Multivariable Model Reference Adaptive Control Using $\boldsymbol{K}_p = \boldsymbol{L}_2\boldsymbol{D}_2\boldsymbol{S}_2$ Factorization[1)]

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Abstract For a class of MIMO plants, using the idea of the high-frequency gain matrix $K_p = L_2D_2S_2$ factorization, the problem of design and analysis for MRAC is further investigated under the single assumption of known signs of the leading principal minors of K_p. By proving the L_p and $L_{2\delta}$ relationship properties between the input and the output and the multivariable swapping lemmas, the relation between all the signals in the closed-loop system and the normalized signal is obtained and the stability and tracking performance of the adaptive system is analyzed rigorously, the proof procedure being more compact.

Key words MRAC, normalizing signal, MIMO systems, $K_p = L_2D_2S_2$ factorization

1 Introduction

For the past 20 years, the study of adaptive control problem for MIMO continuous systems has been focused on by many researchers[1,2]. When using direct MRAC schemes, one drawback is that one has to assume stringent prior knowledge assumption on the high-frequency gain matrix K_p, *i.e.*, there exists a known matrix S_p such that $K_pS_p = (K_pS_p)^{\mathrm{T}} > 0$.

To relax the restriction, many researchers have done much work. Recently, in [3], by introducing three factorizations of K_p, only under the assumption that the signs of the leading principal minors of K_p were known, the authors gave the design and analysis of MRAC for ideal MIMO systems. In the stability analysis of [3], however, the proof of $\tilde{g} \in L_2$ was mistaken, while the property played an essential role in the analysis. In [4], by redefining the normalizing signal, Xie proved the conclusion rigorously.

The purpose of this paper is to further study MRAC problem by using $K_p = L_2D_2S_2$. The research idea originates from the following reasons: 1) Owing to the reason that three factorizations of K_p lead to different adaptive systems, this makes much difference in the analysis of stability. 2) For SISO systems, some important conclusions in [1], such as Lemma 3.3.2, the swapping Lemma A1 and A2, *etc.*, are often used in the analysis of adaptive controllers. While for the MIMO case, these conclusions are no longer applicable, which makes it difficult to extend the existing results for SISO systems to the MIMO case. By proving similar conclusions for MIMO systems, establishing the properties of adaptive laws, and redefining the normalizing signal, the relationship properties between the normalizing signal and all the signals in the closed-loop system can be established. Since the multivariable swapping lemmas are used, there is no need to decompose the sector into the single variable. This leads to the whole proof procedure more compact. 3) Stability of the closed-loop system and convergence of the tracking error are analyzed rigorously.

2 Problem statement

Consider the following MIMO system with zero initial value described by

$$\boldsymbol{y} = G(s)\boldsymbol{u} \tag{1}$$

where $\boldsymbol{u}, \boldsymbol{y} \in R^m$ are the input and output, respectively, s denotes the differential operator.

The control objective is to design an adaptive control law so that all the signals in the closed-loop system are bounded and the output tracks the output $\boldsymbol{y}_m$ of the following reference model

$$\boldsymbol{y}_m = W_m(s)\boldsymbol{r} \tag{2}$$

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where $W_m(s) \in R^{m\times m}[s]$, $\boldsymbol{r} \in R^m$ is any reference signal satisfying $\boldsymbol{r}, \dot{\boldsymbol{r}} \in L_\infty$. Define the tracking error as

$$\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{y}_m(t) \tag{3}$$

For (1), we need the following assumptions:

A1) The transmission zeros of $G(s)$ whose definition is given in [2] have negative real parts, and every element of $G^{-1}(s)$ is analytic in $\mathrm{Re}[s] \geqslant -\frac{\delta_0}{2}$ for some positive constant δ_0.

A2) $G(s)$ is strictly proper, has full rank and its modified left interactor matrix $\xi_m(s)$ whose definition is given in [1] is diagonal and known. $\xi_m(s)$ is defined as the modified left interactor matrix of $G(s)$ if the high frequency gain matrix $K_p = \lim_{s\to\infty} \xi_m(s)G(s)$ is finite and nonsingular.

A3) The observability index ν of $G(s)$ is known.

A4) The signs of leading principal minors of K_p are known.

The reference model (2) satisfies the following assumptions:

M1) All poles and zeros of $W_m(s)$ are stable, and every element is analytic in $\mathrm{Re}[s] \geqslant -\frac{\delta_0}{2}$.

M2) The zero structure at infinity of $W_m(s)$ is the same as that of $G(s)$. We choose $W_m(s) = \xi_m^{-1}(s)$.

Notations: For simplicity, we sometimes denote the time function $x(t)$ by x, the differential operator polynomial $X(s)$ by X, and the 2δ-norm $\|x_t\|_{2\delta}$ by $\|x\|$, where $\|x_t\|_{2\delta} = [\int_0^t e^{-\delta(t-\tau)}|x(\tau)|^2 \mathrm{d}\tau]^{1/2}$ for any $\delta > 0$ and $x(t) \in R^m$; $\|x_t\|_{2\delta} = \sum_{i=1}^{n} \|(\boldsymbol{x}_i)_t\|_{2\delta}$ for any $x = (\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n) \in R^{m\times n}$. c denotes some positive constant independent of μ_1 and μ_2.

3 The design of MRAC using $\boldsymbol{K}_p = \boldsymbol{L}_2\boldsymbol{D}_2\boldsymbol{S}_2$ factorization

Firstly, we give an important lemma on K_p factorization.

Lemma 1[3]**.** Every real matrix $K_p \in R^{m\times m}$ with nonzero leading principal minors $\sigma_1, \cdots, \sigma_m$ can be factored as $K_p = L_2D_2S_2$, where L_2 is a unity lower triangular matrix, S_2 is a positive definite matrix, $D_2 = \varGamma \mathrm{sgn}(D)$, $D = \mathrm{diag}\{\sigma_1, \frac{\sigma_2}{\sigma_1}, \cdots, \frac{\sigma_m}{\sigma_{m-1}}\}$, with $\varGamma$ being an arbitrary positive diagonal matrix.

Next we use Lemma 1 to design a direct MRAC. As in [3], one gets

$$\xi_m(s)\boldsymbol{e} = K_p(\boldsymbol{u} - \theta^{*\mathrm{T}}\boldsymbol{\omega}) = L_2D_2S_2(\boldsymbol{u} - \theta^{*\mathrm{T}}\boldsymbol{\omega}) \tag{4}$$

where $\theta^{*T}\boldsymbol{\omega} = \theta_1^{*\mathrm{T}}\omega_1, +\theta_2^{*\mathrm{T}}\omega_2, +\theta_3^* y + \theta_4^* r$, $\theta^{\mathrm{T}} = (\theta_1^{*\mathrm{T}}, \theta_2^{*\mathrm{T}}, \theta_3^*, \theta_4^*)$, $\boldsymbol{\omega} = (\boldsymbol{\omega}_1^{\mathrm{T}}, \boldsymbol{\omega}_2^{\mathrm{T}}, \boldsymbol{y}^{\mathrm{T}}, \boldsymbol{r}^{\mathrm{T}})^{\mathrm{T}}$, $\theta_1^*, \theta_2^* \in R^{m(\nu-1)\times m}$, $\theta_3^*, \theta_4^* \in R^{m\times m}$, $\boldsymbol{\omega}_1 = \frac{\gamma(s)}{p(s)}\theta^{*\mathrm{T}}\boldsymbol{\omega}$, $\boldsymbol{\omega}_2 = \frac{\gamma(s)}{p(s)}\boldsymbol{y}$, $\gamma(s) = (I, Is, \cdots, Is^{\nu-2})^{\mathrm{T}}$, $p(s) = \lambda_0 + \lambda_1 s + \cdots + s^{\nu-1}$ is an arbitrary Hurwitz polynomial, and $\frac{1}{p(s)}$ is analytic in $Re[s] \geqslant -\frac{\delta_0}{2}$. The adaptive control law is chosen as

$$\boldsymbol{u} = \theta^{\mathrm{T}}\boldsymbol{\omega} \tag{5}$$

where θ is the estimate of θ^*. Set $\varPsi^* = D_2S_2$ and define

$$W(s) = \frac{1}{f(s)} \tag{6}$$

where $f(s)$ is a Hurwitz polynomial with degree equal to the largest relative degree n^* of all elements of $W_m(s)$, and $W(s)$ is analytic in $Re[s] \geqslant -\frac{\delta_0}{2}$. Noting that L_2 and $\varPsi^*$ are constants, by Lemma 1, one has

$$L_2^{-1}W(s)\xi_m(s)\boldsymbol{e} = \varPsi^*(W(s)\boldsymbol{u} - W(s)\theta^{*\mathrm{T}}\boldsymbol{\omega}) \tag{7}$$

Define

$$\boldsymbol{z} = -W(s)\xi_m(s)\boldsymbol{e},\ \boldsymbol{z}_0 = -W(s)\boldsymbol{u},\ \boldsymbol{\phi} = W(s)\boldsymbol{\omega} \tag{8}$$

Similar to [3], one gets

$$\boldsymbol{z} = (0, \boldsymbol{\varLambda}_2^{*\mathrm{T}}\boldsymbol{\zeta}_2, \cdots, \boldsymbol{\varLambda}_m^{*\mathrm{T}}\boldsymbol{\zeta}_m)^{\mathrm{T}} + \varPsi^*(\theta^{*\mathrm{T}}\boldsymbol{\phi} + \boldsymbol{z}_0) \tag{9}$$

where $\zeta_i^{\mathrm{T}} = (z_1, z_2, \cdots, z_{i-1}), i = 2, \cdots, m$. Choose the estimate of z as

$$\hat{\boldsymbol{z}} = (0, \boldsymbol{\varLambda}_2^{\mathrm{T}}\boldsymbol{\zeta}_2, \cdots, \boldsymbol{\varLambda}_m^{\mathrm{T}}\boldsymbol{\zeta}_m)^{\mathrm{T}} + \varPsi(\theta^{\mathrm{T}}\boldsymbol{\phi} + \boldsymbol{z}_0) \tag{10}$$

where Λ, Ψ, θ are the estimates of Λ_i^*, Ψ^*, θ^*, respectively, and define

$$\tilde{\Psi} = \Psi - \Psi^*, \tilde{\theta} = \theta - \theta^*, \tilde{\boldsymbol{\Lambda}}_i = \boldsymbol{\Lambda}_i - \boldsymbol{\Lambda}_i^*, \boldsymbol{\xi} = \theta^{\rm T}\boldsymbol{\phi} + \boldsymbol{z}_0, \aleph^{\rm T} = (0, \tilde{\boldsymbol{\Lambda}}_2^{\rm T}\boldsymbol{\zeta}_2, \cdots, \tilde{\boldsymbol{\Lambda}}_m^{\rm T}\boldsymbol{\zeta}_m) \tag{11}$$

Then the normalized estimation error is defined as

$$\boldsymbol{\varepsilon} = \frac{z - \hat{z}}{\eta_s^2} = -\frac{1}{\eta_s^2}(\aleph + \tilde{\Psi}\boldsymbol{\xi} + \Psi^*\tilde{\theta}^{\rm T}\boldsymbol{\phi}) \tag{12}$$

where $\eta_s^2 = 1 + n_s^2$, $n_s^2 = \|\boldsymbol{u}_t\|_{2\delta_0}^2 + \|\boldsymbol{y}_t\|_{2\delta_0}^2$, δ_0 is the same as that in assumption A1. Set

$$\theta^{*{\rm T}} = \begin{pmatrix} \boldsymbol{\theta}_{11}^{*{\rm T}} & \boldsymbol{\theta}_{12}^{*{\rm T}} & \boldsymbol{\theta}_{13}^* & \boldsymbol{\theta}_{14}^* \\ & \cdots & \cdots & \\ \boldsymbol{\theta}_{m1}^{*{\rm T}} & \boldsymbol{\theta}_{m2}^{*{\rm T}} & \boldsymbol{\theta}_{m3}^* & \boldsymbol{\theta}_{m4}^* \end{pmatrix}, \ \Psi = \begin{pmatrix} \Psi_{11}^* & \Psi_{12}^* & \cdots & \Psi_{1m}^* \\ & \cdots & \cdots & \\ \Psi_{m1}^* & \Psi_{m2}^* & \cdots & \Psi_{mm}^* \end{pmatrix} \tag{13}$$

where $\theta_{i1}^{*{\rm T}}, \theta_{i2}^{*{\rm T}} \in R^{1\times m(\nu-1)}, \theta_{i3}^*, \theta_{i4}^* \in R^{1\times m}, i = 1, \cdots, m$. Since S_2 is constant and positive definite, it is assumed that there exist two known positive constants m_1 and m_2 such that $m_2 I \leqslant S_2 \leqslant m_1 I$. Now, one needs to prove a useful lemma.

Lemma 2. Define $\bar{S}_2 = S_2 \otimes I$, where $\otimes$ expresses the Kronecker product. Then

1) If S_2 is positive definite, then $\bar{S}_2$ is also positive definite; 2) $m_2 I \leqslant \bar{S}_2 \leqslant m_1 I$.

Proof. The proof is omitted due to the limited space.

Let us now assume that there exists known constant M_0 such that $|\boldsymbol{\Lambda}_i| \leqslant M_0$, $|\bar{\boldsymbol{\Psi}}| \leqslant M_0$, $|\bar{\boldsymbol{\theta}}^*| \leqslant m_2 M_0/m_1$, and choose the adaptive laws

$$\dot{\boldsymbol{\Lambda}}_i = \begin{cases} \gamma_{L_i}\boldsymbol{\varepsilon}\boldsymbol{\zeta}, \ \text{if } |\boldsymbol{\Lambda}_i| < M_0, \ \text{or } |\boldsymbol{\Lambda}_i| = M_0, \ \text{and } (\gamma_{L_i}\boldsymbol{\varepsilon}_i\boldsymbol{\zeta}_i)^{\rm T}\boldsymbol{\Lambda}_i \leqslant 0 \\ \left(I - \dfrac{\boldsymbol{\Lambda}_i\boldsymbol{\Lambda}_i^{\rm T}}{\boldsymbol{\Lambda}_i^{\rm T}\boldsymbol{\Lambda}_i}\right)\gamma_{L_i}\boldsymbol{\varepsilon}_i\boldsymbol{\zeta}_i, \ \text{otherwise} \end{cases}$$

$$\dot{\bar{\boldsymbol{\Psi}}} = \begin{cases} \gamma_\Psi\boldsymbol{\varepsilon} \otimes \boldsymbol{\xi}, \ \text{if } |\bar{\boldsymbol{\Psi}}| < M_0, \ \text{or } |\bar{\boldsymbol{\Lambda}}| = M_0 \text{ and } (\gamma_\Psi\boldsymbol{\varepsilon} \otimes \boldsymbol{\xi})^{\rm T}\bar{\Psi} \leqslant 0 \\ \left(I - \dfrac{\bar{\boldsymbol{\Psi}}\bar{\boldsymbol{\Psi}}^{\rm T}}{\bar{\boldsymbol{\Psi}}^{\rm T}\bar{\boldsymbol{\Psi}}}\right)\gamma_\Psi\boldsymbol{\varepsilon} \otimes \boldsymbol{\xi}, \ \text{otherwise} \end{cases} \tag{14}$$

$$\dot{\bar{\boldsymbol{\theta}}} = \begin{cases} [\Gamma\text{sgn}(D)\boldsymbol{\varepsilon}] \otimes \boldsymbol{\phi}, \ \text{if } |\bar{\boldsymbol{\theta}}| < M_0, \ \text{or } |\bar{\boldsymbol{\theta}}| = M_0 \text{ and } ([\Gamma\text{sgn}(D)\boldsymbol{\varepsilon}] \otimes \boldsymbol{\phi})^{\rm T}\bar{\boldsymbol{\theta}} \leqslant 0 \\ \left(I - \dfrac{\bar{\boldsymbol{\theta}}\bar{\boldsymbol{\theta}}^{\rm T}}{\bar{\boldsymbol{\theta}}^{\rm T}\bar{\boldsymbol{\theta}}}\right)\Gamma\text{sgn}(D)\boldsymbol{\varepsilon} \otimes \boldsymbol{\phi}, \ \text{otherwise} \end{cases}$$

where $\bar{\boldsymbol{\Psi}}^* = (\boldsymbol{\Psi}_{11}^*, \cdots, \boldsymbol{\Psi}_{1m}^*, \cdots, \boldsymbol{\Psi}_{m1}^*, \cdots, \boldsymbol{\Psi}_{mm}^*)^{\rm T}$, $\bar{\theta}^{*{\rm T}} = (\boldsymbol{\theta}_{11}^{*{\rm T}}, \boldsymbol{\theta}_{12}^{*{\rm T}}, \theta_{13}^*, \theta_{14}^*, \cdots, \boldsymbol{\theta}_{m1}^{*{\rm T}}, \boldsymbol{\theta}_{m2}^{*{\rm T}}, \theta_{m3}^*, \theta_{m4}^*)$, $\bar{\boldsymbol{\Psi}}, \bar{\boldsymbol{\theta}}$ are the estimates of $\bar{\boldsymbol{\Psi}}^*, \bar{\boldsymbol{\theta}}^*$, ε_i is the ith element of ε. $\gamma_{L_i}, \gamma_\Psi, \Gamma > 0, i = 2, 3, \cdots, m$. The initial values are chosen to satisfy $|\boldsymbol{\Lambda}_i(0)| \leqslant M_0$, $\bar{P}P(0)| \leqslant M_0$, $|\bar{\boldsymbol{\theta}}(0)| \leqslant M_0$. □

4 Main results

With (5), one gets

$$\tilde{\boldsymbol{u}} = \boldsymbol{u} - \theta^{*{\rm T}}\boldsymbol{\omega} = \tilde{\theta}^{\rm T}\boldsymbol{\omega} \tag{15}$$

From (15), $\xi_m(s) = W_m^{-1}(s)$,(1), (2) and (4), it follows that the input and output of the closed-loop system are

$$\boldsymbol{u} = G^{-1}(s)W_m(s)(\boldsymbol{r} + L_2\Psi^*\tilde{\boldsymbol{u}}), \ y = W_m(s)(\boldsymbol{r} + L_2\Psi^*\tilde{\boldsymbol{u}}) \tag{16}$$

We first give a few useful lemmas.

Lemma 3[5]**.** Consider the system $\boldsymbol{y} = H(s)\boldsymbol{u}$, where $\boldsymbol{y}, \boldsymbol{u} \in R^m$, and $H(s) = (h_{ik}(s)) \in R^{m\times m}[s]$. Let every element of $H(s)$ be analytic in $\text{Re}[s] \geqslant -\frac{\delta}{2}$, where $\delta > 0$ is any constant.

1) If every element of $H(s)$ is proper, then $\|\boldsymbol{y}_t\|_{2\delta} \leqslant \|H(s)\|_{\infty\delta}\|\boldsymbol{u}_t\|_{2\delta}$.

2) Furthermore, when every element of $H(s)$ is strictly proper, then $|\boldsymbol{y}(t)| \leqslant \|H(s)\|_{2\delta}\|\boldsymbol{u}_t\|_{2\delta}$, where $\|H(s)\|_{\infty\delta} = \sum\limits_{i,k=1}^m \|h_{ik}(s)\|_{\infty\delta}$, $\|H(s)\|_{\infty\delta} = \sum\limits_{i,k=1}^m \|h_{ik}(s)\|_{2\delta}$, $\|h_{ik}(s)\|_{\infty\delta} = \sup_w |h_{ik}(jw - \frac{\delta}{2})|$, $\|h_{ik}(s)\|_{2\delta} = \dfrac{1}{\sqrt{2p-\delta}}\|(s+p)h_{ik}(s)\|_{\infty\delta}, \forall p > \delta/2$.

Lemma 4. (Multivariable swapping) For any $\vartheta : R^+ \mapsto R^{n\times m}$, $\boldsymbol{\varphi} : R^+ \mapsto R^n$, where ϑ is differentiable, $W(s)$ is defined as (6) with a minimal realization $(A, \boldsymbol{B}, \boldsymbol{C})$. Then

$$W(s)(\vartheta^{\rm T}\boldsymbol{\varphi}) = \vartheta^{\rm T}W(s)\boldsymbol{\varphi} - \left(\boldsymbol{C}\int_0^t e^{A(t-\tau)}X(\tau)\dot{\vartheta}(\tau)\text{d}\tau\right)^{\rm T}$$

where $X=(sI-A)^{-1}(B\boldsymbol{\varphi}^{\mathrm{T}})$.

Proof. See Section 5.

Lemma 5. (Multivariable swapping) Let $\vartheta: R^+ \mapsto R^{n\times m}$, $\varphi: R^+ \mapsto R^n$, and ϑ,φ be differentiable. Then

$$\vartheta^{\mathrm{T}}\boldsymbol{\varphi}=F_1(s,\alpha_1,\cdots,\alpha_m)(\dot{\vartheta}^{\mathrm{T}}\boldsymbol{\varphi}+\vartheta^{\mathrm{T}}\dot{\boldsymbol{\varphi}})+F(s,\alpha_1,\cdots,\alpha_m)(\vartheta^{\mathrm{T}}\boldsymbol{\varphi})$$

where $F(s,\alpha_1,\cdots,\alpha_m)=\mathrm{diag}\left\{\dfrac{\alpha_1^k}{(s+\alpha_1)^k},\cdots,\dfrac{\alpha_m^k}{(s+\alpha_m)^k}\right\}$, $F_1(s,\alpha_1,\cdots,\alpha_m)=\dfrac{I-F(s,\alpha_1,\cdots,\alpha_m)}{s}$, $k\geqslant 1$, α_i are arbitrary constants, $i=1,\cdots,m$. Furthermore, for $\alpha_0=\min\{\alpha_1,\cdots,\alpha_m\}>\delta$, $\|F_1(s,\alpha_1,\cdots,\alpha_m)\|_{\infty\delta}\leqslant\frac{c}{\alpha_0}$, where c is a positive constant independent of α_i, $\delta>0$ is any given constant.

Proof. See Section 5.

Define the normalizing signal η_s and the fictitious normalizing signal η_f as follows:

$$\eta_s=(1+\|\boldsymbol{u}_t\|^2_{2\delta_0}+\|\boldsymbol{y}_t\|^2_{2\delta_0})^{1/2},\ \eta_f(t)=(1+\|\boldsymbol{u}_t\|^2_{2\delta}+\|\boldsymbol{y}_t\|^2_{2\delta})^{1/2} \tag{17}$$

for any $\delta\in(0,\delta_0]$, where $n_s(t)$ is defined in Lemma 6.

Lemma 6. All the signals in the closed-loop system have the following properties:

1) $\boldsymbol{\Lambda}_i$, $\bar{\boldsymbol{\Psi}},\bar{\boldsymbol{\theta}}$, $\Psi,\theta\in L_\infty, i=2,\cdots,m$;

2) $\boldsymbol{\omega}_i/\eta_f, \|\boldsymbol{\omega}_i\|/\eta_f, \|\boldsymbol{\omega}\|/\eta_f, \eta_s/\eta_f, \boldsymbol{y}/\eta_f, \boldsymbol{u}/\eta_f, \boldsymbol{\omega}/\eta_f\in L_\infty, i=1,2$.

3) $W_1(s)\boldsymbol{\omega}/\eta_f, \boldsymbol{\xi}/\eta_f, \boldsymbol{z}_i/\eta_f, \boldsymbol{\zeta}_i/\eta_f, \boldsymbol{\phi}/\eta_f, \boldsymbol{\aleph}/\eta_f\in L_\infty, i=1,\cdots,m$, where every element of $W_1(s)$ is strictly proper and analytic in $Re[s]\geqslant-\frac{\delta_0}{2}$.

4) $\|\dot{\boldsymbol{y}}\|/\eta_f\in L_\infty$. If $\dot{\boldsymbol{r}}\in L_\infty$, thus $\|\dot{\boldsymbol{\omega}}\|/\eta_f\in L_\infty$.

5) When $\delta=\delta_0$, 1)~4) are satisfied by replacing η_f with η_s.

6) $\dot{\boldsymbol{\Lambda}}_i$, $\dot{\bar{\boldsymbol{\Psi}}},\dot{\bar{\boldsymbol{\theta}}}$, $\dot{\Psi},\dot{\theta},\varepsilon,\varepsilon\eta_s,\varepsilon n_s\in L_\infty\cap L_2$.

7) $\boldsymbol{\xi}/\eta_s, z_i/\eta_s, \boldsymbol{\zeta}_i/\eta_s, \boldsymbol{\aleph}/\eta_s\in L_2, i=1,\cdots,m$, where $n_s^2=\eta_s^2-1, \zeta_1=0$.

Proof. The proof is given by following the same approach as in [4].

We are now in a position to state main results in this paper.

Theorem 1. Consider the direct MRAC based on $K_p=L_2D_2S_2$ consisting of system (1), reference model (2), control law (5), and adaptive laws (14). If assumptions A1)~A4) and M1)~M2) hold, then

1) All the signals in the closed-loop system are uniformly bounded.

2) $\lim\limits_{t\to\infty}\boldsymbol{e}(t)=0$.

Proof. By (15)~(17) and using Lemma 3, one has

$$\eta_f^2\leqslant c+c\|\tilde{\boldsymbol{u}}\|^2\leqslant c+c\|\tilde{\theta}^{\mathrm{T}}\boldsymbol{\omega}\|^2 \tag{18}$$

By Lemma 5, one has

$$\tilde{\theta}^{\mathrm{T}}\omega=F_1(s,\alpha_1,\cdots,\alpha_m)(\dot{\tilde{\theta}}^{\mathrm{T}}\boldsymbol{\omega}+\tilde{\theta}^{\mathrm{T}}\dot{\boldsymbol{\theta}})+F(s,\alpha_1,\cdots,\alpha_m)(\tilde{\theta}^{\mathrm{T}}\boldsymbol{\omega}) \tag{19}$$

where $F=\mathrm{diag}\left\{\dfrac{\alpha_1^{n^*}}{(s+\alpha_1)^{n^*}},\cdots,\dfrac{\alpha_m^{n^*}}{(s+\alpha_m)^{n^*}}\right\}$, $F_1=\dfrac{I-F_2(s,\alpha_1,\cdots,\alpha_m)}{s}$, $\alpha_1,\cdots,\alpha_m>\delta$ are some constants, and n^* is defined as (6). Applying Lemma 4, one has

$$\begin{aligned}\tilde{\theta}\boldsymbol{\omega}&=W^{-1}(s)\left[\tilde{\theta}^{\mathrm{T}}W(s)\boldsymbol{\omega}-(\boldsymbol{C}\int_0^t e^{A(t-\tau)}X(\tau)\dot{\tilde{\theta}}(\tau)\mathrm{d}\tau)^{\mathrm{T}}\right]\\ \theta^{\mathrm{T}}\boldsymbol{\omega}&=W^{-1}(s)\left[\theta^{\mathrm{T}}W(s)\boldsymbol{\omega}-(\boldsymbol{C}\int_0^t e^{A(t-\tau)}X(\tau)\dot{\theta}(\tau)\mathrm{d}\tau)^{\mathrm{T}}\right]\end{aligned} \tag{20}$$

where $W(s)$ is defined as (6). Combining (19) with (20) leads to

$$\tilde{\theta}^{\mathrm{T}}\boldsymbol{\omega}=F_1(\dot{\tilde{\theta}}^{\mathrm{T}}\boldsymbol{\omega}+\tilde{\theta}^{\mathrm{T}}\dot{\boldsymbol{\omega}})+FW^{-1}(s)\left[\tilde{\theta}^{\mathrm{T}}W(s)\boldsymbol{\omega}-(\boldsymbol{C}\int_0^t e^{A(t-\tau)}X(\tau)\dot{\tilde{\theta}}(\tau)\mathrm{d}\tau)^{\mathrm{T}}\right] \tag{21}$$

By Lemma 1, $\Psi^*=D_2S_2$ is nonsingular. Hence from (3.9), it follows that

$$\tilde{\theta}^{\mathrm{T}}\boldsymbol{\phi}=-(\Psi^*)^{-1}(\varepsilon\eta_s^2+\boldsymbol{\aleph}+\Psi\boldsymbol{\xi})+\boldsymbol{\xi} \tag{22}$$

By (5), (8), and (9), one obtains

$$-\boldsymbol{z}_0 = W(s)\boldsymbol{u} = W(s)(\theta^{\mathrm{T}}\boldsymbol{\omega}) = \theta^{\mathrm{T}}\boldsymbol{\phi} - (\boldsymbol{C}\int_0^t e^{A(t-\tau)}X(\tau)\dot{\theta}(\tau)\mathrm{d}\tau)^{\mathrm{T}} \tag{23}$$

which implies by (11) and $\dot{\theta} = \dot{\tilde{\theta}}$. that

$$\boldsymbol{\xi} = (\boldsymbol{C}\int_0^t e^{A(t-\tau)}X(\tau)\dot{\theta}(\tau)\mathrm{d}\tau)^{\mathrm{T}} = (\boldsymbol{C}\int_0^t e^{A(t-\tau)}X(\tau)\dot{\tilde{\theta}}(\tau)\mathrm{d}\tau)^{\mathrm{T}} \tag{24}$$

Combining (10), (11), and (13), one concludes that

$$\tilde{\theta}^{\mathrm{T}}\boldsymbol{\omega} = F_1(\dot{\tilde{\theta}}^{\mathrm{T}}\boldsymbol{\omega} + \tilde{\theta}^{\mathrm{T}}\dot{\boldsymbol{\omega}}) - FW^{-1}(s)(\Psi^*)^{-1}[\boldsymbol{\varepsilon}\eta_s^2 + \boldsymbol{\aleph} + \Psi(\boldsymbol{C}\int_0^t e^{A(t-\tau)}X(\tau)\dot{\tilde{\theta}}(\tau)\mathrm{d}\tau)^{\mathrm{T}}] \tag{25}$$

By taking the 2δ-norm on both sides of (14), and using Lemma 5 and $\Psi \in L_\infty$, there exists a constant c independent of α_0 and μ such that

$$\|\tilde{\theta}^{\mathrm{T}}\boldsymbol{\theta}\| = \frac{c}{\alpha_0}(\|\dot{\tilde{\theta}}^{\mathrm{T}}\boldsymbol{\omega}\| + \|\tilde{\theta}^{\mathrm{T}}\dot{\boldsymbol{\omega}}\|) + c\alpha_0^{n^*}(\|\boldsymbol{\varepsilon}\eta_s^2\| + \|\boldsymbol{\aleph}\| + \|\boldsymbol{C}\int_0^t e^{A(t-\tau)}X(\tau)\dot{\tilde{\theta}}(\tau)\mathrm{d}\tau\|) \tag{26}$$

By Lemma 4, $X = (s\boldsymbol{I} - A)^{-1}\boldsymbol{B}\boldsymbol{\omega}^{\mathrm{T}} = \int_0^t e^{A(t-\tau)}\boldsymbol{B}\boldsymbol{\omega}^{\mathrm{T}}(\tau)\mathrm{d}\tau$. Hence by (17) and Lemmas 3 and 6, one gets

$$\|\boldsymbol{\varepsilon}\eta_s^2\| \leqslant \|\boldsymbol{\varepsilon}\eta_f\| + \|\boldsymbol{\varepsilon}n_s\eta_f\| \tag{27}$$

$$\|\boldsymbol{C}\int_0^t e^{A(t-\tau)}X(\tau)\dot{\tilde{\theta}}(\tau)\mathrm{d}\tau\| \leqslant c\|\dot{\tilde{\theta}}\eta_f\| \tag{28}$$

Using $\|\eta_f\| \geqslant 1$ and Lemma 6 leads to $\|\dot{\tilde{\theta}}^{\mathrm{T}}\boldsymbol{\omega}\| \leqslant c\|\dot{\tilde{\theta}}_f\|$, $\|\tilde{\theta}^{\mathrm{T}}\dot{\boldsymbol{\omega}}\| \leqslant c\|\dot{\boldsymbol{\omega}}\| \leqslant c\eta_f$, $\|\boldsymbol{\varepsilon}\| \leqslant \|\boldsymbol{\varepsilon}\eta_f\|$, $\|\boldsymbol{\varepsilon}n_s^2\| \leqslant \|\boldsymbol{\varepsilon}n_s\eta_f\|$, which together with (26)$\sim$(28) imply that

$$\|\tilde{\theta}^{\mathrm{T}}\boldsymbol{\omega}\| \leqslant \frac{1}{\alpha_0}c\eta_f + c\alpha_0^{n^*}\|\tilde{g}\eta_f\| \tag{29}$$

by choosing $\alpha_1, \cdots, \alpha_m$ to satisfy that $\frac{1}{\alpha_0} < \alpha_0^{n^*}$, where $|\tilde{g}|^2 = |\boldsymbol{\varepsilon}|^2 + |\dot{\tilde{\theta}}|^2 + |\boldsymbol{\varepsilon}n_s|^2 + \frac{|\aleph|^2}{\eta_s^2}$. By Lemma 6, one has $\tilde{g} \in L_2$. By following the similar approach as in [4], the conclusions can be proved. □

5 Proof of Lemmas 4 and 5

Proof of Lemma 4. Set $\boldsymbol{\phi} = W(s)\boldsymbol{\omega}$. Since $f(s)$ is Hurwitz polynomial, $W(s)$ can be realized as $\dot{X} = AX + \boldsymbol{B}\boldsymbol{\omega}^{\mathrm{T}}$, $\boldsymbol{\phi}^{\mathrm{T}} = \boldsymbol{C}X$, $X(0) = 0$, where A is a stable matrix. Obviously, $\tilde{\theta}^{\mathrm{T}}W(s)\boldsymbol{\omega} = \tilde{\theta}^{\mathrm{T}}\boldsymbol{\phi} = \tilde{\theta}^{\mathrm{T}}X^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}$. Similarly, $\boldsymbol{\psi} = W(s)(\tilde{\theta}^{\mathrm{T}}\boldsymbol{\omega})$ can be realized as $\dot{Y} = AY + \boldsymbol{B}\boldsymbol{\omega}^{\mathrm{T}}\tilde{\theta}$, $\boldsymbol{\psi}^{\mathrm{T}} = \boldsymbol{C}Y$, $Y(0) = 0$, from which, $W(s)(\tilde{\theta}^{\mathrm{T}}\boldsymbol{\omega}) = Y^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}$. Obviously, $\frac{\mathrm{d}}{\mathrm{d}t}(X\tilde{\theta} - Y) = A(X\tilde{\theta} - Y) + X\dot{\tilde{\theta}}$, whose solution is $X\tilde{\theta} - Y = \int_0^t e^{A(t-\tau)}X(\tau)\dot{\tilde{\theta}}(\tau)\mathrm{d}\tau$ due to $X(0)\tilde{\theta}(0) - Y(0) = 0$. Thus $\tilde{\theta}^{\mathrm{T}}W(s)\boldsymbol{\omega} - W(s)(\tilde{\theta}^{\mathrm{T}}\boldsymbol{\omega}) = (X\tilde{\theta} - Y)^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}} = (\boldsymbol{C}\int_0^t e^{A(t-\tau)}X(\tau)\dot{\tilde{\theta}}(\tau)\mathrm{d}\tau)^{\mathrm{T}}$. □

Proof of Lemma 5. Set $\vartheta^{\mathrm{T}} = (\boldsymbol{\vartheta}_1, \cdots, \boldsymbol{\vartheta}_m)^{\mathrm{T}}$, $\boldsymbol{\vartheta}_i \in R^n$. Applying Lemma A.2 in [1], one has

$$\tilde{\theta}^{\mathrm{T}}\boldsymbol{\omega} = \begin{pmatrix} \tilde{\theta}_1^{\mathrm{T}}\boldsymbol{\omega} \\ \vdots \\ \tilde{\theta}_m^{\mathrm{T}}\boldsymbol{\omega} \end{pmatrix} = \begin{pmatrix} F_1^{(1)}(s,\alpha_1)(\dot{\tilde{\theta}}_1^{\mathrm{T}}\boldsymbol{\omega} + \tilde{\theta}_1^{\mathrm{T}}\dot{\boldsymbol{\omega}}) \\ \vdots \\ F_1^{(m)}(s,\alpha_m)(\dot{\tilde{\theta}}_m^{\mathrm{T}}\boldsymbol{\omega} + \tilde{\theta}_m^{\mathrm{T}}\dot{\boldsymbol{\omega}}) \end{pmatrix} + \begin{pmatrix} F^{(1)}(s,\alpha_1)(\tilde{\theta}_1^{\mathrm{T}}\boldsymbol{\omega} \\ \vdots \\ F^{(m)}(s,\alpha_m)(\tilde{\theta}_m^{\mathrm{T}}\boldsymbol{\omega} \end{pmatrix} =$$

$$F_1(s,\alpha_1,\cdots,\alpha_m)(\dot{\tilde{\theta}}^{\mathrm{T}}\boldsymbol{\omega} + \tilde{\theta}^{\mathrm{T}}\dot{\boldsymbol{\omega}}) + F(s,\alpha_1,\cdots,\alpha_m)(\tilde{\theta}^{\mathrm{T}}\boldsymbol{\omega})$$

and $\|F_1^{(i)}(s,\alpha_i)\|_{\infty\delta} \leqslant \frac{c}{\alpha_i}$ for $\alpha_i > \delta$, where $F^{(i)}(s,\alpha_i) = \frac{\alpha_i^k}{(s+\alpha_i)^k}$, $F_1^{(i)}(s,\alpha_i) = \frac{1 - F^{(i)}(s,\alpha_i)}{s}$, Thus for $\alpha_0 > \delta$, $\|F_1(s,\alpha_1,\cdots,\alpha_m)\|_{\infty\delta} \leqslant \sum_{i=1}^{m}\frac{c}{\alpha_i} \leqslant \frac{c}{\alpha_0}$, $c > 0$ is a constant independent of α_i and α_0. □

6 Conclusion

The design and analysis of multivariable MRAC based on $K_p = L_2 D_2 S_s$ factorization are studied by proving the L_p and $L_{2\delta}$ relationship properties between the input and the output, and the swapping lemmas 4 and 5 for MIMO systems, and by relating all the signals in the closed-loop system with the normalizing signal and proving $\tilde{g} \in L_2$. Compared with the existing results, the proof procedure is more compact.

References

1 Ioannou P A, Sun J. Robust Adaptive Control. New Jersey: John Wiley & Sons, Inc., 1996

2 Tao G, Ioannou P A. Robust model reference adaptive control for multivariable plants. *International Journal of Adaptive Control and Signal Processing*, 1988, **2**(3): 217~248

3 Imai A K, Costa R R, Hsu L, Tao G, Kokotovi P. Multivariable MRAC using high frequency gain matrix factorization. *IEEE Transactions on Automatic Control*, 2004, **49**(7): 1152~1157

4 Xie Xue-Jun, Zhang Zheng-Qiang. Robust direct model reference adaptive control using $K_p = LDU$ factorization for multivariable plants. *International Journal of Control*, 2005, **78**(10): 762~772

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