Fuzzy H_∞ Filter Design for a Class of Nonlinear Systems with Time Delays *via* LMI[1)]

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Abstract This paper develops fuzzy H_∞ filter for state estimation approach for nonlinear discrete-time systems with multiple time delays and unknown bounded disturbances. We design a stable fuzzy H_∞ filter based on the Takagi-Sugeno (T-S) fuzzy model, which assures asymptotic stability and a prescribed H_∞ index for the filtering error system. Sufficient condition for the existence of such a filter is established by solving the linear matrix inequality (LMI) problem. The LMI problem can be efficiently solved with global convergence using the interior point algorithm. Simulation examples are provided to illustrate the design procedure of the proposed method.

Key words Nonlinear systems, H_∞ filter, state estimation, Takagi-Sugeno fuzzy model, time delays, linear matrix inequality (LMI)

1 Introduction

Robust H_∞ control and filtering approaches have been an attractive research topic over the past few years[1~7]. However, it is difficult to design an efficient filter for signal estimation of nonlinear systems. As is well known, time delays usually exist in many dynamic systems. Since time delays usually result in unsatisfactory performance and are frequently a source of instability, their presence must be taken into account in realistic filter design[8]. In [8], the problem of robust H_∞ filtering of linear discrete-time systems with multiple time delays was addressed. However, the system considered was linear and the filter designed was irrelevant with time delays, which resulted in the filter was much conservative. In [9], an H_∞ filtering method for nonlinear discrete-time systems was proposed without considering the presence of state delays in the state-space model. This paper deals with the fuzzy H_∞ filtering problem for nonlinear discrete-time systems with time delays in the state variables based on [9], and the filter designed relies on time delays.

2 The modeling of a class of nonlinear discrete-time systems with time delays using the T-S fuzzy model

Consider a class of nonlinear discrete-time systems with multiple time delays and unknown bounded disturbances:

$$\begin{aligned} &\boldsymbol{x}(k+1) = F(\boldsymbol{x}(k)) + F_d(\boldsymbol{x}(k-d_2), \boldsymbol{x}(k-d_2), \cdots, \boldsymbol{x}(k-d_q)) + B\boldsymbol{w}(k) \\ &\boldsymbol{y}(k) = H(\boldsymbol{x}(k)) + H_d(\boldsymbol{x}(k-d_1), \boldsymbol{x}(k-d_2), \cdots, \boldsymbol{x}(k-d_q)) + G\boldsymbol{v}(k) \\ &\boldsymbol{s}(k) = L\boldsymbol{x}(k) + \sum_{j=1}^{q} L_{dj}\boldsymbol{x}(k-d_j) \end{aligned} \tag{1}$$

where $\boldsymbol{x} = [x_1(k), x_2(k), \cdots, x_n(k)]^{\mathrm{T}} \in R^{n\times 1}$ denotes the state vector; $\boldsymbol{y}(k) \in R^{m\times 1}$ denotes the measurements vector; $\boldsymbol{s}(k) \in R^{p\times 1}$ denotes the signal to be estimated; $\boldsymbol{w}(k) \in R^{n\times 1}$ and $\boldsymbol{v}(k) \in R^{m\times 1}$ assumed to be bounded external disturbance and measurement noise, respectively; $d_1, d_2, \cdots, d_q$ denote bounded time delays of the state; $B \in R^{n\times n}, G \in R^{m\times n}, L \in R^{p\times n}$ and $L_{dj} \in R^{p\times n}$ are constant matrices; $F(\boldsymbol{x}(k))$, $F_d(\boldsymbol{x}(k-d_1), \boldsymbol{x}(k-d_2), \cdots, \boldsymbol{x}(k-d_q)) \in R^{n\times 1}$, $H(\boldsymbol{x}(k))$, $H_d(\boldsymbol{x}(k-d_1), \boldsymbol{x}(k-d_2), \cdots, \boldsymbol{x}(k-d_q)) \in R^{m\times 1}$ are vector fields with $F(0) = H(0) = 0$, $F_d(0, 0, \cdots 0) = H_d(0, 0, \cdots 0) = 0$.

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The initial conditions are given by $\boldsymbol{x}(k)=\boldsymbol{0}$ for $k=-d_{\max},-d_{\max}+1,\cdots,-1$, and $\boldsymbol{x}(0)=\boldsymbol{x}_0$, where $d_{\max}=\max\limits_{1\leqslant j\leqslant q}(d_j)$.

For the sake of convenience, define $[\boldsymbol{x}(k-d_1),\boldsymbol{x}(k-d_2),\cdots,\boldsymbol{x}(k-d_q)]^{\rm T}\doteq\boldsymbol{x}(k,\boldsymbol{d},q)$, $F_d(\boldsymbol{x}(k-d_1),\boldsymbol{x}(k-d-2),\cdots,\boldsymbol{x}(k-d_q))\doteq F_d(\boldsymbol{x}(k,\boldsymbol{d},q))$, $H_d(\boldsymbol{x}(k-d_1),\boldsymbol{x}(k-d_2),\cdots,\boldsymbol{x}(k-d_q))\doteq H_d(\boldsymbol{x}(k,\boldsymbol{d},q))$.

The ith rule of the T-S fuzzy model for the nonlinear discrete-time systems with multiple time delays in (1) is proposed as the following form.

$$\begin{aligned}&\text{Rule } i: \text{If } x_1(k) \text{ is } F_{i1}\cdots \text{ and } x_g(k) \text{ is } F_{ig}, \text{ then}\\ &\boldsymbol{x}(k+1)=A_i\boldsymbol{x}(k)+\sum_{j=1}^{q}A_{ji}^d\boldsymbol{x}(k-d_j)+B\boldsymbol{w}(k)\\ &\boldsymbol{y}(k)=C_i\boldsymbol{x}+\sum_{j=1}^{q}C_{ji}^d\boldsymbol{x}(k-d_j)+G\boldsymbol{v}(k)\\ &\boldsymbol{s}(k)=L\boldsymbol{x}(k)+\sum_{j=1}^{q}L_{dj}\boldsymbol{x}(k-d_j)\end{aligned}\tag{2}$$

where $i=1,2,\cdots,L$; F_{ij} is the fuzzy set; $A_i,A_{ji}^d\in R^{n\times n}$, $C_i,C_{ji}^d\in R^{m\times n}$, and $k=0,\cdots,k_f-1$.

The fuzzy system in (2) has singleton fuzzifier, product inference, and centroid defuzzifier. Its state dynamics and the output equation are of the following form, respectively:

$$\boldsymbol{x}(k+1)=\sum_{i=1}^{L}h_i(\boldsymbol{x}(k))[A_i\boldsymbol{x}(k)+\sum_{j=1}^{q}A_{ji}^d\boldsymbol{x}(k-d_j)]+B\boldsymbol{w}(k)\tag{3}$$

$$\boldsymbol{y}(k)=\sum_{i=1}^{L}h_i(\boldsymbol{x}(k))[C_i\boldsymbol{x}(k)+\sum_{j=1}^{q}C_{ji}^d\boldsymbol{x}(k-d_j)]+G\boldsymbol{v}(k)\tag{4}$$

where $\mu_i(\boldsymbol{x}(k))=\prod\limits_{j=1}^{g}F_{ij}(x_j(k))$, $h_i(\boldsymbol{x}(k)=\mu_i(\boldsymbol{x}(k))/\sum\limits_{i=1}^{L}\mu_i(\boldsymbol{x}(k))$, $F_{ij}(x_j(k))$ is the membership function of $x_j(k)$ in F_{ij} and $\mu_i(\boldsymbol{x}(k))\geqslant 0,\sum\limits_{i=1}^{L}h_i(\boldsymbol{x}(k))=1$.

Therefore, the nonlinear system in (1) can be described as

$$\begin{aligned}&\boldsymbol{x}(k+1)=\sum_{i=1}^{L}h_i(\boldsymbol{x}(k))[A_i\boldsymbol{x}(k)+\sum_{j=1}^{q}A_{ji}^d\boldsymbol{x}(k-d_j)]+B\boldsymbol{w}(k)+\Delta F(\boldsymbol{x}(k))+\Delta F_d(\boldsymbol{x}(k,\boldsymbol{d},q))\\ &\boldsymbol{y}(k+1)=\sum_{i=1}^{L}h_i(\boldsymbol{x}(k))[C_i\boldsymbol{x}(k)+\sum_{j=1}^{q}C_{ji}^d\boldsymbol{x}(k-d_j)]+G\boldsymbol{v}(k)+\Delta H(\boldsymbol{x}(k))+\Delta H_d(\boldsymbol{x}(\boldsymbol{x}(k,\boldsymbol{d},q))\end{aligned}\tag{5}$$

where

$$\begin{aligned}&\Delta F(\boldsymbol{x}(k))=F(\boldsymbol{x}(k))-\sum_{i=1}^{L}h_i(\boldsymbol{x}(k))A_i\boldsymbol{x}(k),\ \Delta H(\boldsymbol{x}(k))=H(\boldsymbol{x}(k)\\ &\Delta F_d(\boldsymbol{x}(k,\boldsymbol{d},q))=F_d(\boldsymbol{x}(k,\boldsymbol{d},q))-\sum_{i=1}^{L}h_i(\boldsymbol{x}(k))\sum_{j=1}^{q}A_{ji}^d\boldsymbol{x}(k-d_j),\ \Delta H_d(\boldsymbol{x}(k,\boldsymbol{d},q)=\\ &\qquad H_d(\boldsymbol{x}(k,\boldsymbol{d},q))-\sum_{i=1}^{L}h_i(\boldsymbol{x}(k))\sum_{j=1}^{q}C_{ji}^d\boldsymbol{x}(k-d_j)\end{aligned}$$

Here, $\Delta F(\boldsymbol{x}(k)),\Delta F_d(\boldsymbol{x}(k,\boldsymbol{d})),\Delta H(\boldsymbol{x}(k)),\Delta H_d(\boldsymbol{x}(k,\boldsymbol{d}))$ denote the approximation errors between the system in (1) and the T-S fuzzy model in (3) and (4). We assume there exist $\varOmega,\varOmega_d$, $\varPsi$ and $\varPsi_d$ such that the following inequalities hold for all $\boldsymbol{x}(k)$

$$\Delta F(\boldsymbol{x}(k))^{\rm T}\Delta F(\boldsymbol{x}(k))\leqslant\boldsymbol{x}(k)^{\rm T}\varOmega\boldsymbol{x}(k),\quad \Delta H(\boldsymbol{x}(k))^{\rm T}\Delta H(\boldsymbol{x}(k))\leqslant\boldsymbol{x}(k)^{\rm T}\varPsi\boldsymbol{x}(k)$$

$$\Delta F_d(\boldsymbol{x}(k,\boldsymbol{d},q))^{\mathrm{T}}\Delta F_d(\boldsymbol{x}(k,\boldsymbol{d},q)) \leqslant \sum_{j=1}^{q}\boldsymbol{x}^{\mathrm{T}}(k-d_j)\Omega_{dj}\boldsymbol{x}(k-d_j)$$

$$\Delta H_d(\boldsymbol{x}(k,\boldsymbol{d},q))^{\mathrm{T}}\Delta H_d(\boldsymbol{x}(k,\boldsymbol{d},q)) \leqslant \sum_{j=1}^{q}\boldsymbol{x}^{\mathrm{T}}(k-d_j)\Psi_{dj}\boldsymbol{x}(k-d_j) \tag{6}$$

where $\Omega, \Omega_d = \mathrm{diag}\{\Omega_{d1}, \Omega_{d2}, \cdots, \Omega_{dq}\}$, Ψ and $\Psi_d = \mathrm{diag}\{\Psi_{d1}, \Psi_{d2}, \cdots, \Psi_{dq}\}$ are positive symmetric matrices.

3 Fuzzy H_∞ filtering design

The lth rule of fuzzy H_∞ filtering based on the T-S fuzzy model is given in the following form:

Rule l : If $\hat{x}_1(k)$ is $F_{i1}\cdots$ and $\hat{x}_g(k)$ is F_{lg}, *then*

$$\hat{\boldsymbol{x}}(k+1) = A_l\boldsymbol{x}(k) + \sum_{j=1}^{q}A_{jl}^{d}\hat{\boldsymbol{x}}(k-d_j) + K_l[y(k)-\hat{\boldsymbol{y}}(k)],\ \hat{\boldsymbol{s}}(k) = L\hat{\boldsymbol{x}}(k) + \sum_{j=1}^{q}L_{dj}\hat{\boldsymbol{x}}(k-d_j) \tag{7}$$

where K_l is the fuzzy estimator gain for the lth estimation rule ($l = 1,2,\cdots,L$); $\hat{\boldsymbol{x}}(k) = 0$ for $k = -d_{\max}, -d_{\max}+1, \cdots, -1$ and $\hat{x}(0) = \hat{x}_0$ are the initial estimates; and $\boldsymbol{y} = \sum_{s=1}^{L}h_s(\boldsymbol{x}(k))(C_s\hat{\boldsymbol{x}}(k) + \sum_{j=1}^{q}C_{js}^{d}\hat{\boldsymbol{x}}(k-d_j))$.

The overall fuzzy estimator in (7) is written as

$$\begin{aligned}\hat{\boldsymbol{x}} =& h_{ils}[A_l\boldsymbol{x}(k) + \sum_{j=1}^{q}A_{jl}^{d}\hat{\boldsymbol{x}}(k-d_j) + K_lC_i\boldsymbol{x}(k) + K_l\sum_{j=1}^{q}C_{ji}^{d}\boldsymbol{x}(k-d_j) + K_lG\boldsymbol{v}(k) + \\ & K_l\Delta h(\boldsymbol{x}(k)) + K_l\Delta H_d(\boldsymbol{x}(k,\boldsymbol{d},q)) - K_lC_s\hat{\boldsymbol{x}}(k) - K_l\sum_{j=1}^{q}C_{js}^{d}\boldsymbol{x}(k-d_j)]\end{aligned} \tag{8}$$

where $h_{ils} \doteq \sum_{i=1}^{L}h_i(\boldsymbol{x}(k))\sum_{l=1}^{L}h_l\hat{\boldsymbol{x}}(k)\sum_{s=1}^{L}h_s(\boldsymbol{x}(k))$.

From (5) and (8), the augmented filtering error system can be written in the following form.

$$\begin{aligned}&\boldsymbol{\eta}(k+1) = \tilde{A}\boldsymbol{\eta}(k) + \sum_{j=1}^{q}\tilde{A}_{dj}\eta(k-d_j) + \tilde{E}\Delta\boldsymbol{\Gamma}(k) + \tilde{E}_d\Delta\boldsymbol{\Gamma}_d(k,\boldsymbol{d},q) + \tilde{G}\varpi(k)\\ &\boldsymbol{e}(k) = \boldsymbol{s}(k) - \hat{\boldsymbol{s}} = \tilde{L}\boldsymbol{\eta}(k) + \sum_{j=1}^{q}\tilde{L}_{dj}\eta(k-d_j)\end{aligned} \tag{9}$$

where

$$\boldsymbol{\eta}(k) = \begin{bmatrix}\boldsymbol{x}\\ \hat{\boldsymbol{x}}(k)\end{bmatrix},\ \tilde{A}_{ils} = \begin{bmatrix}A_i & 0\\ K_lC_i & A_l - K_lC_s\end{bmatrix},\ \tilde{A}_{jils}^{d} = \begin{bmatrix}A_{ji}^{d} & 0\\ K_lC_{ji}^{d} & A_{jl}^{d} - K_lC_{js}^{d}\end{bmatrix},\ \tilde{E}_l = \begin{bmatrix}I & 0\\ 0 & K_l\end{bmatrix}$$

$$\tilde{E}_l^{d} = \begin{bmatrix}I & 0\\ 0 & K_l\end{bmatrix},\ \Delta\Gamma(k) = \begin{bmatrix}\Delta F(\boldsymbol{x}(k))\\ \Delta H(\boldsymbol{x}(k))\end{bmatrix},\ \Delta\Gamma_d(k,\boldsymbol{d},q) = \begin{bmatrix}\Delta F_d(\boldsymbol{x}(k,\boldsymbol{d},q))\\ \Delta H_d(\boldsymbol{x}(k,\boldsymbol{d},q))\end{bmatrix},\ \tilde{G}_l = \begin{bmatrix}B & 0\\ 0 & K_lG\end{bmatrix}$$

$$\varpi(k) = \begin{bmatrix}\boldsymbol{w}(k)\\ \boldsymbol{v}(k)\end{bmatrix},\ \tilde{A} = h_{ils}\tilde{A}_{ils},\ \tilde{A}_{dj} = h_{ils}\tilde{A}_{jils}^{d},\ \tilde{E} = h_{ils}\tilde{E}_l,\ \tilde{E}_d = h_{ils}\tilde{E}_l^{d},\ \tilde{G} = h_{ils}\tilde{G}_l$$

$$\tilde{L} = h_{ils}[L \quad -L],\ \tilde{L}_{dj} = h_{ils}[L_{dj} \quad -L_{dj}]$$

Next, we define H_∞ index of the H_∞ filter. Given a scalar $\gamma > 0$ and initial estimation error, the H_∞ index of the robust H_∞ filter is as follows [9].

$$\sum_{k=1}^{k_f}\boldsymbol{e}^{\mathrm{T}}(k)Q\boldsymbol{e}(k) \leqslant \gamma^2\Big[\boldsymbol{e}^{\mathrm{T}}(0)P_0\boldsymbol{e}(0) + \sum_{k=0}^{k_f-1}(\boldsymbol{w}^{\mathrm{T}}(k)W_w\boldsymbol{w}(k) + \boldsymbol{v}(k)^{\mathrm{T}}W_v\boldsymbol{v}(k))\Big] \tag{10}$$

where $Q \in R^{q\times q}, P_0 \in R^{q\times q}, W_w \in R^{n\times n}$, and $W_v \in R^{m\times n}$ are positive-definite weighting matrices.

The fuzzy H_∞ filter problem is to design an asymptotically stable nonlinear filter of the form of (7) such that the filtering error system in (9) is globally asymptotically stable and satisfies (10).

Define $\boldsymbol{\xi}(k,\boldsymbol{d}) = [\boldsymbol{\eta}^{\mathrm{T}}(k) \quad \boldsymbol{\eta}^{\mathrm{T}}(k-d_1) \quad \cdots \quad \boldsymbol{\eta}^{\mathrm{T}}(k-d_q)]^{\mathrm{T}}$. Using (9), (10) becomes

$$\sum_{k=1}^{k_f} \boldsymbol{e}^{\mathrm{T}}(k)Q\boldsymbol{e}(k) = \sum_{k=1}^{k_f} \boldsymbol{\xi}^{\mathrm{T}}(k,\boldsymbol{d})\tilde{Q}\boldsymbol{\xi}(k,\boldsymbol{d}) \leqslant \gamma^2\Big[\boldsymbol{\xi}^{\mathrm{T}}(0)\tilde{P}_0\boldsymbol{\xi}(0) + \sum_{k=0}^{k_f-1}(\boldsymbol{w}^{\mathrm{T}}(k)W_w\boldsymbol{w}(k) + \boldsymbol{v}(k)^{\mathrm{T}}W_v\boldsymbol{v}(k))\Big] \tag{11}$$

where $\tilde{Q} = \begin{bmatrix}\tilde{Q}_{11} & \tilde{Q}_{12}\\ \tilde{Q}_{21} & \tilde{Q}_{22}\end{bmatrix}$, $\tilde{Q}_{11} = \tilde{L}^{\mathrm{T}}Q\tilde{L}$, $\tilde{Q}_{12} = [\tilde{L}^{\mathrm{T}}Q\tilde{L}_{d1} \quad \tilde{L}^{\mathrm{T}}Q\tilde{L}_{d2} \quad \cdots \quad \tilde{L}^{\mathrm{T}}Q\tilde{d}q]$, $\tilde{Q}_{21} = \tilde{Q}_{21}^{\mathrm{T}}$, $\tilde{Q}_{22} = [\tilde{L}_{d1} \quad \tilde{L}_{d2} \quad \cdots \quad \tilde{L}_{dq}]^{\mathrm{T}}Q[\tilde{L}_{d1} \quad \tilde{d}2 \quad \cdots \quad \tilde{L}_{dq}]$, and $\tilde{P}_0$ is an initial weighting matrix.

Theorem 1. For nonlinear system (1) and a prescribed real number $\gamma > 0$, if there exist a positive scalar $\tau > 0$ and symmetric positive definite matrices $P, P_j \in R^{2n\times 2n}$, such that the matrix inequalities

$$\begin{bmatrix}\Lambda_{11} & * & *\\ \Lambda_{21} & -P^{-1} & *\\ \Lambda_{31} & 0 & -Q^{-1}\end{bmatrix} < 0 \tag{12}$$

hold for $h_i(x(k))\cdot h_l(\boldsymbol{x}(k))\cdot h_s(\boldsymbol{x}(k)) \neq 0$ and $i,l,s = 1,2,\cdots,L$, where $\tilde{A}_d = [\tilde{A}_{d1} \quad \tilde{A}_{d2} \quad \cdots \quad \tilde{A}_{dq}]$, $S = \mathrm{diag}\{P_1,P_2,\cdots,P_q\}$, $\Theta = \mathrm{diag}\{\Omega + \Psi, 0\}$, $\Theta_d = \mathrm{diag}\{\Theta_{d1},\Theta_{d2},\cdots,\Theta_{dq}\}$, $W = \mathrm{diag}\{W_w, W_v\}$, $\Theta_{dj} = \mathrm{diag}\{\Omega_{dj} + \Psi_{dj}, 0\}$, $\Lambda_{11} = \mathrm{diag}\{-P + \sum_{j=1}^{q}P_j + \tau\Theta, -S + \tau\Theta_d, -\tau I, -\tau I, -\gamma^2 W\}$, $\Lambda_{21} = [\tilde{A} \quad \tilde{A}_d \quad \tilde{E} \quad \tilde{E}_d \quad \tilde{G}]$, $\Lambda_{31} = [\tilde{L} \quad \tilde{L}_d \quad 0 \quad 0 \quad 0]$, then the filtering error system (9) is asymptotically stable and satisfies the H_∞ index (10).

Proof. For positive definite symmetric matrices $P \in R^{2n\times 2n}$, $P_j \in R^{2n\times 2n}$, choose the following Lyapunov function candidate for system (9)

$$V(\boldsymbol{\eta}(k)) = \boldsymbol{\eta}^{\mathrm{T}}(k)P\boldsymbol{\eta}(k) + \sum_{j=1}^{q}\sum_{l=k-d_j}^{k-1}\boldsymbol{\eta}^{\mathrm{T}}(l)P_j\boldsymbol{\eta}(l) \tag{13}$$

Along the trajectories of system (9) with $\varpi(k) = 0$, the corresponding forward difference is given by

$$\begin{aligned}
&\Delta V(k) = V(\boldsymbol{\eta}(k+1)) - V(\boldsymbol{\eta}(k)) =\\
&\boldsymbol{\eta}^{\mathrm{T}}(k+1)P\boldsymbol{\eta}(k+1) + \sum_{j=1}^{q}\sum_{l=k-d_j}^{k-1}\boldsymbol{\eta}^{\mathrm{T}}(l)P_j\boldsymbol{\eta}(l) - \boldsymbol{\eta}^{\mathrm{T}}(k)P\boldsymbol{\eta}(k) =\\
&\Xi^{\mathrm{T}}P^{\Xi} + \sum_{j=1}^{q}[\boldsymbol{\eta}^{\mathrm{T}}(k)P_j\boldsymbol{\eta}(k) - \boldsymbol{\eta}^{\mathrm{T}}(k-d_j)P_j\boldsymbol{\eta}(k-d_j)] - \boldsymbol{\eta}^{\mathrm{T}}(k)P\boldsymbol{\eta}(k) + \tau\Delta\boldsymbol{\Gamma}^{\mathrm{T}}(k)\Delta\boldsymbol{\Gamma}^{\mathrm{T}}(k)+\\
&\tau\Delta\boldsymbol{\Gamma}_d^{\mathrm{T}}(k,\boldsymbol{d},q)\Delta\boldsymbol{\Gamma}_d(k,\boldsymbol{d},q) - \tau\Delta\boldsymbol{\Gamma}^{\mathrm{T}}(k) - \tau\Delta\boldsymbol{\Gamma}_d^{\mathrm{T}}(k,\boldsymbol{d},q)\Delta\boldsymbol{\Gamma}_d(k,\boldsymbol{d},q)
\end{aligned} \tag{14}$$

where $\Xi = [\tilde{A}\boldsymbol{\eta}(k) + \sum_{j=1}^{q}\tilde{A}_{dj}\boldsymbol{\eta}(k-d_j) + \tilde{E}\Delta\boldsymbol{\Gamma}(k) + \tilde{E}_d\Delta\boldsymbol{\Gamma}_d(k,\boldsymbol{d},q)]$.

By inequality (6), (14) becomes

$$\begin{aligned}
&\Delta V(k) \leqslant \Xi^{\mathrm{T}}P\Xi + \sum_{j=1}^{q}[\boldsymbol{\eta}^{\mathrm{T}}(k)P_j\boldsymbol{\eta}(k) - \boldsymbol{\eta}^{\mathrm{T}}(k-d_j)P_j\boldsymbol{\eta}(k-d_j)] - \boldsymbol{\eta}^{\mathrm{T}}(k)P\boldsymbol{\eta}(k) + \tau\boldsymbol{\eta}^{\mathrm{T}}(k)\Theta\boldsymbol{\eta}(k)+\\
&\tau\sum_{j=1}^{q}\boldsymbol{\eta}^{\mathrm{T}}(k-d_j)\Theta_{dj}\boldsymbol{\eta}(k-d_j) - \tau\Delta\boldsymbol{\Gamma}^{\mathrm{T}}(k) - \tau\Delta\boldsymbol{\Gamma}_d^{\mathrm{T}}(k,\boldsymbol{d},q)\Delta\boldsymbol{\Gamma}_d(k,\boldsymbol{\eta},q) = -\boldsymbol{\xi}(k,\boldsymbol{d})M\boldsymbol{\xi}(k,\boldsymbol{d})
\end{aligned} \tag{15}$$

where $\tau > 0$, $\boldsymbol{\xi}(k,\boldsymbol{d}) = [\boldsymbol{\eta}^{\mathrm{T}}(k) \quad \boldsymbol{\eta}^{\mathrm{T}}(k,\boldsymbol{d},q) \quad \Delta\boldsymbol{\Gamma}^{\mathrm{T}}(k) \quad \Delta_d^{\mathrm{T}}(k,\boldsymbol{d},q)]^{\mathrm{T}}$, $\boldsymbol{\eta}^{\mathrm{T}}(k,\boldsymbol{d},q) \doteq [\boldsymbol{\eta}^{\mathrm{T}}(k-d_1) \quad \boldsymbol{\eta}^{\mathrm{T}}(k-d_2) \quad \cdots \quad \boldsymbol{\eta}^{\mathrm{T}}(k-d_q)]^{\mathrm{T}}$, $M = -[\tilde{A} \quad \tilde{A}_d \quad \tilde{E} \quad \tilde{E}_d]^{\mathrm{T}}P[\tilde{A} \quad \tilde{A}_d \quad \tilde{E} \quad \tilde{E}_d] - \mathrm{diag}\{-P + \sum_{j=1}^{q}P_j + \tau\Theta, -S + \tau\Theta_d, -\tau I, -\tau I\}$ and $\tilde{A}_d^{\mathrm{T}}, \tilde{E}_d^{\mathrm{T}}, S, \Theta, \Theta_{dj}$ are defined as in inequality (12).

The sufficient condition for $\Delta V(k) \leqslant -\zeta^{\mathrm{T}}(k,\boldsymbol{d})M\zeta(k,\boldsymbol{d}) < 0$ is as follows.

$$[\tilde{A} \quad \tilde{A}_d \quad \tilde{E} \quad \tilde{E}_d]^{\mathrm{T}} P [\tilde{A} \quad \tilde{A}_d \quad \tilde{E} \quad \tilde{E}] + \mathrm{diag}\{-P + \sum_{j=1}^{q} P_j + \tau\Theta, -S + \tau\Theta_d, -\tau I, -\tau I\} < 0 \tag{16}$$

According the Schur complements[9], inequality (16) becomes

$$\begin{bmatrix} \bar{\Lambda}_{11} & * \\ \bar{\Lambda}_{21} & -P^{-1} \end{bmatrix} < 0 \tag{17}$$

where $\bar{\Lambda}_{11} = \mathrm{diag}\{-P + \sum_{j=1}^{q} P_j + \tau\Theta, -S + \tau\Theta_d, -\tau I, -\tau I\}$, $\bar{\Lambda}_{21} = [\tilde{A} \quad \tilde{A}_d \quad \tilde{E} \quad \tilde{E}_d]$.

The inequality (12) can be easily verified to ensure that inequality (17) holds. Therefore, it can be concluded that the filtering error system is globally asymptotically stable.

In order to find a sufficient condition for the filtering error system (9) satisfying condition (10), arbitrary nonzero $\varpi(k) \in l_2[0, +\infty)$ is considered as follows

$$\begin{aligned} &\Delta V(k) + \boldsymbol{e}(k)Q\boldsymbol{e}(k) - \gamma^2(\boldsymbol{w}^{\mathrm{T}}(k)W_w\boldsymbol{w}(k) + \boldsymbol{v}^{\mathrm{T}}(k)W_v\boldsymbol{v}(k)) \leqslant [\Xi + \tilde{G}\varpi(k)]^{\mathrm{T}} P [\Xi + \tilde{G}\varpi(k)] + \\ &\sum_{j=1}^{q} [\boldsymbol{\eta}^{\mathrm{T}}(k)P_j\boldsymbol{\eta}(k) - \boldsymbol{\eta}^{\mathrm{T}}(k-d_j)P_j\boldsymbol{\eta}(k-d_j)] - \boldsymbol{\eta}^{\mathrm{T}}(k)P\boldsymbol{\eta}(k) - \boldsymbol{\xi}^{\mathrm{T}}(k,\boldsymbol{d})\tilde{Q}\boldsymbol{\xi}(k,\boldsymbol{d}) - \\ &\gamma^2(\boldsymbol{w}^{\mathrm{T}}(k)W_w\boldsymbol{w}(k) + \boldsymbol{v}^{\mathrm{T}}(k)W_v(k)) + \tau\boldsymbol{\eta}^{\mathrm{T}}(k)\Theta\boldsymbol{\eta}(k) + \tau\sum_{j=1}^{q}\boldsymbol{\eta}^{\mathrm{T}}(k-d_j)\Theta_{dj}\boldsymbol{\eta}(k-d_j) - \\ &\tau\Delta\boldsymbol{\Gamma}^{\mathrm{T}}(k) - \tau\Delta\boldsymbol{\Gamma}_d^{\mathrm{T}}(k,\boldsymbol{d},q)\Delta\boldsymbol{\Gamma}_d(k,\boldsymbol{d},q) \leqslant -\boldsymbol{\chi}^{\mathrm{T}}(k,\boldsymbol{d})N\boldsymbol{\chi}(k,\boldsymbol{d}) \end{aligned} \tag{18}$$

where $Z = [\tilde{A} \quad \tilde{A}_d \quad \tilde{E} \quad \tilde{E}_d \quad \tilde{G}]$, $N = -Z^{\mathrm{T}}PZ - \mathrm{diag}\{-P + \sum_{j=1}^{q} P_j + \tau\Theta, -S + \tau\Theta_d, -\tau I, -\tau I, -\gamma^2 W\}$, $-[\tilde{L} \quad \tilde{L}_d^{\mathrm{T}} \quad 0 \quad 0 \quad 0]^{\mathrm{T}} Q [\tilde{L} \quad \tilde{L}_d^{\mathrm{T}} \quad 0 \quad 0 \quad 0]$, $\boldsymbol{\chi}(k,\boldsymbol{d}) = [\zeta^{\mathrm{T}}(k) \quad \varpi^{\mathrm{T}}(k)]^{\mathrm{T}}$, $\tilde{L}_d = [\tilde{L}_{d1} \quad \cdots \quad \tilde{L}_{dq}]$.

Defining $N > 0$, and according to the Schur complements, we can obtain inequality (12).

According to the inequalities (12) and (14), (18) becomes

$$\begin{aligned} &\sum_{k=1}^{k_f} \boldsymbol{e}^{\mathrm{T}}(k)Q\boldsymbol{e}(k) = \sum_{k=1}^{k_f} \boldsymbol{\xi}^{\mathrm{T}}(k,\boldsymbol{d})\tilde{Q}\boldsymbol{\xi}(k,\boldsymbol{d}) = -\boldsymbol{\xi}^{\mathrm{T}}(0,\boldsymbol{d})\tilde{Q}(0)\boldsymbol{\xi}(0,\boldsymbol{d}) + \\ &\boldsymbol{\xi}^{\mathrm{T}}(k_f,\boldsymbol{d})\tilde{Q}\boldsymbol{\xi}(k_f,\boldsymbol{d}) + \sum_{k=0}^{k_f-1} \boldsymbol{e}^{\mathrm{T}}(k)Q\boldsymbol{e}(k) + \sum_{k=0}^{k_f-1} \Delta V(k) - \boldsymbol{\eta}^{\mathrm{T}}(k_f)P\boldsymbol{\eta}(k_f) + \boldsymbol{\eta}^{\mathrm{T}}(0)P\boldsymbol{\eta}(0) - \\ &\sum_{k=0}^{k_f}\sum_{j=1}^{q} (\boldsymbol{\eta}^{\mathrm{T}}(k)P_j\boldsymbol{\eta}(k) - \boldsymbol{\eta}^{\mathrm{T}}(k-d_j)P_j\boldsymbol{\eta}(k-d_j)) + \sum_{j=1}^{q} (\boldsymbol{\eta}^{\mathrm{T}}(k)P_j\boldsymbol{\eta}(k) - \boldsymbol{\eta}^{\mathrm{T}}(k-d_j)P_j\boldsymbol{\eta}(k-d_j)) \leqslant \\ &\boldsymbol{\eta}^{\mathrm{T}}(0)P\boldsymbol{\eta}(0) + \boldsymbol{\xi}^{\mathrm{T}}(k_f,\boldsymbol{d})\Pi\boldsymbol{\xi}\boldsymbol{\xi}(k_f,\boldsymbol{d}) + \sum_{k=0}^{k_f-1} \boldsymbol{e}^{\mathrm{T}}(k)Q\boldsymbol{e}(k) + \sum_{k=0}^{k_f-1} \Delta V(k) - \\ &\sum_{k=0}^{k_f}\sum_{j=1}^{q} (\boldsymbol{\eta}^{\mathrm{T}}(k)P_j\boldsymbol{\eta}(k) - \boldsymbol{\eta}^{\mathrm{T}}(k-d_j)P_j\boldsymbol{\eta}(k-d_j)) \end{aligned} \tag{19}$$

where $\Pi = \tilde{Q} - \Lambda^{\mathrm{T}}P\Lambda + \Lambda^{\mathrm{T}}\sum_{j=1}^{q} P_j\Lambda - \Upsilon^{\mathrm{T}}\sum_{j=1}^{q} P_j\Upsilon$, $\Lambda = [I \quad 0 \quad \cdots \quad 0]$, $\Upsilon = [0 \quad I \quad \cdots \quad I]$.

Letting $d_{\min} = \min_{1 \leqslant j \leqslant q}(d_j)$ and according to $\boldsymbol{x}(k) = 0$, $\hat{\boldsymbol{x}}(k) = 0$, $k = -d_{\max}, -d_{\max}+1, \cdots, -1$, we have

$$\begin{aligned} &\sum_{k=0}^{k_f}\sum_{j=1}^{q} (\boldsymbol{\eta}^{\mathrm{T}}(k)P_j\boldsymbol{\eta}(k) - \boldsymbol{\eta}^{\mathrm{T}}(k-d_j)P_j\boldsymbol{\eta}(k-d_j)) = \sum_{l=0}^{k_f-d_{\min}}\sum_{j=1}^{q} \boldsymbol{\eta}^{\mathrm{T}}(l)P_j\boldsymbol{\eta}(l) + \\ &\sum_{l=k_f-d_{\min}+1}^{k_f}\sum_{j=1}^{q} \boldsymbol{\eta}^{\mathrm{T}}(l)P_j\boldsymbol{\eta}(l) - \sum_{l=0}^{k_f-d_j}\sum_{j=1}^{q} \boldsymbol{\eta}^{\mathrm{T}}(l)P_j\boldsymbol{\eta}(l) \geqslant \sum_{l=0}^{k_f-d_{\min}}\sum_{j=1}^{q} \boldsymbol{\eta}^{\mathrm{T}}(l)P_j\boldsymbol{\eta}(l) + \end{aligned}$$

$$\sum_{l=k_f-d_{\min}+1}^{k_f}\sum_{j=1}^{q}\boldsymbol{\eta}^{\mathrm{T}}(l)P_j\boldsymbol{\eta}(l)-\sum_{l=0}^{k_f-d_{\min}}\sum_{j=1}^{q}\boldsymbol{\eta}^{\mathrm{T}}(l)P_j\boldsymbol{\eta}(l)=\sum_{l=k_f-d_{\min}+1}^{k_f}\sum_{j=1}^{q}\boldsymbol{\eta}^{\mathrm{T}}(l)P_j\boldsymbol{\eta}(l)\geqslant 0 \tag{20}$$

Let $\Pi<0$, $\tilde{P}_0=(1/\gamma^2)\Lambda^{\mathrm{T}}P\Lambda$. By (18) and (20), (19) becomes

$$\sum_{k=1}^{k_f}\boldsymbol{e}^{\mathrm{T}}(k)Q\boldsymbol{e}(k)\leqslant\boldsymbol{\eta}^{\mathrm{T}}(0)P\boldsymbol{\eta}(0)+\gamma^2\sum_{k=0}^{k_f-1}(\boldsymbol{w}^{\mathrm{T}}(k)W_w\boldsymbol{w}(k)+\boldsymbol{v}^{\mathrm{T}}(k)W_v\boldsymbol{v}(k))=$$
$$\gamma^2\Big\{\boldsymbol{\xi}^{\mathrm{T}}(0)\tilde{P}_0\boldsymbol{\xi}(0)+\sum_{k=0}^{k_f-1}(\boldsymbol{w}^{\mathrm{T}}(k)W_w\boldsymbol{w}(k)+\boldsymbol{v}^{\mathrm{T}}(k)W_v\boldsymbol{v}(k))\Big\} \tag{21}$$

In addition, $\Pi<0$ is equivalent to $\tilde{Q}+\mathrm{diag}\{-P+\sum_{j=1}^{q}P_j,-S\}<0$, thus yielding $\begin{bmatrix}\tilde{\Lambda}_{11} & *\\ \tilde{\Lambda}_{21} & -Q^{-1}\end{bmatrix}<0$, where $\tilde{\Lambda}=\mathrm{diag}\{-P+\sum_{j=1}^{q}P_j,-S\}$, $\tilde{\Lambda}_{21}=[\tilde{L}\quad\tilde{L}_d]$. Comparing $-P+\sum_{j=1}^{q}P_j+\tau\Theta<0$ with $P+\sum_{j=1}^{q}P_j<0$, and $-S+\tau\Theta_d<0$ with $-S<0$, it is clear that any solution to the former will also satisfy the latter due to the fact that $\tau\Theta>0$ and $\tau\Theta_d>0$. Therefore, according to Schur complements, LMIs of (12) ensure that $\Pi<0$ holds.

Hence, the fuzzy H_∞ filter index is achieved with a prescribed γ^2. This completes the proof. □

In order to obtain fuzzy H_∞ filter, the most important work is to find feasible solution $P=P^{\mathrm{T}}$, and P_j from (12). since $\sum_{i=1}^{L}h_i(\boldsymbol{x}(k))\sum_{l=1}^{L}h_l(\boldsymbol{x}(k))\sum_{s=1}^{L}h_s(\boldsymbol{x}(k)=1$, letting $P=\mathrm{diag}\{\tilde{P},\tilde{P}\}$ and pre and postmultiplying (12) by $\mathrm{diag}\{I,I,I,I,I,P,I\}$ yield

$$\begin{bmatrix}\Sigma_{11} & * & *\\ \Sigma_{21} & -\begin{bmatrix}\tilde{P} & 0\\ 0 & \tilde{P}\end{bmatrix} & *\\ \Sigma_{31} & 0 & -Q^{-1}\end{bmatrix}<0 \tag{22}$$

where $\Sigma_{11}=\mathrm{diag}\{-\mathrm{diag}\{\tilde{P},\tilde{P}\}+\sum_{j=1}^{q}P_j+\tau\Theta,-S+\tau\Theta_d,-\tau I,-\tau I,-\gamma^2W\}$, $\Sigma_{31}=[L\quad -L\quad L_{dj}\quad -L_{dj}\quad 0\quad 0\quad 0]$, $\Sigma_{21}=\begin{bmatrix}\tilde{P}A_i & 0 & \tilde{P}A_{ji}^d & 0 & \tilde{P} & 0 & \tilde{P} & 0 & \tilde{P}B & 0\\ M_lC_i & \tilde{P}A_l-M_lC_s & M_lC_{ji}^d & \tilde{P}A_{jl}^d-M_lC_{js}^d & 0 & M_l & 0 & M_l & 0 & M_lG\end{bmatrix}$, $M_l=\tilde{P}K_l$.

In fact, any feasible solution to (22) yields a suitable robust filter. To obtain a better robust filtering performance against disturbances, the attenuation level γ^2 can be reduced to the minimum possible value such that (10) is satisfied, *i.e.*,

$$\min_{P,P_j,K_l}\delta,\ \text{subject to (22) with }\gamma^2=\delta \tag{23}$$

The design procedure for fuzzy H_∞ filter is summarized as follows.

Step 1. Construct the T-S fuzzy model in (2) for the nonlinear system in (1) using the method such as in [10∼13];

Step 2. Give $\Omega,\Omega_d,\Psi,\Psi_d,Q$;

Step 3. Solve the LMI problem in (23) to obtain P,P_jK_l;

Step 4. Obtain the fuzzy H_∞ filter constructed as in (7).

4 Simulation

Consider the following discrete-time nonlinear system with time delays:

$$\boldsymbol{x}_1(k+1)=0.25x_1(k)+0.03x_2(k)-0.67x_2^3(k)-0.025x_1(k-d_1)-0.5x_2(k-d_2)-0.1x_2^3(k-d_2)+w(k)$$
$$x_2(k+1)=0.1x_1(k),\ y(k)=x_2(k)+0.2x_1(k-d_1)+0.1w(k),\ s(k)=-2x_2(k) \tag{24}$$

We assume that $w(k)$ are normally distributed with zero mean and variance 0.1, and the initial condition is: $(x_1(0),x_2(0),\hat{x}_1(0),x_2(0))=(0,0,0,0)$. We also assume that x_1 and $x_2\in[-1.5.1.5]$. Let $d_1=1,d_2=1$. The design purpose is to construct fuzzy H_∞ filter to estimate $s(k)$.

Using the same procedure as in [14], the nonlinear term can be represented as

$$\begin{aligned} &-0.67x_2^3(k) = N_{11}\cdot 0\cdot x_2(k) - (1-N_{11})\cdot 1.5075x_2(k) \\ &-0.1x_2^3(k-d_2) = N_{11}\cdot 0\cdot x_2(k-d_2) - (1-N_{11})\cdot 0.225x_2(k) \end{aligned} \tag{25}$$

By solving the equation, N_{11} and N_{12} are obtained as follows.

$$N_{11}(x_2(k)) = 1 - \frac{x_2^2(k)}{2.25},\ N_{12}(x_2(k)) = 1 - N_{11}(x_2(k)) = 1 - \frac{x_2^2(k)}{2.25}$$

N_{11}, N_{12} can be interpreted as membership functions of fuzzy set. By using these fuzzy sets, the nonlinear system can be presented by the following T-S fuzzy model:

Rule 1: if $x_2(k)$ is N_{11}, then $x(k+1) = A_1x(k) + A_{d1}x(k-d) + Bw(k), y(k) = C_1x(k) + Gw(k)$

Rule 2: if $x_2(k)$ is N_{12}, then $x(k+1) = A_2x(k) + A_{d2}x(k-d) + Bw(k), y(k) = C_2x(k) + Gw(k)$

where $A_1 = \begin{bmatrix} 0.25 & 0.7 \\ 0.1 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0.25 & -0.8075 \\ 0.1 & 0 \end{bmatrix}$, $A_{d1} = \begin{bmatrix} -0.025 & -0.5 \\ 0 & 0 \end{bmatrix}$, $A_{d2} = \begin{bmatrix} -0.025 & -0.725 \\ 0 & 0 \end{bmatrix}$ $\boldsymbol{x}(k) = [x_1(k) \quad x_2(k)]^{\mathrm{T}}$, $\boldsymbol{B} = [1 \quad 0]^{\mathrm{T}}$, $\boldsymbol{C}_1 = \boldsymbol{C}_2 = [0 \quad 1]$.

We select $\Omega = 0.00001I$, $\Omega_d = 0.00001I$, $\Psi = 0.00001I$ and $\Psi_d = 0.00001I$. Solve the LMI problem in (23). We obtain $\gamma = 9.9652\times 10^{-6}$, and the fuzzy estimation gains as follows: $\boldsymbol{K}_1 = [0 \quad 0.5022]^{\mathrm{T}}$, $\boldsymbol{K}_2 = [0 \quad 0.4549]^{\mathrm{T}}$.

Fig. 1 shows the estimation signal $\hat{s}(k)$ for $s(k)$ by the proposed fuzzy H_∞ filter. Fig. 2 shows estimation $\hat{s}(k)$ for $s(k)$ when noises $w(k) = 0.1\sin(k)$. From the results of this simulation, it is shown that the proposed fuzzy H_∞ filter can obtain better estimation. The proposed fuzzy H_∞ filter can be obtained without any information about external disturbances and measurement noise, as long as they are bounded.

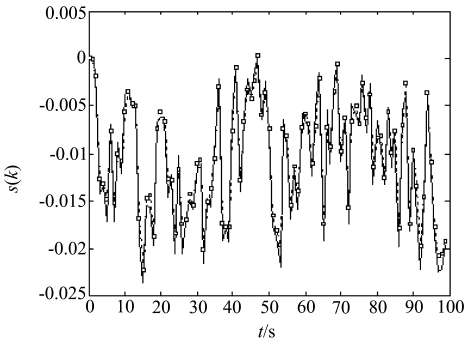

Fig. 1 Estimation signal $\hat{s}(k)$ (dashdot line with squared marks) for $s(k)$ (solid line) when $w(k)$ is normally distributed with zero mean and variance 0.1

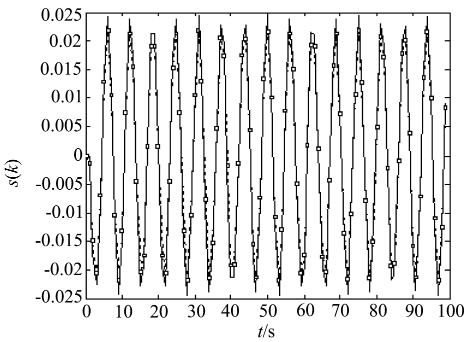

Fig. 2 Estimation signal $\hat{s}(k)$ (dashdot line with squared marks) for $s(k)$ (solid line) when $w(k) = 0.1\sin(k)$

5 Conclusions

In this paper, based on the T-S fuzzy model, fuzzy H_∞ filter is designed for a class of nonlinear discrete-time systems with multiple state time delays. No statistical assumption on the external disturbances and measurement noise is needed. The proposed fuzzy H_∞ filter for the nonlinear system can tolerate the approximation error based on the model error bounds, which can be regarded as the worst-case approximation error. The problem of fuzzy H_∞ filter design is converted into the linear matrix inequalities problem which can be efficiently solved using interior point algorithm. The proposed design procedure is very simple.

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