LMI Condition of Quadratic D-stability for a Class of Uncertain Linear Systems[1)]

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Abstract A necessary and sufficient condition of the quadratic $\mathcal{D}$-stability for a class of uncertain linear systems is presented in terms of linear matrix inequality (LMI) technology. Finally, the validity and less conservatism of the obtained results in this paper are illustrated by a benchmark example.

Key words Uncertain systems, quadratic $\mathcal{D}$-stability, linear matrix inequality (LMI)

1 Introduction

The analysis and synthesis problem of pole location and $\mathcal{D}$-stability for dynamic systems attracts many researchers since the stability, the rapidity of convergence and the steadily of switch systems are usually considered in practice. Gutman S. and Jury E.I.[1] gave a necessary and sufficient condition for a matrix whose eigenvalues are in a polynomial region in the complex plane. The results in [1] were a summary of prophase works in this area which provided a good theoretic foundation for designing control systems. Chilali M. and Gahinet P.[2] obtained a necessary and sufficient condition for a matrix whose eigenvalues are in an LMI region in the complex plane based on which an H_∞ design method was set up by LMI approach. Recently, the robust $\mathcal{D}$-stability analysis and synthesis problem has also become an attractive area of research. Chilali M. *et al.*[3] studied the robust $\mathcal{D}$-stability problem with LMI region pole replacement constraint for a class of structure uncertain and parameter uncertain systems. Peaucelle D. *et al.*[4] and Valter J. *et al.*[5] researched the robust $\mathcal{D}$-stability problem constrained by some quadratic matrix inequality region pole replacement for state feedback control systems with convex polynomial uncertainty. Mao W. J. *et al.*[6] studied the quadratic stability problem for a class of dynamic systems with interval uncertainty. In this paper, we study the state feedback problem with quadratic matrix inequality region stability constraint for a class of linear systems with norm bounded uncertainty. We shall give a sufficient and necessary condition for feedback systems to be quadratic $\mathcal{D}$-stability based on which a designing method of state feedback controller is obtained. Finally, the validity and less conservatism of the obtained results in this paper are illustrated by a benchmark example.

Throughout this paper, R^n denotes the n-dimensional real Euclidean space and C denotes complex plane. $M^{m\times n}$ denotes the set of all $m\times n$ matrices and M^{T} means the transposition of matrix M. Matrix inequality $X>0$ ($X\geqslant 0$) means X is a symmetrical positive definite matrix (symmetric semi-positive definite matrix). 0 and I denote zero matrix and unit matrix, respectively. The sign $*$ in a matrix denotes the symmetrical element. The operation sign $\otimes$ denotes the Kronecker production[1~5] of two matrices, *i.e.*, $A\otimes B=[A_{ij}B]$, where $A=[A_{ij}]_{m\times n}$.

2 Description of problem

Consider the following linear system

$$\delta[\boldsymbol{x}(t)]=A(t)\boldsymbol{x}(t)+B(t)\boldsymbol{u}(t) \tag{1}$$

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where $\boldsymbol{x}(t) \in R^n$ is the state vector, $\boldsymbol{u}(t) \in R^p$ is the input vector of the system, $\delta[\cdot]$ denotes the derived number to time for continuous time systems or the difference for discrete time system. The parameter matrices $A(t) \in M^{n\times n}$, $B(t) \in M^{n\times p}$ of system belong to

$$\mathcal{A} = \{A_0 + DE(t)F : E^{\mathrm{T}}(t)E(t) \leqslant I\} \tag{2a}$$

$$\mathcal{B} = \{B_0 + MH(t)N : H^{\mathrm{T}}(t)H(t) \leqslant I\} \tag{2b}$$

respectively, where A_0, D, F, B_0, M and N are real matrices with appropriate dimensions, $E(t)$ and $H(t)$ are real time varying matrices with appropriate dimensions. In the case of no confusion, we omit the time variable t in relative expressions in the sequel statements.

In this paper, we consider quadratic matrix inequality region in complex plane[4,5]:

$$\mathcal{D} = \{z \in C : R_{11} + R_{12}z + R_{12}^{\mathrm{T}}z^* + R_{22}zz^* < 0\} \tag{3}$$

where $R_{11}, R_{12}, R_{22} \in M^{d\times d}$, R_{11}, R_{22} are symmetric matrices, $R_{22} = LL^{\mathrm{T}}$ is semi-positive definite matrix and d is called rank of region $\mathcal{D}$. Let $R_{\mathcal{D}} = \begin{bmatrix} R_{11} & R_{12} \\ R_{12}^{\mathrm{T}} & R_{22} \end{bmatrix}$ and call it the matrix of region $\mathcal{D}$. It is easy to see that $\mathcal{D}$ is a convex symmetric region with respect to the real axis.

Note 1. There are two special quadratic matrix inequality regions. One is $\mathcal{D}(\alpha) = \{z \in C : \mathrm{Re}(z) < -\alpha\}$ $(\alpha \geqslant 0)$ which is used to analysis α-decaying degree of system and whose rank and region matrix are 1 and $R_{\mathcal{D}(\alpha)} = \begin{bmatrix} 2\alpha & 1 \\ 1 & 0 \end{bmatrix}$, respectively. The other is open disk plate $\mathcal{D}(c, r) = \{z \in C : |z + c| < r, r > 0\}$ whose rank and region matrix are 1 and $R_{\mathcal{D}(c,r)} = \begin{bmatrix} c^2 - r^2 & c \\ c & 1 \end{bmatrix}$, respectively. Furthermore, the typical regions used in the usual stability analysis are the left-hand side of complex plane $\mathcal{D}(0)$ (continuous time system) and the disk $\mathcal{D}(0, 1)$ (discrete time systems). These two regions are all regions with rank 1.

Definition 1. Matrix A is said to be $\mathcal{D}$-stable if all its eigenvalues lie in the region $\mathcal{D}$ defined in (3).

$\mathcal{D}$-stability can be characterized by LMI approach.

Theorem 1[1,2]. $A \in R^{n\times n}$ is $\mathcal{D}$-stable if and only if there exists a symmetric positive definite matrix $P \in M^{n\times n}$, such that

$$R_{11} \otimes P + R_{12} \otimes (PA) + R_{12}^{\mathrm{T}} \otimes (A^{\mathrm{T}}P) + R_{22} \otimes (A^{\mathrm{T}}PA) < 0 \tag{4}$$

From Theorem 1, we can define quadratic $\mathcal{D}$-stability and robust $\mathcal{D}$-stability of a dynamic system as follows.

Definition 2. Uncertain linear system (1) with zero input is said to be robust $\mathcal{D}$-stable if for every $A(t) \in \mathcal{A}$, there exists a varying symmetric positive definite matrix P with respect to $A(t)$, such that

$$R_{11} \otimes P + R_{12} \otimes (PA) + R_{12}^{\mathrm{T}} \otimes (A^{\mathrm{T}}P) + R_{22} \otimes (A^{\mathrm{T}}PA) < 0 \tag{5}$$

Uncertain linear system (1) with zero input is said to be quadratic $\mathcal{D}$-stable if there exists an invariable symmetric positive definite matrix $P \in R^{n\times n}$ such that inequality (5) holds.

It is obvious that quadratic $\mathcal{D}$-stability implies robust $\mathcal{D}$-stability for linear system (1) with zero input. By Schur complement lemma, inequality (5) is equivalent to the following LMI

$$\begin{bmatrix} R_{11} \otimes P + R_{12} \otimes (PA) + R_{12}^{\mathrm{T}} \otimes (A^{\mathrm{T}}P) & * \\ L^{\mathrm{T}} \otimes (PA) & -I_d \otimes P \end{bmatrix} < 0 \tag{6}$$

The aim of this paper is to design a state feedback controller

$$\boldsymbol{u}(t) = K\boldsymbol{x}(t) \tag{7}$$

such that the closed-loop system

$$\delta[\boldsymbol{x}(t)] = (A(t) + B(t)K)\boldsymbol{x}(t) \tag{8}$$

is quadratic $\mathcal{D}$-stable. That is to seek the gain matrix K of controller and a symmetric positive definite matrix P, such that for every $A(t) \in \mathcal{A}$, $B(t) \in \mathcal{B}$, the inequality

$$R_{11} \otimes P + R_{12} \otimes (P(A+BK)) + R_{12}^{\mathrm{T}} \otimes ((A^{\mathrm{T}} + K^{\mathrm{T}}B^{\mathrm{T}})P) + R_{22} \otimes ((A^{\mathrm{T}} + K^{\mathrm{T}}B^{\mathrm{T}})P(A+BK)) < 0 \quad (9)$$

or

$$\begin{bmatrix} R_{11} \otimes P + R_{12} \otimes (P(A+BK)) + R_{12}^{\mathrm{T}} \otimes ((A^{\mathrm{T}} + K^{\mathrm{T}}B^{\mathrm{T}})P) & * \\ L^{\mathrm{T}} \otimes (P(A+BK)) & -I_d \otimes P \end{bmatrix} < 0 \quad (10)$$

holds.

To prove the main consults in this paper, we introduce the following lemma firstly.

Lemma 1[7]. Let Y, H, E be matrices with appropriate dimensions and Y a symmetrical matrix. Then for every matrix F with $F^{\mathrm{T}}F \leqslant I$,

$$Y + HFE + E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}} < 0 \quad (11)$$

if and only if there exists a constant $\varepsilon > 0$, such that

$$Y + \varepsilon HH^{\mathrm{T}} + \varepsilon^{-1}E^{\mathrm{T}}E < 0 \quad (12)$$

3 Main results

In this section, we give a necessary and sufficient condition guaranteeing uncertain linear system (1) to be quadratic $\mathcal{D}$-stable. Since there is not time-delay in system (1), the methods to prove the main results in this paper are applicable to both continuous systems and discrete systems. So we consider only the case of continuous systems in the sequel theorems.

Theorem 2. The n-dimension uncertain dynamic system (1) with zero input is quadratic $\mathcal{D}$-stable if and only if there exists a symmetric positive definite matrix $X \in R^{n\times n}$ and a scalar quantity $\lambda > 0$, such that the following inequality holds

$$\begin{bmatrix} R_{11}\otimes X + R_{12}\otimes(A_0X) + R_{12}^{\mathrm{T}}\otimes(XA_0^{\mathrm{T}}) + \lambda(R_{12}R_{12}^{\mathrm{T}})\otimes(DD^{\mathrm{T}}) & * & * \\ L^{\mathrm{T}} \otimes (A_0X) + \lambda(L^{\mathrm{T}}R_{12}^{\mathrm{T}}) \otimes (DD^{\mathrm{T}}) & -I_d\otimes X + \lambda(L^{\mathrm{T}}L)\otimes(DD^{\mathrm{T}}) & * \\ I_d \otimes (FX) & 0 & -\lambda I \end{bmatrix} < 0 \quad (13)$$

Proof. From Definition 2, uncertain linear system (1) with zero input is quadratic $\mathcal{D}$-stable if there exists an invariable symmetric positive definite matrix $P \in R^{n\times n}$, such that inequality (6) holds for every $A(t) \in \mathcal{A}$. Substituting $A_0 + DE(t)F$ to A in (6) and making appropriate transmutation, one obtains

$$\Phi + \begin{bmatrix} R_{12} \otimes (PD) \\ L^{\mathrm{T}} \otimes (PD) \end{bmatrix} (I_d \otimes E)[I_d \otimes F \quad 0] + \left(\begin{bmatrix} R_{12} \otimes (PD) \\ L^{\mathrm{T}} \otimes (PD) \end{bmatrix} (I_d \otimes E)[I_d \otimes F \quad 0] \right)^{\mathrm{T}} < 0 \quad (14)$$

where $\Phi = \begin{bmatrix} R_{11} \otimes P + R_{12} \otimes (PA_0) + R_{12}^{\mathrm{T}} \otimes (A_0^{\mathrm{T}}P) & * \\ L^{\mathrm{T}} \otimes (PA_0) & -I_d \otimes P \end{bmatrix}$. From Lemma 1 and $E^{\mathrm{T}}E \leqslant I$, inequality (14) is equivalent to that there exists a real number $\lambda > 0$ such that

$$\Phi + \lambda \begin{bmatrix} R_{12} \otimes (PD) \\ L^{\mathrm{T}} \otimes (PD) \end{bmatrix} [R_{12}^{\mathrm{T}} \otimes (D^{\mathrm{T}}P) \quad L \otimes (D^{\mathrm{T}}P)] + \lambda^{-1} \begin{bmatrix} I_d \otimes F^{\mathrm{T}} \\ 0 \end{bmatrix} [I_d \otimes F \quad 0] < 0 \quad (15)$$

Performing the congruence transformation $diag\{I_d \otimes P^{-1}, I_d \otimes P^{-1}\}$ in (15), letting $P^{-1} = X$ and making use of Schur complement lemma, we know that the resulting inequality is equivalent to (13). This completes the proof. □

Inequality (13) is an LMI with X and λ in which the solutions of variables can be easily found in terms of Matlab tool box.

For $\mathcal{D}$-stablility of uncertain system (1), we have the following result.

Theorem 3. Closed-loop system (8) is quadratic $\mathcal{D}$-stable if and only if there exist a symmetric positive definite matrix $X \in R^{n\times n}$ a matrix $Z \in R^{p\times n}$ and two real numbers $\lambda > 0$ and $\delta > 0$, such that

$$\begin{bmatrix} \Theta_{11} & * & * & * \\ \Theta_{21} & -I_d \otimes X + \lambda(L^{\mathrm{T}}L) \otimes (DD^{\mathrm{T}}) + \delta(L^{\mathrm{T}}L) \otimes (MM^{\mathrm{T}}) & * & * \\ I_d \otimes (FX) & 0 & -\lambda I & * \\ I_d \otimes (NZ) & 0 & 0 & -\delta I \end{bmatrix} < 0 \tag{16}$$

where

$$\Theta_{11} = R_{11} \otimes X + R_{12} \otimes (A_0X + B_0Z) + R_{12}^{\mathrm{T}} \otimes (XA_0^{\mathrm{T}} + Z^{\mathrm{T}}B_0^{\mathrm{T}})$$
$$\Theta = L^{\mathrm{T}} \otimes (A_0X + B_0Z) + \lambda(L^{\mathrm{T}}R_{12}^{\mathrm{T}}) \otimes (DD^{\mathrm{T}}) + \delta(L^{\mathrm{T}}R_{12}^{\mathrm{T}}) \otimes (MM^{\mathrm{T}})$$

If there exists feasible solution (X, Z) in LMI (16), then the gain matrix of the state feedback controller is $K = ZX^{-1}$.

Proof. By Theorem 1, closed-loop system (8) is quadratic $\mathcal{D}$-stable if there exists an invariable symmetric positive definite matrix $P \in R^{n\times n}$, such that inequality (10) holds for every $A(t) \in \mathcal{A}$ and $B(t) \in \mathcal{B}$. Substituting $A_0 + DE(t)F$ to A and $B_0 + MH(t)N$ to B in (10) and making appropriate transmutation, one obtains

$$\Psi + \Psi_1 + \Psi_1^{\mathrm{T}} + \Psi_2 + \Psi_2^{\mathrm{T}} < 0 \tag{17}$$

where

$$\Psi = \begin{bmatrix} R_{11} \otimes P + R_{12} \otimes (P(A_0 + B_0K)) + R_{12}^{\mathrm{T}} \otimes ((A_0^{\mathrm{T}} + K^{\mathrm{T}}B_0^{\mathrm{T}})P) & * \\ L^{\mathrm{T}} \otimes (P(A_0 + B_0K)) & -I_d \otimes P \end{bmatrix}$$
$$\Psi_1 = \begin{bmatrix} R_{12} \otimes (PD) \\ L^{\mathrm{T}} \otimes (PD) \end{bmatrix} (I_d \otimes E)[I_d \otimes F \quad 0],\ \ \Psi_2 = \begin{bmatrix} R_{12} \otimes (PM) \\ L^{\mathrm{T}} \otimes (PM) \end{bmatrix} (I_d \otimes H)[I_n \otimes (NK) \quad 0]$$

By Lemma 1 and $E^{\mathrm{T}}E \leqslant I, H^{\mathrm{T}}H \leqslant I$, inequality (17) is equivalent to that there exist real numbers $\lambda > 0$ and $\delta > 0$ such that

$$\Psi + \lambda \begin{bmatrix} R_{12} \otimes (PD) \\ L^{\mathrm{T}} \otimes (PD) \end{bmatrix} [R_{12}^{\mathrm{T}} \otimes (D^{\mathrm{T}}P) \quad L \otimes (D^{\mathrm{T}}P)] + \lambda^{-1} \begin{bmatrix} I_d \otimes F^{\mathrm{T}} \\ 0 \end{bmatrix} [I_d \otimes F \quad 0] +$$
$$\delta \begin{bmatrix} R_{12} \otimes (PM) \\ L^{\mathrm{T}} \otimes (PM) \end{bmatrix} [R_{12}^{\mathrm{T}} \otimes (M^{\mathrm{T}}P) \quad L \otimes (M^{\mathrm{T}}P)] + \delta^{-1} \begin{bmatrix} I_d \otimes (K^{\mathrm{T}}N^{\mathrm{T}}) \\ 0 \end{bmatrix} [I_d \otimes (NK) \quad 0] < 0 \tag{18}$$

Performing the congruence transformation $diag\{I_d \otimes P^{-1}, I_d \otimes P^{-1}\}$ in (18), one gets

$$\begin{bmatrix} R_{11} \otimes P^{-1} + R_{12} \otimes ((A_0 + B_0K)P^{-1}) + R_{12}^{\mathrm{T}} \otimes (P^{-1}(A_0^{\mathrm{T}} + K^{\mathrm{T}}B_0^{\mathrm{T}})) & * \\ L^{\mathrm{T}} \otimes ((A_0 + B_0K)P^{-1}) & -I_d \otimes P^{-1} \end{bmatrix} +$$
$$\lambda \begin{bmatrix} R_{12} \otimes D \\ L^{\mathrm{T}} \otimes D \end{bmatrix} [R_{12}^{\mathrm{T}} \otimes D^{\mathrm{T}} \quad L \otimes D^{\mathrm{T}}] + \lambda^{-1} \begin{bmatrix} I_d \otimes (P^{-1}F^{\mathrm{T}}) \\ 0 \end{bmatrix} [I_d \otimes (FP^{-1}) \quad 0] +$$
$$\delta \begin{bmatrix} R_{12} \otimes M \\ L^{\mathrm{T}} \otimes M \end{bmatrix} [R_{12}^{\mathrm{T}} \otimes M^{\mathrm{T}} \quad L \otimes M^{\mathrm{T}}] + \delta^{-1} \begin{bmatrix} I_d \otimes (P^{-1}K^{\mathrm{T}}N^{\mathrm{T}}) \\ 0 \end{bmatrix} [I_d \otimes (NKP^{-1}) \quad 0] < 0 \tag{19}$$

Letting $P^{-1} = X, KX = Z$ in (19) , one knows that inequality (19) is equivalent to inequality (16) by Schur complement lemma. Furthermore, if there exists feasible solution (X, Z) in LMI (16), then the gain matrix of state feedback controller is $K = ZX^{-1}$. This completes the proof. □

Inequality (16) is an LMI with X and λ in which the solutions of variables can be easily found in terms of Matlab tool box. Therefore Inequality (16) is a sufficient and necessary condition with which the quadratic $\mathcal{D}$-stability of closed-loop system (8) can be easily checked. And it is since inequality (16) is a sufficient and necessary condition that the design method of state feedback controller for system (1) in this paper possesses less conservatism.

4 Numeral example

In this section, we give a numeral example to illustrate how to make use of Theorem 2 to check the quadratic $\mathcal{D}$-stability of system (1).

Consider the flight control problem of helicopter studied by Kosmidou. *et al.* in [8] in which the dynamic model is as follows.

$$\dot{\boldsymbol{x}}(t) = (A_0 + r_1(t)A_1 + r_2(t)A_2)\boldsymbol{x}(t) + (B_0 + s_1(t)B_1)\boldsymbol{u}(t),\ \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{20}$$

where

$$A_0 = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -0.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.2855 & -0.7070 & 1.3229 \\ 0 & 0 & 1 & -1.0880 \end{bmatrix},\ B_0 = \begin{bmatrix} 0.4422 & 0.1711 \\ 3.0447 & -7.5922 \\ -5.5200 & 4.9900 \\ 0 & 0 \end{bmatrix},\ \boldsymbol{x}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.2192 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.2031 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 & 0 \\ 1.0673 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$r_1(t), r_2(t)$ and $s_1(t)$ are uncertain parameters in system which satisfy

$$-1 \leqslant r_1(t) \leqslant 1, \quad -1 \leqslant r_2(t) \leqslant 1, \quad -1 \leqslant s_1(t) \leqslant 1$$

System (20) can be expressed as

$$\dot{\boldsymbol{x}}(t) = (A_0 + DEF)\boldsymbol{x}(t) + (B_0 + MHN)\boldsymbol{u}(t) \tag{21}$$

where

$$D = M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ E = H = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & s_1 \end{bmatrix},\ F = \begin{bmatrix} 0 & 0.2192 & 0 & 0 \\ 0 & 0 & 0 & 0.2031 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ N = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1.0673 & 0 \end{bmatrix}$$

Let state feedback control law be (7). Consider the following region whose rank is not equal 1:

$$\mathcal{D} = (0.9) \cap \mathcal{D}(1, 1.1) = \{z \in C : \mathrm{Re}(z) < -0.9, |z+1| < 1.1\} : R = \left[\begin{array}{cc|cc} 1.8 & 0 & 1 & 0 \\ 0 & \text{-}0.21 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]$$

Making use of Theorem 3, one can obtain the gain matrix of state feedback controller guaranteeing uncertain linear system (21) to be $\mathcal{D}$-stable

$$K = \begin{bmatrix} -0.8037 & 0.1006 & 0.2745 & 0.1726 \\ -0.2854 & 0.1038 & 0.0171 & -0.4497 \end{bmatrix}$$

For convenience of emulation, we let uncertain parameters in system be $r_1(t) = \sin 5t$, $r_2(t) = \cos 5t$ and $s_1(t) = \sin t$ $(0 \leqslant t \leqslant 2\pi)$. For region $\mathcal{D}$ mentioned above, we may paint the state responding curve and the poles distributing graph of the closed-loop system, respectively, as following Fig. 1 and Fig. 2.

It is easy to see from Fig. 1 that uncertain linear state feedback system (20) is robust stable and from Fig. 2 that the closed-loop poles of uncertain linear state feedback system (20) lie in the region .

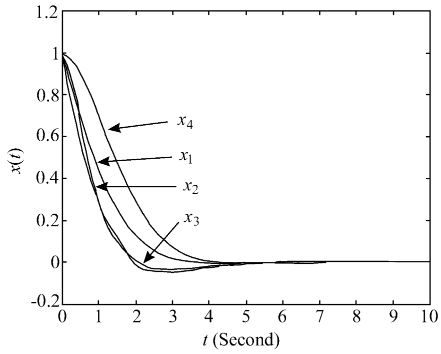

Fig. 1 State responding curve of closed-loop system to quadratic D-stability

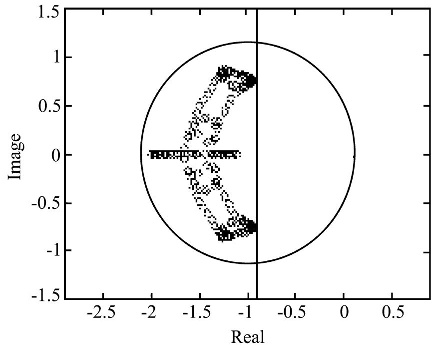

Fig. 2 Pole distribution of closed-loop system to quadratic stability

5 Conclusion

Quadratic matrix inequality regions are wide which includes LMI regions. Therefore, the quadratic $\mathcal{D}$-stability problem of state feedback system studied in this paper possesses universality. Furthermore, since inequality (16) is a sufficient and necessary condition that guarantees the closed-loop system (8) quadratic $\mathcal{D}$-stable, the design method of state feedback controller for system (1) in this paper possesses less conservatism. A benchmark example is given to illustrate the validity of the obtained results in this paper.

References

1 Gutman S, Jury E I. A general theory for matrix root-clustering in subregions of the complex plane. *IEEE Transactions on Automatic Control*, 1981, **26**(4): 853∼863

2 Chilali M, Gahinet P. H_∞ design with pole placement constraints: an LMI approach, *IEEE Transactions on Automatic Control*, 1996, **41**(3): 358∼367

3 Chilali M, Gahinet P, Apkarian P. Robust pole placement in LMI regions, *IEEE Transactions on Automatic Control*, 1999, **44**(12): 2257∼2270

4 Peaucelle D, Arzelier D, Bachelier O, Bernussou J. A new robust D-stability condition for real convex polytopic uncertainty, *Systems and Control Letters*, 2000, **40**(1): 21∼30

5 Valter J, Leite S, Pedro L, Peres D. An improved LMI condition for robust D-stability of uncertain polytopic systems, *IEEE Transactions on Automatic Control*, 2003, **48**(3): 500∼504

6 Mao W J, Chu J. Quadratic stabilization of dynamic interval systems, *IEEE Transactions on Automatic Control*, 2003, **48**(6): 1007∼1012

7 Xie L, Fu M, De Souza C E, H_∞ control and quadratic stabilization of systems with parameter uncertainty via output feedback, *IEEE Transactions on Automatic Control*, 1992, **37**(8): 1253∼1256

8 Kosmidou O I, Abou-Kandil H, Bertrand P. A game theoretic approach for guaranteed cost control. In: Proceedings of European Control Conference, Grenoble, France: 1991. 2220∼2225

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