Adaptive Neural Tracking Control for Unknown Output Feedback Nonlinear Time-delay Systems[1)]

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Abstract An adaptive output feedback neural network tracking controller is designed for a class of unknown output feedback nonlinear time-delay systems by using backstepping technique. Neural networks are used to approximate unknown time-delay functions. Delay-dependent filters are introduced for state estimation. The domination method is used to deal with the smooth time-delay basis functions. The adaptive bounding technique is employed to estimate the upper bound of the neural network reconstruction error. Based on Lyapunov-Krasoviskii functional, the semi-global uniform ultimate boundedness (SGUUB) of all the signals in the closed-loop system is proved. The arbitrary output tracking accuracy is achieved by tuning the design parameters and the neural node number. The feasibility is investigated by an illustrative simulation example.

Key words Nonlinear time-delay systems, neural network, backstepping

1 Introduction

Recently, neural network (NN) control has made great progress[1~6]. Due to their inherent approximation capability, NN control has the special advantage in cases where system modeling is difficult. In contrast to the early NN control approaches[1~2], where the optimization theory was used to design the parameter adaptive laws, the main advantage of adaptive NN control[3~6] is that the parameter adaptive law is derived based on the Lyapunov synthesis and therefore the stability of the closed-loop systems is guaranteed. By using backstepping technique, several important adaptive NN controllers have been proposed for some classes of uncertain nonlinear systems[3~6]. However, the nonlinear systems with unknown time-delay functions were not considered in [3~6]. On the other hand, the study for time-delay systems has also obtained some important results[7~9]. In [9], an adaptive NN control scheme was presented for a class of unknown strict feedback nonlinear time-delay systems. However, the NNs were only used to approximate the delay-independent unknown functions and the time-delay unknown functions were assumed to be bounded by the known upper bound functions. Due to the complexity and uncertainty of the controlled systems, this assumption is difficult to be satisfied.

In this paper, we propose an adaptive NN tracking control scheme for a class of unknown nonlinear time-delay systems. The NNs are employed to approximate the unknown time-delay functions and therefore the requirement on time-delay terms of the systems is relaxed. Finally, we provide a simulation example to illustrate the feasibility of the proposed approach.

2 Problem statement and preliminaries

Consider the following unknown output feedback nonlinear time-delay system

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(y) + h_i(y(t-\tau)), \quad 1 \leqslant i \leqslant n-1 \\ \dot{x}_n = u + f_n(y) + h_n(y(t-\tau)) \\ y = \boldsymbol{e}_i^{\mathrm{T}} \boldsymbol{x} \end{cases} \tag{1}$$

where $\boldsymbol{x} = [x_1, \cdots, x_n]^{\mathrm{T}} \in R^n$, $u \in R$, $y \in R$ represent the system state, control input and output, respectively. $f_i(\cdot), h_i(\cdot), (1 \leqslant i \leqslant n)$ are unknown smooth functions. $\boldsymbol{e}_i \in R^n$ denotes the n dimensional column vector whose elements are zero except that the i-th element is equal to one. Time delay τ is assumed known. The initial condition $y(t) = Q(t)$ is assumed to be bounded in the interval $[-\tau, 0]$.

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Only the output is available for measurement. The objective of this paper is to design an adaptive NN controller to track a given reference signal $y_r(t)$.

On a compact set $D \subset R$, $f_i(y)$ and $h_i(y(t-\tau))$ can be approximated by the following linearly parameterized NNs, respectively.

$$\begin{cases} f_i(y) = \boldsymbol{\rho}_i(y)^{\mathrm{T}}\boldsymbol{\theta}_{f_i} + \varepsilon_{f_i}(y), \quad i = 1,\cdots,n \\ h_i(y(t-\tau)) = \boldsymbol{\xi}_i(y(t-\tau))^{\mathrm{T}}\boldsymbol{\theta}_{h_i} + \varepsilon_{h_i}(y(t-\tau)), \quad i=1,\cdots,n \end{cases} \tag{2}$$

where $\boldsymbol{\rho}_i(\cdot) : D \to R^{p_i}$ and $\boldsymbol{\xi}_i(\cdot) : D \to R^{q_i}$ are smooth basis functions. $\varepsilon_{f_i}(\cdot)$ and $\varepsilon_{h_i}(\cdot)$ are the approximation errors. p_i and q_i are the NN node numbers. The optimal weights $\boldsymbol{\theta}_{f_i}$ and $\boldsymbol{\theta}_{h_i}$ are defined as $\boldsymbol{\theta}_{f_i} := \arg \min_{\hat{\boldsymbol{\theta}}_{f_i} \in R^{p_i}} \{\sup_{y\in D} |f_i(y) - \boldsymbol{\rho}_i(y)^{\mathrm{T}}\hat{\boldsymbol{\theta}}_{f_i}|\}$, and $\boldsymbol{\theta}_{h_i} := \arg \min_{\hat{\boldsymbol{\theta}}_{h_i} \in R^{q_i}} \{\sup_{y(t-\tau)\in D} |h_i(y(t-\tau)) - \boldsymbol{\xi}_i(y(t-\tau))^{\mathrm{T}}\hat{\boldsymbol{\theta}}_{h_i}|\}$, respectively.

Defining the NN reconstruction errors as $\upsilon_i(y, y(t-\tau)) := \varepsilon_{f_i}(y) + \varepsilon_{h_i}(y(t-\tau)), i = 1,\cdots,n$, and substituting (2) into (1), we have

$$\begin{cases} \dot{\boldsymbol{x}} = A\boldsymbol{x} + \phi(y)\boldsymbol{\theta}_1 + \Phi(y(t-\tau))\boldsymbol{\theta}_2 + \boldsymbol{\upsilon}(y, y(t-\tau)) + \boldsymbol{e}_n u \\ y = \boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{x} \end{cases} \tag{3}$$

where

$$\boldsymbol{\theta}_1 = [\boldsymbol{\theta}_{f_1}^{\mathrm{T}},\cdots,\boldsymbol{\theta}_{f_n}^{\mathrm{T}},\ \boldsymbol{\theta}_2 = [\boldsymbol{\theta}_{h_1}^{\mathrm{T}},\cdots,\boldsymbol{\theta}_{h_n}^{\mathrm{T}}]^{\mathrm{T}},\ \boldsymbol{\upsilon}(y, y(t-\tau)) = [\upsilon_1,\cdots,\upsilon_n]^{\mathrm{T}}$$

$$A = \begin{bmatrix} 0 & & \\ \vdots & I_{n-1} & \\ 0 & \cdots & 0 \end{bmatrix}_{n\times n},\ \phi(\cdot) = \begin{bmatrix} \boldsymbol{\rho}_1^{\mathrm{T}}(\cdot) & & \\ & \ddots & \\ & & \boldsymbol{\rho}_n^{\mathrm{T}}(\cdot) \end{bmatrix}_{n\times p},\ \Phi(\cdot) = \begin{bmatrix} \boldsymbol{\xi}_1^{\mathrm{T}}(\cdot) & & \\ & \ddots & \\ & & \boldsymbol{\xi}_n^{\mathrm{T}}(\cdot) \end{bmatrix}_{n\times q}$$

$p = p_1 + \cdots + p_n$, and $q = q_1 + \cdots + q_n$. We make the following assumptions.

Assumption 1[3]**.** On the compact set D, the NN reconstruction errors are assumed bounded with $|\upsilon_i(y, y(t-\tau))| \leqslant \bar{\upsilon}$, $i = 1,\cdots,n$, where $\bar{\upsilon}$ is an unknown constant.

Assumption 2[10]**.** The reference signal $y_r(t)$, and its first n derivatives are known and bounded uniformly in the interval $[-\tau,\infty)$. According to mean value theorem, we have

$$|\Phi_{i,j}(y) - \Phi_{i,j}(y_r)| \leqslant |y - y_r| s_{i,j}(y - y_r), \quad 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant q \tag{4}$$

where $X_{i,j}$ denotes the (i,j)-th element of matrix X. $s_{i,j}(\cdot)$ are smooth functions.

Throughout this paper, $\hat{*}$ denotes the estimate of unknown parameter $*$ with $\tilde{*} := * - \hat{*}$.

3 Adaptive NN controller design

For the system given by (3), define the time-delay filters as follows

$$\begin{cases} \dot{\boldsymbol{\xi}} = A_0\boldsymbol{\xi} + \boldsymbol{k}y, & \dot{\Xi} = A_0\Xi + \phi(y) \\ \dot{\Omega} = A_0\Omega + \Phi(y(t-\tau)), & \dot{\boldsymbol{\lambda}} = A_0\boldsymbol{\lambda} + \boldsymbol{e}u \end{cases} \tag{5}$$

where, the vector $\boldsymbol{k} = [k_1,\cdots,k_n]^{\mathrm{T}}$ is chosen so that matrix $A_0 = A - \boldsymbol{k}\boldsymbol{e}_1^{\mathrm{T}}$ is stable. This is to say that there exists a matrix $P > 0$ such that $PA_0 + A_0^{\mathrm{T}}P = -I$. The observer is designed as $\hat{\boldsymbol{x}} = \boldsymbol{\xi} + \Xi\boldsymbol{\theta}_1 + \Omega\boldsymbol{\theta}_2 + \lambda$ and the observer error $\boldsymbol{\varepsilon} = \boldsymbol{x} - \hat{\boldsymbol{x}}$ would be governed by

$$\dot{\boldsymbol{\varepsilon}} = A_0\boldsymbol{\varepsilon} + \boldsymbol{\upsilon} \tag{6}$$

It can be shown from (6) that the error system with state $\boldsymbol{\varepsilon}$ is input state stable with respect to the NN reconstruction error $\boldsymbol{\upsilon}$. From (5), the unavailable state x_2 is rewritten as $x_2 = \xi_2 + \Xi_{(2)}\boldsymbol{\theta}_1 + \Omega_{(2)}\boldsymbol{\theta}_2 + \lambda_2 + \varepsilon_2$, where $X_{(i)}$ denotes the i-th row of matrix X. Substituting x_2 into the first equation of system (3), we obtain

$$\dot{y} = \lambda_2 + \xi_2 + \boldsymbol{w}_{\boldsymbol{\theta}_1}^{\mathrm{T}}\boldsymbol{\theta}_1 + \boldsymbol{w}_{\boldsymbol{\theta}_2}^{\mathrm{T}}\boldsymbol{\theta}_2 + \Lambda_1 + \delta \tag{7}$$

where $\boldsymbol{w}_{\boldsymbol{\theta}_1}^{\mathrm{T}} = \Xi_{(2)} + \phi_{(1)}(y)$, $\boldsymbol{w}_{\boldsymbol{\theta}_2}^{\mathrm{T}} = \Omega_{(2)} + \Phi_{(1)}(y_r(t-\tau))$, $\Lambda_1 = [\Phi_1(y(t-\tau)) - \Phi_{(1)}(y_r(t-\tau))]\boldsymbol{\theta}_2$, $\delta = \varepsilon_2 + \upsilon_1$. Similarly to [6], we conclude that $\delta = \delta_\varepsilon + \delta_\upsilon$, where $\|\delta_\varepsilon| \leqslant l(\|\varepsilon(0)\|, t)$ with $l(x,t)$ strictly increasing in $x \in R^+$, $l(0,t)=0$, and $l(x,t)$ is a decreasing function of t and $\lim\limits_{t\to\infty} l(x,t)=0, \forall x \in R^+$. $|\delta_\upsilon| \leqslant \psi$, where ψ is an unknown constant.

From (7) and the fourth equality of (5), system (3) can be rewritten as follows

$$\begin{cases} \dot{y} = \lambda_2 + \xi_2 + \boldsymbol{w}_{\boldsymbol{\theta}_1}^{\mathrm{T}}\boldsymbol{\theta}_1 + \boldsymbol{w}_{\boldsymbol{\theta}_2}^{\mathrm{T}}\boldsymbol{\theta}_2 + \varLambda_1 + \delta_\varepsilon + \delta_\upsilon \\ \dot{\lambda}_i = \lambda_{i+1} - k_i\lambda_1, \quad 2 \leqslant i \leqslant n-1 \\ \dot{\lambda}_n = u - k_n\lambda_1 \end{cases} \tag{8}$$

For system (8), we define a change of coordinates,

$$z_1 = y - y_r,\ z_i = \lambda_i - y_r^{(i-1)} - \alpha_{i-1}, \quad i = 2, \cdots, n \tag{9}$$

Based on the backstepping technique, the stabilizing functions are given by

$$\begin{cases} \alpha_1 = -a_{11}z_1 - \xi_2 - \boldsymbol{w}_{\boldsymbol{\theta}_1}^{\mathrm{T}}\hat{\boldsymbol{\theta}}_1 - \boldsymbol{w}_{\boldsymbol{\theta}_2}^{\mathrm{T}}\hat{\boldsymbol{\theta}}_2 - \hat{\psi}\beta_1\tanh\left(\dfrac{\beta_1 z_1}{\varsigma}\right) \\ \alpha_i = -z_{i-1} - a_{ii}z_i + \dfrac{\partial\alpha_{i-1}}{\partial y}(\boldsymbol{w}_{\boldsymbol{\theta}_1}^{\mathrm{T}}\hat{\boldsymbol{\theta}}_1 + \boldsymbol{w}_{\boldsymbol{\theta}_2}^{\mathrm{T}}\hat{\boldsymbol{\theta}}_2) - \Delta_i - \hat{\psi}\beta_i\tanh\left(\dfrac{Z_i\beta_i}{\varsigma}\right) - \sum\limits_{j=2}^{i-1}\sigma_{j,i}z_j \end{cases} \tag{10}$$

where $2 \leqslant i \leqslant n$.

$$\beta_1 = -1,\ \beta_i = -\frac{\partial\alpha_{i-1}}{\partial y},\ \alpha_n = u - y_r^{(n)},\ z_{n+1} = 0,\ \varTheta := \|\boldsymbol{\theta}_2\|^2$$

$$a_{11} = c_1 + d_1 + W(z_1) + \frac{1}{2}\hat{\varTheta},\ a_{ii} = c_i + (d_i + \frac{1}{2}\hat{\varTheta})\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{q}\left(\frac{\partial\alpha_{i-1}}{\partial\varOmega_{j,k}}\right)^2,\ 2 \leqslant i \leqslant n$$

$$\begin{aligned} \Delta_i = & -k_i\lambda_1 - \frac{\partial\alpha_{i-1}}{\partial y}(\lambda_2 + \xi_2) - \sum_{k=1}^{2}\frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{\theta}}_k}\varGamma_{\boldsymbol{\theta}_k}(\boldsymbol{\tau}_{\boldsymbol{\theta}_k,i} - \iota\hat{\boldsymbol{\theta}}_k) - \frac{\partial\alpha_{i-1}}{\partial\hat{\varTheta}}\gamma_\varTheta(\tau_{\varTheta,i} - \iota\hat{\varTheta}) - \\ & \frac{\partial\alpha_{i-1}}{\partial\hat{\psi}}\gamma_\psi(\tau_{\psi,i} - \iota\hat{\psi}) - \frac{\partial\alpha_{i-1}}{\partial\boldsymbol{\xi}}(A_0\boldsymbol{\xi} + \boldsymbol{k}y) - \frac{\partial\alpha_{i-1}}{\partial\varXi}(A_0\varXi + \phi(y)) - \frac{\partial\alpha_{i-1}}{\partial\varOmega}(A_0\varOmega + \varPhi(y_r(t-\tau))) - \\ & \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\lambda_j}(-k_j\lambda_1 + \lambda_{j+1}) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(j-1)}}y_r^{(j)} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(j-1)}(t-\tau)}y_r^{(j)}(t-\tau), \quad 2 \leqslant i \leqslant n \end{aligned}$$

$$\sigma_{i,j} = \sum_{k=1}^{2}\frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{\theta}}_k}\varGamma_{\boldsymbol{\theta}_k}\frac{\partial\alpha_{i-1}}{\partial y}\boldsymbol{w}_{\boldsymbol{\theta}_k} - \frac{1}{2}\frac{\partial\alpha_{i-1}}{\partial\hat{\varTheta}}\gamma_\varTheta\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_j - \frac{\partial\alpha_{i-1}}{\partial\hat{\psi}}\gamma_\psi\beta_j\tanh\left(\frac{z_j\beta_j}{\varsigma}\right),\quad 2 \leqslant i \leqslant n$$

$c_i, d_i, \varsigma, \iota > 0$, $1 \leqslant i \leqslant n$, are the design parameters. $\varGamma_{\boldsymbol{\theta}_1}, \varGamma_{\boldsymbol{\theta}_2}, \gamma_\psi, \gamma_\varTheta > 0$ are the adaptive gains. $W(z_1) = \frac{1}{2}n\sum\limits_{j=1}^{q}s_{1,j}^2(z_1) + \frac{1}{2}(n-1)\sum\limits_{j=1}^{n}\sum\limits_{k=1}^{q}s_{j,k}^2(z_1)$.

The tuning functions $\boldsymbol{\tau}_{\boldsymbol{\theta}_1,i}, \boldsymbol{\tau}_{\boldsymbol{\theta}_2,i}, \tau_{\psi,i}, \tau_{\varTheta,i}$ are design as

$$\begin{cases} \boldsymbol{\tau}_{\boldsymbol{\theta}_1,1} = \boldsymbol{w}_{\boldsymbol{\theta}_1}z_1, \quad \boldsymbol{\tau}_{\boldsymbol{\theta}_1,i} = \boldsymbol{\tau}_{\boldsymbol{\theta}_1,i-1} - \dfrac{\partial\alpha_{i-1}}{\partial y}\boldsymbol{w}_{\boldsymbol{\theta}_1}z_i, & 2 \leqslant i \leqslant n \\ \boldsymbol{\tau}_{\boldsymbol{\theta}_2,1} = \boldsymbol{w}_{\boldsymbol{\theta}_2}z_1, \quad \boldsymbol{\tau}_{\boldsymbol{\theta}_2,i} = \boldsymbol{\tau}_{\boldsymbol{\theta}_2,i-1} - \dfrac{\partial\alpha_{i-1}}{\partial y}\boldsymbol{w}_{\boldsymbol{\theta}_2}z_i, & 2 \leqslant i \leqslant n \\ \tau_{\varTheta,1} = \frac{1}{2}z_1^2, \quad \tau_{\varTheta,i} = \tau_{\varTheta,i-1} + \frac{1}{2}\left(\dfrac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_i^2, & 2 \leqslant i \leqslant n \\ \tau_{\psi,1} = z_1\beta_1\tanh\left(\dfrac{\beta_1 z_1}{\varsigma}\right), \quad \tau_{\psi,i} = \tau_{\psi,i-1} + z_i\beta_i\tanh\left(\dfrac{z_i\beta_i}{\varsigma}\right), & 2 \leqslant i \leqslant n \end{cases} \tag{11}$$

The parameter adaptive laws are given by

$$\begin{cases} \dot{\hat{\boldsymbol{\theta}}}_1 = \varGamma_{\boldsymbol{\theta}_2,n}(\boldsymbol{\tau}_{\boldsymbol{\theta}_1,n} - \iota\hat{\boldsymbol{\theta}}_1), \quad \dot{\hat{\boldsymbol{\theta}}}_2 = \varGamma_{\boldsymbol{\theta}_2}(\boldsymbol{\tau}_{\boldsymbol{\theta}_2,n} - \iota\hat{\boldsymbol{\theta}}_2) \\ \dot{\hat{\varTheta}} = \gamma_\varTheta(\tau_{\varTheta,n} - \iota\hat{\varTheta}), \quad \dot{\hat{\psi}} = \gamma_\psi(\tau_{\psi,n} - \iota\hat{\psi}) \end{cases} \tag{12}$$

The control law is given by

$$u = \alpha_n + y_r^{(n)} \tag{13}$$

Substituting (10) into (9) and according ot (5) and (8), the derivative of z_i is given by

$$\begin{cases} \dot{z}_1 = -a_{11}z_1 + z_2 + \delta_\varepsilon + \delta_\upsilon + \boldsymbol{w}_{\boldsymbol{\theta}_2}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_1 + \boldsymbol{w}_{\boldsymbol{\theta}_2}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_2 - \hat{\psi}\beta_1 \tanh\left(\frac{\beta_1 z_1}{\varsigma}\right) + \Lambda_1 \\ \dot{z}_i = -z_{i-1} - a_{ii}z_i + z_{i+1} - \sum_{j=2}^{i-1}\sigma_{j,i}z_j + \sum_{j=i+1}^{n}\sigma_{i,j}z_j - \frac{\partial\alpha_{i-1}}{\partial y}\delta_\varepsilon - \frac{\partial\alpha_{i-1}}{\partial y}\delta_\upsilon - \\ \quad \frac{\partial\alpha_{i-1}}{\partial y}\boldsymbol{w}_{\boldsymbol{\theta}_1}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_1 - \frac{\partial\alpha_{i-1}}{\partial y}\boldsymbol{w}_{\boldsymbol{\theta}_2}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_2 - \hat{\psi}\beta_i \tanh\left(\frac{Z_i\beta_i}{\varsigma}\right) + \Lambda_i, \quad 2 \leqslant i \leqslant n \end{cases} \tag{14}$$

where $\Lambda_i = -\frac{\partial\alpha_{i-1}}{\partial y}\Lambda_1 - \frac{\partial\alpha_{i-1}}{\partial \Omega}[\Phi(y(t-\tau)) - \Phi(y_r(t-\tau))],\ 2 \leqslant i \leqslant n.$

4 The main results

Theorem 1. Under Assumptions 1 and 2, the closed-loop adaptive system consisting of plant (1), filters (5), the adaptive laws (12) and the control law (13) has the following properties: All the signals are SGUUB and the tracking error satisfies

$$\lim_{t\to\infty}\left[\left(\int_0^t |z_1(\sigma)|^2 \mathrm{d}\sigma\right)\Big/ t\right] \leqslant \ell/c_0 \tag{15}$$

where $\ell = n\kappa\psi\varsigma + \frac{1}{2}\iota(\|\boldsymbol{\theta}_1\|^2 + \|\boldsymbol{\theta}_2\|^2 + \Theta^2 + \psi^2),\ c_0 = \min\{c_1, \cdots, c_n\}$.

Proof. Define the error vector $\boldsymbol{z} = [z_1, \cdots, z_n]^{\mathrm{T}}$ and select a Lyapunov functional as

$$V = \frac{1}{2}[\boldsymbol{z}^{\mathrm{T}}\boldsymbol{z} + \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_{\boldsymbol{\theta}_1}^{-1}\tilde{\boldsymbol{\theta}}_1 + \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\Gamma_{\boldsymbol{\theta}_2}^{-1}\tilde{\boldsymbol{\theta}}_2 + \gamma_\Theta^{-1}\tilde{\Theta}^2 + \gamma_\psi^{-1}\tilde{\psi}^2] + \int_{t-\tau}^{t} S(z_1(\sigma))\mathrm{d}\sigma \tag{16}$$

where a nonnegative function $S(z_1(\sigma))$ is chosen as $S(z_1(\sigma)) = z_1^2(\sigma)W(z_1(\sigma))$. Along (12) and (14), the derivative of V is given by

$$\dot{V} = -\sum_{i=1}^{n} a_{ii}z_i^2 - \sum_{i=1}^{n} z_i\frac{\partial\alpha_{i-1}}{\partial y}\delta_\varepsilon - \sum_{i=1}^{n} z_i\frac{\partial\alpha_{i-1}}{\partial y}\delta_\upsilon - \sum_{i=1}^{n} z_i\beta_i\tanh\left(\frac{z_i\beta_i}{\varsigma}\right)\psi + \sum_{i=1}^{n} z_i\Lambda_i - \\ \frac{1}{2}\tilde{\Theta}\left[\sum_{i=1}^{n}\left(\frac{\partial\alpha_{i-1}}{\partial y}z_i\right)^2\right] + S(z_i) - S(z_1(t-\tau)) + \iota\left(\sum_{k=1}^{2}\tilde{\boldsymbol{\theta}}_k^{\mathrm{T}}\hat{\boldsymbol{\theta}}_k + \tilde{\psi}\hat{\psi} + \tilde{\Theta}\hat{\Theta}\right) \tag{17}$$

We use inequality $|\eta| \leqslant \eta\tanh(\eta/\varsigma) + \kappa\varsigma$[3], $\kappa = 0.2785$ to deal with $z_i\frac{\partial\alpha_{i-1}}{\partial y}\delta_\upsilon$, and employ (4) and Young′s inequality to deal with $z_i\Lambda_i$. Observing $\hat{\Theta} + \tilde{\Theta} = \|\boldsymbol{\theta}_2\|^2$ and

$$\sum_{k=1}^{2}\tilde{\boldsymbol{\theta}}_k^{\mathrm{T}}\hat{\boldsymbol{\theta}}_k + \tilde{\psi}\hat{\psi} + \tilde{\Theta}\hat{\Theta} \leqslant \frac{1}{2}\left(-\sum_{k=1}^{2}\|\tilde{\boldsymbol{\theta}}_k\|^2 - \tilde{\psi}^2 - \tilde{\Theta}^2 + \sum_{k=1}^{2}\|\boldsymbol{\theta}_k\|^2 + \psi^2 + \Theta^2\right)$$

we have

$$\dot{V} \leqslant -\sum_{i=1}^{n} c_i z_i^2 - \frac{1}{2}\iota\left(\sum_{k=1}^{2}\|\tilde{\boldsymbol{\theta}}_k\|^2 + \tilde{\psi}^2 + \tilde{\Theta}^2\right) + \sum_{i=1}^{n} + \frac{1}{4d_i}\delta_\varepsilon^2 + \frac{1}{2}\iota\left(\sum_{k=1}^{2}\|\boldsymbol{\theta}_k\|^2 + \psi^2 + \Theta^2\right) + n\psi\kappa\varsigma \tag{18}$$

Define $\boldsymbol{\pi} := [\boldsymbol{z}^{\mathrm{T}}, \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}, \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}, \tilde{\psi}, \tilde{\Theta}]^{\mathrm{T}}$, and we obtain

$$\dot{V}(\boldsymbol{\pi}) \leqslant -\bar{c}\|\boldsymbol{\pi}\|^2 + \sum_{i=1}^{n} + \frac{1}{4d_i}\delta_\varepsilon^2 + \ell \tag{19}$$

where $\bar{c} = \min\{c_0, \frac{1}{2}\iota\}$.

For $\lim_{t\to\infty}\delta_\varepsilon = 0$, integrating (18) from 0 to t, we obtain (15). In the light of the Lyapunov stability theory, (19) implies that π is bounded. □

5 Simulation study

Consider the following unknown time-delay nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 + (\sin(y^3(t-\tau)) + y(t-\tau))/(y^2(t-\tau)+1) \\ \dot{x}_2 = u + \sin(y)e^{-y^4}, \ y = x_1 \end{cases}$$

where the time delay $\tau = 5$. The reference signal is chosen as $y_r(t) = \sin(0.5t)\sin(0.2t)$.

In simulation, the initial conditions are set to $x_1(\sigma) = 0.2$, for $-\tau \leqslant \sigma \leqslant 0$, $x_2(0) = -0.6$. The other initial condition is set to be zero, $k = [1, 1]^{\mathrm{T}}$, $c_1 = c_2 = 0.02$, $d_1 = d_2 = 0.5$, $\Gamma_{\theta_1} = 0.5I$, $\Gamma_{\theta_2} = 0.8I$, $\gamma_\psi = \gamma_\Theta = 0.003$, $\varsigma = 10^{-5}$, $\iota = 0.001$. The basis function is chosen as $\phi_i(z) = \phi_i^0(z)\Big/\sum_{j=1}^{N}\phi_j^0(z), (i = 1, \cdots, N)$, where $\phi_i^0(z) = e^{-(z-\eta_i)^2/\bar{\varsigma}}$, $\bar{\varsigma} = \dfrac{1}{100\ln(2)}$, $N = 11$, η_i is the center of the ith basis function, and $\boldsymbol{D} = [-1, 1]$. Simulation results are shown in Figs 1 and 2.

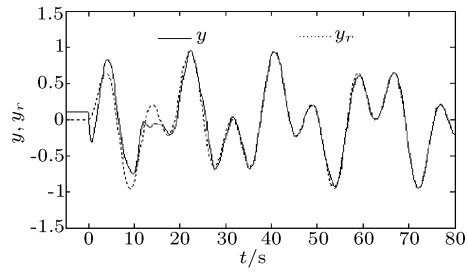

Fig. 1 The output $y(t)$ and the reference signal $y_r(t)$

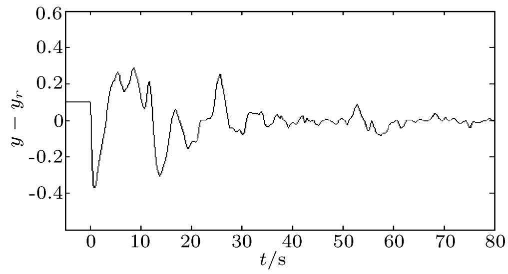

Fig. 2 The tracking error $y(t) - y_r(t)$

6 Conclusion

This paper extends the adaptive NN control approaches to a class of unknown output feedback nonlinear time-delay systems. However, the time delay is assumed to be known. How to relax the assumption is a problem to be resolved later.

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