Reliability Analysis of Partially Repairable Systems[1)]

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Abstract The reliability analysis of a system with repairable failures and non-repairable failures is presented. It is assumed that the system has n repairable failure modes and m non-repairable failure modes. As one repairable failure mode takes place, the system will be repaired after the failure mode is detected, otherwise, it would never work again when attaining one non-repairable failure mode. Thus, the system brings about new reliability indices for having both repairable failures and non-repairable failures. The definitions of the new reliability indices are given, and the calculating methods for them are derived by using probability analysis and the supplementary variable technique.

Key words Rreliability, repairable failure, non-repairable failure

1 Introduction

In the reliability study of a system, the system is usually classified into two types: repairable and non-repairable[1~6]. By non-repairable it is meant that a unit of the system will not be repaired after suffering a failure and the system will not work again; by repairable it is meant that a unit of the system will be repaired after suffering a failure and the system will run again when repaired.

In actual projects, however, many systems have both repairable units and non-repairable units; or in other words, they have both repairable failures and non-repairable failures. In this paper, a system is called a partially repairable system if it has both repairable failures and non-repairable failures. Thus, the system brings about new reliability indices for having both repairable failures and non-repairable failures. By using probability analysis and the supplementary variable technique[3~6], we study the partially repairable system and derive the new reliability indices.

2 Model and assumptions

The followings are assumed:

1) The system has one work mode (W), n repairable failure modes ($RF_1, RF2_, \cdots, RF_n$) and m non-repairable failure modes ($NRF_1, NRF_2, \cdots, NRF_m$). When the system is in W, it can transfer to RF_i ($i = 1, 2, \cdots, n$) with a constant failure rate λ_{1i} or to NRF_i ($i = 1, 2, \cdots, m$) with λ_{2i}.

2) While the system is in RF_i, it is inspected to make sure of the mode where the system is in, and the inspection is perfect. The duration of each inspection, X_i is a random variable. Once we make the decision that the system is in RF_i by inspection, the system is repaired immediately, and the repairs are perfect.

3) The system would never work again when attaining one non-repairable failure mode. The repair time distributions and the distribution of X_i are continuous. All random variables are mutually independent. Initially the system is in W.

Notations

- $^-$: $\bar{f} = 1 - f$.
- *: $f^*(s) = \int_0^\infty f(t)\exp(-st)\mathrm{d}t$ (Laplace transform).
- (0,0): The state of the system. It means that the system is in W.
- $(1, i)$ $(i = 1, 2, \cdots, n)$: The states of the system. i means that the system is in RF_i and inspected.
- $(2, i)$ $(i = 1, 2, \cdots, n)$: The states of the system. i means that the system is in RF_i and repaired.
- $(3, i)$ $(i = 1, 2, \cdots, m)$: The states of the system. i means that the system is in NRF_i.

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- $h_i(t), H_i(t), \alpha_i(x), \alpha_i^{-1}$: $h_i(t)$ and $\alpha_i(x)$ are p.d.f. and the failure rate of X_i, respectively, and

$$H_i(t) = \int_0^t h_i(x)\mathrm{d}x = 1 - \exp\Big[-\int_0^t \alpha_i(x)\mathrm{d}x\Big],\ \alpha_i^{-1} = \int_0^\infty t\mathrm{d}H_i(t),\ i = 1, 2, \cdots, n$$

- $g_i(t), G_i(t), \mu_i(y), \mu_i^{-1}$: $g_i(t)$ and $\mu_i(y)$ are p.d.f. and the failure rate of the repair time when the system is in $(2, i)$, respectively, and

$$G_i(t) = \int_0^t g_i(y)\mathrm{d}y = 1 - \exp\Big[-\int_0^t \mu_i(y)\mathrm{d}y\Big],\ \mu_i^{-1} = \int_0^\infty t\mathrm{d}G_i(t),\ i = 1, 2, \cdots, n$$

- $P_{00}(t)$: p.d.f. that the system is in (0,0) at time t.
- $P_{1i}(t, x)$: p.d.f. (with respect to X_i) that the system is in $(1, i)$ at time t and has an elapsed X_i of x, $i = 1, 2, \cdots, n$.
- $P_{2i}(t, y)$: p.d.f. (with respect to repair time) that the system is in $(1, i)$ at time t and has an elapsed repair time of y, $i = 1, 2, \cdots, n$.

The possible states of the system and the transitions among them are shown in Fig. 1.

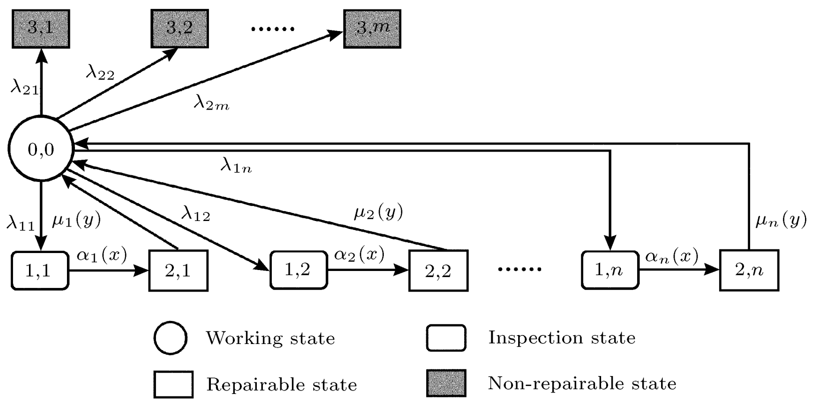

Fig. 1 Transition diagram

According to Fig. 1, we can get the following differential equations for the system:

$$\Big[\frac{\mathrm{d}}{\mathrm{d}t} + \sum_{i=1}^n \lambda_{1i} + \sum_{i=1}^m \lambda_{2i}\Big] P_{00}(t) = \sum_{i=1}^n \int_0^\infty \mu_i(y) P_{2i}(t, y)\mathrm{d}y \tag{1}$$

$$\Big[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha_i(x)\Big] P_{1i}(t, x) = 0,\ i = 1, 2, \cdots, n \tag{2}$$

$$\Big[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_i(y)\Big] P_{2i}(t, y) = 0,\ i = 1, 2, \cdots, n \tag{3}$$

$$P_{1i}(t, 0) = \lambda_{1i} P_{00}(t),\ i = 1, 2, \cdots, n \tag{4}$$

$$P_{2i}(t, 0) = \int_0^\infty \alpha_i(x) P_{1i}(t, x)\mathrm{d}x,\ i = 1, 2, \cdots, n \tag{5}$$

$$P_{00}(0) = 1 \tag{6}$$

$$P_{1i}(0, x) = 0,\ i = 1, 2, \cdots, n \tag{7}$$

$$P_{2i}(0, y) = 0,\ i = 1, 2, \cdots, n \tag{8}$$

The solution of (1)$\sim$(8) in terms of the Laplace transform is

$$P_{00}(s) = \frac{1}{s + \sum\limits_{i=1}^n \lambda_{1i} + \sum\limits_{i=1}^m \lambda_{2i} - \sum\limits_{i=1}^n \lambda_{1i} h_i^*(s) g_i^*(s)} \tag{9}$$

$$P_{1i}^*(s,x)=\frac{\lambda_{1i}\bar{H}_i(x)e^{-sx}}{s+\sum\limits_{i=1}^{n}\lambda_{1i}+\sum\limits_{i=1}^{m}\lambda_{2i}-\sum\limits_{i=1}^{n}\lambda_{1i}h_i^*(s)g_i^*(s)},\quad i=1,2,\cdots,n \tag{10}$$

$$P_{2i}^*(s,y)=\frac{\lambda_{1i}h_i^*(s)\bar{G}_i(y)e^{-sy}}{s+\sum\limits_{i=1}^{n}\lambda_{1i}+\sum\limits_{i=1}^{m}\lambda_{2i}-\sum\limits_{i=1}^{n}\lambda_{1i}h_i^*(s)g_i^*(s)},\quad i=1,2,\cdots,n \tag{11}$$

3 Reliability indices of the system

According to the assumptions of the partially repairable system and the ideas of reliability indices, using the method in [3~6] we can get new reliability indices of the system.

3.1 Time to non-repairable failure

There is a period of random time from starting work of the system to entering non-repairable failure. The random time T is called the time to non-repairable failure. Obviously, T is a random variable, its c.d.f. $F(t)$ is called the cumulative distribution function of the time to non-repairable failure, *i.e.*,

$$F(t)=P\{T\leqslant t\}=1-P\{T>t\}$$

where $P\{T>t\}=\bar{F}(t)$ is the probability that "the system has not entered non-repairable failure at time t". According to Fig. 1, the probability

$$\bar{F}(t)=P_{00}(t)+\sum_{i=1}^{n}\Big[\int_0^\infty P_{1i}(t,x)\mathrm{d}x+\int_0^\infty P_{2i}(t,y)\mathrm{d}y\Big] \tag{12}$$

Taking the Laplace transforms on both sides of (12) and using (9)~(11), we have

$$\bar{F}^*(s)=\frac{1+\sum\limits_{i=1}^{n}\lambda_{1i}[\bar{H}_i^*(s)+h_i^*(s)\bar{G}_i^*(s)]}{s+\sum\limits_{i=1}^{n}\lambda_{1i}+\sum\limits_{i=1}^{m}\lambda_{2i}-\sum\limits_{i=1}^{n}\lambda_{1i}h_i^*(s)g_i^*(s)} \tag{13}$$

The expected value of T is denoted as $E(T)$ and is called the mean time to non-repairable failure (MTTNRF). By the definition of $E(T)$, we have

$$E(T)=\bar{F}^*(0)=\frac{1+\sum\limits_{i=1}^{n}\lambda_{1i}[\alpha_i^{-1}+\mu_i^{-1}]}{\sum\limits_{i=1}^{m}\lambda_{2i}} \tag{14}$$

3.2 Time to first failure

There is a period of random time from starting work of the system to entering either repairable or non-repairable failure. The random time T_0 is called the time to first failure. Obviously, T_0 is a random variable, its c.d.f. $F_0(t)$ is called the cumulative distribution function of the time to first failure, *i.e.*,

$$F_0(t)=P\{T_0\leqslant t\}=1-P\{T_0>t\}$$

where $P\{T_0>t\}=\bar{F}_0(t)$ is the probability that "the system has entered neither of the 2 failures at time t". According to Fig. 1, the probability

$$\bar{F}_0(t)=\prod_{i=1}^{n}e^{-\lambda_{1i}t}\cdot\prod_{i=1}^{m}e^{-\lambda_{2i}t}e^{-\lambda_{2i}t}=e^{-\left(\sum\limits_{i=1}^{n}\lambda_{1i}+\sum\limits_{i=1}^{m}\lambda_{2i}\right)t} \tag{15}$$

i.e., the cumulative distribution function of the time to first failure

$$F_0(t)=1-e^{-\left(\sum\limits_{i=1}^{n}\lambda_{1i}+\sum\limits_{i=1}^{m}\lambda_{2i}\right)t} \tag{16}$$

The expected value of T_0 is denoted as $E(T_0)$ and is called the mean time to first failure (MTTFF). By the definition of $E(T_0)$, we have

$$E(T_0) = \frac{1}{\sum_{i=1}^{n} \lambda_{1i} + \sum_{i=1}^{m} \lambda_{2i}} \tag{17}$$

3.3 Availability

The probability that the system is at work at time t is denoted as $A(t)$, and is said to be the instantaneous availability at time t. In accordance with the definition of $A(t)$ and the characteristics of the system, we obtain

$$A(t) = P_{00}(t) \tag{18}$$

Taking the Laplace transforms on both sides of (18) and using (9), we have

$$A^*(s) = P^*_{00}(s) = \frac{1}{s + \sum_{i=1}^{n} \lambda_{1i} + \sum_{i=1}^{m} \lambda_{2i} - \sum_{i=1}^{n} \lambda_{1i} h^*_i(s) g^*_i(s)} \tag{19}$$

The mean working time T_1 from starting work of the system to entering non-repairable failure is called the mean working time (MWT) of the system. Applying probability analysis, definite integral and the limiting theorem of the Laplace transform, we get

$$T_1 = \int_0^{\infty} A(t)\mathrm{d}t = \lim_{t\to\infty} \int_0^t A(t)\mathrm{d}t = \lim_{s\to 0} sL[\int_0^t A(t)\mathrm{d}t] = \lim_{s\to 0} A^*(s) = \min_{s\to 0} P^*_{00}(s) = \frac{1}{\sum_{i=1}^{m} \lambda_{2i}} \tag{20}$$

Denote the quotient $A = T_1/E(t)$ as the mean availability (MA) of the system, which may be used instead of the steady state availability of a common repairable system. Obviously, the mean availability

$$A = \frac{T_1}{E(t)} = \frac{1}{1 + \sum_{i=1}^{n} \lambda_{1i} [\alpha_i^{-1} + \mu_i^{-1}]} \tag{21}$$

3.4 Repairable failure frequency

State $(1, i)$ $(i = 1, 2, \cdots, n)$ is called RF_i state of the system. According to Fig. 1, the system suffers RF_i if and only if it transfers from state (0,0) into state $(1, i)$. Denoting $W_{1i}(t)$ as the instantaneous RF_i frequency at time t, we obtain

$$W_{1i}(t) = \lambda_{1i} P_{00}(t), \quad i = 1, 2, \cdots, n \tag{22}$$

Taking the Laplace transforms on both sides of (22) and using (9), we have

$$W^*_{1i}(s) = \frac{\lambda_{1i}}{s + \sum_{i=1}^{n} \lambda_{1i} + \sum_{i=1}^{m} \lambda_{2i} - \sum_{i=1}^{n} \lambda_{1i} h^*_i(s) g^*_i(s)}, \quad i = 1, 2, \cdots, n \tag{23}$$

Let M_{1i} represent the mean numbers which the system suffers RF_i from starting work of the system to entering non-repairable failure. Applying probability analysis, definite integral and the limiting theorem of the Laplace transform, we get

$$M_{1i} = \int_0^{\infty} W_{1i}(t)\mathrm{d}t = \lim_{t\to\infty} \int_0^t W_{1i}(t)\mathrm{d}t = \lim_{s\to 0} sL[\int_0^t W_{1i}(t)\mathrm{d}t] = \lim_{s\to 0} W^*_{1i}(s) = \frac{\lambda_{1i}}{\sum_{i=1}^{m} \lambda_{2i}} \tag{24}$$

Denote the quotient $W_{1i} = M_{1i}/E(t)$ as the mean RF_i frequency of the system, which may be used to replace of the steady state failure frequency of a common repairable system. Obviously, the mean RF_i frequency of the system

$$W_{1i} = \frac{M_{1i}}{E(T)} = \frac{\lambda_{1i}}{1 + \sum_{i=1}^{n} \lambda_{1i}[\alpha_i^{-1} + \mu_i^{-1}]}, \quad i = 1, 2, \cdots, n \tag{25}$$

3.5 Repair frequency of RF_i

According to Fig. 1, RF_i is repaired if and only if the system transfers from state $(1, i)$ into state $(2, i)$ $(i = 1, 2, \cdots, n)$. Denoting $W_{2i}(t)$ as the instantaneous repair frequency of RF_i at time t, we obtain

$$W_{2i}(t) = \int_0^\infty \alpha_i(x) P_{1i}(t, x) \mathrm{d}x, \quad i = 1, 2, \cdots, n \tag{26}$$

Taking the Laplace transforms on both sides of (26) and using (10), we have

$$W_{2i}^*(s) = \frac{\lambda_{1i} h_i^*(s)}{s + \sum_{i=1i}^{n} \lambda_{1i} + \sum_{i=1}^{m} \lambda_{2i} - \sum_{i=1}^{n} \lambda_{1i} h_i^*(s) g_i^*(s)}, \quad i = 1, 2, \cdots, n \tag{27}$$

Let M_{2i} represent the mean repair numbers of RF_i from starting work of the system to entering non-repairable failure. Then

$$M_{2i} = \int_0^\infty W_{2i}(t)\mathrm{d}t = \lim_{t\to\infty} \int_0^t W_{2i}(t)\mathrm{d}t = \lim_{s\to 0} sL[\int_0^t W_{2i}(t)\mathrm{d}t] = \lim_{s\to 0} W_{2i}^*(s) = \frac{\lambda_{1i}}{\sum_{i=1}^{m} \lambda_{2i}} \tag{28}$$

Denote the quotient $W_{2i} = M_{2i}/E(t)$ as the mean repair frequency of RF_i, which may be used instead of the steady state repair frequency of a common repairable system. Obviously, the mean repair frequency of RF_i

$$W_{2i} = \frac{M_{2i}}{E(T)} = \frac{\lambda_{1i}}{1 + \sum_{i=1}^{n} \lambda_{1i}[\alpha_i^{-1} + \mu_i^{-1}]}, \quad i = 1, 2, \cdots, n \tag{29}$$

3.6 Inspection frequency

According to Fig. 1, the system is inspected if and only if the system transfers from state (0,0) into state $(1, i)$ $(i = 1, 2, \cdots, n)$. Denoting $W_0(t)$ as the instantaneous inspection frequency at time t, we obtain

$$W_0(t) = P_{00}(t) \sum_{i=1}^{n} \lambda_{1i} \tag{30}$$

Taking the Laplace transforms on both sides of (30), we have

$$W_0^*(s) = \frac{\sum_{i=1}^{n} \lambda_{1i}}{s + \sum_{i=1}^{n} \lambda_{1i} + \sum_{i=1}^{m} \lambda_{2i} - \sum_{i=1}^{n} \lambda_{1i} h_i^*(s) g_i^*(s)} \tag{31}$$

Let M_0 represent the mean inspection numbers from starting work of the system to entering non-repairable failure. Then

$$M_0 = \frac{\sum_{i=1}^{n} \lambda_{1i}}{\sum_{i=1}^{m} \lambda_{2i}} \tag{32}$$

Denote the quotient $W_0 = M_0/E(t)$ as the mean inspection frequency from starting work of the system to entering non-repairable failure, which may be used instead of the steady state inspection frequency of a common repairable system. Obviously, the mean inspection frequency

$$W_0 = \frac{\sum_{i=1}^{n} \lambda_{1i}}{1 + \sum_{i=1}^{n} \lambda_{1i}[\alpha_i^{-1} + \mu_i^{-1}]} \tag{33}$$

The above mentioned reliability indices are all very important in the partially repairable systems and can not be spared in the reliability analysis and optimization research of the systems. In addition, those new reliability indices are different from the corresponding indices of both common repairable systems and common non-repairable systems.

4 Conclusions

In actual projects many systems are partially repairable ones that have both repairable failures and non-repairable failures. New reliability indices should be applied to the study of this type of system. In this paper, the definitions of the new reliability indices are given, and the calculating methods for them are derived by using probability analysis and the supplementary variable technique. The definitions and the calculation methods are important for the study of reliability and optimization research of a partially repairable system.

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