Feedback Stabilization by Robust Passivity of General Nonlinear Systems with Structural Uncertainty[1)]

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Abstract The general nonlinear system with structural uncertainty is dealt with and necessary conditions for it to be robust passivity are derived. From these necessary conditions, sufficient conditions of zero state detectability are deduced. Based on passive systems theory and the technique of feedback equivalence, sufficient conditions for it to be locally (globally) asymptotically stabilized *via* smooth state feedback are developed. A smooth state feedback control law can be constructed explicitly to locally (globally) stabilize the equilibrium of the closed-loop system. Simulation example shows the effectiveness of the method.

Key words Structural uncertainty, general nonlinear systems, robust passivity, zero state detectability

1 Introduction

At an early stage, passive or dissipative systems theory was primarily used to analyze stability of nonlinear systems. In recent 15 years, great developments of synthesis techniques that combine the theory of passive systems with geometric nonlinear control theory have been made[1~7]. In particular, using the passivity-based synthesis approach, Byrnes[1] presented a fairly complete solution to the fundamental question of when an affine nonlinear system is feedback equivalent to a passive system via state feedback. In the passivity-based synthesis framework, the aim of control action is to render a nonlinear system passive.

Lin[2,3] extended the idea and method developed in [1] to general nonlinear control systems. It is very natural to expect that the results of Lin[2] can be extended to general nonlinear systems with structural uncertainty. Indeed, this is precisely the point of view to be pursued in this paper. We will show how the theory of passive systems developed in [1,2] can be extended to general nonlinear systems with structural uncertainty, and how the problem of asymptotic stabilization of this kind of nonlinear systems can be solved by passive systems theory and the technique of feedback equivalence.

2 System description and preliminaries

Consider a nonlinear system with structural uncertainty described by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) + \Delta \boldsymbol{f}(\boldsymbol{x}) \tag{1a}$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{u}) \tag{1b}$$

where $\boldsymbol{x} \in R^n$, $\boldsymbol{u} \in R^m$, and $\boldsymbol{y} \in R^m$ are the state, input and output of the system, respectively. $\boldsymbol{f} : R^n \times R^m \rightarrow R^n$, $\boldsymbol{h} : R^n \times R^m \rightarrow R^m$ are assumed to be smooth and satisfy $\boldsymbol{f}(\boldsymbol{0}, \boldsymbol{0}) = \boldsymbol{0}$, $\boldsymbol{h}(\boldsymbol{0}, \boldsymbol{0}) = \boldsymbol{0}$, $\Delta \boldsymbol{f} : R^n \rightarrow R^n$ represents the structural uncertainty characterized by

$$\Delta \boldsymbol{f}(\boldsymbol{x}) = e(\boldsymbol{x})\boldsymbol{\delta}(\boldsymbol{x}), \quad \Delta \boldsymbol{f}(\boldsymbol{0}) = \boldsymbol{0} \tag{2}$$

where $e : R^n \rightarrow R^{n\times m}$, with $e(\boldsymbol{0}) = \boldsymbol{0}$ being a known matrix whose entries are given smooth functions, and $\boldsymbol{\delta} : R^n \rightarrow R^m$ is an unknown vector-valued function. It is assumed that $\boldsymbol{\delta}(\boldsymbol{x})$ is constrained to a given smooth function $\boldsymbol{n} : R^n \rightarrow R^m$, *i.e.*,

$$\varGamma = \{\boldsymbol{\delta}(\boldsymbol{x}) : \|\boldsymbol{\delta}(\boldsymbol{x})\| \leqslant \|\boldsymbol{n}(\boldsymbol{x})\|\} \tag{3}$$

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where $\|\cdot\|$ stands for the Euclidean norm. If $\boldsymbol{\delta}(\boldsymbol{x}) \in \Gamma$, then $\boldsymbol{\delta}(\boldsymbol{x})$ or $\Delta \boldsymbol{f}(\boldsymbol{x})$ is said to be admissible.

Definition 1. An uncertain system of the form (1) is said to be robust passivity if there exists a C^0 nonnegative function $V: R^n \rightarrow R^+$ with $V(\mathbf{0}) = 0$ such that for any $\boldsymbol{u} \in R^m$, $\boldsymbol{x}_0 \in R^n$ and for all admissible $\Delta \boldsymbol{f}(\boldsymbol{x})$,

$$V(\boldsymbol{x}(t)) - V(\boldsymbol{x}_0) \leqslant \int_0^t \boldsymbol{y}^{\mathrm{T}}(\tau)\boldsymbol{u}(\tau)\mathrm{d}\tau \tag{4}$$

where $\boldsymbol{x}(t) = \boldsymbol{\phi}(t, \boldsymbol{x}_0, \boldsymbol{u})$ is the solution to (1a) with $\boldsymbol{x}(0) = \boldsymbol{x}_0$.

If V is $C^r (r \geqslant 1)$, the passive inequality (4) is equivalent to

$$\frac{\mathrm{d}V(\boldsymbol{x}(t))}{\mathrm{d}t} \leqslant \boldsymbol{y}^{\mathrm{T}}(t)\boldsymbol{u}(t) \tag{5}$$

for any $\boldsymbol{u} \in R^m$, and for all admissible $\Delta \boldsymbol{f}(\boldsymbol{x})$.

Definition 2. An input-output nonlinear system with structural uncertainty (1) is said to be locally zero-state detectable if there is a neighborhood N of $\boldsymbol{x} = \mathbf{0}$ such that $\forall \boldsymbol{x} \in N$,

$$\boldsymbol{h}(\boldsymbol{\phi}(t, \boldsymbol{x}; \boldsymbol{u}), \boldsymbol{u})|_{\boldsymbol{u}=0} = 0 \quad \forall t \geqslant 0 \Rightarrow \lim_{t \to \infty} \boldsymbol{\phi}(t, \boldsymbol{x}; \mathbf{0}) = 0$$

If $N = R^n$, system (1) is zero-state detectable.

Throughout this paper, let $\boldsymbol{f}_0(\boldsymbol{x})$, $g^0(\boldsymbol{x})$ and $\boldsymbol{g}_i^0(\boldsymbol{x})$ denote the vector fields defined by

$$\boldsymbol{f}_0(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x}, \mathbf{0}) \in R^n, \quad \boldsymbol{g}_i^0(\boldsymbol{x}) = \boldsymbol{g}_i(\boldsymbol{x}, \mathbf{0}) = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}_i}(\boldsymbol{x}, \mathbf{0}) \in R^n, \ 1 \leqslant i \leqslant m \tag{6}$$

$$g_0(x) = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}}(\boldsymbol{x}, \mathbf{0}) = [\boldsymbol{g}_1^0(\boldsymbol{x}), \cdots, \boldsymbol{g}_m^0(\boldsymbol{x})] \in R^{n \times m} \tag{7}$$

With the vector fields $\boldsymbol{f}_0, \boldsymbol{g}_1^0, \cdots, \boldsymbol{g}_m^0$, we introduce the distribution

$$D = span\{\boldsymbol{ad}_{f0}^k \boldsymbol{g}_i^0 : 0 \leqslant k \leqslant n-1, 1 \leqslant i \leqslant m\} \tag{8}$$

Two sets Ω and S associated with D are defined respectively by

$$\Omega = \{\boldsymbol{x} \in N \subseteq R^n : L_{f0}^k V(\boldsymbol{x}) = 0, k = 1, \cdots, r\} \tag{9}$$

$$S = \{\boldsymbol{x} \in N \subseteq R^n : L_{f0}^k L_\tau V(\boldsymbol{x}) = 0, \forall \tau \in D, k = 0, 1, \cdots, r-1\} \tag{10}$$

Let

$$g(\boldsymbol{x}, \boldsymbol{u}) = \int_0^1 \left.\frac{\partial f(\boldsymbol{x}, \boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}}\right|_{\alpha = \theta u} \mathrm{d}\theta = (\boldsymbol{g}_1(\boldsymbol{x}, \boldsymbol{u}), \cdots, \boldsymbol{g}_m(\boldsymbol{x}, \boldsymbol{u})) \tag{11}$$

Then the nonlinear system (1a) can be always represented as

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_0(\boldsymbol{x}) + \Delta \boldsymbol{f}(\boldsymbol{x}) + \sum_{i=1}^m \boldsymbol{g}_i(\boldsymbol{x}, \boldsymbol{u})u_i \tag{12}$$

Similarly, (12) can be further decomposed as

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_0(\boldsymbol{x}) + \Delta \boldsymbol{f}(\boldsymbol{x}) + g_0(\boldsymbol{x})\boldsymbol{u} + \sum_{i=1}^m u_i R_i(\boldsymbol{x}, \boldsymbol{u})\boldsymbol{u} \tag{13}$$

with $R_i(\boldsymbol{x}, \boldsymbol{u}): R^n \times R^m \rightarrow R^{n \times m}$ being a smooth map for $1 \leqslant i \leqslant m$.

Since a smooth nonlinear system of the form (1a) is equivalent to either system (12) or (13), we will use them interchangeably when referring to a smooth nonlinear plant.

3 Main results

Lemma 1. Let $\Omega_1 = \{\boldsymbol{x} \in R^n : L_{\boldsymbol{f}_0(\boldsymbol{x})} V(\boldsymbol{x}) = 0\}$. Necessary conditions for system (1) to be robust passivity with a C^2 storage function V are that

1) $L_{\boldsymbol{f}_0} V(\boldsymbol{x}) \leqslant -\|(L_e V(\boldsymbol{x}))\| \|\boldsymbol{n}(\boldsymbol{x})\|$, $\forall \boldsymbol{x} \in R^n$;

2) $L_{g_0} V(\boldsymbol{x}) = \boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}, \mathbf{0})$, $\forall \boldsymbol{x} \in \Omega_1$;

3) $\sum_{i=1}^{n} \frac{\partial^2 f_i}{\partial \boldsymbol{u}^2}(\boldsymbol{x}, \boldsymbol{0}) \cdot \frac{\partial V}{\partial x_i} \leqslant \frac{\partial \boldsymbol{h}^{\mathrm{T}}}{\partial \boldsymbol{u}}(\boldsymbol{x}, \boldsymbol{0}) + \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{u}}(\boldsymbol{x}, \boldsymbol{0}),\ \forall \boldsymbol{x} \in \Omega_1$

where $f_i(\boldsymbol{x}, \boldsymbol{u})$ is the ith component of the vector function $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$.

Proof. Consider an auxiliary function $F : R^n \times R^m \to R$ defined by

$$F(\boldsymbol{x}, \boldsymbol{u}) = \frac{\partial V}{\partial \boldsymbol{x}}(\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) + \Delta \boldsymbol{f}(\boldsymbol{x})) - \boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}, \boldsymbol{u})\boldsymbol{u}$$

Since system (1) is robust passivity, $F(\boldsymbol{x}, \boldsymbol{u}) \leqslant 0$, $\forall \boldsymbol{u} \in R^m$ and for all admissible $\boldsymbol{\delta}(\boldsymbol{x}) \in \Gamma$. When $\boldsymbol{u} = 0$, it leads to $\frac{\partial V}{\partial \boldsymbol{x}}(\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{0}) + \Delta \boldsymbol{f}(\boldsymbol{x})) \leqslant 0$. Using (2), we have the following inequality

$$\frac{\partial V}{\partial \boldsymbol{x}} \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{0}) \leqslant -\frac{\partial V}{\partial \boldsymbol{x}} e(\boldsymbol{x}) \boldsymbol{\delta}(\boldsymbol{x}) \tag{14}$$

In view of the symmetry of the set Γ, 1) follows immediately from (14).

We can deduce $F(\boldsymbol{x}, \boldsymbol{0}) = L_{\boldsymbol{f}_0 + \Delta \boldsymbol{f}} V(\boldsymbol{x}) \equiv 0$ from 1) and $0 = L_{\boldsymbol{f}_0} V(\boldsymbol{x})$ for $\forall \boldsymbol{x} \in \Omega_1$. Since system (1) is passive, $F(\boldsymbol{x}, \boldsymbol{u}) \leqslant 0 \equiv F(\boldsymbol{x}, \boldsymbol{0})$ for $\forall \boldsymbol{x} \in \Omega_1$, $\boldsymbol{u} \in R^m$. In view of (6) and (7), 2) and 3) follow immediately.

Theorem 1. Consider a robust passive system of the form (1) with a C^1 storage function V, which is positive definite. Suppose (1) is locally zero-state detectable. Let $\boldsymbol{s} : R^m \to R^m$ be a smooth function such that $\boldsymbol{s}(\boldsymbol{0}) = \boldsymbol{0}$ and $\boldsymbol{y}^{\mathrm{T}} \boldsymbol{s}(\boldsymbol{y}) > 0$ for each nonzero $\boldsymbol{y}$. The control law

$$\boldsymbol{u} = -\boldsymbol{s}(\boldsymbol{y}) \tag{15}$$

asymptotically stabilizes the equilibrium $\boldsymbol{x} = 0$. If (1) is zero-state detectable and V is proper, the control law (15) globally asymptotically stabilizes the equilibrium $\boldsymbol{x} = 0$.

The proof of this theorem is exactly similar to that of Theorem 3.2 given in [1], so it is omitted.

Now a question arises immediately: when is the robust passive system (1) zero-state detectable? To begin with, let

$$\hat{\Omega} = \bigcup_{\boldsymbol{x}_0 \in N \subseteq R^n} (\varpi\text{-limit set of } \boldsymbol{\phi}(t, \boldsymbol{x}_0; 0)), \qquad \Omega_2 = \{\boldsymbol{x} \in R^n : L_{f_0} V(\boldsymbol{x}) = -\|L_e V(\boldsymbol{x})\| \|\boldsymbol{n}(\boldsymbol{x})\|\}$$

where $\boldsymbol{\phi}(t, \boldsymbol{x}_0; \boldsymbol{0})$ is the trajectory of $\dot{\boldsymbol{x}} = \boldsymbol{f}_0(\boldsymbol{x}) + \Delta \boldsymbol{f}(\boldsymbol{x})$ with $\boldsymbol{x}_0 \in N$.

Theorem 2. Consider the robust passive system of the form (1) with a $C^r (r \geqslant 1)$ storage function $V(\boldsymbol{x})$, which is positive definite and proper. Suppose $\Omega_2 = \Omega_1$. Then system (1) is zero-state detectable if $\hat{\Omega} \cap S = \{\boldsymbol{0}\}$. In the local case, the conclusion remains valid without the hypothesis that V is proper.

Proof. Let $\boldsymbol{x}(t) = \boldsymbol{\phi}(t, \boldsymbol{x}_0; \boldsymbol{0})$ be the trajectory of the unforced dynamic system $\dot{\boldsymbol{x}} = \boldsymbol{f}_0(\boldsymbol{x}) + \Delta \boldsymbol{f}(\boldsymbol{x})$ of (1a) starting from $\boldsymbol{x}_0 \in R^n$. Since system (1) is robust passive with a C^2 storage function V, $L_{\boldsymbol{f}_0 + \Delta \boldsymbol{f}} V(\boldsymbol{x}) \leqslant 0$, for all admissible $\Delta \boldsymbol{f}(\boldsymbol{x})$. Let r^0 be its ϖ-limit set. By [8], r^0 is nonempty, compact and invariant. Since $V(\boldsymbol{x}(t))$ is positive definite and nonincreasing along the trajectory $\boldsymbol{x}(\boldsymbol{t}) = \boldsymbol{\phi}(t, \boldsymbol{x}_0; \boldsymbol{0})$, $\lim_{t \to \infty} V(\boldsymbol{x}(t))$ exists. Let $\lim_{t \to \infty} V(\boldsymbol{x}(t)) = c \geqslant 0$. By continuity of V, $V(\bar{\boldsymbol{x}}) = c$ for every point $\bar{\boldsymbol{x}} = \lim_{t \to \infty} \boldsymbol{\phi}(\boldsymbol{t}, \boldsymbol{x}_0; \boldsymbol{0}) \in r^0$.

Let $\boldsymbol{\phi}(t, \bar{\boldsymbol{x}}; \boldsymbol{0})$ be the corresponding trajectory of $\dot{\boldsymbol{x}} = \boldsymbol{f}_0(\boldsymbol{x}) + \Delta \boldsymbol{f}(\boldsymbol{x})$ starting from $\bar{\boldsymbol{x}} \in r^0 \subseteq \hat{\Omega}$. $\boldsymbol{\phi}(t, \bar{\boldsymbol{x}}; \boldsymbol{0}) \in r^0 \subseteq \hat{\Omega}$ by the invariance. It follows that $V(\boldsymbol{\phi}(t, \bar{\boldsymbol{x}}; \boldsymbol{0})) = V(\bar{\boldsymbol{x}}) = c$, $\forall t \geqslant 0$. Hence $\dot{V}(\boldsymbol{\phi}(t, \bar{\boldsymbol{x}}; \boldsymbol{0})) = L_{\boldsymbol{f}_0 + \Delta \boldsymbol{f}} V(\boldsymbol{\phi}(t, \bar{\boldsymbol{x}}; \boldsymbol{0})) = 0$, $\forall t \geqslant 0$. By 1) in Lemma 1, it holds that $\boldsymbol{\phi}(t, \bar{\boldsymbol{x}}_0; \boldsymbol{0}) \in \Omega_2$. By assumption of $\Omega_1 = \Omega_2$, it follows that $\hat{\Omega} \subseteq \Omega_1 \subseteq \Omega$.

Let $\bar{\boldsymbol{x}}(t) = \boldsymbol{\phi}(t, \bar{\boldsymbol{x}}_0; \boldsymbol{0})$ be a trajectory of $\dot{\boldsymbol{x}} = \boldsymbol{f}_0(\boldsymbol{x}) + \Delta \boldsymbol{f}(\boldsymbol{x})$ yielding $\boldsymbol{y}(t) = \boldsymbol{h}(\bar{\boldsymbol{x}}(t), \boldsymbol{0}) = \boldsymbol{0}$, $\forall t \geqslant 0$. From the above, we get $\forall \boldsymbol{\phi}(t, \bar{\boldsymbol{x}}_0; \boldsymbol{0}) \in r^0 \subseteq \hat{\Omega} \subseteq \Omega_1$, then for any $t \geqslant 0$,

$$L_{\boldsymbol{f}_0} V(\bar{\boldsymbol{x}}(t)) = L_{\boldsymbol{f}_0} V(\boldsymbol{\phi}(t, \bar{\boldsymbol{x}}_0; \boldsymbol{0})) = 0 \tag{16}$$

In addition, since $\bar{\boldsymbol{x}} = \boldsymbol{\phi}(t, \bar{\boldsymbol{x}}_0; \boldsymbol{0}) \in \Omega_1$, from 2) in Lemma 1

$$L_{g_0} V(\bar{\boldsymbol{x}}(t)) = \boldsymbol{h}^{\mathrm{T}}(\bar{\boldsymbol{x}}(t), \boldsymbol{0}) = \boldsymbol{0}, \quad \forall t \geqslant 0 \tag{17}$$

From (16) and (17) we deduce that for any $\boldsymbol{\tau} = [\boldsymbol{f}_0, g_0] \in D$,

$$L_{\boldsymbol{\tau}} V(\bar{\boldsymbol{x}}(t)) = L_{\boldsymbol{f}_0} L_{g_0} V(\bar{\boldsymbol{x}}(t)) - L_{g_0} L_{\boldsymbol{f}_0} V(\bar{\boldsymbol{x}}(t)) = 0, \quad \forall t \geqslant 0$$

Using the inductive argument, it can be shown that for any $t \geqslant 0$,

$$L_{\boldsymbol{f}_0}^k L_{\boldsymbol{\tau}} V(\bar{\boldsymbol{x}}(t)) = 0, \quad \forall \tau \in D,\ 0 \leqslant k \leqslant r-1 \tag{18}$$

Using (18), we conclude that $\bar{\boldsymbol{x}}(t) \in S$. Hence $\bar{\boldsymbol{x}}(t) \in \hat{\Omega} \cap S$. If $\hat{\Omega} \cap S = \{\mathbf{0}\}$, then $\bar{\boldsymbol{x}}(t) = \mathbf{0}$. Thus $V(\bar{\boldsymbol{x}}(t)) = 0$. In addition, since $V(\boldsymbol{x})$ is positive definite with $V(\mathbf{0}) = 0$, $\lim_{t\to\infty} \boldsymbol{\phi}(t, \boldsymbol{x}_0; \mathbf{0}) = \mathbf{0}$. By Definition 2, system (1) is zero-state detectable. In the local case, the conclusion remains valid without the hypothesis that V is proper. □

As a consequence of Theorem 2, we have ($\hat{\Omega} \subseteq \Omega$) the following corollary.

Corollary 1. Consider the robust passive system of the form (1) is zero-state detectable if $\Omega_2 = \Omega_1$ and $\Omega \cap S = \{\mathbf{0}\}$.

To present sufficient conditions for a class of nonlinear systems with structural uncertainty to be locally (globally) asymptotically stabilized, we make the following assumption. It is weaker than the hypothesis that system (1) is robust passive.

Assumption 1. There exists a $C^r (r \geqslant 1)$ function $V : R^n \to R^+$, $V(\mathbf{0}) = 0$ which is positive definite and defined on some neighborhood N of $x = 0$ such that

$$L_{\boldsymbol{f}(\boldsymbol{x},\mathbf{0})} V(\boldsymbol{x}) \leqslant -\|L_e V(\boldsymbol{x})\| \|\boldsymbol{n}(\boldsymbol{x})\|, \quad \forall \boldsymbol{x} \in N \tag{19}$$

Theorem 3. Suppose the nonlinear system (1a) satisfies Assumption 1, and $S \cap \Omega = \{\mathbf{0}\}$, $\Omega_1 = \Omega_2$. Then (1a) is locally asymptotically stabilized at the equilibrium $\boldsymbol{x} = \mathbf{0}$ by a smooth state feedback. In particular, a local smooth state feedback control law $\boldsymbol{u} = \boldsymbol{\alpha}(\boldsymbol{x})$, $\boldsymbol{u}(\mathbf{0}) = \mathbf{0}$ can be solved uniquely from the equation

$$\boldsymbol{u} + (L_{g(x,u)} V(\boldsymbol{x}))^{\mathrm{T}} = \boldsymbol{u} + \left(\frac{\partial V}{\partial \boldsymbol{x}} g(\boldsymbol{x}, \boldsymbol{u})\right)^{\mathrm{T}} = 0,\ \boldsymbol{u}(\mathbf{0}) = \mathbf{0} \tag{20}$$

where $g(\boldsymbol{x}, \boldsymbol{u})$ is defined by (11). If V is proper, and $N = R^n$, $S \cap \Omega = \{\mathbf{0}\}$, $\Omega_1 = \Omega_2$, and (20) has a solution well-defined on R^n, then (1a) is globally asymptotically stabilized by $\boldsymbol{u} = \boldsymbol{\alpha}(\boldsymbol{x})$.

Proof. Since system (12) satisfies Assumption 1 it is easy to obtain that

$$\dot{V} = L_{\boldsymbol{f}_0 + \Delta \boldsymbol{f}} V(\boldsymbol{x}) + (L_{g(\boldsymbol{x},\boldsymbol{u})} V(\boldsymbol{x}))\boldsymbol{u} \leqslant (L_{g(\boldsymbol{x},\boldsymbol{u})} V(\boldsymbol{x}))\boldsymbol{u} \tag{21}$$

Choose a dummy output

$$\boldsymbol{y} = (L_{g(\boldsymbol{x},\boldsymbol{u})} V(\boldsymbol{x}))^{\mathrm{T}} \tag{22}$$

The input-output system (12)∼(22) is robust passive. Moreover, since $S \cap \Omega = \{\mathbf{0}\}$ and $\Omega_1 = \Omega_2$, it follows from Corollary 1 that the robust passive system (12)∼(22) is locally zero-state detectable. By Theorem 1, the output feedback control law $\boldsymbol{u} = -\boldsymbol{y} = -(L_{g(\boldsymbol{x},\boldsymbol{u})} V(\boldsymbol{x}))^{\mathrm{T}}$ locally asymptotically stabilizes the equilibrium $\boldsymbol{x} = 0$ of system (12)∼(22). In turn, it is implied that $\boldsymbol{u} = \boldsymbol{\alpha}(\boldsymbol{x})$ locally asymptotically stabilizes the equilibrium $\boldsymbol{x} = \mathbf{0}$ of the nonlinear system (12) or (1a), provided that there exists a C^1 solution $\boldsymbol{u} = \boldsymbol{\alpha}(\boldsymbol{x})$ with $\boldsymbol{\alpha}(\mathbf{0}) = \mathbf{0}$, locally defined on a neighborhood of $\boldsymbol{x} = \mathbf{0}$, such that (20) is satisfied. If V is proper, and for any $\boldsymbol{x} \in R^n$, $S \cap \Omega = \{\mathbf{0}\}$, $\Omega_1 = \Omega_2$ and (20) is globally solvable, then Theorem 3 also holds globally.

4 Example

Consider a nonlinear system with structural uncertainty

$$\dot{x}_1 = x_2 - 2x_1 e^{x_2^2} + (x_1 - x_2^2)\delta(\boldsymbol{x}) + x_2^2 u, \quad \dot{x}_2 = -x_1 + x_1 x_2 \delta(\boldsymbol{x}) + x_1 u^3, \quad \|\delta(\boldsymbol{x})\| \leqslant e^{x_2^2} \tag{23}$$

Choose $V(\boldsymbol{x}) = \frac{1}{2}(x_1^2 + x_2^2)$, $\|\boldsymbol{n}(\boldsymbol{x})\| = e^{x_2^2}$. Then $L_{\boldsymbol{f}_0} V(\boldsymbol{x}) = -2x_1^2 e^{x_2^2}$, $L_e V(\boldsymbol{x}) = x_1^2$, $L_{\boldsymbol{f}_0} V(\boldsymbol{x}) \leqslant -\|L_e V(\boldsymbol{x})\| \|\boldsymbol{n}(\boldsymbol{x})\|$, $\Omega_1 = \{\boldsymbol{x} \in R^2 : x_1 = 0\}$, $\Omega_2 = \{\boldsymbol{x} \in R^2 : L_{\boldsymbol{f}_0} V(\boldsymbol{x}) = -\|L_e V(\boldsymbol{x})\| \|\boldsymbol{n}(\boldsymbol{x})\|\} = \{\boldsymbol{x} \in R^2 : x_1 = 0\}$. So $\Omega_1 = \Omega_2$. It is easy to verify that the other conditions of Theorem 3 hold. In view of $u + x_1 x_2^2 + x_1 x_2 u^2 = 0$, and $u(0) = 0$, then the control

$$u = \frac{-1 + \sqrt{1 - 4x_1^2 x_2^3}}{2x_1 x_2} \tag{24}$$

locally asymptotically stabilizes the equilibrium $\boldsymbol{x} = \mathbf{0}$. Fig. 1 shows the states $x_1(t)$ and $x_2(t)$ under the state feedback (24) and the control u for the initial state $\boldsymbol{x}(0) = (0.6 \quad 0.7)^{\mathrm{T}}$ and $\delta(\boldsymbol{x}) = e^{x_2^2} \sin x_1$. For any initial state in the neighborhood of $\boldsymbol{x} = \mathbf{0}$, it has the same simulation result.

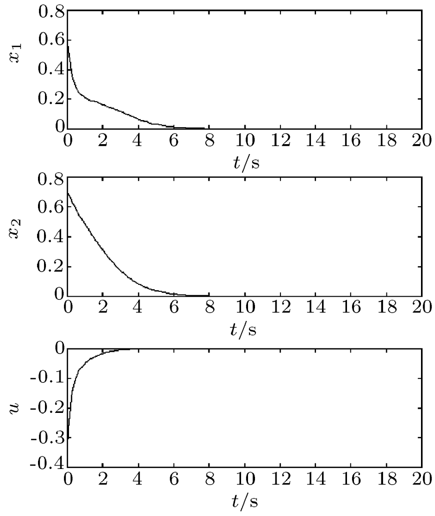

Fig. 1 The state $\boldsymbol{x}(t)$ and the control $u(t)$ of the closed system

5 Conclusions

This paper deals with the general nonlinear system with structural uncertainty. Necessary conditions for it to be robust passivity are derived. From these necessary conditions, sufficient conditions of zero state detectability are deduced. Based on passive systems theory and the technique of feedback equivalence, sufficient conditions for it to be locally (globally) asymptotically stabilized via smooth state feedback are developed. A smooth state feedback control law can be constructed explicitly. Simulation example shows the effectiveness of the method.

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