Backstepping-based Decentralized PID Controller Design for MIMO Processes[1)]

ZHANG Yan LI Shao-Yuan
(*Institute of Automation, Shanghai Jiaotong University, Shanghai* 200030)
(E-mail: syli@sjtu.edu.cn)

Abstract A novel decentralized PID controller design procedure based on backstepping principles is presented to operate multiple-input multiple-output (MIMO) dynamic processes. The first key feature of the design procedure is that a whole MIMO control system is decomposed into multiple control loops, therefore the sub-controllers can be efficiently flexibly designed in parallel prototype. The second key feature is that the decentralized controller has equivalency to those designed by backstepping approach. As a complementary support to the design procedure, the sufficient condition of the whole closed-loop system stability is analyzed *via* the small gain theorem and it can be proven that the process tracking performance is improved. The simulation results of the Shell benchmark control problem are provided to verify the effectiveness and practicality of the proposed decentralized PID control.

Key words Backstepping, decentralized PID control, small gain theorem, Shell benchmark control problem

1 Introduction

Decentralized PID control[1,2] for MIMO processes is popular in the chemical and process industries because of its relatively simple structure and fewer tuning parameters. In addition, controller design focuses on single control loop and has little affect on other control loops, which is convenient to maintenance and amendment. However, the tuning procedure often involves trial and error experiments and requires an experienced operator, which is time consuming. With respect to the structural feature of decentralized control, there are some rules in PID controller′s parameters tuning for the adjustment of controller and the performance analysis of the control system. [3] and [2] presented effective decentralized PID control algorithms for interacting two-input two-output (TITO) and MIMO processes, respectively, however, they all lack the stability analysis.

Backstepping[4] is a recursive and systematic design scheme first presented by Kokotovic in 1991. Its designing idea is to decompose a complex system into multiple small-scale subsystems, then to design recursively control Lyapunov function (CLF)[5] and virtual control variable for each subsystem, and finally to obtain the original control law, realizing the global regulation and tracking for a class of feedback linearizable nonlinear systems[6]. Some researches have been focused on the application of backstepping method to decentralized control. [7] proposed an adaptive backstepping-based scheme for designing a totally decentralized adaptive stabilizers for a class of large-scale systems with guaranteed transient performance. Unfortunately, the industrial PID control application examples of backstepping method are very few[8∼9]. [8] developed a backstepping-based adaptive PID control scheme that the robustness and transient performance are better than those of the conventional PID control. Therefore, combining backstepping with decentralized PID control will be of great theoretical and applicable value.

Decentralized PID controller design scheme and procedure are proposed for MIMO processes on the basis of backstepping approach. MIMO processes are decomposed into multiple loops and controllers are designed in parallel. First, CLF and virtual control variable based on backstepping are derived recursively for each loop and a multivariable controller can be obtained. Then, a decentralized PID controller can be derived *via* selection of the auxiliary control variable. By introducing small gain theorem the sufficient condition of the whole closed-loop system stability is acquired and the tracking performance is improved. The simulation study of the Shell benchmark control problem illustrates that the decentralized PID control scheme is effective for MIMO processes.

1) Supported by National Natural Science Foundation of P. R. China (60474051), Program for New Century Excellent Talents in University, and the Specialized Research Fund for the Doctoral Program of Higher Education of P. R. China (20020248028)
Received January 26, 2004; in revised form June 8, 2005

2 Problem statement

Consider an $N \times N$ process controlled by decentralized controller, the block diagram of general decentralized control system is shown in Fig. 1, where $\boldsymbol{y}(t) = [y_1 \cdots y_N]^{\mathrm{T}} \in R^N$ is the vector of process outputs, $\boldsymbol{u}(t) = [u_1 \cdots u_N]^{\mathrm{T}} \in R^N$ is the vector of process inputs, $\boldsymbol{y}_r(t) = [y_{r1} \cdots y_{rN}]^{\mathrm{T}} \in R^N$ is the vector of loop reference signals, and $\boldsymbol{e}(t) = [e_1 \cdots e_N]^{\mathrm{T}} \in R^N$ is the vector of loop errors.

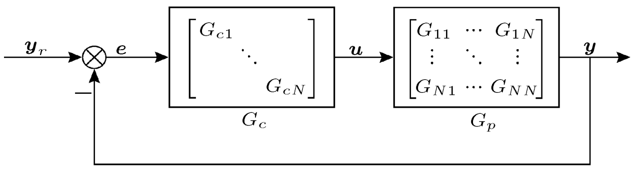

Fig. 1 MIMO decentralized control system

The control objective of MIMO decentralized control system is to decompose the MIMO process into multiple control loops according to the control inputs and design multi-loop PID controllers, making the process outputs $y_i(t)(i = 1, \cdots N)$ track the loop reference signals $y_{ri}(t)$ respectively and making the whole control system asymptotically stable. In decentralized control the controller G_c is a diagonal matrix given by $G_c = \mathrm{diag}\{G_{c1}, \cdots, G_{cN}\}$, where G_{ci} is considered as a PID controller. The process transfer matrix $G_p = [G_{ij}]_{N\times N}$, where G_{ij} is a first-order or second-order transfer function model in general.

3 Backstepping-based decentralized PID control

3.1 Configuration analysis of control system

Taking the 3×3 system as an example, the simple block diagram of a 3×3 decentralized control system is shown in Fig. 2, where $u_i(i = 1, 2, 3)$ is the control input (*i.e.*, the controller output) for the ith sub-process, y_{ii} is the output response to the input u_i, the control effect on the ith sub-process from other control inputs is denoted as y_{ci}, and $Y_{ci}(s) = \sum\limits_{j=1, j\neq i}^{3} G_{ij}(s)U_j(s)$, where $U_i(s)$ and $Y_{ci}(s)$ are the Laplace transforms of u_i and y_{ci}, y_i is the output response of the ith control loop, and $Y_i(s) = Y_{ii}(s) + Y_{ci}(s)$, where $Y_i(s)$ and $Y_{ii}(s)$ are the Laplace transforms of y_i and y_{ii}. The controllers for all loops are designed in parallel and PID controller is derived based on backstepping approach for each loop.

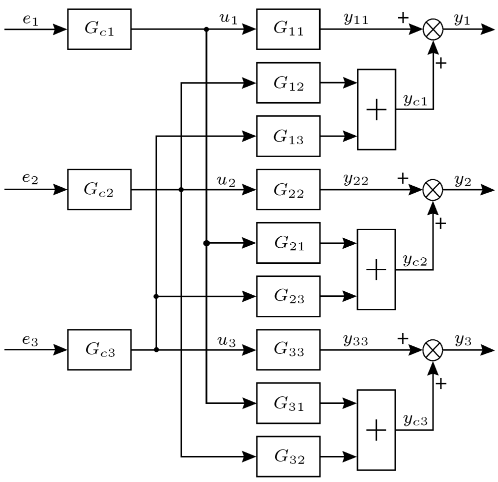

Fig. 2 3×3 decentralized control system

3.2 Backstepping-based multivariable control

For the $i(i = 1, \cdots, N)$th control loop $Y_{ri}(s)$ and $E_i(s)$ are the Laplace transforms of y_{ri} and e_i, the transfer function of the ith sub-process is

$$G_{ii}(s)=\frac{Y_{ii}(s)}{U_i(s)}\triangleq\frac{B_i(s)}{A_i(s)}=\frac{s^{m_i}+b_{i,m_i-1}s^{m_i-1}+\cdots+b_{i0}}{s^{n_i}+a_{i,n_i-1}s^{n_i-1}+\cdots+a_{i0}},\quad m_i<n_i \tag{1}$$

Decentralized PID control for MIMO process is presented on the basis of the work of Benaskeur and Desbiens[8,9]. An integral action is inserted into the control input, $V_i(s)=sB_i(s)U_i(s)$, v_i is seen as a new control input and $V_i(s)$ is the Laplace transform of v_i, then the transfer function of the ith sub-process can be written as

$$G^*_{ii}(s)=\frac{Y_{ii}(s)}{V_i(s)}=\frac{1}{sA_i(s)}=\frac{1}{s^{n_i+1}+a_{i,n_i-1}s^{n_i}+\cdots+a_{i0}s} \tag{2}$$

and its state-space representation is

$$\begin{cases}\dot{x}_1=x_2\\ \quad\vdots\\ \dot{x}_{n_i}=x_{n_i+1}\\ \dot{x}_{n_i+1}=-a_{i0}x_2-\cdots-a_{i,n_i-1}x_{n_i+1}+v_i\\ y_{ii}=x_1\end{cases} \tag{3}$$

Since backstepping method is applicable to the controller design of lower-triangle nonlinear systems or accurate linearization systems[4], (3) is a special lower-triangle linear system, backstepping can be used recursively in the controller design for each loop. The design procedure for the ith control loop is as follows.

Step 1. The first error variable is defined as

$$z_1=y_i-y_{ri}+\varepsilon_i=y_{ii}+y_{ci}-y_{ri}+\varepsilon_i=x_i+y_{ci}-y_{ri}+\varepsilon \tag{4}$$

where ε_i is an auxiliary control variable used in the decentralized controllers design in Section 3.3.

Choosing the first CLF, $V_1=\frac{1}{2}z_1^2$, and its derivative is

$$\dot{V}_1=z_1(x_2+\dot{y}_{ci}-\dot{y}_{ri}+\dot{\varepsilon}_i) \tag{5}$$

$x_2+\dot{y}_{ci}$ is taken as the first virtual control variable, and its desired value is

$$\alpha_1=(x_2+\dot{y}_{ci})_d=-c_1z_1+\dot{y}_{ri}-\dot{\varepsilon}_i \tag{6}$$

where $c_1>0$ is the backstepping parameter to be designed. With the above choice, (5) becomes negative definite.

Step $l(l=2,\cdots,n_i)$. The $l(l=2,\cdots,n_i)$th error variable is

$$z_l=x_l+y_{ci}^{(l-1)}-\alpha_{l-1} \tag{7}$$

Then $\dot{z}_{l-1}=x_l+y_{ci}^{(l-1)}-\dot{\alpha}_{l-2}=z_l+\alpha_{l-1}-\dot{\alpha}_{l-2}=-z_{l-2}-c_{l-1}z_{l-1}+z_l$.

Defining the $l(l=2,\cdots,n_i)$th CLF, $V_l=\frac{1}{2}\sum\limits_{h=1}^{l}z_h^2$,

$$\dot{V}_l=\sum_{h=1}^{l}z_h\dot{z}_h=-\sum_{h=1}^{l-1}c_hz_h^2+z_l(z_{l-1}+x_{l+1}+y_{ci}^{(l)}-\dot{\alpha}_{l-1}) \tag{8}$$

The $l(l=2,\cdots,n_i)$th virtual control variable is

$$\alpha_l=(x_{l+1}+y_{ci}^{(l)})_d=-z_{l-1}-c_lz_l+\dot{\alpha}_{l-1} \tag{9}$$

where $c_l>0$.

Step (n_i+1). The (n_i+1)th error variable is

$$z_{n_i+1}=x_{n_i+1}+y_{ci}^{(n_i)}-\alpha_{n_i} \tag{10}$$

Then $\dot{z}_{n_i} = x_{n_i+1} + y_{ci}^{(n_i)} - \dot{\alpha}_{n_i-1} = z_{n_i+1} + \alpha_{n_i} - \dot{\alpha}_{n_i-1} = -z_{n_i-1} - c_{n_i} z_{n_i} + z_{n_i+1}$

Choosing the (n_i+1)th CLF, $V_{n_i+1} = \frac{1}{2}\sum_{h=1}^{n_i+1} z_h^2$,

$$\dot{V}_{n_i+1} = \sum_{h=1}^{n_i+1} z_h \dot{z}_h = -\sum_{h=1}^{n_i} c_h z_h^2 + z_{n_i+1}(z_{n_i} + \dot{x}_{n_i+1} + y_{ci}^{(n_i+1)} - \dot{\alpha}_{n_i}) \tag{11}$$

Select $\dot{z}_{n_i+1} = -z_{n_i} - c_{n_i+1} z_{n_i+1} (c_{n_i+1} > 0)$.

The (n_i+1)th virtual control variable is

$$\alpha_{n_i+1} = (\dot{x}_{n_i+1} + y_{ci}^{(n_i+1)})_d = -z_{n_i} - c_{n_i+1} z_{n_i+1} + \dot{\alpha}_{n_i} \tag{12}$$

In the s-plane, (12) can be rewritten as

$$L_{n_i+1}(s) = |sI_{n_i+1} - C_{n_i+1}|[Y_{ri}(s) - F_i(s)] - (|sI_{n_i+1} - C_{n_i+1}| - s^{n_i+1})[Y_{ii}(s) + Y_{ci}(s)] \tag{13}$$

where $L_{n_i+1}(s)$, $F_i(s)$ are the Laplace transforms of α_{n_i+1}, ε_i, and

$$C_{n_i+1} = \begin{bmatrix} -c_1 & 1 & 0 & \cdots & 0 \\ -1 & -c_2 & 1 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & -1 & -c_{n_i} & 1 \\ 0 & \cdots & 0 & -1 & -c_{n_i+1} \end{bmatrix}.$$

From (3), the new control input is

$$\begin{aligned} V_i(s) = \sum_{h=0}^{n_i-1} a_{i,h} s^{h+1} X_1(s) + sX_{n_i+1}(s) = \sum_{h=0}^{n_i-1} a_{i,h} s^{h+1} Y_{ii}(s) + L_{n_i+1}(s) - s^{n_i+1} Y_{ci}(s) = \\ \sum_{h=0}^{n_i-1} a_{i,h} s^{h+1} Y_{ii}(s) + D_i(s)[Y_{ri}(s) - F_i(s)] - (D_i(s) - s^{n_i+1}) Y_{ii}(s) - D_i(s) Y_{ci}(s) \end{aligned} \tag{14}$$

where $D_i(s) \triangleq |sI_{n_i+1} - C_{n_i+1}|$.

The loop error is defined as $E_i(s) = Y_{ri}(s) - [Y_{ii}(s) + Y_{ci}(s)]$, (14) can be derived

$$\begin{aligned} V_i(s) = \left(s^{n_i+1} + \sum_{h=0}^{n_i-1} a_{i,h} s^{h+1} - D_i(s)\right) Y_{ii}(s) + D_i(s)[Y_{ri}(s) - F_i(s)] - D_i(s) Y_{ci}(s) = \\ (D_i(s) - sA_i(s)) E_i(s) + sA_i(s) Y_{ri}(s) - [D_i(s) F_i(s) + sA_i(s) Y_{ci}(s)] \end{aligned} \tag{15}$$

Since $Y_{ci}(s) = \sum_{\substack{j=1 \\ j\neq i}} G_{ij}(s) U_j(s)$, the control input of the $i(i = 1, \cdots, N)$th loop is

$$U_i(s) = \frac{V_i(s)}{sB_i(s)} = \frac{D_i(s) - sA_i(s)}{sB_i(s)} E_i(s) + G_{ii}^{-1}(s) Y_{ri}(s) - \frac{D_i(s)}{sB_i(s)} F_i(s) - G_{ii}^{-1}(s) \sum_{j=1, j\neq i}^{N} G_{ij}(s) U_j(s) \tag{16}$$

It can be seen that the controller design for the ith loop is dependent on the control inputs in other loops, which is a multivariable control structure.

3.3 Decentralized PID controller design

If $D_i(s)F_i(s) + sA_i(s)Y_{ci}(s) = 0$ is held, the effect of other control inputs $V_j(s)(j = 1, \cdots, N, j \neq i)$ on $V_i(s)$ can be eliminated. The auxiliary control variable $F_i(s)$ can be chosen as

$$F_i(s) = -\frac{sA_i(s)}{D_i(s)} Y_{ci}(s) \tag{17}$$

then (16) can become

$$U_i(s) = \frac{D_i(s) - sA_i(s)}{sB_i(s)} E_i(s) + G_{ii}^{-1}(s) Y_{ri}(s) \triangleq G_{ci}(s) E_i(s) + G_{ii}^{-1}(s) Y_{ri}(s) \tag{18}$$

It can be observed from (18) that the control input (controller output) of the $i(i=1,\cdots,N)$th loop only depends on the error and reference signal in the same loop and is not related to other control inputs, which realizes the decentralized control. The block diagram of the ith control loop is shown in Fig. 3.

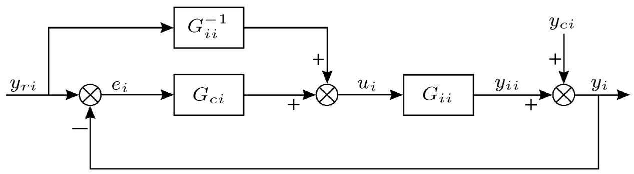

Fig. 3 Control structure of the ith loop

Since the majority of industrial plants can be seen approximately as first-order or second-order models, the decentralized PID controller design for first-order and second-order models are paid more attention.

1) For the first-order model, *i.e.*, $n_i=1$, $m_i=0$, $D_i(s)=|sI_2-C_2|=s^2+(c_1+c_2)s+c_1c_2+1$, the control input of the ith loop is

$$\begin{aligned}U_i(s)=&\frac{(c_1+c_2-a_{i0})+(c_1c_2+1)/s}{b_{i0}}E_i(s)+G_{ii}^{-1}(s)Y_{ri}(s)\triangleq\\&(K_P+K_I/s)E_i(s)+G_{ii}^{-1}(s)Y_{ri}(s)\triangleq G_{PI}(s)E_i(s)+G_{ii}^{-1}(s)Y_{ri}(s)\end{aligned}\tag{19}$$

where PI-type controller parameters are

$$\begin{cases}K_P=(c_1+c_2-a_{i0})/b_{i0}\\K_I=(c_1c_2+1)/b_{i0}\end{cases}\tag{20}$$

2) For the second-order model, *i.e.*, $n_i=2$, $m_i=1$, $D_i(s)=|sI_3-C_3|=s^3+(c_1+c_2+c_3)s^2+(c_1c_2+c_2c_3+c_1c_3+2)s+c_1c_2c_3+c_1+c_3$, and the control input of the ith loop under the assumption $b_{i1}=0$, $b_{i0}\neq 0$, is

$$U_i(s)\triangleq(K_P+K_I/s+K_Ds)E_i(s)+G_{ii}^{-1}(s)Y_{ri}(s)\triangleq G_{PID}(s)E_i(s)+G_{ii}^{-1}(s)Y_{ri}(s)\tag{21}$$

where PID-type controller parameters are

$$\begin{cases}K_P=(c_1c_2+c_2c_3+c_1c_3+2-a_{i0})/b_{i0}\\K_I=(c_1c_2c_3+c_1+c_3)/b_{i0}\\K_D=(c_1+c_2+c_3-a_{i1})/b_{i0}\end{cases}\tag{22}$$

It can be drawn that decentralized PID controller parameters are equivalent to the designing parameters based on the backstepping method. For the first-order transfer function plant model backstepping designing parameters are equivalent to a PI-type controller and for the second-order plant model backstepping designing parameters are equivalent to a PID-type controller. The conventional PID controller parameters K_P, K_I, K_D can be converted into backstepping designing parameters $c_i(c_i>0)$.

4 Stability analysis

To guarantee system stability, the following assumptions are presented for the plant models[9]:

1) $B_i(s)(i=1,\cdots,N)$ is a Hurwitz polynomial;

2) The interaction transfer function $G_{ij}(s)(i,j=1,\cdots,N,i\neq j)$ is strictly stable.

For the ith control loop, error variables defined in (4), (7) and (10) make the derivative of CLF $\dot V_{n_i+1}$ negative definite, then V_{n_i+1} is bounded, $z_l(l=1,\cdots,n_i+1)$ is bounded[4], and

$$\lim_{t\to\infty}z_l(t)=0,\quad l=1,\cdots,n_i+1\tag{23}$$

In addition, the following expression is held according to final value theorem

$$\lim_{t\to\infty}\varepsilon_i(t)=\lim_{s\to 0}sF_i(s)=-\lim_{s\to 0}\frac{s^2A_i(s)}{D_i(s)}Y_{ci}(s)=0\tag{24}$$

From (4), (23) and (24),

$$\lim_{t\to\infty} z_1(t) = \lim_{t\to\infty}[y_i(t) - y_{ri}(t) + \varepsilon_i(t)] = \lim_{t\to\infty}[y_i(t) - y_{ri}(t)] = 0 \tag{25}$$

Hence, the process output $y_i(t)$ tracks asymptotically the loop reference signal $y_{ri}(t)$, the ith sub-process is asymptotically stable. The controller G_c is to be designed for the process $\bar{G}_p = \mathrm{diag}\{G_{11}, \cdots, G_{NN}\}$ such that the block diagonal closed-loop system with the transfer matrix $\bar{G}_h = \mathrm{diag}\{\bar{G}_{h1}, \cdots, \bar{G}_{hN}\}$ is stable, where $\bar{G}_{hi} = G_{ii}G_{ci}(1 + G_{ii}G_{ci})^{-1}$.

It should be noticed that the small gain theorem[10] can be used to obtain the sufficient condition for determining the stability of the full closed-loop system $G_h = G_pG_c(I + G_pG_c)^{-1}$, which the closed-loop process is stable if the spectral radius (the maximum eigenvalue of a matrix) or the infinite norm

$$\rho(E_p(j\omega)\bar{G}_h(j\omega)) < 1 \text{ or } \|E_p(j\omega)\bar{G}_h(j\omega)\|_\infty < 1, \quad \forall\omega \tag{26}$$

where $E_p = (G_p - \bar{G}_p)\bar{G}_p^{-1}$ is the multiplicative error between the full MIMO and decentralized models.

For TITO processes, $E_p = \begin{bmatrix} 0 & G_{12}G_{22}^{-1} \\ G_{21}G_{11}^{-1} & 0 \end{bmatrix}$, $\bar{G}_{hi} = 1 - \dfrac{sA_i}{Q_i}(i = 1, 2)$, then the sufficient condition of the stability is

$$\left\|\frac{G_{12}G_{21}}{G_{11}G_{22}}\left(1 - \frac{sA_1}{Q_1}\right)\left(1 - \frac{sA_2}{Q_2}\right)\right\|_\infty < 1 \tag{27}$$

For $N \times N(N \geqslant 3)$ processes, the sufficient condition of the stability is

$$\left\|\begin{bmatrix} 0 & G_{12}G_{22}^{-1}\bar{G}_{h2} & \cdots & G_{1N}G_{NN}^{-1}\bar{G}_{hN} \\ G_{21}G_{11}^{-1}\bar{G}_{h1} & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & G_{N-1,N}G_{NN}^{-1}\bar{G}_{hN} \\ G_{N1}G_{11}^{-1}\bar{G}_{h1} & \cdots & G_{N,N-1}G_{N-1,N-1}^{-1}\bar{G}_{hN-1} & 0 \end{bmatrix}\right\|_\infty < 1 \tag{28}$$

5 Simulation study

The Shell heavy oil fractionator is a multivariable control problem with three product draws and three side circulating loops. Product specifications for the top and side draw streams are determined by economics and operating requirements. There is no product specification for the bottom draw, but there is an operating constraint on the temperature in the lower part of the column. The three circulating loops remove heat to achieve the desired product separation. The heat exchangers in these loops reboil columns in other parts of the plant. The bottom loop has an enthalpy controller which regulates heat removal in the loop by adjusting steam make. Its heat duty can be used as a manipulated variable to control the column. The heat duties of the other two loops act as disturbances to the column.

Prett and Morari[11] presented a model for a heavy oil fractionator as the benchmark process for the Shell standard control problem

$$\boldsymbol{y} = G_p(s)\boldsymbol{u} + G_d(s)\boldsymbol{d} \tag{29}$$

where $G_p(s)$ and $G_d(s)$ are the process and disturbance transfer function matrix (See [11]), respectively, $\boldsymbol{u} = [u_1 \quad u_2 \quad u_3]^{\mathrm{T}}$ are input variables to control the process, $\boldsymbol{d} = [d_1 \quad d_2]^{\mathrm{T}}$ are unmeasured but bounded disturbances entering the column, with $|d_1| \leqslant 0.5$ and $|d_2| \leqslant 0.5$, $\boldsymbol{y} = [y_1 \quad y_2 \quad y_3]^{\mathrm{T}}$ are output variables.

The control objective of the whole system is to maintain the draw composition from the top (y_1) and the side (y_2) of the column at specification. In order to test the performance of the control scheme, the closed-loop system is subject to disturbance patterns $\boldsymbol{d}^1 = [0.5 \quad 0.5]^{\mathrm{T}}$ and $\boldsymbol{d}^2 = [-0.5 \quad -0.5]^{\mathrm{T}}$[12]. Use sampling time of 4 minutes and simulation time of 400 minutes.

By examining the elements of $G_p(s)$ it is observed that the best pairing of input and output variables is to control y_1 with u_1, y_2 with u_2, and y_3 with u_3, accordingly three control loops can be derived. The Matlab based simulation results (Figs. 4~7) show the system output responses, the control input signals, and the corresponding sufficient conditions for the stability of the closed-loop system $\|E_p\bar{G}_h\|_\infty$, under the disturbance patterns $\boldsymbol{d}^1$ and $\boldsymbol{d}^2$, respectively.

It can be observed from Figs. 4∼7 that the system output responses are rapidly stable and all control input signals are within the saturation and rate limit constraints. Since the decentralized controllers have $\|E_p\bar{G}_h\|_\infty < 1$ for all frequencies, the closed-loop stability is guaranteed. In addition, the designing parameters for each loop under decentralized control can be designed and tuned separately, which is superior to centralized control and can be simple to implement.

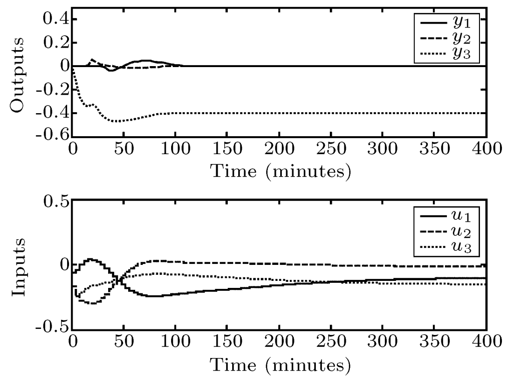

Fig. 4 System output responses and control input signals under the disturbance pattern $\boldsymbol{d}^1$

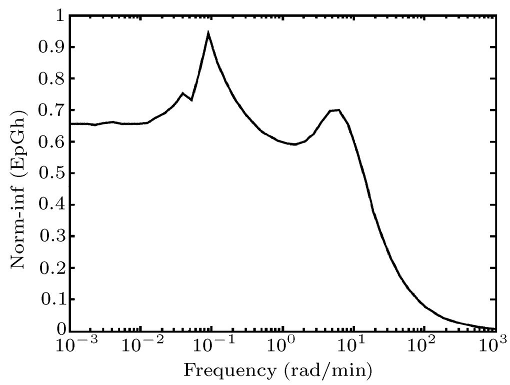

Fig. 5 $\|E_p\bar{G}_h\|_\infty$ with the disturbance pattern $\boldsymbol{d}^1$

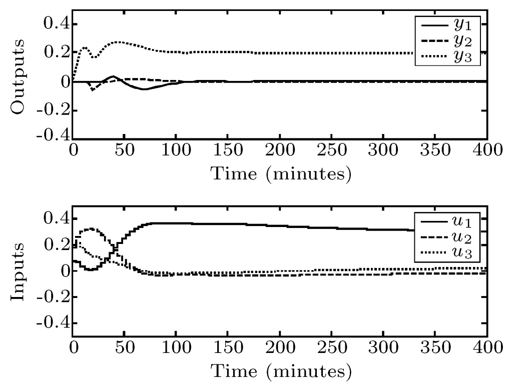

Fig. 6 System output responses and control input signals under the disturbance pattern $\boldsymbol{d}^2$

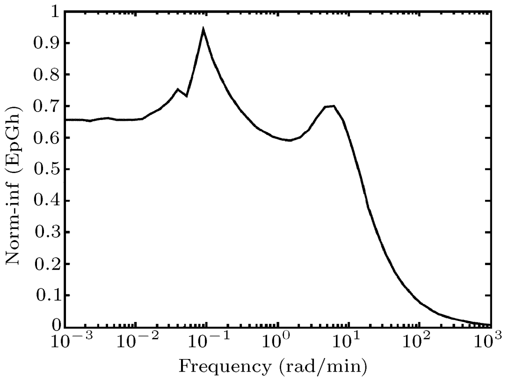

Fig. 7 $\|E_p\bar{G}_h\|_\infty$ with the disturbance pattern $\boldsymbol{d}^2$

Backstepping designing parameters for each control loop are as follows

$$\begin{cases} c_1^1 = 6.2627,\ c_2^1 = 4.8485,\ c_3^1 = 3.4343 \\ c_1^2 = 6.2627,\ c_2^2 = 4.8485,\ c_3^2 = 3.4343 \\ c_1^3 = 3.6667,\ c_2^3 = 1.6667 \end{cases} \tag{30}$$

6 Conclusion

A new backstepping-based decentralized PID control scheme is presented for linear MIMO processes. The MIMO control system is decomposed into multiple control loops and controllers are designed in parallel prototype. It has been proved that the controller parameters obtained with backstepping design for the first-order transfer function models are equivalent to a PI-type controller and for the second-order plant model backstepping design parameters are equivalent to a PID-type controller. In addition, the sufficient stability conditions are derived via the small gain theorem for TITO and MIMO closed-loop systems, respectively. The major advancement of the approach is that multivariable control for a MIMO process can be converted to designing PID controller for multiple SISO sub-processes. Thus it can significantly reduce the computational complexity while keeping satisfactory performance.

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ZHANG Yan Received her master degree from Shandong University in 2002 and now she is a Ph. D. candidate at Shanghai Jiaotong University. Her research interests include optimization and control of complex systems.

LI Shao-Yuan Professor of Shanghai Jiaotong University. His research interests include model predictive control, fuzzy systems, and neural networks.