Robust Dissipative Control for Linear Multi-variable Systems[1)]

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Abstract Robust quadratic dissipative control for a class of linear multi-variable systems with parameter uncertainties is considered, where the uncertainties are expressed in a linear fractional form. For the nominal system without uncertainties, the equivalence between quadratic dissipativeness and positive realness is established, and conditions are derived for linear systems to be quadratic dissipative. As for uncertain systems, it is shown that the robust quadratic dissipative control problem for the uncertain system can be reduced to the corresponding problem for a related system without uncertainties. The control problem concerned can be solved using LMI approach. The results of the paper unify existing results on H_∞ control and positive real control and provide a more flexible and less conservative control design method.

Key words Dissipative, dissipative control, positive real, uncertain systems, LMI

1 Introduction

The notion of dissipative systems plays important roles in system and control theory and its applications extensively in stability analysis, nonlinear control, adaptive control system design and so on[1,2]. For last decades, passive control and H_∞ control have received much attention[3~7]. However, passive control may lead to conservative results because only the phase information of systems is taken into account, while H_∞ control can also get conservative results because only the gain information of systems is used for the control. By contrast, dissipative control may lead to less conservative results since dissipativeness is a generalization of passivity and H_∞ performance, and provides a flexible tradoff between gain and phase. This is the reason why dissipative control problems have attracted much interest in recent years.

In this paper, we will address quadratic dissipative analysis and control problems for linear systems with norm-bounded parameter uncertainties. First we will investigate conditions under which the uncertain system is robust quadratic dissipative. Then we will design both state-feedback and output feedback controllers such that the closed-loop system is robust quadratic dissipative.

The rest of the paper is organized as follows. Section 2 gives system description and preliminaries. In Section 3, we present the necessary and sufficient conditions for the uncertain system to be robust quadratic dissipative. Robust quadratic dissipative control problems will be discussed in Section 4. Finally some concluding remarks are drawn in Section 5.

2 System description and preliminaries

Notation: In the following, if not explicitly stated, matrices are assumed to have compatible dimensions. The identity and zero matrices are denoted by I and 0, respectively. The notation $M > 0 (< 0)$ stands for M is positive definite (negative definite) and $G^*(s)$ means the complex conjugate transpose of $G(s)$.

Consider the uncertain linear system described by

$$(\Sigma_\Delta): \dot{\boldsymbol{x}}(t) = A_\Delta \boldsymbol{x}(t) + B_\Delta \boldsymbol{\omega}(t),\ \boldsymbol{x}(0) = 0$$
$$\boldsymbol{z}(t) = C_\Delta \boldsymbol{x}(t) + D_\Delta \boldsymbol{\omega}(t)$$

where $\boldsymbol{x}(t) \in R^n$ is the state, $\boldsymbol{\omega}(t) \in R^m$ the exogenous input and $\boldsymbol{z}(t) \in R^p$ the controlled output. $A_\Delta, B_\Delta, C_\Delta$ and D_Δ are uncertain parameter matrices satisfying the following assumptions

A1. $$\begin{bmatrix} A_\Delta & B_\Delta \\ C_\Delta & D_\Delta \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} - \begin{bmatrix} H \\ H_1 \end{bmatrix} \Delta(t) [E \quad E_1] \tag{1}$$

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where $\Delta(t)$ depends on uncertain matrix $F(t) \in R^{i\times j}$:

$$\Delta(t) = F(t)[I + JF(t)]^{-1},\ F^{\mathrm{T}}(t)F(t) \leqslant I,\ J^{\mathrm{T}}J < I \tag{2}$$

and $A, B, C, D, H, H_1, E, E_1, J$ are known constant matrices.

In the following the parameter uncertainties are said to be admissible if both (1) and (2) hold.

Now let′s recall the notion of quadratic dissipativeness. Consider the nominal system of system (Σ_Δ), that is, system (Σ_Δ) satisfying $F(t) = 0$:

$$\begin{aligned}(\Sigma_1): \dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + B\boldsymbol{\omega}(t), \quad \boldsymbol{x}(0) = 0\\ \boldsymbol{z}(t) &= C\boldsymbol{x}(t) + D\boldsymbol{\omega}(t)\end{aligned}$$

with its transfer function matrix

$$G(s) = D + C(sI - A)^{-1}B$$

It is assumed that system (Σ_1) is controllable and observable. Given $Q \in R^{p\times p}, S \in R^{p\times m}$ and $R \in R^{m\times m}$ with $Q = Q^{\mathrm{T}}$ and $R = R^{\mathrm{T}}$. Denote

$$M(s) = G^*(s)QG(s) + S^{\mathrm{T}}G(s) + G^*(s)S + R$$

Then, based on the definition of quadratic dissipativeness in [8], we give a notation of strictly quadratic dissipativeness as follows.

Definition 1. Assume that A is asymptotically stable. System (Σ_1) is said to be $(Q, S, R)-$*dissipative* if $M(j\omega) \geqslant 0, \forall 0 \leqslant \omega < \infty$ and system (Σ_1) is said to be $strictly(Q, S, R) - dissipative$ if $M(j\omega) > 0, \forall 0 \leqslant \omega \leqslant \infty$. For short $(Q, S, R) - dissipative$ and $strictly(Q, S, R) - dissipative$ are referred to as quadratic dissipative and strictly quadratic dissipative, respectively.

When $Q = R = 0, S = I, p = m$, Definition 1 reduces to the following definition:

Definition 2. Assume that A is asymptotically stable. System (Σ_1) is said to be strictly positive real if $G(j\omega) + G^*(j\omega) \geqslant 0, 0 \leqslant \omega < \infty$ and system (Σ_1) is said to be extended strictly positive real $(ESPR)$ if $G(j\omega) + G^*(j\omega) > 0,\ 0 \leqslant \omega \leqslant \infty$.

The $strictly(Q, S, R) - dissipative$ includes not only extended strictly positive realness but also the standard H_∞ performance as special cases. When $Q = -I, S = 0$ and $R = I$, the $strictly(Q, S, R) - dissipative$ reduces to the standard H_∞ performance. Noting that $Q \leqslant 0$ holds in these two cases, in the following we suppose

A2. $$\bar{Q} = -Q \geqslant 0$$

Definition 2 is a frequency domain characterization of ESPR systems. The state space characterization of ESPR systems can be given by

Lemma 1[9]. Given system (Σ_1) with $p = m$, the following statements are equivalent:

a) (Σ_1) is ESPR;

b) There exists $0 < P \in R^{n\times n}$ such that $\begin{bmatrix} A^{\mathrm{T}}P + PA & C^{\mathrm{T}} - PB \\ C - B^{\mathrm{T}}P & -(D^{\mathrm{T}} + D)\end{bmatrix} < 0$.

[8] obtained a state space characterization of quadratic dissipative systems by spectral factorization. For the strictly quadratic dissipative case we will give a corresponding result, which is essential to deal with strictly quadratic dissipative control problems. Augmenting system (Σ_1), a system is given as follows.

$$\begin{aligned}(\Sigma_2): \dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + [B \quad 0]\bar{\boldsymbol{\omega}}(t)\\ \boldsymbol{z}(t) &= \begin{bmatrix} S^{\mathrm{T}}C \\ -\bar{Q}^{1/2}C \end{bmatrix}\boldsymbol{x}(t) + \begin{bmatrix} R/2 + S^{\mathrm{T}}D & 0 \\ -\bar{Q}^{1/2}D & I_p/2 \end{bmatrix}\bar{\omega}(t)\end{aligned}$$

we have

Lemma 2. Consider systems (Σ_1) and (Σ_2), the following statements are equivalent:

a) (Σ_1) is $strictly(Q, S, R) - dissipative$

b) (Σ_2) is ESPR

c) There exists $0 < P \in R^{n\times n}$ such that

$$\begin{bmatrix} A^{\mathrm T}P + PA & PB - C^{\mathrm T}S & C^{\mathrm T}\bar{Q}^{1/2} \\ B^{\mathrm T}P - S^{\mathrm T}C & -D^{\mathrm T}S - S^{\mathrm T}D - R & D^{\mathrm T}\bar{Q}^{1/2} \\ \bar{Q}^{1/2}C & \bar{Q}^{1/2}D & -I \end{bmatrix} < 0 \tag{3}$$

Proof. The transfer function matrix of (Σ_2) is

$$T(s) = \begin{bmatrix} R/2 + S^{\mathrm T}G(s) & 0 \\ -\bar{Q}^{1/2}G(s) & I_p/2 \end{bmatrix}$$

Hence, b) $\Leftrightarrow$ A is asymptotically stable and

$$\begin{bmatrix} R + S^{\mathrm T}G(j\omega) + G^*(j\omega)S & -G^*(j\omega)\bar{Q}^{1/2} \\ -\bar{Q}^{1/2}G(j\omega) & I_p \end{bmatrix} > 0,\ \forall 0 \leqslant \omega \leqslant \infty$$

$\Leftrightarrow$ A is asymptotically stable and

$$G^*(j\omega)QG(j\omega) + S^{\mathrm T}G(j\omega) + G^*(j\omega)S + R > 0,\ \forall 0 \leqslant \omega \leqslant \infty$$

$\Leftrightarrow$ a).

By Lemma1, b) $\Leftrightarrow$ c). This completes the proof. □

By establishing equivalence between the strictly quadratic dissipativeness of system (Σ_1) and the extended strictly positive realness of another associated with (Σ_1), Lemma 2 gives a state space characterization of strictly quadratic dissipative systems, which will plays a key role in next sections.

3 Robustly dissipative analysis problem

Now we investigate conditions under which the uncertain system (Σ_Δ) is strict quadratic dissipative for all admissible uncertainties.

Definition 3. System (Σ_Δ) satisfying assumption A.1 is said to be *robustly strict* $(Q,S,R) -$ *dissipative* if there exists $0 < P \in R^{n\times n}$ such that the following LMI holds:

$$\begin{bmatrix} A_\Delta^{\mathrm T}P + PA_\Delta & PB_\Delta - C_\Delta^{\mathrm T}S & C_\Delta^{\mathrm T}\bar{Q}^{1/2} \\ B_\Delta^{\mathrm T}P - S^{\mathrm T}C_\Delta & -D_\Delta^{\mathrm T}S - S^{\mathrm T}D_\Delta - R & D_\Delta^{\mathrm T}\bar{Q}^{1/2} \\ \bar{Q}^{1/2}C_\Delta & \bar{Q}^{1/2}D_\Delta & -I \end{bmatrix} < 0 \tag{4}$$

for all admissible uncertainties.

For short *robustly strict*$(Q,S,R) -$ *dissipative* is said to be robustly strict quadratic dissipative. Now that condition (4) is difficult to verify because of the uncertainties, we will reduce the robustly dissipative analysis problem of (Σ_Δ) to that of a system without uncertainties. Introducing a system associated with (Σ_Δ):

$$\begin{aligned} (\Sigma_\varepsilon): \dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + B_\varepsilon\bar{\boldsymbol{\omega}}(t) = A\boldsymbol{x}(t) + [B\ \varepsilon H]\bar{\boldsymbol{\omega}}(t),\ \boldsymbol{x}(0) = 0 \\ \boldsymbol{z}(t) &= C_\varepsilon\boldsymbol{x}(t) + D_\varepsilon\bar{\boldsymbol{\omega}}(t) = \begin{bmatrix} \varepsilon^{-1}E \\ C \end{bmatrix}\boldsymbol{x}(t) + \begin{bmatrix} \varepsilon^{-1}E_1 & J \\ D & \varepsilon H_1 \end{bmatrix}\bar{\boldsymbol{\omega}}(t) \end{aligned}$$

where $\varepsilon > 0$ will be specified later.
Denoting

$$\widehat{Q} = \begin{bmatrix} -I & 0 \\ 0 & Q \end{bmatrix},\ \widehat{S} = \begin{bmatrix} 0 & 0 \\ S & 0 \end{bmatrix},\ \widehat{R} = \begin{bmatrix} R & 0 \\ 0 & I \end{bmatrix},\ \widehat{Q}_- = -\widehat{Q}$$

then, we have

Theorem 1. System (Σ_Δ) is *robustly strict*$(Q,S,R) -$ *dissipative* if and only if there exists a scalar $\varepsilon > 0$ such that (Σ_ε) is *strictly*$(\widehat{Q},\widehat{S},\widehat{R}) -$ *dissipative.*

Proof. By Definition 3, (Σ_Δ) is *robustly strict*$(Q,S,R) -$ *dissipative* if and only if there exists $0 < P \in R^{n\times n}$ such that

$$\begin{bmatrix} A_\Delta^{\mathrm T}P + PA_\Delta & PB_\Delta - C_\Delta^{\mathrm T}S & C_\Delta^{\mathrm T}\bar{Q}^{1/2} \\ B_\Delta^{\mathrm T}P - S^{\mathrm T}C_\Delta & -D_\Delta^{\mathrm T}S - S^{\mathrm T}D_\Delta - R & D_\Delta^{\mathrm T}\bar{Q}^{1/2} \\ \bar{Q}^{1/2}C_\Delta & \bar{Q}^{1/2}D_\Delta & -I \end{bmatrix} < 0$$

for all admissible uncertainties. Observing (1), it is easy to see that

$$\begin{bmatrix} A^{\mathrm{T}}P+PA & PB-C^{\mathrm{T}}S & C^{\mathrm{T}}\bar{Q}^{1/2} \\ B^{\mathrm{T}}P-S^{\mathrm{T}}C & -D_k^{\mathrm{T}}S-S^{\mathrm{T}}D-R & D_k^{\mathrm{T}}\bar{Q}^{1/2} \\ \bar{Q}^{1/2}C & \bar{Q}^{1/2}D_k & -I \end{bmatrix} -$$

$$\begin{bmatrix} PH \\ -S^{\mathrm{T}}H_1 \\ \bar{Q}^{1/2}H_1 \end{bmatrix} \Delta(t)[E \quad E_1 \quad 0] - \left(\begin{bmatrix} PH \\ -S^{\mathrm{T}}H_1 \\ \bar{Q}^{1/2}H_1 \end{bmatrix} \Delta(t)[E \quad E_1 \quad 0] \right)^{\mathrm{T}} < 0$$

This LMI is equivalent to

$$\begin{bmatrix} A^{\mathrm{T}}P+PA & PB-C^{\mathrm{T}}S & C^{\mathrm{T}}\bar{Q}^{1/2} \\ B^{\mathrm{T}}P-S^{\mathrm{T}}C & -D^{\mathrm{T}}S-S^{\mathrm{T}}D-R & D^{\mathrm{T}}\bar{Q}^{1/2} \\ \bar{Q}^{1/2}C & \bar{Q}^{1/2}D & -I \end{bmatrix} +$$

$$\begin{bmatrix} \varepsilon PH & \varepsilon^{-1}E^{\mathrm{T}} \\ -\varepsilon S^{\mathrm{T}}H_1 & \varepsilon^{-1}E_1^{\mathrm{T}} \\ \varepsilon\bar{Q}^{1/2}H_1 & 0 \end{bmatrix} \begin{bmatrix} I & -J^{\mathrm{T}} \\ -J & I \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon PH & \varepsilon^{-1}E^{\mathrm{T}} \\ -\varepsilon S^{\mathrm{T}}H_1 & \varepsilon^{-1}E_1^{\mathrm{T}} \\ \varepsilon\bar{Q}^{1/2}H_1 & 0 \end{bmatrix}^{\mathrm{T}} < 0$$

for some $\varepsilon > 0$ (See [3]). By Schur complements, it follows that

$$\begin{bmatrix} A^{\mathrm{T}}P+PA & PB-C^{\mathrm{T}}S & \varepsilon PH & \varepsilon^{-1}E^{\mathrm{T}} & C^{\mathrm{T}}\bar{Q}^{1/2} \\ B^{\mathrm{T}}P-S^{\mathrm{T}}C & -D^{\mathrm{T}}S-S^{\mathrm{T}}D-R & -\varepsilon S^{\mathrm{T}}H_1 & \varepsilon^{-1}E_1^{\mathrm{T}} & D^{\mathrm{T}}\bar{Q}^{1/2} \\ \varepsilon H^{\mathrm{T}}P & -\varepsilon H_1^{\mathrm{T}}S & -I & J^{\mathrm{T}} & \varepsilon H_1^{\mathrm{T}}\bar{Q}^{1/2} \\ \varepsilon^{-1}E & \varepsilon^{-1}E_1 & J & -I & 0 \\ \bar{Q}^{1/2}C & \bar{Q}^{1/2}D & \varepsilon\bar{Q}^{1/2}H_1 & 0 & -I \end{bmatrix} < 0$$

i.e.,

$$\begin{bmatrix} A^{\mathrm{T}}P+PA & PB_\varepsilon - C_\varepsilon^{\mathrm{T}}\widehat{S} & C_\varepsilon^{\mathrm{T}}\widehat{Q}_-^{1/2} \\ B_\varepsilon^{\mathrm{T}}P - \widehat{S}^{\mathrm{T}}C_\varepsilon & -D_\varepsilon^{\mathrm{T}}\widehat{S} - \widehat{S}^{\mathrm{T}}D_\varepsilon - \widehat{R} & D_\varepsilon^{\mathrm{T}}\widehat{Q}_-^{1/2} \\ \widehat{Q}_-^{1/2}C_\varepsilon & \widehat{Q}_-^{1/2}D_\varepsilon & -I \end{bmatrix} < 0$$

Thus, (Σ_ε) is *strictly*$(\widehat{Q},\widehat{S},\widehat{R})$ − *dissipative*. □

In view of Theorem 1, the robustly strict quadratic dissipative analysis of system (Σ_Δ) is simplified to that of (Σ_ε) without uncertainties. The latter can be solved using LMI approach.

Theorem 2. (Σ_ε) is *strictly*$(\widehat{Q},\widehat{S},\widehat{R})$ − *dissipative* if and only if there exists a scalar $\mu > 0$ and a matrix $X > 0$ such that

$$\begin{bmatrix} XA^{\mathrm{T}}+AX & B-XC^{\mathrm{T}}S & \mu H & XE^{\mathrm{T}} & XC^{\mathrm{T}}\bar{Q}^{1/2} \\ B^{\mathrm{T}}-S^{\mathrm{T}}CX & -D^{\mathrm{T}}S-S^{\mathrm{T}}D-R & -\mu S^{\mathrm{T}}H_1 & E_1^{\mathrm{T}} & D^{\mathrm{T}}\bar{Q}^{1/2} \\ \mu H^{\mathrm{T}} & -\mu H_1^{\mathrm{T}}S & -\mu I & \mu J^{\mathrm{T}} & \mu H_1^{\mathrm{T}}\bar{Q}^{1/2} \\ EX & E_1 & \mu J & -\mu I & 0 \\ \bar{Q}^{1/2}CX & \bar{Q}^{1/2}D & \mu\bar{Q}^{1/2}H_1 & 0 & -I \end{bmatrix} < 0 \tag{5}$$

Proof. By Lemma 2, (Σ_ε) is *strictly*$(\widehat{Q},\widehat{S},\widehat{R})$ − *dissipative* if and only if there exists a scalar $\varepsilon > 0$ and a matrix $P > 0$ such that

$$\begin{bmatrix} A^{\mathrm{T}}P+PA & PB-C^{\mathrm{T}}S & \varepsilon PH & \varepsilon^{-1}E^{\mathrm{T}} & C^{\mathrm{T}}\bar{Q}^{1/2} \\ B^{\mathrm{T}}P-S^{\mathrm{T}}C & -D^{\mathrm{T}}S-S^{\mathrm{T}}D-R & -\varepsilon S^{\mathrm{T}}H_1 & \varepsilon^{-1}E_1^{\mathrm{T}} & D^{\mathrm{T}}\bar{Q}^{1/2} \\ \varepsilon H^{\mathrm{T}}P & -\varepsilon H_1^{\mathrm{T}}S & -I & J^{\mathrm{T}} & \varepsilon H_1^{\mathrm{T}}\bar{Q}^{1/2} \\ \varepsilon^{-1}E & \varepsilon^{-1}E_1 & J & -I & 0 \\ \bar{Q}^{1/2}C & \bar{Q}^{1/2}D & \varepsilon\bar{Q}^{1/2}H_1 & 0 & -I \end{bmatrix} < 0$$

Pre-multiply and post-multiply the last matrix by $\mathrm{diag}(P^{-1}, I, \varepsilon I, \varepsilon I, I)$ and let $\varepsilon^2 = \mu$ and $P^{-1} = X$. The desired result follows immediately. □

4 Robustly dissipative control problem

Consider the following uncertain system

$$(\Sigma_{\Delta s}): \dot{\boldsymbol{x}}(t) = A_\Delta \boldsymbol{x}(t) + B\boldsymbol{\omega}(t) + B_{1\Delta}\boldsymbol{u}(t), \quad \boldsymbol{x}(0) = 0$$

$$z(t) = Cx(t) + D_\Delta \omega(t) + D_{12\Delta} u(t)$$

where $u(t) \in R^q$ is the control input, $B_{1\Delta} = B_1 - H\Delta(t)E_2$, $D_{12} = D_{12} - H_1\Delta(t)E_2$, B_1, D_{12} and E_2 are known matrices, and the rest matrices are the same as those in system (Σ_Δ).

4.1 State-feedback design

Suppose the state can be measured, the robustly dissipative control problem via state feedback can be stated as follows: design a state feedback controller $u(t) = Kx(t)$ for uncertain system (Σ_Δ) such that the closed-loop system is robustly strict quadratic dissipative. It is expected to reduce the robustly dissipative control problem via state feedback to that for a system without uncertainties. To this end, an auxiliary system associated with (Σ_Δ) is introduced.

$$(\Sigma_{\varepsilon s}) : \dot{x}(t) = Ax(t) + B_\varepsilon \bar{\omega}(t) + B_1 u(t),\ x(0) = 0$$
$$z(t) = C_\varepsilon x(t) + D_\varepsilon \bar{\omega}(t) + D_{1\varepsilon} u(t)$$

where $D_{1\varepsilon}^{\mathrm{T}} = [\varepsilon^{-1}E_2^{\mathrm{T}} \quad D_{12}^{\mathrm{T}}]$.

Theorem 3. There exists a state-feedback controller $u(t) = Kx(t)$ for system $(\Sigma_{\Delta s})$ such that the resulting closed-loop system is *robustly strict*$(Q, S, R) - $*dissipative* if and only if there exists a scalar $\varepsilon > 0$ such that the closed-loop system composed of system $(\Sigma_{\varepsilon s})$ and the controller is *strictly* $(\widehat{Q}, \widehat{S}, \widehat{R}) - $*dissipative*.

Proof. Applying $u(t) = Kx(t)$ to $(\Sigma_{\Delta s})$, we get the closed-loop system

$$\dot{x}(t) = \tilde{A}_\Delta x(t) + B_\triangle \bar{\omega}(t), \quad x(0) = 0, \quad z(t) = \tilde{C}_\Delta x(t) + D_\Delta \bar{\omega}(t)$$

where

$$\tilde{A}_\Delta = \tilde{A} - H\Delta(t)\tilde{E},\ \tilde{C}_\Delta = \tilde{C} - H_1\Delta(t)\tilde{E}$$

and

$$\tilde{A} = A + B_1K,\ \tilde{C} = C + D_{12}K,\ \tilde{E} = E + E_2K$$

Applying $u(t) = Kx(t)$ to $(\Sigma_{\varepsilon s})$, we obtain

$$(\Sigma_{\varepsilon k}) : \dot{x}(t) = \tilde{A}x(t) + B_\varepsilon \bar{\omega}(t)$$
$$z(t) = \tilde{C}_\varepsilon x(t) + D_\varepsilon \bar{\omega}(t)$$

where $\tilde{C}_\varepsilon^{\mathrm{T}} = [\varepsilon^{-1}\tilde{E}^{\mathrm{T}} \quad \tilde{C}^{\mathrm{T}}]$.

Consider the two closed-loop systems above and the desired result is readily obtained using Theorem 1. □

By Theorem 3, in order to solve the robustly dissipative control problem for uncertain system $(\Sigma_{\Delta s})$, we only need to solve that for $(\Sigma_{\varepsilon s})$.

Theorem 4. There exists a state-feedback controller $u(t) = Kx(t)$ for system $(\Sigma_{\varepsilon s})$such that the resulting closed-system is *strictly*$((\widehat{Q}, \widehat{S}, \widehat{R}) - $*dissipative* if and only if there exists a scalar $\mu > 0$ and matrices $W \in R^{n\times n}$ and $0 < X \in R^{n\times n}$ such that

$$\begin{bmatrix} XA^{\mathrm{T}}+W^{\mathrm{T}}B_1^{\mathrm{T}}+AX+B_1W & B-(CX+D_{12}W)^{\mathrm{T}}S & \mu H & (EX+E_2W)^{\mathrm{T}} & (CX+D_{12}W)^{\mathrm{T}}\bar{Q}^{1/2} \\ B^{\mathrm{T}}-S^{\mathrm{T}}(CX+D_{12}W) & -D^{\mathrm{T}}S-S^{\mathrm{T}}D-R & -\mu S^{\mathrm{T}}H_1 & E_1^{\mathrm{T}} & D^{\mathrm{T}}\bar{Q}^{1/2} \\ \mu H^{\mathrm{T}} & -\mu H_1^{\mathrm{T}}S & -\mu I & \mu J^{\mathrm{T}} & \mu H_1^{\mathrm{T}}\bar{Q}^{1/2} \\ EX+E_2W & E_1 & \mu J & -\mu J & 0 \\ \bar{Q}^{1/2}(CX+D_{12}W) & \bar{Q}^{1/2}D & \mu\bar{Q}^{1/2}H_1 & 0 & -I \end{bmatrix} < 0 \tag{6}$$

Moreover, if X^*, W^* and μ^* is a solution to (6), $u(t) = W^*(X^*)^{-1}x(t)$ is a desired controller.

Proof. (Similar to the proof of Theorem 2, it is omitted.) □

4.2 Dynamic output feedback design

Consider the following system

$$(\Sigma_{\Delta f}) : \dot{x} = A_\Delta x(t) + B_\Delta \omega(t) + B_{1\Delta} u(t), \quad x(0) = 0$$
$$z(t) = C_\Delta x(t) + D_\Delta \omega(t) + D_{12\Delta} u(t)$$
$$y(t) = C_{1\Delta} x(t) + D_{21\Delta} \omega(t) + D_{22\Delta} u(t)$$

where $\boldsymbol{y}(t) \in R^r$ is the measured output

$$[C_{1\Delta} \quad D_{21\Delta} \quad D_{22\Delta}] = [C_1 \quad D_{21} \quad D_{22}] - H_2\Delta(t)[E \quad E_1 \quad E_2]$$

H_2, C_1, D_{21} and D_{22} are known matrices, and the rest matrices are the same in $(\Sigma_{\Delta s})$.

In this section, we will address the robustly dissipative control problem via output feedback, that is, design of a dynamic output feedback controller of the form

$$\begin{aligned}(\Sigma_f): \dot{\boldsymbol{\xi}}(t) &= F_0\boldsymbol{\xi}(t) + G_0\boldsymbol{y}(t)\\ \boldsymbol{u}(t) &= K_0\boldsymbol{\xi}(t)\end{aligned}$$

for $(\Sigma_{\Delta f})$ such that the resulting closed-loop system is *robustly strict*$(Q,S,R)-$*dissipative*.

Similar to the case of state feedback design, an auxiliary system related to $(\Sigma_{\Delta f})$ is constructed

$$\begin{aligned}(\Sigma_{\varepsilon f})\ \dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + B_\varepsilon\bar{\boldsymbol{\omega}}(t) + B_1\boldsymbol{u}(t)\\ \boldsymbol{z}(t) &= C_\varepsilon\boldsymbol{x}(t) + D_\varepsilon\bar{\boldsymbol{\omega}}(t) + D_{1\varepsilon}\boldsymbol{u}(t)\\ \boldsymbol{y}(t) &= C_1\boldsymbol{x}(t) + D_{2\varepsilon}\bar{\boldsymbol{\omega}}(t) + D_{22}\boldsymbol{u}(t)\end{aligned}$$

where $D_{2\varepsilon} = [D_{21} \quad \varepsilon H_2]$, and the other matrices are defined as in $(\Sigma_{\varepsilon s})$.

The following theorem gives the main result of this paper.

Theorem 5. Dynamic output feedback controller (Σ_f) for $(\Sigma_{\Delta f})$ achieves *robustly strict*$(Q,S,R)-$*dissipative* if and only if the controller (Σ_f) for $(\Sigma_{\varepsilon f})$ achieves *strictly*$(\widehat{Q},\widehat{S},\widehat{R})-$*dissipative* for some $\varepsilon > 0$.

Proof. On the one hand, the closed-loop system of $(\Sigma_{\Delta f})$ together with (Σ_f) is

$$\begin{aligned}(\Sigma_{\Delta c}): \dot{\boldsymbol{\eta}}(t) &= \bar{A}_\Delta\boldsymbol{\eta}(t) + \bar{B}_\Delta\boldsymbol{\omega}(t), \quad \boldsymbol{x}(0) = 0\\ \boldsymbol{z}(t) &= \bar{C}_\Delta\boldsymbol{\eta}(t) + D_\Delta\boldsymbol{\omega}(t)\end{aligned}$$

where

$$\begin{aligned}&\boldsymbol{\eta}(t) = [\boldsymbol{x}^{\mathrm{T}}(t) \quad \boldsymbol{\xi}^{\mathrm{T}}(t)]^{\mathrm{T}}, \quad \bar{C}_\Delta = \bar{C} - H_1\Delta(t)\bar{E}, \quad \bar{C} = [C \quad D_{12}K_0]\\ &\bar{E} = [E \quad E_2K_0], \quad [\bar{A}_\Delta \quad \bar{B}_\Delta] = [\bar{A} \quad \bar{B}] - \bar{H}\Delta(t)[\bar{E} \quad E_1]\\ &\bar{A} = \begin{bmatrix} A & B_1K_0\\ G_0C_1 & F_0 + G_0D_{22}K_0\end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B\\ G_0D_{21}\end{bmatrix}, \quad \bar{H} = \begin{bmatrix} H\\ G_0H_2\end{bmatrix}\end{aligned}$$

on the other hand, the closed-loop system composed of $(\Sigma_{\varepsilon f})$ and (Σ_f) is

$$\begin{aligned}(\Sigma_{\varepsilon c}): \dot{\boldsymbol{\eta}}(t) &= \bar{A}\boldsymbol{\eta}(t) + \bar{B}_\varepsilon\bar{\boldsymbol{\omega}}(t), \quad \boldsymbol{x}(0) = 0\\ \boldsymbol{z}(t) &= \bar{C}_\varepsilon\boldsymbol{\eta}(t) + D_\varepsilon\boldsymbol{\omega}(t)\end{aligned}$$

where $\bar{B}_\varepsilon = [\bar{B} \quad \varepsilon\bar{H}]$, $\bar{C}_\varepsilon^{\mathrm{T}} = [\varepsilon^{-1}\bar{E}^{\mathrm{T}} \quad \bar{C}^{\mathrm{T}}]$.

For systems $(\Sigma_{\Delta c})$ and $(\Sigma_{\varepsilon c})$ using Theorem 1, the desired result is readily obtained. □

Theorem 5 provides necessary and sufficient conditions for robustly dissipative control problems via output feedback. From Theorem 5 together with Theorem 3, it can be concluded that robustly dissipative control problems can be converted to those without uncertainties.

5 Conclusions

In this paper we have studied strictly quadratic dissipative analysis and control problems for a class of uncertain systems. Necessary and sufficient conditions have been obtained for the uncertain systems to be robustly strict quadratic dissipative. It has been shown that robust strict quadratic dissipative control problems for the uncertain systems can be reduced to those for some related systems without uncertainties. The latter can be solved using an LMI approach. As a byproduct, it has been shown that strictly quadratic disspativeness of a linear system is equivalent to extended strictly positive realness of a related system. By the equivalence, strictly quadratic disspativeness characterization of linear systems has been derived. The proof, compared with that of [8], is simple.

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