An HMM Based Terrain Elevation Matching Algorithm[1)]

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Abstract Terrain-aided navigation (TAN) uses terrain height variations under an aircraft to render the position estimate to bound the inertial navigation system (INS) error. This paper proposes a new terrain elevation matching(TEM) model, *viz.* Hidden-Markov-model(HMM) based TEM (HMMTEM) model. With the given model, an HMMTEM algorithm using Viterbi algorithm is designed and implemented to estimate the position error in INS. The simulation results show that HMMTEM algorithm can better improve the positioning precision of autonomous navigation than SITAN algorithm.

Key words TAN, TEM, HMM, Viterbi algorithm, INS

1 Introduction

Terrain-aided navigation (TAN) is an autonomous, all-weather, and low-altitude navigation technique, which performs terrain elevation matching (TEM) between terrain elevation profile under the aircraft flight track and the reference map stored on-board to get the aircraft position and correct the INS (Inertial navigation system) errors. Essentially, TEM is a non-linear estimation problem. Due to the unstructured, nonlinear terrain, the local approximation schemes, like the extended Kalman Filter (EKF), fail in this application.

The two most typical TAN algorithms are TERCOM (Terrain contour matching) algorithm and SITAN (Sandia inertial terrain aided navigation) algorithm. TERCOM is batch oriented and correlates gathered terrain elevation profiles with the map periodically[1]. SITAN is recursive and uses a modified version of an EKF in its original formulation[2]. These two methods have all worked successfully in some specific application. In 1997, Bergman[3] introduced the Bayesian approach to TEM, and the simulation results showed that the Bayesian approach can resolve TEM robustly.

In this paper, we propose a new HMM based TEM algorithm, *viz.* HMMTEM (Hidden Markov models terrain elevation matching) to resolve TEM. The simulation shows that our new method excels SITAN algorithm in positioning precision and some other performance.

2 HMM-based terrain elevation matching model

HMM (Hidden Markov models) is a kind of statistical signal model derived from Markov chain[4]. Since the middle of 1980′s, it has been more and more applied to speech recognition[5] and image processing[6].

Let HMM have N hidden states, with every state a probability density function (PDF) for its measurements. The HMM can be expressed by the state transition probability matrix $A_{N\times N}=\{a_{ij}\}$, the PDF matrix of measurements $B_{N\times 1}=\{b_j(y)\}$ and the initial states PDF vector $\boldsymbol{\pi}=\{\pi_i\}$. Generally, HMM is expressed in formula $\lambda=(A,\boldsymbol{\pi},B)$ for brief.

Assume the drifts of INS positioning error are a first-order Markov chain, which means the current positioning error of the INS depends only on errors of the last time step. This hypothesis closely approximates the position error drifting approximately, because the INS positioning error normally drifts very slowly and continuously, with little drift within a short interval. By taking INS positioning errors as hidden states, and the terrain elevation value measured in real-time as the measurement, the HMM based TEM model can be generated. In this model, the model parameters, such as the state

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transition probability, depend on the specific INS characteristics and the sampling intervals, but have nothing to do with flight tracks of the aircraft.

2.1 State transition probability matrix *A*

Let the INS horizontal position error (x_e, y_e) be the state DX of HMM, *viz.* $DX = [x_e\ y_e]'$, where x_e is the x-direction error and y_e is the y-direction error.

Suppose (x_e, y_e) satisfies: $-3\sigma < x_e < 3\sigma$, $-3\sigma < y_e < 3\sigma$, $\sigma > 0$. Then the value- range of the state DX is a square S with side of 6σ in length. If square S is divided into $N = \text{INT}(6\sigma/\delta)^*\text{INT}(6\sigma/\delta)$ smaller square S with side length of δ, then the value-range of the state DX is separated into N discrete states: $\theta_i, i = 1, 2, \cdots, N$.

Let the number of states in HMM model be N. Then the state transition probability matrix is $A_{N^*N} = \{a_{ij}\}$, where

$$a_{ij} = p(DX(k+1) = \theta_j | DX(k) = \theta_i) = p(\theta_j|\theta_i) \tag{1}$$

According to the INS positioning error which drifts slowly, continuously and has little variations within a short interval, suppose the state at time k $DX(k) = \theta_i$. Then the state at time $k+1$ is $DX(k+1)$, with the value-range being the neighborhood of θ_i, marked as $S(\theta_i)$. It has high probability that $DX(k+1)$ equals θ_j, which is near θ_i. Furthermore, $DX(k+1) = \theta_i$ has the highest probability, and a_{ij} must satisfy $\sum\limits_{j=1}^{N} a_{ij} = 1$ due to HMM requirement.

As the sampling interval (normally 0.25 seconds) in terrain elevation matching is relatively short, we presume that state θ_i can only transit to its neighborhood $S(\theta_i)$ within a sampling interval, as depicted in Fig. 1.

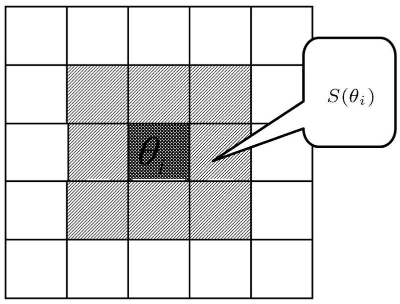

Fig. 1 Illustration of transition of INS error

The state transition probability is

$$a_{ij} = \begin{cases} p, 1 > p > 0, & j = i \\ \dfrac{1-p}{8}, & \theta_j \in S(\theta_i) - \{\theta_i\}\ , \quad i, j = 1, 2, \cdots, N,\ i \neq j \\ 0, & \theta_j \notin S(\theta_j) \end{cases}$$

where the value of p relates to the INS performances and the sampling interval.

2.2 PDF matrix of the measurements: *B*

The PDF matrix of the measurements is B:

$$B = \{b_j(y) = P(y|\theta_j),\ j = 1, \cdots, N\}, \quad y(k) = h(X(k) - DX(k)) + v(k) \tag{2}$$

Formula (2) is a measurement equation, where $y(k)$ is the measured terrain elevation at time k, $X(k)$ is the position indicated by INS, $X(k) - DX(k)$ is the corrected position from $X(k)$ and the state $DX(k)$, $h(X(k) - DX(k))$ is the terrain elevation at position of $X(k) - DX(k)$ in the reference map, and $v(k)$ is the measuring noise.

Formula (2) can be rewritten as

$$v_j(k) = y(k) - h(X(k) - \theta_j), \quad j = 1, 2, \cdots, N \tag{3}$$

Then the PDF of y, which is the measurement of the state θ_j, is

$$b_j(y) = p(y|\theta_j) = p_{v_j(k)}(y - h(X(k) - \theta_j)) \tag{4}$$

Suppose there are no bias errors on the radar altimeter, barometric altimeter and the reference map, and that the PDF of $v_j(k)$ is of normal distribution[7] and satisfies the following conditions

$$v_j(k) \sim N(0, \sigma_h) \tag{5}$$

where σ_h has relation to the random error of the radar altimeter, the barometric altimeter and the reference map.

In practical computing, the probability of $y = y(k)$, which is a measurement of state θ_j, is substituted by the probability of $y \in [y(k) - \sigma_h, y(k) + \sigma_h]$, *i.e.*,

$$P(y(k)|\theta_j) = P(v_j(k)) = \int_{v_j(n)-\sigma_h}^{v_j(k)+\sigma_h} p_{v_j(k)}(v)\mathrm{d}v, \quad j = 1, 2, \cdots, N \tag{6}$$

2.3 Initial probability of state $\boldsymbol{\pi}$

The initial probability of states is expressed as $\boldsymbol{\pi} = (\pi_1, \cdots, \pi_N)$, where π_i represents the initial probability of the state being θ_i, viz. $\pi_i = P(DX(0) = \theta_i), i = 1, \cdots, N$.

In HMM based terrain elevation matching, it is supposed that the INS positioning error is within a square of side length of 6σ at the beginning of the terrain matching. So discretizing the square, we can get the probability vector of the HMM initial state as

$$\boldsymbol{\pi} = (\pi_1, \cdots, \pi_N), \quad \pi_i = \frac{1}{N}, \quad i = 1, \cdots, N \tag{7}$$

3 HMMTEM algorithm

The aim of TEM is using all the current measurements $Y_k = \{y(i)\}_{i=1}^k$ to estimate the current positioning errors of the aircraft, namely $DX(k)$. From the point view of HMM, it means finding a state sequence $Q^* = \{DX^*(i)\}_{i=1}^k$ which is optimal at some sense, with a given measurement sequence $Y_k = \{y(i)\}_{i=1}^k$ and a model $\lambda = (\pi, A, B)$. The above optimal state sequence is the state sequence that makes $P(\{DX(i)\}_{i=1}^k, Y_k|\lambda)$ maximum. The most used approach to this problem is Viterbi algorithm[7].

Let $S_k(i)$ be the maximum probability of $Y_k = \{y(i)\}_{i=1}^k$ at time k along a path $DX(1), DX(2), \cdots, DX(k)$, where $DX(k) = \theta_i$. Then

$$S_k(i) = \max_{\{DX(i)\}_{i=1}^{k-1}} P(\{DX(i)\}_{i=1}^{k-1}, DX(k) = \theta_i, Y_k = \{y(i)\}_{i=1}^k|\lambda) \tag{8}$$

The process of getting the optimal state sequence Q^* is as follows.

Step 1. initialization

$$S_1(i) = \pi_i b_i(y(1)), \quad 1 \leqslant i \leqslant N \tag{9}$$

$$\varphi_1(i) = 0, \quad 1 \leqslant i \leqslant N \tag{10}$$

Step 2. recursion

$$S_k(j) = \max_{1\leqslant i\leqslant N}[S_{k-1}(i)a_{ij}]b_j(y(k)), \quad 1 \leqslant j \leqslant N \tag{11}$$

$$\varphi_k(j) = \arg\max_{1\leqslant i\leqslant N}[S_{k-1}(i)a_{ij}], \quad 1 \leqslant j \leqslant N \tag{12}$$

Step 3. termination

$$P^* = \max_{1\leqslant i\leqslant N}[S_k(i)] \tag{13}$$

$$DX^*(k) = \arg\max_{1\leqslant i\leqslant N}[S_k(i)] \tag{14}$$

Step 4. acquisition of state sequence

$$DX^*(t) = \varphi_{t+1}(DX(t+1)), \quad t = k-1, k-2, \cdots, 1 \tag{15}$$

In TEM, suppose Y_k is known at time k, and the optimal state $DX^*(k)$ and $S_k(j), 1 \leqslant j \leqslant N$ at time k has been computed. After getting the new measurement at time $k+1$, the optimal state $DX^*(k+1)$ and $S_{k+1}(j), 1 \leqslant j \leqslant N$ can be computed using formulae (11), (12) and (14). Then the position error of INS, (x_e, y_e) at time $k+1$, can be evaluated from $DX^*(k+1)$.

HMMTEM Algorithm:

1) Initialization: set N, $k=0$, $\pi_i = 1/N$, $b_i(y(0)) = 1$, $S_0(i) = 1/N$, $i = 1, 2, \cdots, N$

2) Sampling : $k = k+1$

Sampling current position $X(k)$ from the INS, the radar altitude and the barometric altitude

3) Computing $b_j(y_j(k)), j = 1, 2, \cdots, N$ with formulae (5) and (6)

4) Computing $S_k(j), j = 1, 2, \cdots, N$ with formula (11)

5) Computing $DX^*(k)$ with formula (14), getting INS horizontal position error

6) go to 2)

4 Simulation results

In order to test the performance of the HMMTEM algorithm, we make Monte-Carlo simulation 50 times for the HMMTEM algorithm and SITAN algorithm with the same flying path, the same initial positioning errors and the same measuring noises.

The terrain profile under the flying path is shown in Fig. 2, and simulation results of the DRMS (distance-root-mean-square) of horizontal errors are shown in Fig. 3.

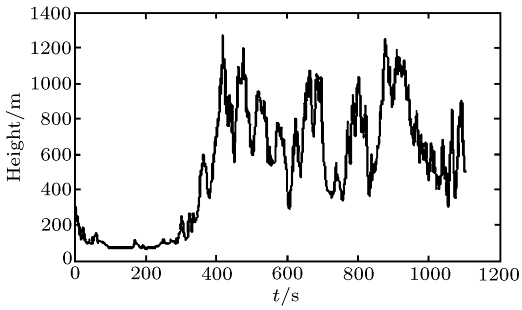

Fig. 2 Terrain profile under flight path

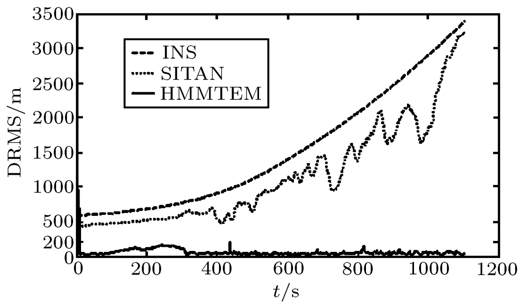

Fig. 3 DRMS of position error

In Fig. 2, the first third of the terrain profile under the flying path is flat, but the other two thirds of the terrain profile are very rough. In Fig. 3, the DRMS of INS horizontal position errors increase along time. Although DRMS in SITAN algorithm is less than that in INS, but its increasing trendline is similar to the INS and incline to divergence. In contrast, the DRMS in HMMTEM does not increase with time and converges quickly. At the same time, the DRMS in HMMTEM is hardly bigger than 200 meters, and is almost all less than 100 meters, with the mean of 50.16 meters, CEP of 32.73 meters.

From the above comparison, it is obvious that the simulation results of HMMTEM algorithm excel that of SITAN algorithm, and even in the first third flight path above flat terrain, the DRMS of horizontal position error in HMMTEM does not exceed 200 meters.

5 Conclusions

In this paper, we have applied HMM to TAN problem and proposed the HMMTEM algorithm.

Compared with SITAN algorithm, HMMTEM algorithm has two distinct advantages. The first is that HMMTEM algorithm does not need terrain linearization, so it can still hold convergency within searching bound even with large initial errors, while SITAN algorithm may probably diverge with big initial errors due to the influences of stochastic terrain linearization. The second is that HMMTEM algorithm can work under a number of noise distributions, but SITAN algorithm only works under Gaussian noise distribution because it bases on EKF.

The shortcoming of HMMTEM algorithm is that it needs more computing than SITAN, but it can be overcome with computer hardware development.

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