance learning algorithm for BAM (SVBAM)

From the analysis in the last section, it can be expected to decrease the appearing probability of sensitive neurons and the failure association rate subsequently through decreasing the variance of BAM weights. In this paper, we propose a novel method (small variance learning algorithm for BAM, SVBAM) based on QLBAM, which is expected to decrease the variance of BAM weights and improve the anti-noise capability in BAM. The new algorithm also includes two steps.

**Step 1.** This step is the same as QLBAM, in which Hebbian learning algorithm is employed to obtain the initial weight matrix.

**Step 2.** Multiply $\Delta W_{nm}$ obtained in PRLAB by an adjusting vector, as shown in the following equality

$$\Delta W_{nm} = \Delta W_{nm}^P P(W_{nm}) \tag{20}$$

where $\Delta W_{nm}^P$ is obtained by PRLAB in (7) and (11).

$$P(W_{nm}) = 1 - \text{sign}(|W_{nm} + \Delta W_{nm}^P| - |W_{nm}|) h(|W_{nm}|) \tag{21}$$

In (21), $h(|W_{nm}|)$ is a positive monotonically increasing function defined in $[0, \max(|W_{nm}|)]$. The adjust factor defined as (21) makes it more difficult for the weights to move far away from the center ($u = 0$), but easier to move near to the mean center. So it leads to a smaller variance of weights than QLBAM. It should be noted that $h(|W_{nm}|)$ should keep the normal distribution character of weight matrix, and the mean of weights should be zero still. The function of $P$ and the performance of this novel algorithm are demonstrated in the following section.

## 5 Results of simulations

In the above section, a novel learning algorithm is presented based on the analysis in Section 3. Now, in this section, several experiments are conducted to study SVBAM performance, and compare it with QLBAM. There are great sorts of functions that satisfy the requirements in Section 4. In this paper, $h(|W_{nm}|)$ is a linear function of $|W_{nm}|$ as shown below.

$$h(|W_{nm}|) = 1 - \cos\left(\frac{\gamma |W_{nm}|}{\max(|W_{nm}|)}\right) \tag{22}$$

The first experiment aims to confirm the conclusion about NSN and anti-noise ability of BAM. From Fig. 2, it is very clear that the more unstable neurons, the lower perfect recall rate (the higher failure association rate). This means one can improve the anti-noise capability in BAM by decreasing the number of unstable neurons.

The second experiment is to demonstrate the anti-noise capability of SVBAM versus QLBAM ($\xi = 0.4, \lambda = 1.9$). Fig. 3 shows the result, through which we can draw a conclusion that SVBAM has a better anti-noise performance than QLBAM. Better anti-noise performance means fewer unstable neurons in SVBAM.

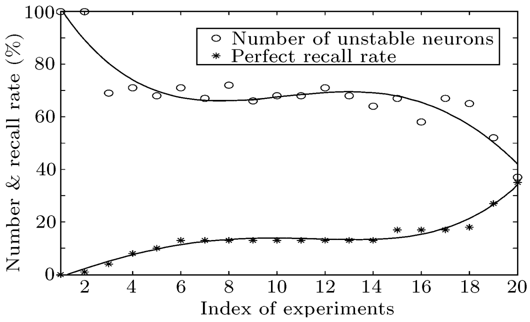


Fig. 2 Perfect recall rate decreases with the number of unstable neurons increasing

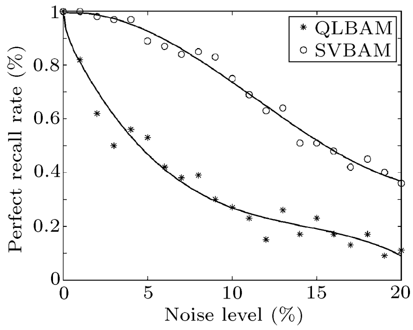


Fig. 3 Perfect recall rate of QLBAM and SVBAM

## 6 Conclusion

In this paper, we analyze the reason of failure association in BAM, and define the unstable neurons and sensitive neurons which are closely related to anti-noise capacity of BAM. Based on the proof of the number of sensitive neurons (NSN) satisfying $\chi$ distribution, this paper draws the conclusion that sensitivity to noise in BAM relates to the variance of weights besides the minimum absolute value of net inputs(MAV). Then a novel learning algorithm (SVBAM) is proposed which can obtain a small variance weight matrix, and can improve anti-noise capacity in BAM subsequently. Several experiments are conducted to confirm the conclusion of this paper. The novel algorithm has better anti-noise capability than QLBAM.

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