Analysis of Sliding Mode in Planar Switched Systems[1)]

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Abstract This paper discusses the phenomena of sliding mode in dynamic behavior of planar switched systems. We propose an approach to estimate the domain in which the sliding motion may occur. The relationship between the chattering angle and the switching time is given and utilized to check the existence of sliding mode with chattering. Finally, the application of the proposed results is illustrated by an example.

Key words Planar switched systems, sliding mode, autonomous switching, chattering

1 Introduction

Switched systems, a special hybrid system, are composed of a set of subsystems. The switching among subsystems is designated by a rule driven by time or events. Switched systems are commonly found in engineering practice, such as in computer disk driver system, the highway supervisory system, the constrained robotics and so on. Recently, more attention has been attracted onto the stability analysis[1] and stabilization[2,3], the controllability and reachability[4] of switched systems, with little research on the dynamic behavior of switched systems. However, due to the speciality of switched systems, the dynamic behavior of different switched systems may present different modes. Even if these switched systems are all stable, sometimes sliding motion might occur. For example, consider the following two switched systems: A: $A_1 = \begin{bmatrix} -0.5 & -2 \\ 2 & -1 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 3 \\ -0.3 & 0.2 \end{bmatrix}$; B: $A_1 = \begin{bmatrix} -0.5 & -2 \\ 2 & -1 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & -3 \\ 0.3 & 0.2 \end{bmatrix}$. The switching rule is: if $x_1^2 - x_2^2 > 0$, subsystem 2 is active; else, subsystem 1 is active. The switching time is 0.05 seconds, the initial condition is $[1.3 \quad -1.6]^{\mathrm{T}}$. The trajectories and the response of the two switched systems A and B are shown in Fig. 1 and Fig. 2, respectively.

Figs. 1 and 2 show that although being asymptotic stable, these two switched systems have much different dynamic behavior: Switched systems A has a shorter adjusting time, but with chattering on the trajectory. Especially, if the switching acts fast enough, sliding motion will appear. Switched systems B, without chattering, has a longer adjusting time as well as overshoot.

In recent years, some studies on the sliding mode control of switched systems have been reported[5,6]. Akar and Ozguner[5] proposed a sliding control method to make the switched systems exponentially stable by input/output feedback. Zhang and Hu[6] addressed the sliding mode control for planar switched systems under an arbitrary switching sequence. However, the aforementioned contributions only concentrated on designing a sliding mode controller, not concerning the sliding mode in the dynamic behavior of switched systems themselves. This paper investigates the sliding motion of switched systems without the control input.

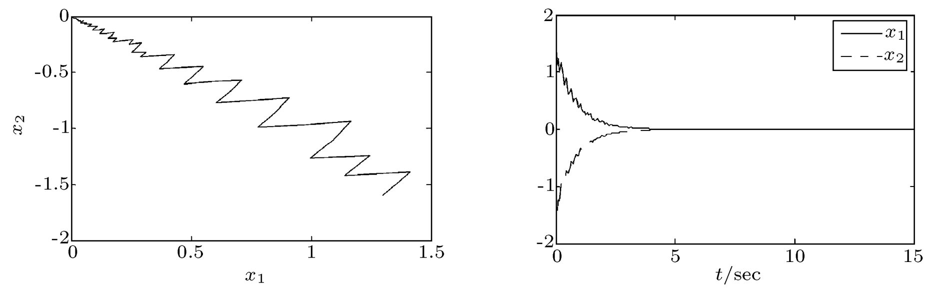

Fig. 1 Trajectory and response of switched systems A

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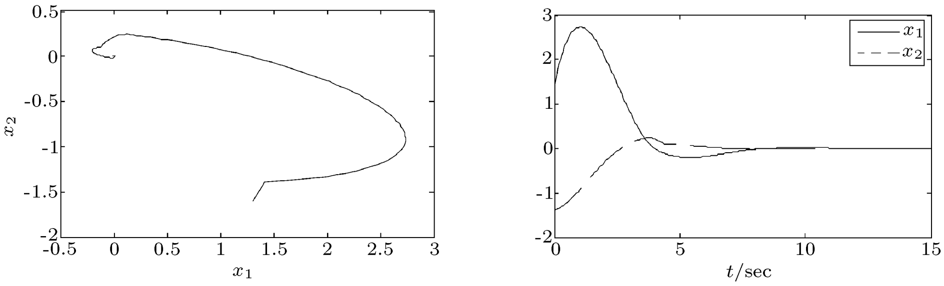

Fig. 2 Trajectory and response of switched systems B

2 Problem formulation

Consider planar switched systems as

$$\dot{\boldsymbol{x}} = A_\sigma \boldsymbol{x} = \begin{bmatrix} a_{i1} & a_{i2} \\ a_{i3} & a_{i4} \end{bmatrix} \boldsymbol{x} \tag{1}$$

where $\boldsymbol{x} \in R^2$ is a state vector, $\sigma \in \{1, 2, \cdots, N\}$, N is the number of subsystems.

Definition 1. Let $\vec{\boldsymbol{x}}$ stands for the vector pointing from zero to x a state on trajectory. If the vector Ax coincides with $\vec{\boldsymbol{x}}$ after being rotated angle $\omega(x)$ counterclockwise, then $\omega(x)$ is called the rotating angle at point x. Moreover, if $0 < \omega(x) < \pi$, this rotating direction at point x is called clockwise rotation; and $\pi < \omega(x) < 2\pi$, called counterclockwise rotation, as illustrated in Fig. 3.

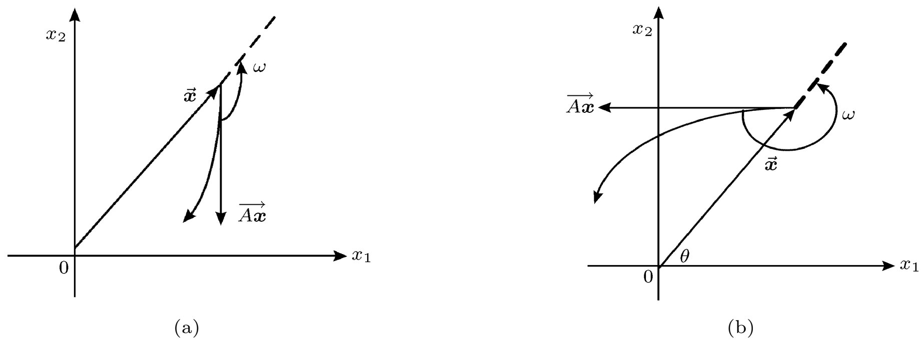

Fig. 3 (a) Illustration of clockwise rotation, (b) Illustration of counterclockwise rotation

Substituting, $x_1 = \rho\cos\theta$, $x_2 = \rho\sin\theta$ into (1) yields

$$\dot{\rho} = \rho[a_{i1}\cos^2\theta + a_{i4}\sin^2\theta + (a_{i2} + a_{i3})\sin\theta\cos\theta] \tag{2}$$

$$\dot{\theta} = [-a_{i2}\sin^2\theta + a_{i3}\cos^2\theta + (a_{i4} - a_{i1})\sin\theta\cos\theta] \tag{3}$$

Obviously, if $\dot{\theta} > 0(\dot{\theta} < 0)$, the trajectory at point $x = [x_1, x_2]^{\mathrm{T}}$ will rotate counterclockwise (clockwise, respectively). We can see from Fig. 1 that the chattering would arise if the rotating directions of the subsystem trajectories are opposite in the two sides of the switching curve.

3 Main results

This section deals with the problem that for switched systems described by (1), if the switching time (the time consumed by the switching action) is small enough, then in what regions the sliding motion might occur. Obviously, a sufficient condition for this sliding motion is $\dot{\theta}_i(x) \cdot \dot{\theta}_j(x) < 0$, where $\dot{\theta}_i(x)$ denotes the derivative of the angle to time for subsystem i.

For a planar system, $\dot{x} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} x$, the rotating direction of a trajectory is tabled as below:

Remark 1. In Table 1, $\Delta = (a_1 - a_4)^2 + 4a_2a_3$, $\Omega_1 \sim \Omega_4$ are defined as follows: The plane is divided into four sectors by two lines $k_1 : m_1x_1 + n_1x_2 = 0$, $k_2 : m_2x_1 + n_2x_2 = 0$, called alteration lines of rotating direction. The region Ω_1 represents the sector which is bordered by the lines k_1, k_2

and is situated in $\{x \in R^2 | x_1 \in (0, +\infty)\} \cup \{x \in R^2 | x_1 = 0, x_2 \in (0, +\infty)\}$. Similarly the other three sectors is denoted counterclockwise as $\Omega_2, \Omega_3, \Omega_4$. The parameters of k_1, k_2 are given as: when $(a_4 - a_1) > 0$, $m_1 = (a_4 - a_1) + \sqrt{\Delta}$, $m_2 = 2a_3$, $n_1 = -2a_2$, $n_2 = (a_4 - a_1) + \sqrt{\Delta}$; when $(a_4 - a_1) \leqslant 0$, $m_1 = -(a_4 - a_1) + \sqrt{\Delta}$, $m_2 = -2a_3$, $n_1 = 2a_2$, $n_2 = -(a_4 - a_1) + \sqrt{\Delta}$.

Table 1 Rotating direction of planar linear systems

	$\Delta < 0$	$\Delta = 0$	$\Delta > 0$		
Region of clockwise rotating Φ_c	$a_2 > 0$ whole plane	$a_2 > 0$ whole plane			when $a_2 > 0$, in regions Ω_2, Ω_4
					when $a_2 < 0$, in regions Ω_1, Ω_3
		$a_2 = 0$, $a_3 < 0$ whole plane	$a_2 = 0$	$a_3 \neq 0$	when $a_4 - a_1 > 0$, in regions Ω_2, Ω_4
					when $a_4 - a_1 < 0$, in regions Ω_1, Ω_3
				$a_3 = 0$	when $a_4 - a_1 > 0$, in quadrants 2,4
					when $a_4 - a_1 < 0$, in quadrants 1,3
Region of counterclock-wise rotating Φ_{cc}	$a_2 < 0$ whole plane	$a_2 < 0$ whole plane			when $a_2 < 0$, in regions Ω_2, Ω_4
					when $a_2 > 0$, in regions Ω_1, Ω_3
		$a_2 = 0$, $a_3 > 0$ whole plane	$a_2 = 0$	$a_3 \neq 0$	when $a_4 - a_1 > 0$, in regions Ω_1, Ω_3
					when $a_4 - a_1 < 0$, in regions Ω_2, Ω_4
				$a_3 = 0$	when $a_4 - a_1 > 0$, in quadrants 1,3
					when $a_4 - a_1 < 0$, in quadrants 2,4

For switched systems (1), if there are $\dot{\rho}_i < 0$ and $\dot{\rho}_j < 0$ on the switching line ($\dot{\rho}_i$ denotes the derivative of polar radius to time t of subsystem i), the sliding mode will converge to zero along the switching line. Table 2 shows the radius change for the planar linear system.

Table 2 Change of polar radius for planar linear system

	$\Delta_\rho < 0$	$\Delta_\rho = 0$	$\Delta_\rho > 0$		
Region of radius decreasing Σ_s	$a_4 < 0$ whole plane	$a_4 < 0$ whole plane			when $a_4 < 0$, in regions Ψ_2, Ψ_4
					when $a_4 > 0$, in regions Ψ_1, Ψ_3
		$a_4 = 0$, $a_1 < 0$ whole plane	$a_4 = 0$	$a_1 \neq 0$	when $a_2 + a_3 > 0$, in regions Ψ_2, Ψ_4
					when $a_2 + a_3 < 0$, in regions Ψ_1, Ψ_3
				$a_1 = 0$	when $a_2 + a_3 > 0$, in quadrants 2,4
					when $a_2 + a_3 < 0$, in quadrants 1,3
Region of radius increasing Σ_{us}	$a_4 > 0$ whole plane	$a_4 > 0$ whole plane			when $a_4 > 0$, in regions Ψ_2, Ψ_4
					when $a_4 < 0$, in regions Ψ_1, Ψ_3
		$a_4 = 0$, $a_1 > 0$ whole plane	$a_4 = 0$	$a_1 \neq 0$	when $a_2 + a_3 > 0$, in regions Ψ_1, Ψ_3
					when $a_2 + a_3 < 0$, in regions Ψ_2, Ψ_4
				$a_1 = 0$	when $a_2 + a_3 > 0$, in quadrants 1,3
					when $a_2 + a_3 < 0$, in quadrants 2,4

Remark 2. In Table 2, $\Delta_\rho = (a_2 + a_3)^2 - 4a_1a_4$. Similar to remark 1, Ψ_1 denotes the sector which is bordered by lines $r_1 : p_1x_1 + q_1x_2 = 0$, $r_2 : p_2x_1 + q_2x_2 = 0$, and is situated in $\{x \in R^2 | x_1 \in (0, +\infty)\} \cup \{x \in R^2 | x_1 = 0, x_2 \in (0, +\infty)\}$. The other three sectors are denoted counterclockwise as Ψ_2, Ψ_3, Ψ_4, and the parameters of r_1, r_2 are given as: when $(a_2 + a_3) > 0$, $p_1 = (a_2 + a_3) + \sqrt{\Delta_\rho}$, $p_2 = 2a_1$, $q_1 = 2a_4$, $q_2 = (a_2 + a_3) + \sqrt{\Delta_\rho}$; when $(a_2 + a_3) \leqslant 0$, $p_1 = -(a_2 + a_3) + \sqrt{\Delta_\rho}$, $p_2 = -2a_1$, $q_1 = -2a_4$, $q_2 = -(a_2 + a_3) + \sqrt{\Delta_\rho}$.

Remark 3. It is useful for us to obtain regions $\Psi_c, \Psi_{cc}, \Sigma_s, \Sigma_{us}$. For example, if the region Σ_s of different subsystems are coincided partially and the rotating directions of the trajectories of those subsystems are opposite in such coincided region, then any curve inside the coincided region can be tracked by using appropriate switching sequence.

Theorem 1. For the switched systems (2)∼(3), if (4)∼(6) are satisfied, then the system trajectory starting from the initial point x_0 will occur sliding mode and converges to zero.

$$l \subset \Sigma_{is} \cap \Sigma_{js} \tag{4}$$

$$l \subset \Psi_{ic} \cap \Psi_{jcc} \text{ or } l \subset \Psi_{icc} \cap \Psi_{jc} \tag{5}$$

$$x_0 \in \Psi_{ic} \cap \Psi_{jcc} \text{ or } x_0 \in \Psi_{icc} \cap \Psi_{jc} \tag{6}$$

where switching curve $l : S(x) = 0$ satisfies $S(0) = 0$, Σ_{is}, Ψ_{ic} and Ψ_{icc} represent the decreasing region of polar radius, the clockwise rotating region and the counterclockwise rotating region for subsystem i, respectively. The switching rule is given as: If $l \subset \Psi_{ic} \cap \Psi_{jcc}$, then when $x(t)$ lies in the counterclockwise side of curve l, subsystem i functions, then when $x(t)$ lies in the clockwise side of curve l, subsystem j

functions. If $l \subset \Psi_{icc} \cap \Psi_{jc}$, when $x(t)$ lies in the counterclockwise side of curve l, subsystem j functions, when $x(t)$ lies in the clockwise side of curve l, subsystem i functions.

Proof. The polar radius will reduce along the trajectory if (4) holds. Noting (5)∼(6), the trajectory leaving from x_0 will arrive at the switching curve and then slide to zero.

For a practical switched system, the switching time, expressed as t^*, usually cannot be infinite small. Due to the existence of t^*, chattering may take place, an occurrence which might alter the sliding mode. Hence, it is necessary to figure out the relationship between switching time t^* and chattering angle which is useful in checking whether the sliding mode could retain. □

Definition 2. For switched systems (1), if the sliding motion with chattering occurs on switching line l, then the angle that the trajectory, departing from l, rotated after t^* is called the chattering angle, where t^* is the switching time. Fig. 4 provides the illustration of the chattering angle.

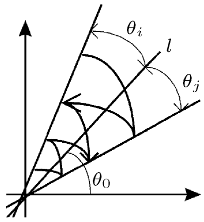

Fig. 4 Chattering effect

Theorem 2. For switched systems (2)∼(3), if (7)∼(9) hold, then the trajectory starting from x_0 will occur chattering sliding motion and converge to zero.

$$\text{sector } [\theta_0+\theta_i, \theta_0+\theta_j] \subset \Sigma_{is} \cap \Sigma_{js} \tag{7}$$

$$\text{sectoror } [\theta_0+\theta_i, \theta_0+\theta_j] \subset \Psi_{ic} \cap \Psi_{jcc} \text{ or } [\theta_0+\theta_i, \theta_0+\theta_j] \subset \Psi_{icc} \cap \Psi_{jc} \tag{8}$$

$$x_0 \in \Psi_{ic} \cap \Psi_{jcc} \text{ or } x_0 \in \Psi_{icc} \cap \Psi_{jc} \tag{9}$$

where θ_0 is the sloping angle of the switching line $l: \ S(x) = Cx = 0$, $C = [c_1 \quad c_2]$; θ_i and θ_j are the chattering angles, the definitions of Σ_{is}, Ψ_{ic} and Ψ_{icc} are the same as Theorem 1.

The procedure of computing the chattering angles θ_i, θ_j is given below: Assume that trajectory of subsystem i rotates counterclockwise and trajectory of subsystem j rotates clockwise on the switching line. Obviously, $\dot{\theta}_i > 0$. Calculating the integral of (3), it follows that

$$\int_{\theta_0}^{\theta_0+\theta_i} \frac{\mathrm{d}\theta}{-a_{i2}\sin^2\theta + a_{i3}\cos^2\theta + (a_{i4}-a_{i1})\sin\theta\cos\theta} = \int_{t_1}^{t_2} \mathrm{d}t$$

Let $r = tg\theta$, $s_0 = tg\theta_0$, $s_i = tg(\theta_i + \theta_0)$. We have

$$t_2 - t_1 = t^* = \int_{s_0}^{s_i} \frac{\mathrm{d}r}{-a_{i2}r^2 + (a_{i4}-a_{i1})r + a_{i3}} \tag{10}$$

Solving (10) and simplifying, it follows that

$$\theta_i = \mathrm{arg}tg\left(\frac{F_i(s_0) - s_0}{1 + F_i(s_0)s_0}\right) \tag{11}$$

where $F_i(s_0)$ is given as follows.

a) when $a_{i2} = 0$

$$F_i(s_0) = \begin{cases} \dfrac{\sqrt{-\Delta_i}G_i(s_0) - \Delta_i tg(t^*\sqrt{-\Delta_i}/2)}{2a_{i2}(G_i(s_0)tg(t^*\sqrt{-\Delta_i}/2) - \sqrt{-\Delta_i})} + \dfrac{a_{i4}-a_{i1}}{2a_{i2}}, & \Delta_i < 0 \\ \dfrac{G_i(s_0)}{a_{i2}(t^*G_i(s_0) - 2)} + \dfrac{a_{i4}-a_{i1}}{2a_{i2}}, & \Delta_i = 0 \\ \dfrac{\sqrt{\Delta_i}(G_i(s_0) + \sqrt{\Delta_i})}{a_{i2}(\exp(\sqrt{\Delta_i}t^*)(G_i(s_0) - \sqrt{\Delta_i}) - G_i(s_0) - \sqrt{\Delta_i})} + \dfrac{a_{i4}-a_{i1}+\sqrt{\Delta_i}}{2a_{i2}}, & \Delta_i > 0 \end{cases} \tag{12a}$$

here $\Delta_i = (a_{i1} - a_{i4})^2 + 4a_{i2}a_{i3}$, $G_i(s_0) = -2a_{i2}s_0 + (a_{i4} - a_{i1})$.

b) when $a_{i2} = 0$

$$F_i(s_0) = \exp(t^*(a_{i4} - a_{i1}))s_0 + \frac{[\exp(t^*(a_{i4} - a_{i1})) - 1]a_{i3}}{a_{i4} - a_{i1}} \tag{12b}$$

Similarly, we can obtain chattering angle θ_j. □

Remark 4. If $[\theta_0, \theta_0 + \theta_i]$ include the radials $\theta = \pi/2$, $3\pi/2$, the procedure of computing is similar to that in the above, but some revision should be made as below.

1) $s_i = ctg(\theta_0 + \theta_i)$, $s_j = ctg(\theta_0 + \theta_j)$.

2) Parameters a_{i1}, a_{i2}, a_{i3}, a_{i4} in those formulations are shifted to a_{i4}, a_{i3}, a_{i2}, a_{i1} accordingly.

Furthermore, if the polar radius of system trajectory increases and decreases alternately, and besides, the amount of decreasing surpasses that of increasing in one chattering, the chattering sliding motion can still keep stable.

Theorem 3. For switched systems (2)∼(3), if (13)∼(15) are satisfied, then the trajectory starting from the initial point x_0 will occur sliding mode and converges to zero.

$$\Theta_i(s_i, s_j) - \Theta_j(s_i, s_j) < 0 \tag{13}$$

$$\text{Sector } [\theta_0 + \theta_i, \theta_0 + \theta_j] \subset \Phi_{ic} \cap \Psi_{jcc} \text{ or } [\theta_0 + \theta_i, \theta_0 + \theta_j] \subset \Phi_{icc} \cap \Phi_{jc} \tag{14}$$

$$x_0 \in \Psi_{ic} \cap \Psi_{jcc} \text{ or } x_0 \in \Psi_{icc} \cap \Psi_{jc} \tag{15}$$

where $s_i = tg(\theta_0 + \theta_i)$, $s_j = tg(\theta_0 + \theta_j)$, the definition of switching line l, angles θ_0, θ_i, θ_j and the switching rule are the same as those in Theorem 2.

The procedure of computing $\Theta_i(s_i, s_j)$, $\Theta_j(s_i, s_j)$ is given as below.

By Theorem 2, we can obtain the chattering θ_i, θ_j. It follows from (2)∼(3) that

$$\int_{\rho_1}^{\rho_2} \frac{\mathrm{d}\rho}{\rho} = \int_{\theta_0+\theta_j}^{\theta_0+\theta_j} \frac{a_1\cos^2\theta + a_4\sin^2\theta + (a_2 + a_3)\sin\theta\cos\theta}{-a_2\sin^2\theta + a_3\cos^2\theta + (a_4 - a_1)\sin\theta\cos\theta}\mathrm{d}\theta \tag{16}$$

Let $tg(\theta_0 + \theta_i) = s_i$, $tg(\theta_0 + \theta_j) = s_j$,

$$\int_{\rho_1}^{\rho_2} \frac{\mathrm{d}\rho}{\rho} = \int_{s_j}^{s_i} \frac{a_{i4}t^2 + (a_{i2} + a_{i3})t + a_{i1}}{(1 + t^2)(-a_{i2}t^2 + (a_{i4} - a_{i1})t + a_{i3})}\mathrm{d}t = \\ \int_{s_j}^{s_i} \left(\frac{mt + n}{1 + t^2} + \frac{pt + q}{-a_{i2}t^2 + (a_{i4} - a_{i1})t + a_{i3}}\right)\mathrm{d}t$$

where m, n, p, q are yielded by solving the following equation:

$$\begin{bmatrix} -a_{i2} & 0 & 1 & 0 \\ a_{i4} - a_{i1} & -a_{i2} & 0 & 1 \\ a_{i3} & a_{i4} - a_{i1} & 1 & 0 \\ 0 & a_{i3} & 0 & 1 \end{bmatrix} \begin{bmatrix} m \\ n \\ p \\ q \end{bmatrix} = \begin{bmatrix} 0 \\ a_{i4} \\ a_{i2} + a_{i3} \\ a_{i1} \end{bmatrix} \tag{17}$$

Integrate

$$\int_{s_j}^{s_i} \frac{mt + n}{1 + t^2}\mathrm{d}t = \left[\frac{m}{2}\ln(1 + t^2) + n\arg\mathrm{tg}(t)\right]\Big|_{s_j}^{s_i} \triangleq F_i(t)\Big|_{s_j}^{s_i} \tag{18}$$

$$\int_{s_j}^{s_i} \frac{pt + q}{-a_{i2}t^2 + (a_{i4} - a_{i1})t + a_{i3}}\mathrm{d}t \triangleq H_i(t)\Big|_{s_j}^{s_i} = \\ \begin{cases} \left(H_{i1}(t) + \dfrac{p(a_{i4} - a_{i1}) + 2a_{i2}q}{2a_{i2}}H_{i2}(t)\right)\Big|_{s_j}^{s_i}, & \text{if } a_{i2} \neq 0 \\ \left(\dfrac{pt}{a_{i4} - a_{i1}} + \dfrac{q(a_{i4} - a_{i1}) - a_{i3}p}{(a_{i4} - a_{i1})^2}\ln H_{i3}(t)\right)\Big|_{s_j}^{s_i}, & \text{if } a_{i2} = 0 \end{cases} \tag{19}$$

where

$$H_{i1}(t) = \frac{p}{-2a_2}\ln(-a_{i2}t^2 + (a_{i4} - a_{i1})t + a_{i3}) \tag{20}$$

$$H_{i2}(t)=\begin{cases}\dfrac{2}{-\sqrt{\Delta_i}}\arg\operatorname{tg}\dfrac{-2a_{i2}t+(a_{i4}-a_{i1})}{\sqrt{-\Delta_i}}, & \Delta_i<0\\ \dfrac{1}{\sqrt{\Delta_i}}\ln\dfrac{-a_{i2}t+(a_{i4}-a_{i1})-\sqrt{\Delta_i}}{-a_{i2}t+(a_{i4}-a_{i1})+\sqrt{\Delta_i}}, & \Delta_i>0\\ \dfrac{-2}{-2a_{i2}+(a_{i4}-a_{i1})}, \Delta_i=0\end{cases} \tag{21}$$

$$H_{i3}(t)=(a_{i4}-a_{i1})t+a_{i3} \tag{22}$$

Let

$$\Theta_k(s_i,s_j)=F_k(s_i)-F_k(s_j)+H_k(s_i)-H_k(s_j) \tag{23}$$

where $k=i,j$.

Therefore,

$$\rho_2=\rho_1\exp(\Theta_i(s_i,s_j)) \tag{24}$$

Similarly

$$\rho_3=\rho_2\exp(-\Theta_j(s_i,s_j)) \tag{25}$$

Then

$$\rho_3/\rho_1=\exp(\Theta_i(s_i,s_j)-\Theta_j(s_i,s_j)) \tag{26}$$

Hence, if $\Theta_i(s_i,s_j)-\Theta_j(s_i,s_j)<0$ is satisfied, the trajectory can move to zero in sliding mode.

Remark 5. If $[\theta_0+\theta_i,\theta_0+\theta_j]$ include the radials $\theta=\pi/2,3\pi/2$, the procedure of computing should be revised as in Remark 4.

Remark 6. Theorem 3 can be also used to study the stability as well as the limit circle for autonomous planar switched systems. Although somewhat tedious and complicated in form, the formulations in Theorem 2 and 3 can be solved conveniently by programming.

4 Numerical example

For switched systems $A_1=\begin{bmatrix}-1 & -2\\ 1 & -2\end{bmatrix}$, $A_2=\begin{bmatrix}3 & 6\\ -2 & 1\end{bmatrix}$, $A_3=\begin{bmatrix}-3 & 4\\ -2 & 1\end{bmatrix}$, $\boldsymbol{x}_0=[2.5 \quad -6.5]^{\mathrm{T}}$, the curve l to be traced is: when $0<x_1\leqslant 1$, $x_2=-x_1$; when $x_1>1$, $x_2=-x_1^2$.

From the analysis in section 3, we know that the sliding mode can appear on curve l if the following switched rule is designated: when $x_1\leqslant 1$ and $x_2<-x_1$, subsystem 1 operates; when $x_1\leqslant 1$ and $x_2>-x_1$, subsystem 3 operates; when $x_1>1$ and $x_2<-x_1^2$, subsystem 1 operates; when $x_1>1$ and $x_2>-x_1^2$, subsystem 2 operates.

The trajectory and response of the switched systems are shown in Fig. 5. It is evident that sliding mode appears on the switching curve, thus the proposed results are validated.

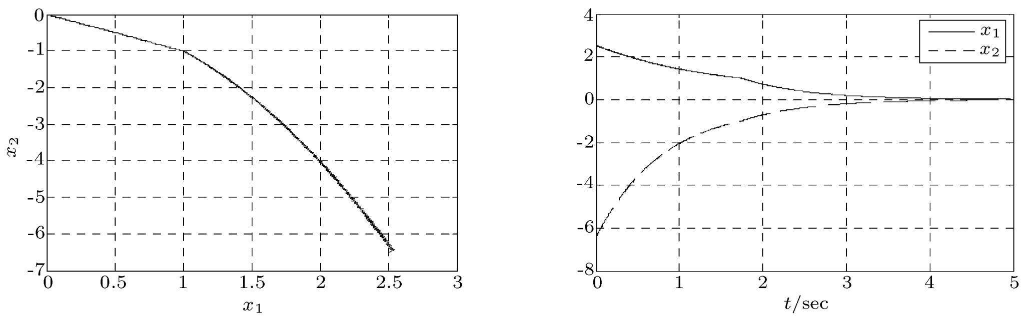

Fig. 5 Trajectory and response of switched systems

5 Conclusions

This paper analyzes the sliding mode for the planar switched linear systems without control input. The region in which the sliding mode might occur is given. In addition, we provide formulations to reveal the relationship between the chattering angle and switching time, which can be used to judge whether the chattering sliding mode would occur on switching curve.

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