State Estimation of a Class of Hybrid Systems in the Presence of Unknown Mode Transitions[1)]

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Abstract Since the state of hybrid systems is determined by interacting continuous and discrete dynamics, the state estimation of hybrid systems becomes a challenging problem. It is more complicated when the discrete mode transition information is not available, and the modes of hybrid systems are nonlinear stochastic dynamic systems. To address this problem, this paper proposes a novel hybrid strong tracking filter (HSTF) for state estimation of a class of hybrid nonlinear stochastic systems with unknown mode transition, the method for designing HSTF is presented. The HSTF can estimate the continuous state and discrete mode accurately with unknown mode transition information, and the estimation of hybrid states is robust against the initial state. Simulation results illustrate the effectiveness of the proposed approach.

Key words Hybrid systems, hybrid state estimate, hybrid strong tracking filter

1 Introduction

Hybrid systems have been used to describe complex dynamic systems that involve both continuous and discrete states. Recently, the research of hybrid systems has become an intensive study domain in both control and computer science communities[1]. The state estimation of hybrid system has become a crucial and challenging research topic because it needs estimating the discrete mode and the continuous state simultaneously. Several approaches have been introduced for the hybrid system state estimation recently[2~7]. Some approaches require that the mode transition satisfy some stochastic rule, such as Markov chain, or the mode transition information be available, such as the condition of the mode transition and the successor mode.

Usually, hybrid system mode transitions are caused either by the interior states or the external events or inputs. In fact, the mode transition information is often not available, including: 1) the transition mechanism is unknown; 2) the transition instant is unknown. For example, the mode transitions caused by the exterior events are unpredictable and its probability distribution is unknown. In addition, the mode transitions caused by the interior states usually do not accord with the certain probability distribution (*e.g.* a Markov chain). The state estimation of the hybrid system becomes more complicated when the discrete mode transition information is not available and the modes of hybrid systems are nonlinear stochastic dynamic systems. To address this problem, this paper proposes a novel hybrid strong tracking filter (HSTF) for the state estimation of a class of hybrid nonlinear stochastic systems with unknown mode transitions.

2 Problem formulation

The system considered in this paper is a class of hybrid nonlinear stochastic systems, which involves both continuous states and discrete modes. The discrete modes are described by a finite set. The continuous state evolves according to a nonlinear stochastic differential equation that depends on the discrete modes, and mode transitions may occur when certain conditions are satisfied. The hybrid system has two kinds of mode transitions: 1) the controlled mode transitions trigged by external events or inputs; 2) the autonomous mode transitions trigged by internal continuous states. Moreover, we assume that the mode transitions do not reset the continuous states.

Definition 1. A class of hybrid nonlinear stochastic systems is described as follows:

$$HS = (Q, X, U, Y, f, h, g, T_c, T_s) \tag{1}$$

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where $Q = \{1, 2, \cdots, n_q\}$ is a finite set of discrete modes, and $n_q = |Q|$; $X \subseteq \mathbb{R}^{n_s}$ is the continuous state space; $U \subseteq \mathbb{R}^{n_u}$ is the continuous input space; $Y \subseteq \mathbb{R}^{n_y}$ is the continuous output space; $f : \mathbb{R}^{n_u} \times \mathbb{R}^{n_x} \times Q \to \mathbb{R}^{n_x}$, $h : \mathbb{R}^{n_s} \times Q \to \mathbb{R}^{n_y}$ and $g : \mathbb{R}^{n_v} \times Q \to \mathbb{R}^{n_x}$ specify the continuous evolution of the hybrid system; T_c and T_s are the finite set of controlled mode transitions and autonomous mode transitions, respectively. In this paper, we address the state estimation of hybrid system in the presence of unknown mode transitions, so that T_c and T_s are unknown.

For $q(k+1) = i, i \in Q$, the continuous dynamics is described by nonlinear stochastic difference equations:

$$\begin{cases} \boldsymbol{x}(k+1) = f_i(k, \boldsymbol{u}(k), \boldsymbol{x}(k)) + g_i(k)\boldsymbol{\xi}(k) \\ \boldsymbol{y}(k+1) = h_i(k+1, \boldsymbol{x}(k+1)) + \boldsymbol{\omega}(k+1) \end{cases} \tag{2}$$

where f_i and h_i are nonlinear functions and are assumed to have continuous derivatives with respect to the continuous $\boldsymbol{x}$; g_i is the known process noise matrices; f_i, h_i and g_i depend on the discrete mode $i \in Q$. The process noise $\boldsymbol{\xi}(k) \in \mathbb{R}^{n_\xi}$ and the measurement noise $\boldsymbol{\omega}(k) \in \mathbb{R}^{n_\omega}$ are independent Gaussian white sequence with the statistics:

$$\begin{cases} E\{\boldsymbol{\xi}(k)\} = E\{\boldsymbol{\omega}(k)\} = 0, \quad k \geqslant 0 \\ E\{\boldsymbol{\xi}(k)\boldsymbol{\xi}^{\mathrm{T}}(j)\} = R_\xi(k)\delta_{k,j}, \quad \forall k, j \geqslant 0 \\ E\{\boldsymbol{\omega}(k)\boldsymbol{\omega}^{\mathrm{T}}(j)\} = R_\omega(k)\delta_{k,j}, \quad \forall k, j \geqslant 0 \\ E\{\boldsymbol{\xi}(k)\boldsymbol{\omega}^{\mathrm{T}}(j)\} = 0, \quad \forall k, j \geqslant 0 \end{cases} \tag{3}$$

$x(0)$ is a random variable that is independent of $\boldsymbol{\xi}(k)$ and $\boldsymbol{\omega}(k)$, with the statistics:

$$\begin{cases} E\boldsymbol{x}(0) = \boldsymbol{x}_0 \\ E\{(\boldsymbol{x}(0) - \boldsymbol{x}_0)(\boldsymbol{x}(0) - \boldsymbol{x}_0)^{\mathrm{T}}\} = P_0 \end{cases} \tag{4}$$

Problem 1. The state estimation of hybrid system in the presence of unknown mode transitions:

Consider the hybrid system described by definition 1 in the presence of unknown mode transitions, the discrete mode set $Q = \{1, 2, \cdots, n_q\}$ is known, and for each mode $i \in Q, f_i, g_i$ and h_i are known. Given the sequence of continuous inputs $\{\boldsymbol{u}(k)\}$ and the sequence of continuous outputs $\{\boldsymbol{y}(k)\}$, the objective now is to estimate the discrete mode $q(k)$ and the continuous state $\boldsymbol{x}(k)$.

The main challenging aspects of this problem are: 1) The discrete mode transition information is not available, and the available information is the continuous input and output; 2) The mode transitions include the autonomous mode transitions trigged by the continuous state and the controlled mode transitions trigged by unknown external events; 3) For each mode, it is a nonlinear stochastic system.

3 Hybrid strong tracking filter

In this section, a novel hybrid strong tracking filter (HSTF) is proposed for the state estimation of the class of hybrid nonlinear stochastic systems (1) $\sim$ (4) with unknown mode transitions, which combines the strong tracking filter[8] and Bayesian approach. The HSTF is introduced as follows:

Let $Y^k \triangleq \{\boldsymbol{y}(0), \boldsymbol{y}(1), \cdots, \boldsymbol{y}(k)\}$; the hybrid system continuous state estimation is:

$$\hat{\boldsymbol{x}}(k+1|k+1) \triangleq E[\boldsymbol{x}(k+1)|Y^{k+1}] \tag{5}$$

From (5), we have

$$\hat{\boldsymbol{x}}(k+1|k+1) = E[\boldsymbol{x}(k+1)|Y^{k+1}] \sum_{i=1}^{n_q} \Pr(q(k+1) = i|Y^{k+1}) E[\boldsymbol{x}(k+1)|Y^{k+1}, q(k+1) = i] \tag{6}$$

where $\Pr(q(k+1) = i|Y^{k+1})$ denotes the posterior probability distribution of the discrete mode, which will be used for the discrete mode estimation.

Let $\hat{\boldsymbol{x}}_i(k+1|k+1)$ denote the mode conditional estimation of the continuous states under $q(k+1) = i, \forall i \in \{1, \cdots, n_q\}$:

$$\hat{\boldsymbol{x}}_i(k+1|k+1) \triangleq E[\boldsymbol{x}(k+1)|Y^{k+1}, q(k+1) = i] = \int \boldsymbol{x}(k+1) p(\boldsymbol{x}(k+1)|Y^{k+1}, q(k+1) = i) \mathrm{d}\boldsymbol{x}(k+1) \tag{7}$$

Corresponding to $q(k+1)=i, \forall i \in \{1,\cdots,n_q\}$, the continuous dynamics is described as follows:

$$\begin{cases} \boldsymbol{x}(k+1) = f_i(k,\boldsymbol{u}(k),\boldsymbol{x}(k)) + g_i(k)\boldsymbol{\xi}(k) \\ \boldsymbol{y}(k+1) = h_i(k+1,\boldsymbol{x}(k+1)) + \boldsymbol{\omega}(k+1) \end{cases} \tag{8}$$

Thus the mode conditional estimation of the continuous states can be computed by

$$\hat{\boldsymbol{x}}_i(k+1|k+1) = \hat{\boldsymbol{x}}_i(k+1|k) + K_i(k+1)\boldsymbol{\gamma}_i(k+1) \tag{9}$$

where $\hat{\boldsymbol{x}}_i(k+1|k) \triangleq E[\boldsymbol{x}(k+1)|q(k+1)=i, Y^k]$ denotes the mode conditional prediction of the continuous states:

$$\hat{\boldsymbol{x}}_i(k+1|k) = f_i(k,\boldsymbol{u}(k),\hat{\boldsymbol{x}}(k|k)) \tag{10}$$

$K_i(k+1)$ is the mode conditional gain:

$$K_i(k+1) = \frac{P(k+1|k)\bar{h}_i^{\mathrm{T}}(k+1,\hat{\boldsymbol{x}}(k+1|k))}{\bar{h}_i(k+1,\hat{\boldsymbol{x}}(k+1|k))P(k+1|k)\bar{h}_i^{\mathrm{T}}(k+1,\hat{\boldsymbol{x}}(k+1|k)) + R_\omega(k+1)} \tag{11}$$

where

$$\bar{h}_i(k+1,\hat{\boldsymbol{x}}(k+1|k)) = \left.\frac{\partial h_i(k+1,\boldsymbol{x}(k+1))}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}(k+1)=\hat{\boldsymbol{x}}(k+1|k)} \tag{12}$$

In (11) and (12), $\hat{\boldsymbol{x}}(k+1|k)$ is the continuous state prediction:

$$\hat{\boldsymbol{x}}(k+1|k) \triangleq E[\boldsymbol{x}(k+1)|Y^k] \tag{13}$$

$P(k+1|k)$ is the covariance matrix of the prediction error:

$$P(k+1|k) = E[(\boldsymbol{x}(k+1)-\hat{\boldsymbol{x}}(k+1|k))(\boldsymbol{x}(k+1)-\hat{\boldsymbol{x}}(k+1|k))^{\mathrm{T}}|Y^k] \tag{14}$$

From (13), we have

$$\hat{\boldsymbol{x}}(k+1|k) = \sum_{i=1}^{n_q}\Pr(q(k+1)=i|Y^k)E[\boldsymbol{x}(k+1)|Y^k,q(k+1)=i] = \sum_{i=1}^{n_q}\Pr(q(k+1)=i|Y^k)\hat{\boldsymbol{x}}_i(k+1|k) \tag{15}$$

where $\Pr(q(k+1)=i|Y^k)$ is the prediction probability of the discrete mode:

$$\Pr(q(k+1)=i|Y^k) = \sum_{j=1}^{n_q}\Pr(q(k+1)=i|Y^k,q(k)=j)\Pr(q(k)=j|Y^k) \tag{16}$$

where

$$\Pr(q(k+1)=i|Y^k,q(k)=j) = \frac{\Pr(Y^k|q(k+1)=i,q(k)=j)\Pr(q(k+1)=i|q(k)=j)}{\Pr(Y^k|q(k)=j)} \tag{17}$$

Because the observation sequence Y^k is uncorrelated with $q(k+1)$,

$$\Pr(Y^k|q(k+1)=i,q(k)=j) = \Pr(Y^k|q(k)=j) \tag{18}$$

Since the discrete mode transition information is not available, we assume that

$$\Pr(q(k+1)=i|q(k)=j) = \frac{1}{n_q} \tag{19}$$

Substituting (17) $\sim$ (19) into (16) yields

$$\Pr(q(k+1)=i|Y^k) = \frac{1}{n_q} \tag{20}$$

Substituting (10), (20) into (15), we have

$$\hat{\boldsymbol{x}}(k+1|k) = \frac{1}{n_q}\sum_{i=1}^{n_q} f_i(k,\boldsymbol{u}(k),\hat{\boldsymbol{x}}(k|k)) \tag{21}$$

From (14), we obtain

$$P(k+1|k) = E[(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}(k+1|k))(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}(k+1|k))^{\mathrm{T}}|Y^k] =$$

$$\sum_{i=1}^{n_q} \Pr(q(k+1) = i|Y^k) E[(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}(k+1|k))(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}(k+1|k))^{\mathrm{T}}|Y^k, q(k+1) = i] =$$

$$\frac{1}{n_q}\sum_{i=1}^{n_q} E[(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}_i(k+1|k) + \hat{\boldsymbol{x}}_i(k+1|k) - \hat{\boldsymbol{x}}(k+1|k)) \cdots (\boldsymbol{x}(k+1) -$$

$$\hat{\boldsymbol{x}}_i(k+1|k) + \hat{\boldsymbol{x}}_i(k+1|k) - \hat{\boldsymbol{x}}(k+1|k))^{\mathrm{T}}|Y^k, q(k+1) = i] =$$

$$\frac{1}{n_q}\sum_{i=1}^{n_q} (E[(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}_i(k+1|k))(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}_i(k+1|k))^{\mathrm{T}}|Y^k, q(k+1) = i] +$$

$$(\hat{\boldsymbol{x}}_i(k+1|k) - \hat{\boldsymbol{x}}(k+1|k))(\hat{\boldsymbol{x}}_i(k+1|k) - \hat{\boldsymbol{x}}(k+1|k))^{\mathrm{T}}) =$$

$$= \frac{1}{n_q}\sum_{i=1}^{n_q} (P_i(k+1|k) + (\hat{\boldsymbol{x}}_i(k+1|k) - \hat{\boldsymbol{x}}(k+1|k))(\hat{\boldsymbol{x}}_i(k+1|k) - \hat{\boldsymbol{x}}(k+1|k))^{\mathrm{T}}) \tag{22}$$

where $P_i(k+1|k)$ is the covariance matrix of the mode conditional prediction error:

$$P_i(k+1|k) \triangleq E[(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}_i(k+1|k))(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}_i(k+1|k))^{\mathrm{T}}|Y^k, q(k+1) = i] \tag{23}$$

$P_i(k+1|k)$ can be computed by

$$P_i(k+1|k) = \lambda_i(k+1)\bar{f}_i(k, \boldsymbol{u}(k), \hat{\boldsymbol{x}}(k|k))P(k|k)\bar{f}_i^{\mathrm{T}}(k, \boldsymbol{u}(k), \hat{\boldsymbol{x}}(k|k)) + g_i(k)R_\xi(k)g_i^{\mathrm{T}}(k) \tag{24}$$

where

$$\bar{f}_i(k, \boldsymbol{u}(k), \hat{\boldsymbol{x}}(k|k)) = \left.\frac{\partial f_i(k, \boldsymbol{u}(k), \boldsymbol{x}(k))}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}(k) = \hat{\boldsymbol{x}}(k|k)} \tag{25}$$

$\lambda_i(k+1) \geqslant 1$ is the time-varying sub-optimal fading factor, which makes the filter have strong robustness against the model uncertainty and strong tracking ability to the suddenly changing states. It can be determined as follows.

$$\lambda_i(k+1) = \begin{cases} \lambda_i^0 & \lambda_i^0 \geqslant 1 \\ 1, & \lambda_i^0 < 1 \end{cases} \tag{26}$$

where

$$\lambda_i^0 = \frac{tr[N_i(k+1)]}{tr[M_i(k+1)]} \tag{27}$$

$$\begin{aligned} M_i(k+1) = {} & \bar{h}_i(k+1, \hat{\boldsymbol{x}}(k+1|k))\bar{f}_i(k, \boldsymbol{u}(k), \hat{\boldsymbol{x}}(k|k))P(k|k)\cdot \\ & \bar{f}_i^{\mathrm{T}}(k, \boldsymbol{u}(k), \hat{\boldsymbol{x}}(k|k))\bar{h}_i^{\mathrm{T}}(k+1, \hat{\boldsymbol{x}}(k+1|k)) \end{aligned} \tag{28}$$

$$\begin{aligned} N_i(k+1) = {} & V_i(k+1) - \bar{h}_i(k+1, \hat{\boldsymbol{x}}(k+1|k))g_i(k)R_\xi(k)g_i^{\mathrm{T}}(k)\cdot \\ & \bar{h}_i^{\mathrm{T}}(k+1, \hat{\boldsymbol{x}}(k+1|k)) - \beta R_\omega(k+1) \end{aligned} \tag{29}$$

where in (29) $\beta \geqslant 1$ is the pre-selected softening factor, which makes the estimate more smooth.

$$V_i(k+1) = \begin{cases} \boldsymbol{\gamma}_i(1)\boldsymbol{\gamma}_i^{\mathrm{T}}(1), & k = 0 \\ \dfrac{\rho S(k) + \boldsymbol{\gamma}_i(k+1)\boldsymbol{\gamma}_i^{\mathrm{T}}(k+1)}{1+\rho}, & k > 0 \end{cases} \tag{30}$$

In (30), $0 < \rho \leqslant 1$ is the pre-selected forgetting factor. Usually, let $\rho = 0.95$. $S(k)$ is the covariance matrix of the output error and can be computed by (38) at the last step.

$\boldsymbol{\gamma}_i(k+1)$ is the mode conditional residual:

$$\boldsymbol{\gamma}_i(k+1) = \boldsymbol{y}(k+1) - \hat{\boldsymbol{y}}_i(k+1) = \boldsymbol{y}(k+1) - \boldsymbol{h}_i(k+1, f_i(k, \boldsymbol{u}(k), \hat{\boldsymbol{x}}(k+1|k))) \tag{31}$$

From (10), (11) and (31), we have the mode conditional estimation of the continuous states $\hat{\boldsymbol{x}}_i(k+1|k+1)$ under $q(k+1) = i, \forall i \in \{1, \cdots, n_q\}$.

$P_i(k+1|k+1)$ is the covariance matrix of the mode conditional estimation error of the continuous states:

$$P_i(k+1|k+1) = [I - K_i(k+1)\bar{h}_i(k+1, \hat{\boldsymbol{x}}(k+1|k))]P(k+1|k) \tag{32}$$

In (6), $\Pr(q(k+1) = i|Y^{k+1})$ can be computed by

$$\Pr(q(k+1) = i|Y^{k+1}) = \frac{L(\boldsymbol{y}(k+1)|q(k+1) = i, Y^k)\Pr(q(k+1) = i|Y^k)}{\sum_{j=1}^{n_q} L(\boldsymbol{y}(k+1)|q(k+1) = j, Y^k)\Pr(q(k+1) = j|Y^k)} \tag{33}$$

where $L(\boldsymbol{y}(k+1)|q(k+1) = i, Y^k)$ is the likelihood function of mode i:

$$L(\boldsymbol{y}(k+1)|q(k+1) = i, Y^k) \triangleq Norm[\boldsymbol{\gamma}_i(k+1); 0, S_i(k+1)] \tag{34}$$

where $Norm[\boldsymbol{\gamma}_i(k+1); 0, S_i(k+1)]$ is the probability density of random variable with normal distribution whose mean is zero and covariance is $S_i(k+1)$ when the value is $\boldsymbol{\gamma}_i(k+1)$. $S_i(k+1)$ is the covariance matrix of the mode conditional residual of the output:

$$\begin{aligned} S_i(k+1) =& E[\boldsymbol{\gamma}_i(k+1)\boldsymbol{\gamma}_i^{\rm T}(k+1)|q(k+1) = i] \approx \\ & \bar{h}_i(k+1, \hat{\boldsymbol{x}}(k+1|k))P_i(k+1|k)\bar{h}_i^{\rm T}(k+1, \hat{\boldsymbol{x}}(k+1|k)) + R_w(k+1) \end{aligned} \tag{35}$$

Remark 1. In (35), $P_i(k+1|k)$ replaces $P(k+1|k)$ in order to increase the mode distinguish ability.

So the continuous state estimation of the hybrid system is

$$\hat{\boldsymbol{x}}(k+1|k+1) = \sum_{i=1}^{n_q} \Pr(q(k+1) = i|Y^{k+1})\hat{\boldsymbol{x}}_i(k+1|k+1) \tag{36}$$

The covariance matrix of the estimation error of the continuous state is

$$\begin{aligned} & P(k+1|k+1) = E[(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}(k+1|k))(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}(k+1|k))^{\rm T}|Y^{k+1}] = \\ & \sum_{i=1}^{n_q} \Pr(q(k+1) = i|Y^{k+1})E[(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}(k+1|k+1))(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}(k+1|k+1))^{\rm T}|Y^{k+1}, q(k+ \\ & 1) = i] = \sum_{i=1}^{n_q} \Pr(q(k+1) = i|Y^{k+1})E[(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}_i(k+1|k+1) + \hat{\boldsymbol{x}}_i(k+1|k+1) - \\ & \hat{\boldsymbol{x}}(k+1|k+1)) \cdot (\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}_i(k+1|k+1) + \hat{\boldsymbol{x}}_i(k+1|k+1) - \hat{\boldsymbol{x}}(k+1|k+1))^{\rm T}|Y^{k+1}, q(k+ \\ & 1) = i] = \sum_{i=1}^{n_q} \Pr(q(k+1) = i|Y^{k+1})\{E[(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}_i(k+1|k+1))(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}_i(k+1|k+ \\ & 1))^{\rm T}|Y^{k+1}, q(k+1) = i] + (\hat{\boldsymbol{x}}_i(k+1+k+1) - \hat{\boldsymbol{x}}(k+1|k+1))(\hat{\boldsymbol{x}}_i(k+1|k+1) - \hat{\boldsymbol{x}}(k+1|k+ \\ & 1))^{\rm T}\} = \sum_{i=1}^{n_q} \Pr(q(k+1) = i|Y^{k+1})\{P_i(k+1|k+1) + (\hat{\boldsymbol{x}}_i(k+1|k+1) - \hat{\boldsymbol{x}}(k+1|k+1))(\hat{\boldsymbol{x}}_i(k+ \\ & 1|k+1) - \hat{\boldsymbol{x}}(k+1|k+1))^{\rm T}\} \end{aligned} \tag{37}$$

The covariance matrix of the residual of the continuous output is

$$S(k+1) = \sum_{i=1}^{n_q} \Pr(q(k+1) = i|Y^{k+1})\{S_i(k+1) + (\hat{\boldsymbol{y}}_i(k+1) - \hat{\boldsymbol{y}}(k+1))(\hat{\boldsymbol{y}}_i(k+1) - \hat{\boldsymbol{y}}(k+1))^{\rm T}\} \tag{38}$$

where

$$\hat{\boldsymbol{y}}(k+1) = \sum_{i=1}^{n_q} \Pr(q(k+1) = i|Y^{k+1})\hat{\boldsymbol{y}}_i(k+1) \tag{39}$$

From (33), we get the posterior probability distribution of the discrete mode $\Pr(q(k+1)=i|Y^{k+1})$. Based on the posterior probability distribution, the discrete mode estimation is

$$\hat{q}(k+1)=\begin{cases} i, & \exists i\in Q,\ \text{s.t. } \Pr(q(k+1)=i|Y^{k+1})\geqslant\theta \\ \hat{q}(k), & \text{else}\end{cases} \tag{40}$$

where $0.9\leqslant\theta<1$ is a pre-selected mode threshold.

4 Simulation study

A hybrid three-tank system is described as in Fig. 1, which consists of three cylindrical tanks (T1, T2, T3), an input pipe (P1), an output pipe (P2) and two connection pipes (P3, P4). For each pipe, there is a valve (V1~V4) that can be used to change the mode. The discrete mode set $Q=\{1,2,3\}$: 1) Mode 1: V1, V3 and V4 are open. V2 is closed; 2) Mode 2: V2, V3 and V4 are open. V1 is closed; 3) Mode 3: V1, V2 and V4 are open. V3 is closed.

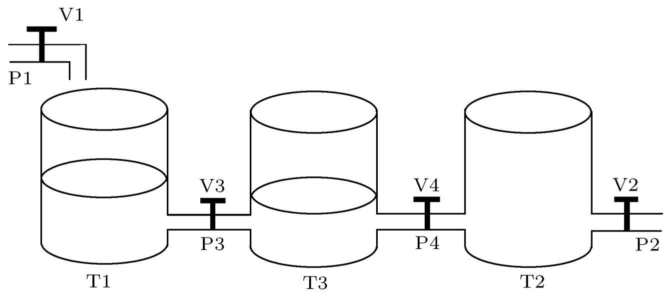

Fig. 1 Hybrid three-tank

The hybrid system can be described by

$$\begin{cases} \boldsymbol{x}(k+1)=f_i(k,\boldsymbol{u}(k),\boldsymbol{x}(k))+\boldsymbol{\xi}(k) \\ \boldsymbol{y}(k+1)=h_i(k+1,\boldsymbol{x}(k+1))+\boldsymbol{\omega}(k+1)\end{cases},\quad i\in\{1,2,3\} \tag{41}$$

$$\boldsymbol{x}=[x_1,x_2,x_3]^{\mathrm{T}},\quad \boldsymbol{y}=[y_1,y_2,y_3]^{\mathrm{T}}$$

$$f_1(\bullet)=\begin{bmatrix} x_1-\frac{1}{A_1}a_3s_3\mathrm{sgn}(x_1-x_3)\sqrt{2g|x_1-x_3|}+\frac{1}{A_1}u \\ x_2+\frac{1}{A_2}a_4s_4\mathrm{sgn}(x_3-x_2)\sqrt{2g|x_3-x_2|} \\ x_3+\frac{1}{A_3}a_3s_3\mathrm{sgn}(x_1-x_3)\sqrt{2g|x_1-x_3|}-\frac{1}{A_3}a_4s_4\mathrm{sgn}(x_3-x_2)\sqrt{2g|x_3-x_2|}\end{bmatrix}$$

$$f_2(\bullet)=\begin{bmatrix} x_1-\frac{1}{A_2}a_3s_3\mathrm{sgn}(x_1-x_3)\sqrt{2g|x_1-x_3|} \\ x_2+\frac{1}{A_2}a_4s_4\mathrm{sgn}(x_3-x_2)\sqrt{2g|x_3-x_2|}-\frac{1}{A_2}a_2s_2\sqrt{2gx_2} \\ x_3+\frac{1}{A_3}a_3s_3\mathrm{sgn}(x_1-x_3)\sqrt{2g|x_1-x_3|}-\frac{1}{A_3}a_4s_4\mathrm{sgn}(x_3-x_2)\sqrt{2g|x_3-x_2}\end{bmatrix}$$

$$f_3(\bullet)=\begin{bmatrix} x_1+\frac{1}{A_1}u \\ x_2+\frac{1}{A_2}a_4s_4\mathrm{sgn}(x_3-x_2)\sqrt{2g|x_3-x_2|}-\frac{1}{A_2}a_2s_2\sqrt{2gx_2} \\ x_3-\frac{1}{A_3}a_4s_4\mathrm{sgn}(x_3-x_2)\sqrt{2g|x_3-x_2}\end{bmatrix}$$

$$h_1(k,\boldsymbol{x}(k))=I_3\times\begin{bmatrix}x_1(k)\\x_2(k)\\x_3(k)\end{bmatrix},\ h_2(k,\boldsymbol{x}(k))=2I_3\times\begin{bmatrix}x_1(k)\\x_2(k)\\x_3(k)\end{bmatrix},\ h_3(k,\boldsymbol{x}(k))=3I_3\times\begin{bmatrix}x_1(k)\\x_2(k)\\x_3(k)\end{bmatrix}$$

where A is the section of each cylindrical tank (154cm^2), $s_i, i=1-4$ is the section of each pipe (0.5cm^2), $a_i, i=1-4$ is the adjust parameter (0.5), g=981cm/s^2, these parameters are taken from [8]. The sampling time is 1s. The process noise and measure noise are both Gaussian white sequences with mean zero and variance 0.5. The initial state is $x_1(0)=15$cm, $x_2(0)=10$cm, and $x_3(0)=10$cm. The input is $\boldsymbol{u}(k)=20$cm^3, $\theta=0.95$. The system runs through three modes: Mode 1 is from 1s to 100s; Mode 2 is from 101s to 200s; Mode 3 is from 201s to 300s. For the HSTF, $\hat{\boldsymbol{x}}(0|0)=[20,5,10]^{\mathrm{T}}$, $P(0|0)=100\times I_3$, $\Pr(q(0)=2|Y^0)=1$, and $\Pr(q(0)=1|Y^0)=\Pr(q(0)=3|Y^0)=0$ are selected.

Fig. 2 and Fig. 3 show the simulation results. In Fig. 2, the left-hand side shows the actual and estimated water levers and the right-hand shows the estimation error. Fig. 3 shows the estimation of the discrete mode. The simulation results show that:

1) The HSTF can estimate the continuous state and discrete mode accurately with unknown mode transition information;

2) The estimation of the continuous state is robust against the initial state errors;

3) The estimation of the discrete mode is robust against the wrong initial probability distribution of the discrete mode.

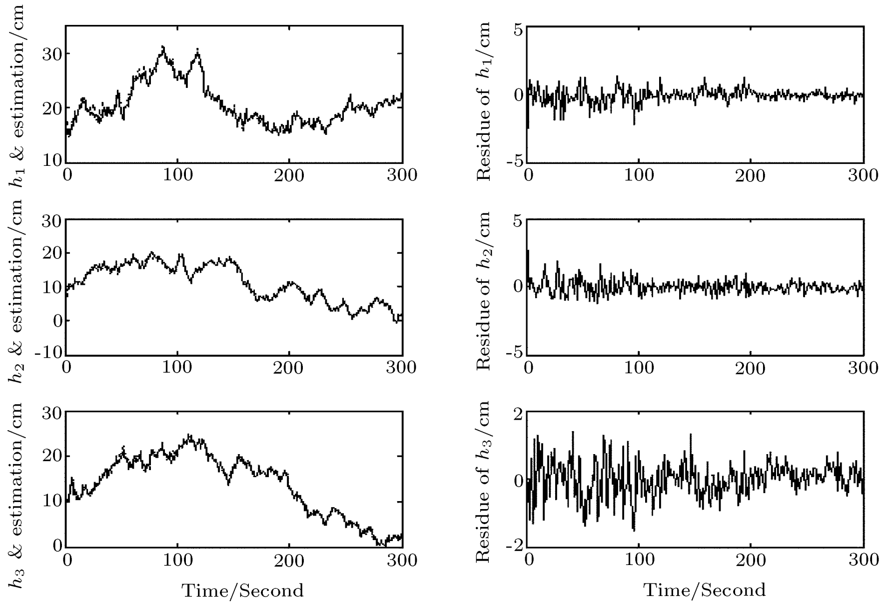

Fig. 2 Continuous states estimation

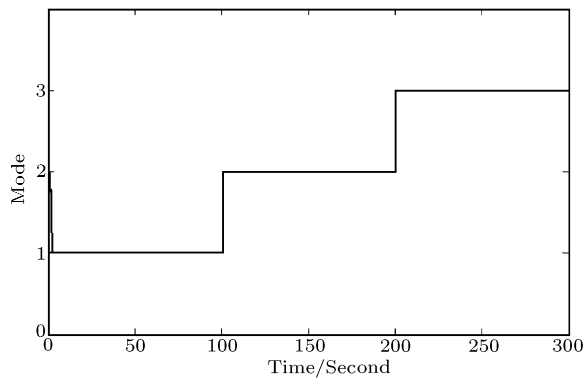

Fig. 3 Discrete mode estimation

5 Conclusions

To address the state estimation of a class of hybrid nonlinear stochastic systems in the presence of unknown mode transitions, a novel HSTF based on the STF and Bayesian approach is proposed. The simulation results show the effectiveness of the HSFT and the HSFT is robust to the initial hybrid states. The convergence of the HSTF will be studied in the future work.

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